reduced variables Z_R and Z_Q. The line of interest to us in reliability analysis is the line corresponding to $g(Z_R, Z_Q) = 0$ because this line separates the safe and failure domains in the space of reduced variables.

5.3.2 General Definition of the Reliability Index

In Chapter 3 (Example 3.1), a version of the reliability index was defined as the inverse of the coefficient of variation. In the context of the present discussion, we will define the reliability index as the *shortest* distance from the origin of reduced variables to the line $g(Z_R, Z_Q) = 0$. This definition, which was introduced by Hasofer and Lind (1974), is illustrated in Figure 5.12.

Using geometry, we can calculate the reliability index (shortest distance) from the following formula:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{5.14}$$

where β is the inverse of the coefficient of variation of the function $g(R, Q) = R - Q$ when R and Q are uncorrelated. For normally distributed random variables R and Q, it can be shown (following the procedure presented in Examples 3.1 and 3.2) that the reliability index is related to the probability of failure by

$$\beta = -\Phi^{-1}(P_f) \quad \text{or} \quad P_f = \Phi(-\beta) \tag{5.15}$$

Table 5.1 provides an indication of how β varies with P_f and vice versa based on Eq. 5.15.

The definition for a two-variable case can be generalized for n variables as follows. Consider a limit state function $g(X_1, X_2, \ldots, X_n)$ where the X_i variables are all

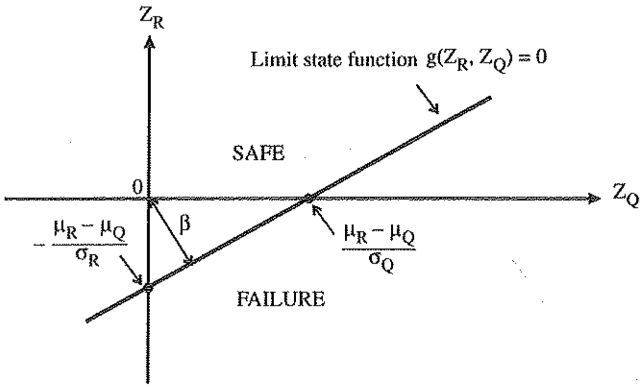

FIGURE 5.12 Reliability index defined as the shortest distance in the space of reduced variables.

index β and probability of failure P_f

P_f	β
10^{-1}	1.28
10^{-2}	2.33
10^{-3}	3.09
10^{-4}	3.71
10^{-5}	4.26
10^{-6}	4.75
10^{-7}	5.19
10^{-8}	5.62
10^{-9}	5.99

uncorrelated. The Hasofer-Lind reliability index is defined as follows:

1. Define the set of reduced variables $\{Z_1, Z_2, \ldots, Z_n\}$ using

$$Z_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}$$

2. Redefine the limit state function by expressing it in terms of the reduced $(Z_1, Z_2, \ldots, Z_n)$.
3. The reliability index is the shortest distance from the origin in the n-dim space of reduced variables to the curve described by $g(Z_1, Z_2, \ldots, Z_n) =$

5.3.3 First-Order Second-Moment Reliability Index

Linear limit state functions

Consider a *linear* limit state function of the form

$$g(X_1, X_2, \ldots, X_n) = a_0 + a_1X_1 + a_2X_2 + \cdots + a_nX_n = a_0 + \sum_{i=1}^{n} a_iX_i$$

where the a_i terms $(i = 0, 1, 2, \ldots, n)$ are constants and the X_i terms are *unc* random variables. If we apply the three-step procedure outlined above for det the Hasofer-Lind reliability index, we would obtain the following expression

$$\beta = \frac{a_0 + \sum_{i=1}^{n} a_i\mu_{X_i}}{\sqrt{\sum_{i=1}^{n} (a_i\sigma_{X_i})^2}}$$