Goal-oriented low-rank approximations for high dimensional stochastic problems

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Abstract:

Due to the need of more realistic numerical simulations, models presenting uncertainties are receiving a growing interest. Various numerical methods attempt to quantify the probabilistic response of some physical phenomena, often modeled by partial differential equations with random coefficients. Here we adopt a functional point of view of the uncertainty, meaning that we look for an approximation of the solution that is a function of the random coefficients (seen as new variables). Low-rank tensor methods appear as an efficient way to solve the resulting high dimensional problems, and can also be interpreted as model reduction (see recent surveys [2, 6, 5]).

Different strategies have been proposed for the solution of equations in tensor format. One can obtain an approximation of the solution in low-rank tensor subsets through the direct minimization of some residual norm. Using this approximation, estimation of quantities of interest (expectation, variance or sensitivity indices) can be computed. However, there is no guaranty that classical minimal residual formulations provide accurate reduced order models for the estimation of quantities of interest.

The basic idea of the present work (inspired from [3, 4]) is to introduce an ideal minimal residual formulation such that the optimality of the approximation is achieved with respect to a specified norm. We propose and analyze in [1] an algorithm that provide a quasi-optimal low-rank approximation of the solution defined by the ideal formulation.

This new approach offers the possibility to choose a norm so that the optimality of the approximation is achieved with respect to some quantity of interest. We investigate different constructions of such "goal-oriented norm" in the case of linear and quadratic quantities of interest. The resulting method can be seen as an optimal goal-oriented model reduction method.

References

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Short biography –

After a master in advanced numerical methods for computational mechanics at the École Normale Supérieure de Cachan, I started my PhD in 2011 in the GeM laboratory at the École Centrale de Nantes (http://gem.ec-nantes.fr/). I am also a teaching assistant for a course of applied mathematics at the third-year Bachelor level (numerical analysis, optimization, probability and statistics). My PhD is funded by the French ministry of research.