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Adaptive particle methods for rare event simulation in a Markovian framework

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Abstract:

In recent decades, standards of quality and safety requirements is increasingly demanding in numerous industrial and scientific areas. Thus, the estimation of probability of rare events is become of great interest. Two popular methods in this regards are the splitting algorithm [6] and a weighted importance resampling algorithm (WIR) [3]. Despite a deep theoretical study of these two algorithms, the practical side has not been handled completely. The choice of a good tuning often proves to be fastidious and may decrease their usefulness. This study aims at answering this point. More precisely, we propose some adaptive methods to make the implementation of the splitting algorithm and the WIR algorithm automatic. Furthermore, we enlarge the area of application of the WIR algorithm to make WIR and the splitting estimating the same kind of probability.

Firstly, we focus on the splitting algorithm. The main goal of this algorithm is to estimate the probability that a Markov process X_t enters a critical set B before some stopping time T. Namely, it gives some numerical approximations of

$$\mathbb{P}(T_B \leq T) = \mathbb{P}(X_t \in B, \text{ for some } t \in [0,T]), \tag{1}$$

where T_B is the first time X_t enters B. This kind of problems typically arises in air traffic management [7], telecommunication networks [4] and electrical grid reliability estimation [8].
The principle of the splitting is to consider a sequence of decreasing supersets of B, $B_1 \supset \cdots \supset B_{m-1} \supset B_m = B$, and to estimate each probability of reaching B_k starting from B_{k-1}. Consequently, we first have to determine the sequence of decreasing supersets B_k. To this aim, the existing algorithms [2, 4] are too restrictive. Indeed, they can only be used with time-homogeneous X_t process and T random. Moreover, the algorithm proposed in [2] is only made for a process X_t that takes its value in $\mathbb{R}$. That is why we propose a general adaptive splitting algorithm for T random or deterministic, X_t possibly time-homogeneous and multi-dimensional. For that purpose, we assume that the rare event is characterized by some exceedance over a given threshold S of a real valued function Φ, $B = \{x,\ \Phi(x) \geq S\}$. For all we know, all the problems addressed in the scientific literature ([4, 7, 8]) can be expressed in such a way. In that case, we derive an adaptive algorithm for the choice of the supersets B_k, as concisely detailed in [5]. To this end, the proposed algorithm is based on some quantile estimation of the random variable of the maxima of a trajectory of $\Phi(X_t)$ over final time T. Thus, the supersets B_k are implicitly defined with Φ. Finally, we give some numerical estimations of the conflict probability between aircraft.

Secondly, we proposed some improvement of the WIR algorithm [3]. The goal of the WIR algorithm is to estimate

$$\mathbb{P}(\Psi(Z_n) \in C), \tag{2}$$

where $(Z_k,\ k = 0, \ldots, n)$ is a Markov chain, n a fixed integer, Ψ a real valued function and C a subset of $\mathbb{R}$. Such problems appears in the estimation of credit portfolio losses [1] and in fiber optic [3].

To estimate the probability (2), WIR consists in a set of N random paths $(Z_{0:n}^{(i)},\ 1 \leq i \leq N)$. The construction of these paths is performed in two steps. First, at each iteration time, the trajectories

which are more likely to reach the rare set C are multiplied and the others are killed. This is made through a selection function. Secondly, the Markov transition kernel of $(Z_k,\, k = 0, \dots, n)$ is applied for the trajectory evolution.

To our knowledge, all the selection functions used in the scientific literature for the WIR algorithm only depend on a real parameter, denoted here by α [3]. Indeed, the choice of α strongly influences the variance of the estimated probability. However, there is no formula neither rules on the choice of some good parameters. We thus propose an algorithm to compute some good parameter for the WIR algorithm. This procedure is motivated by an original characterisation of the parameters that should be used. This characterisation is made with the proportion of the trajectories that reach the rare set at final time. Moreover, we numerically show that this procedure achieves to determine the optimal parameters. Then we test the proposed algorithm with the estimation of outage probability in optic fiber and the estimation of large credit portfolio losses.

At first sight, the splitting and WIR algorithms cannot be used for the same estimation problem. Indeed, the first one estimates the probability (1) that a critical event occurs during the evolution of a Markov process whereas the second one estimates the probability (2) that a critical event occurs at the deterministic final evolution time of a Markov chain. However, if one considers the process of the maxima of $\Phi(X_t)$, namely $M_t = \max\limits_{0 \le s \le t} \Phi(X_s)$ we show that the WIR algorithm can indeed be used for the estimation of (1). Finally, we compare the efficiency of our modified version of the WIR algorithm and the splitting algorithm on the estimation of the conflict probability. We conclude that the WIR algorithm gives better variance than the splitting algorithm.

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Short biography – Damien Jacquemart is a Ph.D student in applied mathematics at ONERA, the French aerospace lab and INRIA. He obtained a Master degree in high education teaching, in 2010. Then, he graduated from the University of Marseille, France with a research Masters degree in probability and statistics, in 2011. His research interests include rare event estimation with aerospace applications. He teaches at ENSAI, the national school of statistics and at the engineering school ENSTA.