# Use of Chaos Polynomials in a Universal Kriging Model: Application to the Numerical Dosimetry

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**Abstract:**

If recent advances in terms of IT resources led to strongly reduce computational time for numerical dosimetry, the calculation of the Specific Absorption Rate that assess the human exposure to electromagnetic fields (EMF) remains very costly (a few hours per calculation). Consequently usual methods as the Monte Carlo cannot be used to study the influence of random input parameters variability on the SAR ( the output variable). Then, optimal metamodeling strategies have to be employed to model the output response depending on the input parameters.

Universal Kriging and Polynomial Chaos are strategies that both aim at modeling the relation between an output process $Y$ an some random input parameters $X = (x_1, ..., x_M) \in \mathbb{R}^M$.

On one side, Universal Kriging theory [1] models the relation between the output and the input parameters by the following expression:

$$Y(X) = \sum_{k=0}^{P-1} \beta_k \psi_k(X) + Z(X) \tag{1}$$

Where $\psi = \{\psi_k, k = 0...P-1\}$ is a collection of regression functions, $\beta = \{\beta_k, k = 0...P-1\}$ are the regression coefficients and $Z$ is a Gaussian process depending on $X$. In practice, the regression functions are chosen with respect to an a priori knowledge about the evolution of the output process. Most of the time, there is no such obvious a priori knowledge and the regression function ensemble is reduced to the unit function. The resulting meta model in generally called Ordinary Kriging model.

On the other side, Polynomial Chaos uses the Wiener polynomial expansion [2] to model the relation between the output variable and the input parameters:

$$Y = \sum_{\boldsymbol{\alpha} \in \mathbb{N}^M} \beta_{\boldsymbol{\alpha}}(X) \psi_{\boldsymbol{\alpha}}(X) \tag{2}$$

Where $\boldsymbol{\alpha} = [\alpha_1 ... \alpha_M]$ is the multi-index, the $\beta_{\boldsymbol{\alpha}}$ are deterministic coefficients to compute and the $\psi_{\boldsymbol{\alpha}}$ are multivariate orthonormal polynomials with respect to the probability measure associated with the random input parameters $X$.

Considering a sparse representation of the Polynomial Chaos expansion using Least Angle Regression technique [3] to select the most influencing polynomials, we propose to use these polynomials as regression functions in the Universal Kriging model. This Universal Kriging enriched with chaos polynomials will be called PC Universal Kriging.

The leave-one-out cross validation is used to assess respective accuracies of sparse Polynomial Chaos, Ordinary Kriging and PC Universal Kriging techniques.

This approach is illustrated with several benchmark functions used in the literature and with a dosimetry example aiming at assessing the fetus exposure to electromagnetic fields. The optimal nature of the PC Universal Kriging approach is illustrated.

As example, for the well-known Ishigami function [4], Figure 1 presents the mean of each metamodeling technique Root Mean Square Errors (RMSE) over 50 different initial LHS designs of experiments that are iteratively increased by the NLHS technique [5]. RMSE is computed using 100000 Monte Carlo points of the generated metamodels. Figure 1 shows that PC Universal Kriging brings about better accuracy in average than the two other techniques for the different numbers of points in the LHS design: slightly better than sparse Polynomial Chaos and much better than Ordinary Kriging for 80 and 160 points.

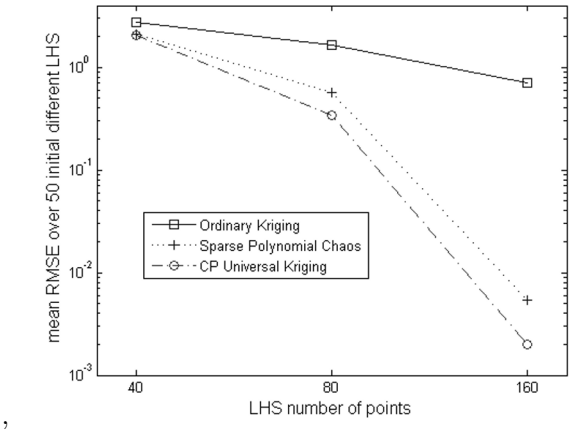


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Figure 1: Accuracy comparison between Spase PC, Ordinary Kriging and PC Universal Krigingdepending on the number of points in the LHS for the Ishigami function

# References

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**Short biography** – From a master in Signal Processing, I am currently involved in the PhD thesis: "Statistical Analysis of People Exposure via the Numerical Dosimetry and the Design of Experiments in Orange Labs and Université Paris-Est". The main purpose of this thesis is to statistically characterize the exposure induced by wireless communication systems.