Getting into the failure domain: application in estimation of extreme quantiles and probabilities

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Abstract:

In the context of reliability analysis, complex models representing physical situations are used to determine failure domain via a given threshold not to be overpassed. These models are often very time consuming and no analytical expression is available. Thus, the evaluation of a failure probability given a threshold or the estimation of a quantile given a targeted probability cannot be done by usual computation tools. Some developments have been done in two different directions : first the use of sequential Monte-Carlo methods allows to decrease the number of necessary calls to the model ([3], [1]) ; in this mood Guyader *et al.* [6] have shown that the optimal algorithm works with limit subsets, *ie.* fixing the current threshold at the minimum of the working population. Another approach is to use the given computational budget to fit a surrogate model and then to use it instead of the real model to evaluate quantities of interest (see [2], [5] or [4] for examples). Both approaches suffer from several limitations.

Sequential Monte-Carlo would still require quite a lot of calls to the limit-state function, while fitted surrogate models can be very far from the original ones without any possibility to control precision. However, in the context of probability or quantile estimation, only the boundary delimiting failure and safety domains is to be well approximated ; especially, no good approximation of the model in both domains is necessary if data points are well classified.

The novelty of our work comes from our understanding of these two approaches in the rare event simulation context as a *run to the failure domain*. This means that sequential Monte-Carlo appears indeed like a move of particles from an initial random state to the failure domain and that meta-modelling needs only pairs of points on each side of the boundary. The basic result is that with a sequential sampling strategy, the theoretical number of samples needed to get into a failure domain of probability measure p follows a Poisson law with parameter $\log(1/p)$. This number is to be compared with a classical Monte-Carlo sampling with a theoretical $1/p$ number of samples needed.

A consequence for sequential Monte-Carlo algorithms is the theoretically full parallelisation of Guyader *et al.* ([6]) optimal algorithm for rare event probability estimation with the limit case of algorithms with only 1 particle. On the other hand quantile estimation requires a small adaptation and we thus present a modified algorithm which enables full parallelisation. This modification is of very little effect as in the test cases it increases the total number of calls by approximately 10% while parallelisation can reduce computational budget by a factor up to population size (*eg.* 100, 1000...). However because of robustness issue in the Metropolis-Hastings generation step, it appears that population size for one algorithm shouldn't be smaller than 10 to 20.

On the other hand we use this *run to the failure domain* approach to derive a new strategy for first Design of Experiment (DoE) in the case of meta-modelling. In fact, it is well noticed that surrogate models require failing data points in their learning database to perform well ([5]). Unlike Space-filling strategies which aim at giving an overall knowledge of the model or Gaussian sampling techniques which "overlearn" the model in the safety domain, we propose to use a sequential strategy to get into the failure

domain quickly. More precisely, by replacing the expensive-to-evaluate model by the cheap surrogate in the Metropolis-Hastings step, we get similar results in terms of number of calls before the failure domain while limiting drastically the total number of calls usually driven by a *burn-in* parameter. Finally, we get a new relation for the size of the first DoE, it is : $N_{DoE} \approx d + 1 + N \log(1/p)$ with N the number of willing failing points in the DoE and d the dimension of the input space. Practically, it appears that given N and d, total number of calls is indeed lower than this theoretical value.

References

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Short biography – I am Clément Walter, 24 years old. I just graduated from Mines ParisTech which I integrated after 2 years of prep classes, section Math and Physics. At Mines Paristech I specialised in geostatistics and used it for coal resources evaluation and drill hole campaign planing. For my 4 months final internship I started working at CEA on rare event simulation and got the opportunity to continue here in a PhD, which I officially started in November.