Crack Detection by SBFEM with Global Optimization Algorithms

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Motivation

Inverse problems increasingly arise in contemporary structural engineering problems:

- SHM of wind turbine blades
- Damage localization in fuselage

Stability and efficiency gains in SBFEM solution:

- Egger et al. (2017), A robust and efficient SBFEM [...], Arch .of Appl. Mech.

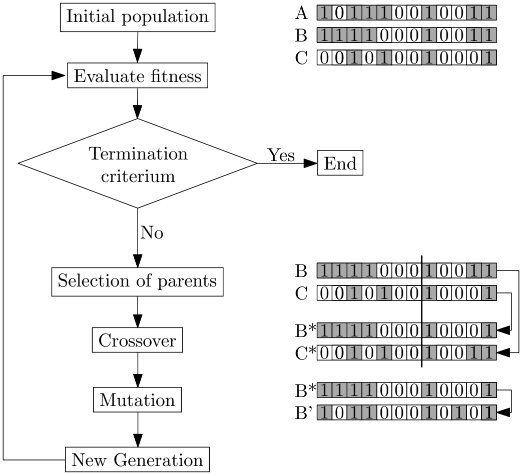
Significant advances in SBFEM tailored **meshers**:

- Liu et al. (2017), Automatic polyhedral mesh generation [...], Comp. Meth. in Appl. Mech. and Eng.

https://www.wind-watch.org

http://www.industrialheating.com

Genetic Algorithm



Positive:

- derivative free
- Can find global minima
- easily parallelizable
- intuative implementation

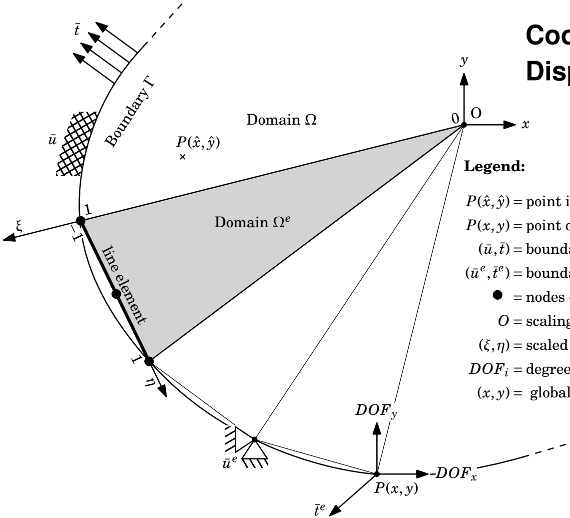
Negative:

- Many function evaluations
- No convergence guarantee
- Symmetry issues possible

SBFEM: Discretization and coordinate system

Coordinates: $x(\xi,\eta) = x_0 + \xi[N(\eta)]\{x\}$

Displacements: $u(\xi,\eta) = [N^u(\eta)]\{u(\xi)\}$



Legend:

$P(\hat{x},\hat{y})$ = point in domain
$P(x,y)$ = point on boundary
$(\bar{u},\bar{t})$ = boundary conditions of domain
$(\bar{u}^e,\bar{t}^e)$ = boundary conditions of elements (i.e. discretized boundary)
● = nodes of line element with coordinates (x_n, y_n)
O = scaling center and origin of normalized radial coordinate ξ
(ξ,η) = scaled boundary coordinates with $0 \le \xi \le 1$ and $-1 \le \eta \le 1$
DOF_i = degree of freedom of each node in global (x,y)-direction
(x,y) = global coordinate system

SBFEM : Scaled boundary finite element equation

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Upon application of the principle of virtual work two equations arise. The first is valid only on the boundary, while the **second term** holds for the domain and is termed **the scaled boundary finite element equation**:

$$\{P\} = [E^0]\{u\}_{,\xi} + [E^1]^T\{u\} \tag{1}$$

$$[E^0]\xi^2\{u(\xi)\}_{,\xi\xi} + [[E^0] + [E^1]^T - [E^1]]\xi\{u(\xi)\}_{,\xi} - [E^2]\{u(\xi)\} = \{0\} \tag{2}$$

with the following substitutions:

$$[E^0] = \int_{\partial\Omega}[B^1(\eta)]^T[D][B^1(\eta)]|J|d\eta \tag{3a}$$

$$[E^1] = \int_{\partial\Omega}[B^1(\eta)]^T[D][B^2(\eta)]|J|d\eta \tag{3b}$$

$$[E^2] = \int_{\partial\Omega}[B^2(\eta)]^T[D][B^2(\eta)]|J|d\eta \tag{3c}$$

SBFEM: Hamiltonian eigen-problem in 2D

Assuming the **general solution as a power series**, we can rewrite the previous equations in modal form:

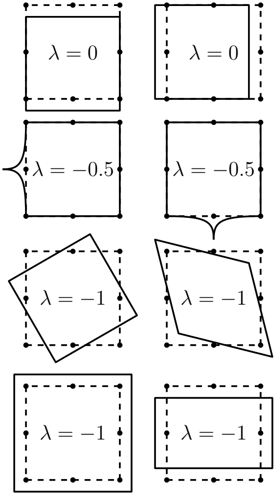
$$\{u(\xi)\} = [\phi]\xi^{\lceil -\lambda \rfloor}\{c\} \tag{4}$$

such that (1) and (2) may be more compactly rewritten as:

$$[Z]\begin{Bmatrix}\phi\\ q\end{Bmatrix} = \lceil \lambda \rfloor \begin{Bmatrix}\phi\\ q\end{Bmatrix} = \begin{bmatrix}\lambda_- & \\ & \lambda_+\end{bmatrix}\begin{bmatrix}[\phi_1] & [\phi_2]\\ [Q_1] & [Q_2]\end{bmatrix} \tag{5}$$

with the **Hamiltonian matrix Z** given as:

$$[Z] = \begin{bmatrix}[E^0]^{-1}[E^1]^T & -[E^0]^{-1}\\ [E^1][E^0]^{-1}[E^1]^T - [E^2] & -[E^1][E^0]^{-1}\end{bmatrix} \tag{6}$$



SBFEM: Generating stiffness matrix

Retaining the bounded response corresponding to the negative eigenvalues:

$$\{u(\xi)\} = [\phi_1]\xi^{\lceil -\lambda_- \rfloor}\{c_1\} \tag{7}$$

and thus on the boundary ($\xi = 1$) after rearranging:

$$\{c_1\} = [\Phi_1]^{-1}\{u(\xi = 1)\} \tag{8}$$

By equation to the modal representation of the external forces on the boundary:

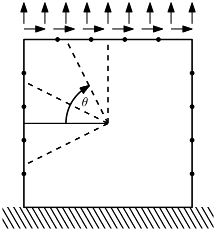
$$\{P_{bounded}\} = [q_1]\{c_1\} = [\mathbf{q_1}][\mathbf{\Phi_1}]^{-\mathbf{1}}\{u(\xi = 1)\} = \mathbf{K_{bounded}}\{u(\xi = 1)\} \tag{9}$$

The stiffness matrix $\mathbf{K}_{bounded}$, though **fully populated**, is **symmetric** and only of dimension $\text{nDOF}_{boundary}$

Numerical example: Introduction

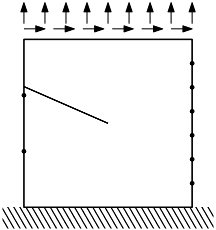
An **edge crack localization** case by GA is considered:

Case 1: crack angle



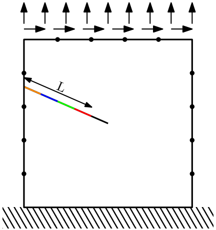
- Crack angles of $[-30, 0, 30, 60, 90]^\circ$
- Two load cases

Case 2: sensor number



- sensors per side [2,3,4,5,6]
- double load case

Case 3: crack length

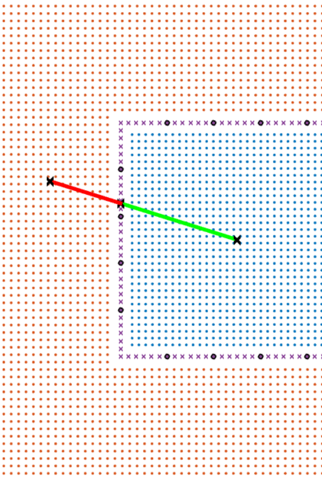


- crack lengths of $L = [\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{2}{15}, \frac{1}{10}]$
- double load case

Numerical example: Method

GA implementation related:

1. Population type as '**bitstring**' leads to **discretization of solution space**
2. **Lower and upper bounds** translate into discretization points
3. Fitness function $f = ||u - u_h|| / ||u||$
   - crack through edge $f \rightarrow inf$
   - both crack tips in same region $f \rightarrow inf$
4. Stopping criteria given by stall generations



Result evaluation related:

- crack defined by intersection with boundary
- Detectability

$$D = \sqrt{(x_{c1} - x_{t1})^2 + (y_{c1} - y_{t1})^2 + (x_{c2} - x_{t2})^2 + (y_{c2} - y_{t2})^2}$$

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Numerical example: Varying crack angles (I)

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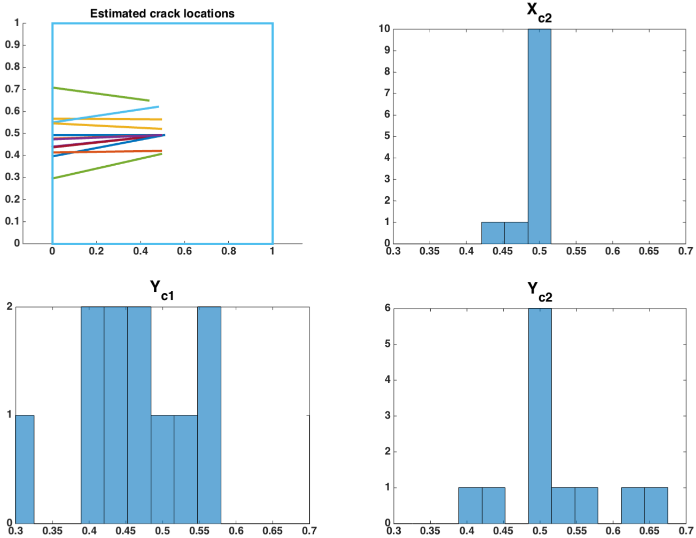
Fittest member of each generation (right) and corresponding crack location estimation (left).

Case of horizontal crack, with a/L = 0.5, using only the tension load case and 4 sensors per side.

Numerical example: Varying crack angles (II)



Numerical example: Varying crack angles (III)

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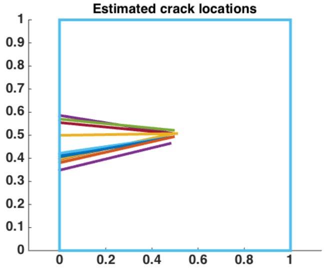
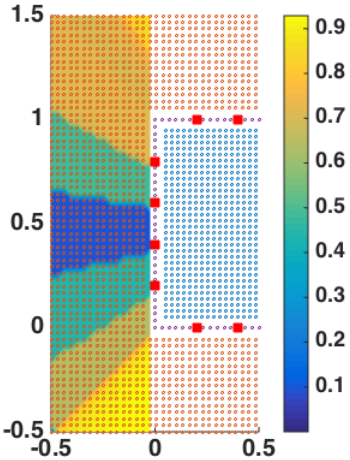
- Ill-posedness of the inverse problem
- Region with tightly spaced fitness function scores
- Stepping in contour plot of the fitness function corresponds to sensor placement location
- Secondary load case helps mitigate issues





Numerical example: Varying crack angles (IV)

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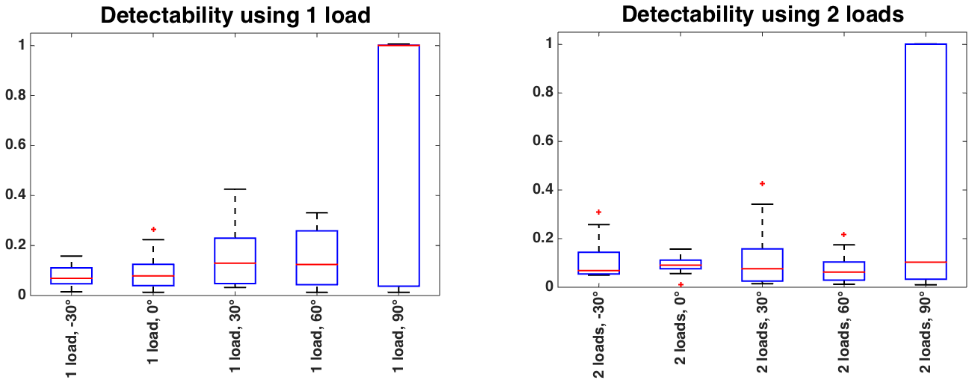
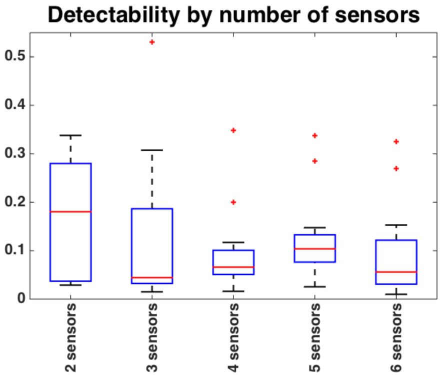
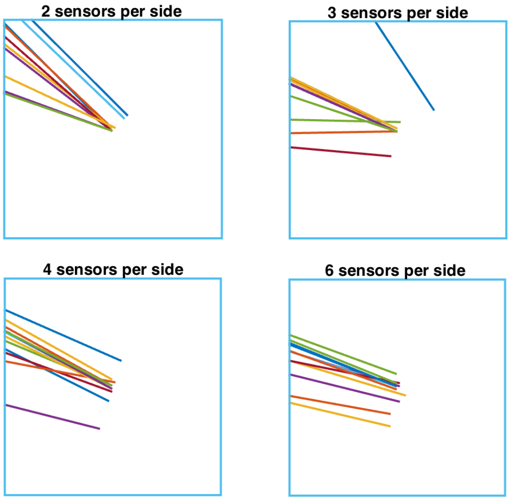


Oberservation: For the load case parallel to the crack direction (1 load) it is impossible to locate the crack accurately, as any vertical crack will "minimize" the optimization function!

Numerical example: Varying sensor amounts





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Numerical example: Varying crack lengths

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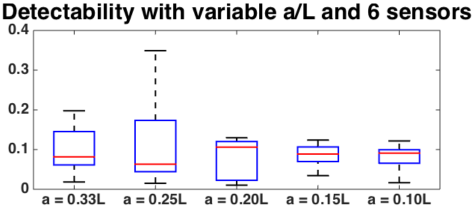

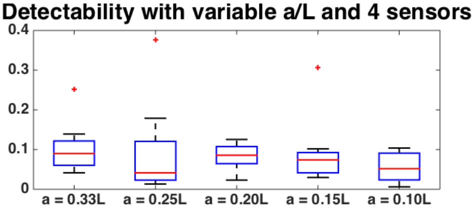

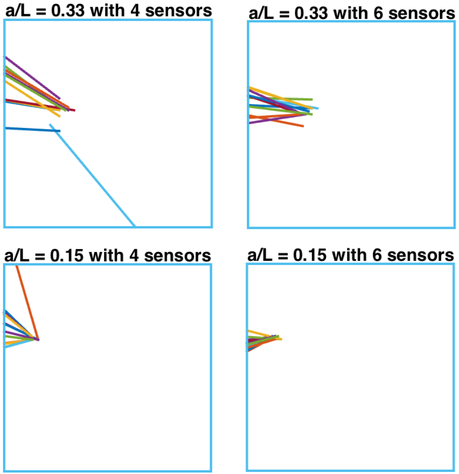

Conclusions

Positive:

- Coupling of global optimization methods with SBFEM
- Crack localization successful
- Detectability within acceptable ranges
- Forward SBFEM evaluation computationally efficient at low computational cost

Negative:

- GA does not always converge
- 1 load case can be insufficient
- Ill-posedness of inverse problem

Outlook:

- More complex geometries and cracks / inclusions
- Multiple cracks in same problem domain
- Better mapping algorithm for binary to decimal based crack representation

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