Mechanics of Building Materials

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Contents

- Introduction to constitutive models for materials
- Fundaments of mechanics of materials
- Cauchy-, hyper- and hypo-elastic material descriptions
- Constitutive Models for Concrete (non-linear elastic)
- Introduction to metal and concrete plasticity
- Introduction to ABAQUS UMAT Programming
- Damage continuum mechanics
- Linear visco-elastic materials
The teaching goals

This introductory course aims to bridge the gap between phenomenological, qualitative comprehension of processes in building materials, their characterization in mechanical testing and the ability to apply those for practical design purposes via constitutive models.

Upon completion of the course you should be able to:

- classify different material behavior (e.g. linear/non-linear elastic, elasto-plastic, creep) with respect to types of constitutive material models (total /incremental strain models, damage / plasticity models, linear visco-elasticity),

- review how incremental strain models (e.g. elasto-plastic) are algorithmically implemented in Finite Element software (UMat of Abaqus),

- formulate the main approach and assumptions to the most import models for building materials and discuss their limitations,

- propose experimental campaigns for obtaining relevant material parameters for non-linear material models.

Tools:

Basics of elasticity theory:
- General principles for formulating material laws
- RVE vs. RUC
- Definition of stress /strain tensors, rotation and principle axis transformation
- Equilibrium conditions / Consistency conditions
- Math tools like Einstein’s summation convention, Kronecker delta Levi-Civita tensor
- Principal stress and shear stress, hydrostatic and deviatoric stress
- Invariants of stress and strain tensors
- Invariant spaces for material laws
- Octahedral, deviatoric, pi-plane, meridian, principal stress planes
- Stiffness and compliance tensor, material symmetries and independent parameters
- Generalized Hooks law with Lamés-constants.
- Strain energy density and complimentary one.
Control questions

1. What are the premises of linear elasticity theory?
2. Why is the stress tensor symmetric?
3. What is an invariant of the stress tensor?
4. What premises lead to the symmetry of the strain tensor?
5. How many independent variables constitute the deformation state in a point?
6. How does the strain state change with respect to transformations?
7. What is anisotropy of elastic material properties?
8. How many independent material constants does a homogeneous, isotropic, elastic body need?
10. Is the MOD for isotropic material larger or smaller than the MOS?
11. How are principal strains and their orientations defined?
12. What are the local equilibrium conditions for the strain stress state in Cartesian coordinates.

Failure models

Material behavior
- Behavior / phenomena for concrete 1-3axial hardening and softening
- Behavior / phenomena for metal1-3axial hardening and softening
  - qualitative behavior with mechanistic explanations.

Failure models
- 1P-models: Rankine, Tresca, vonMises
- 2P-models: Mohr-Coulomb, Drucker-Prager
- Models for concrete 3P-5P:
  - MC with cutoff, Bresler-Pister, Ottosen, Hsieh-Ting-Chen, focus on 5P-Willam-Warnke
- Generalizations for anisotropy
- Smeared Crack Model
  - Assumptions, dependences, approaches, applicability
Comprehension questions:

- What is a constitutive law, what conditions does it have to fulfill?
- What is an RVE, an RUV and what is its purpose?
- Why are there metals with and without yield stress?
- What is the BAUSCHINGER-effect and its explanation?
- What are equivalent stresses for?
- What are yield surfaces?
- What failure hypotheses work for metals, polymers or concrete?

Summary non-linear elastic materials:
Elastic total stress-strain relations

**CAUCHY-type:** \( \sigma_{ij} = F_{ij}(\varepsilon_{kl}) \)

- Stresses and strains are reversible and path independent.
- Reversibility and path independence of the strain energy density \( W(\Omega) \) is not assured \( \Rightarrow \) can produce energy!
- Secant stiffness- and compliance matrix are in general symmetric.
- Uniqueness is not automatically assured \( \Rightarrow \) additional condition needed!
- Typical models have modifications of isotropic linear elastic stress-strain relations via variable secant moduli. Material parameters are well defined and can be obtained from experimental data.

**GREEN (hyper elastic) type:** \( \sigma_{ij} = \partial W / \partial \varepsilon_{ij} \)

- Stresses and strains as well as \( W(\Omega) \) are reversible and path independent.
- Mathematically well defined, parameters are physically hard to interpret and experimentally almost inaccessible.
- Functional shape of \( W(\Omega) \) can be simple assumed to represent phenomena like non-linearity, dilatation, coupling, strain-stress induced anisotropy.
- DRUCKER’s stability postulates are fulfilled, if convexity condition is fulfilled.
- Material secant stiffness matrix and compliance are always symmetric.
Summary non-linear elastic materials:
Incremental stress-strain relations

**Hypo-elastic type:**
\[
\sigma_{ij} = C_{ijkl} (\sigma_{mn}) \dot{\epsilon}_{kl} \\
\dot{\epsilon}_{ij} = D_{ijkl} (\sigma_{mn}) \dot{\sigma}_{kl} \]

- Stresses and strains are only incrementally reversible and path independent.
- Initial conditions have to be given.
- Hypo elastic models can produce energy.
- Material parameter identification requires complicated experimental campaigns. Parameters are hard to fit and have no direct relation to physical properties.
- No clear connection between parameter variation and physical reply.

**Variable Moduli type:**
Models based on curve-fitting techniques.
- Subclass of isotropic hypo-elastic materials that are incrementally isotropic but incremental irreversible.
- No clear stress-strain relation, since different loading and unloading laws act.
- Can produce energy and instability for neutral loading.
- No coupling between hydrostatic and deviatoric part.
- Model assumes coaxially of principal systems for stress and strain increments.

\[
\dot{s}_{ij} = 2 G (I_1, J_2) \dot{\epsilon}_{ij}; \dot{p} = K (I_1) \dot{\epsilon}_{kk} \]

Non-linear-elastic model

**Elastic, Non-linear material models:**
- Cauchy elastic material, hypo-elastic, hyper-elastic material, as well as variable moduli models.
  - How is the non-linearity implemented?
  - What basic assumptions and limitations hold?
  - How are constitutive relations derived?
  - Where are the differences?
  - How are the models applicable (examples for models / materials)?
  - What stability conditions have to be fulfilled?
  - What are advantages and disadvantages of different approaches?
  - Where is the difference in incremental and total material formulations?
**Plasticity theory**

- Approaches for approximation of 1D behavior: Ramberg-Osgood curves et al.
- Recipe for plasticity (1D example) (limit analysis, Drucker’s stability postulates)
- Normality and consistency condition, flow rule (associated / non-associated → stability conditions)
- Hardening rules (kinematic (Ziegler/Prager)/isotropic/mixed),
- History parameters: plastic work, effective plastic strain
- Yield surfaces (Problems with edges)
- J2 Plasticity vs. Drucker-Prager
- Elastic-plastic stiffness tensor, plastic modulus

**Plasticity theory for metals and concrete**

**Metal plasticity:**
- Basic observations
- Basic assumptions of metal plasticity
- Deformation theory vs. Prandtl-Reuss yield theory (incremental theory)
- Hardening module and yield surfaces

**Concrete plasticity:**
- Typical hardening / softening behavior → Conditions to yield surfaces
- Ottosen, Hsie-Ting-Chen and Willam-Warnke
- Peculiarities of concrete plasticity (initial yield surface / hardening rules/ flow rule)
- Non-uniform hardening and multiple plastic hardening
- Models for softening (yield surfaces in strain space)
- Formulation with damage plasticity: Problem, approach, limitations
Plasticity theory for metals, FEM implementation

- What is a Jacobian and why is it important?
- How is the integration procedure for plastic strain increments?
- Why is it necessary to iterate for isotropic hardening and what methods do you know?
- What is radial return mapping?

Continuum damage mechanics (CDM)

- What are basic assumption of the CDM and what is this method good for?
- What are simple damage variables (phenomenological or continuum mechanical ones)?
- What does thermo dynamics have to do with damage mechanics (Clausius-Duhem) and what implications for stability to emerge?
- What is the typical approach for solving problems with CDM?
- Show the analogy between plasticity theory and CDM.
- What is a scalar damage model and how can they be improved?
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Linear Viscoelasticity

- What basic phenomena do we observe for visco-elastic materials and what causes them for polymers, metals or in concrete?
- What is the difference between creep and retardation functions and a relaxation function?
- Define creep modulus, relaxation time and retardation time.
- What 2-parameter models do you know of?
- What is a standard solid and liquid?
- What is the Boltzmann’s super position principle?
- Define storage and loss modulus as well as loss factor.

Thank you for your attention and good exam preparation...