Development of a Dispersometer for the Implementation into Geodetic High-Accuracy Direction Measurement Systems

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Abstract

In the course of the progressive developments of sophisticated geodetic systems which offer a very high accuracy potential strategies for correcting atmosphere-related effects will become increasingly important. These atmosphere-related effects arise in a large span of time scales: systematic deviations caused by a quasi-stationary refractive index gradient environment, generally referred to as refraction in geodetic context, slowly transfer to stochastic deviations resulting from optical turbulence. Refraction corrected optical direction and angle measurements are required in numerous high-accuracy measurement applications. These applications include surveying tasks in connection with civil engineering projects, the alignment of particle accelerator facilities, surveying tasks in context within assembling processes in industrial environments, e.g. aircraft industry, tasks wherein surveying instruments provide the spatial guidance of large machines, etc. A dispersometer, based on the dual-wavelength method by utilizing atmospheric dispersion, constitutes a metrological solution to atmosphere-related effects. Another decisive advantage of a dispersometer is that the envisaged correction of atmosphere-related effects works integrally and is available in real time. The aim of this thesis was to develop this dispersometer to overcome atmospherically induced limitations in very high-accuracy direction and angle measurements.

The dispersometer consists of two modules: the dual-wavelength transmitter and the detection system being composed of the dispersion telescope and a position sensitive detector. By applying the dual-wavelength method, the major challenges in instrumental realization are the generation of coaxial single-mode emission at two spectrally optimized wavelengths and the achievement of optical position sensing accuracy in the order of a few nanometers. The development of the dispersometer is principally made possible by focussing on three key technologies: dual-wavelength generation by frequency conversion, optical fiber technology, and gap-technology. Within this work detailed studies of these three key technologies are performed.

In this work it is demonstrated that a dual-wavelength laser by frequency conversion is clearly suited for the implementation in the dual-wavelength transmitter. Furthermore, a novel technique for achieving coaxial single-mode propagation at two spectrally wide-separated wavelengths by one single-mode fiber is established within this thesis. Due to the application of optical fiber technology it is now possible to couple both beams into one optical channel of a modern geodetic total station. In order to achieve optical position sensing with the accuracy of a few nanometers by using a short-focal-length receiving telescope, gap-technology by utilizing special segmented position sensitive detectors is applied. This thesis contains a complete treatment addressed to this technology. Within the course of dispersometer performance tests, difference position sensing accuracy of $\sigma = 7.3$ nm was achieved. Additionally, the existence of the position sensitive detector inherent dispersion was demonstrated. In combination with the dispersion of the receiving optics, the position sensitive detector inherent dispersion has to be considered for the measurement of the atmospheric dispersion induced displacement between both beams of different wavelengths. As a solution a self-calibration procedure which corrects the dispersion of the complete detection system is described. This self-calibration procedure which utilizes the impact of optical turbulence possesses the decisive advantages that it obviates the need of additional measurements and the dispersion correction can be computed and applied in real time.
A substantial part of this thesis is devoted to dispersometer measurements. Two basic atmospheric conditions which are typical for industrial measurement tasks indoors were simulated. Additionally, a detailed study of the influence of the aperture diameter on the dispersometer measurements was performed.

The optimal aperture diameter for the present instrumental layout and for the prevailing ambient conditions was 30 mm. For theodolite-like and smaller apertures it is confirmed that the accuracy of the refraction angle improves with the square root of the integration time. Due to dispersometer performance by using theodolite-like and possibly smaller apertures in combination with the self-calibration procedure, the implementation of a standard theodolite-telescope is proposed. In a moderately turbulent atmosphere the accuracy of the refraction angle for single-face telescope observation was found to be 0.2 μrad (0.01 mgon) after an integration time of 12 s and a sight length of 17 m.

Summarizing the theoretical investigations, the key technologies involved in the instrumental development, and the experimental results, presented in this dissertation, it can be concluded that the realized dispersometer in combination with a theodolite is capable of the refraction corrected angular measurements, the influences of optical turbulence notwithstanding. The application of optical fiber technology and the envisaged implementation of a standard theodolite-telescope confirm the presumption that the realized dispersometer can be implemented into modern geodetic total stations. Improvements with respect to field-operativeness are expected by an industrial realization of the dispersometer and by implementing the dispersometer into modern geodetic total stations. The integration of blue laser diodes, when meeting the standards of nowadays infrared laser diodes, would significantly enhance efficiency and reduce overall costs. Due to the technologies presented within this thesis such an integration is clearly feasible.
Kurzfassung


Der optimale Aperturdurchmesser für die vorliegende instrumentelle Entwicklung und für die vorherrschenden Umgebungsbedingungen beträgt 30 mm. Es wird für theodolit-ähnliche und kleinere Aperturen bestätigt, dass die Genauigkeit mit der Quadratwurzel der Integrationszeit steigt. Basierend auf dem Leistungsverhalten des Dispersometers für theodolit-ähnliche und kleinere Aperturen im Zusammenspiel mit der Selbstkalibrierung wird die Implementation eines standardmässigen Theodolitfernrohres vorgeschlagen. In einer mässig turbulenten Atmosphäre wurde für die Genauigkeit des Refraktionswinkels für eine Fernrohrlage 0.2 µrad (0.01 mgon) nach einer Integrationszeit von 12 s und einer Visurlänge von 17 m erzielt.

1 Introduction

1.1 Aim of this thesis

The aim of this work was to develop a dispersometer as a metrological solution to overcome atmospherically induced limitations in very high-accuracy direction and angle measurements. It had to be demonstrated that by advancing and implementing recent technologies the instrumental realization of the dispersometer is capable of utilizing atmospheric dispersion. Furthermore, it had to be investigated experimentally that by applying the dual-wavelength method as correction method, atmosphere-related influences on the measurements can be greatly reduced in a large span of time scales. Special emphasize was on the design of the dispersometer in accordance with the established standards of geodetic instruments, in order to enable the implementation of the dispersometer into modern geodetic instruments.

1.2 Approaches for refraction-correction

In the course of the progressive developments of sophisticated geodetic systems utilizing electromagnetic waves in the visible or near infrared range, a more detailed knowledge of the medium in which beam propagation occurs and simultaneously strategies for overcoming atmospherically induced limitations will become important. Although atmosphere-related effects can be observed in several fields related to geodesy, we focus in this thesis on terrestrial high-accuracy direction and angle measurements, leveling, and alignment tasks, because as far as the envisaged applications are concerned, as reported below, the influence of atmosphere-related effects on these observation methods is especially crucial.

The aforementioned atmosphere-related effects arise from density variations in the propagation medium, i.e. in the air, due to spatial and temporal variations in the atmospheric states, e.g. temperature and pressure. Hence, a non-uniform and non-isotropic refractive index field occurs which leads to gradients in the refractive index of air. Consequently, the wavefront of an electromagnetic wave propagating within such a refractive index gradient environment will be tilted. This leads to a systematic discrepancy in angle between the true and the apparent direction at the position of the geodetic instrument, e.g. the theodolite. These angular deviations are called refraction angles in geodetic context. Thus, the detrimental influences of the ambient air are induced by quasi-stationary gradients of the refractive index which appear to have a predominating systematic component.

As far as the complete description of the propagation medium is concerned a contribution towards smaller time scales, i.e. atmospheric turbulence, has to be made. However, larger turbulence structures with long-lasting influence can also cause similar effects, as described above. Unlike the influence of quasi-stationary gradients, the stochastic influences of a turbulent atmosphere can be significantly reduced by extending the integration time. Hence, the accuracy of optical direction and angle measurements is not limited by the precision of the geodetic systems, but by influences caused by the non-uniform and non-isotropic refractive index field of the propagation medium that cannot be averaged out within tolerable integration times.
Refraction-corrected optical direction and angle measurements are required in numerous high-accuracy measurement applications. These applications include construction and monitoring tasks in the field of civil engineering, surveying tasks in connection with large physical installations for basic research, i.e. alignment of particle accelerator facilities, surveying tasks in context within assembling and monitoring processes in industrial environments, e.g. aircraft industry, and tasks wherein surveying instruments provide the spatial guidance of large machines, e.g. controlling of paving machines.

In order to emphasize the detrimental influence of atmosphere-related effects on optical measurements, we will give some examples in the following. Schwarz [1997, 1999] reports on numerical simulations in connection with the planned linear collider with a length of 33 km at DESY (Deutsches Elektronen-Synchrotron), Germany. For the alignment task a horizontal accuracy of $< 0.5$ mm of any point over a range of 576 m (betatron-wavelength) is prescribed. But a constant temperature gradient of 0.1 K/m perpendicular to the axis causes a maximum offset of 4.5 mm for non-refraction-corrected optical measurements. This is nine times the postulated alignment accuracy. For tunnel boring tasks in connection with the ambitious Swiss AlpTransit project, Hennes et al. [1999] determined deviations in direction ranging from 10 to 50 mm for one or two traverse legs of 500 m, even though sophisticated measurement set-ups were considered. These calculations were based on preliminary temperature gradient measurement studies in the Swiss Albula tunnel. A thorough case study of the influence of atmosphere-related effects within the Swiss AlpTransit project can be found in Hennes [1998], emphasizing the importance of substantially reducing atmosphere-related effects on the planned optical measurements. Furthermore, for industrial applications with accuracy demands of $< 10$ μm, a transverse temperature gradient of 0.1 K/m restricts the maximum sight line to $R < 14$ m.

Due to the requirements induced by the envisaged fields of application in which a high degree of automation was already achieved, an effective method for reducing the atmosphere-related influences has to be in real time and should avoid additional instruments, e.g. temperature sensors [Böckem et al., 2000]. Meeting these requirements would enable to incorporate these modules into modern geodetic instruments, e.g. total stations or laser trackers. Consequently, for the first time their accuracy potential could be fully exploited. In order to assure spatial and temporal coincidence between the measured data and the correction values the envisioned approach should work integrally along the actual line of sight.

For more than a century the elimination of atmosphere-related effects in connection with optical measurements has been a central issue in geodetic research, resulting in a vast quantity of methods. However, none of the approaches proposed achieved conclusive elimination or at least substantial reduction of atmosphere-related effects. In order to highlight the dual-wavelength method applied within the instrumental realization of the dispersometer a brief review of the preceding approaches will be given in the following.

Reducing systematic deviations by considering special measurement configurations and procedures is intrinsic to geodetic metrology, e.g. direction and angle measurements in two telescope faces. In connection with atmosphere-related effects, methods considering special measurement configurations and procedures have also been applied, as reported in e.g. [Jordan et al., 1956; Bahnert, 1986; Witte, 1990]. However, these methods often base on idealized assumptions, e.g. symmetrical beam curvature and uniform stratification of air layers, and therefore are often insufficient and inapplicable because model deviations exceed the required accuracy. Furthermore, these methods do often not include the temporal component of the refractive index.
field. In addition, approaches by executing an elaborated measurement procedure provide a substantial lack in operating efficiency, preventing their application for real-time positioning, as described above. However, by carefully selecting observation time and the position of target and instrument with respect to the boundary layer environmental density gradients, the influence on the measurements can be at least reduced which on the other hand compromises efficiency.

A further classical approach is based on the local determination of meteorological parameters. The refractive index of the air can be described by the atmospheric states. Wherein to good approximation the influences of pressure variations and water vapor are negligible, the decisive refractive index gradient which induces the wavefront tilt can be expressed solely by the temperature gradient. Based on this approach Gottwald [1985] and Wilhelm [1993] tried to determine experimentally the refractive index gradient by means of relative temperature measurements. Although the determination of temperature gradients can be achieved with sufficient accuracy by relative temperature measurements using modern high-precision sensors [Hennés et al., 1999], the decisive criterion for the suitability of this method is how well local measurements represent the refractive index gradients along the complete line of sight [Witte, 1990]. Consequently, this method is limited by the finite number of measurement points and the temporal allocation towards the actual geodetic measurement. Additionally, the large effort in determining representative refractive index gradients [Wilhelm, 1993; Hennés et al., 1999] is not bearable in conjunction with highly automated systems.

A purely mathematical approach is given by introducing influences due to atmosphere-related effects to the adjustment process of geodetic networks [Brunner, 1984]. Although the estimation of atmospheric parameters seems to be confident within a redundant configuration, it might be the case that deficiencies of the deterministic model contribute to the estimated parameters of the atmospheric corrections [Gottwald, 1985]. As reported above, the representation of the model in comparison to the real conditions is at least questionable.

A different approach which can be considered as integrally working along the complete line of sight is based on atmospheric turbulence. Herein, the analysis of electromagnetic waves propagating within the turbulent atmosphere leads to the derivation of atmospheric turbulence parameters. From these turbulence parameters the vertical refractive index gradient can be derived by application of an atmospheric model [Casott and Deussen, 2000; Böckem et al., 2000]. In principle, with this approach correction of atmosphere-related effects can be achieved in real time. Brunner [1979, 1980] introduced the fundamental theories of this approach, based on works on optical turbulence, e.g. [Tatarskii, 1971], and boundary-layer meteorology, e.g. similarity relations derived by Monin and Obukhov [1954]. However, a thorough analysis of experimental data was not possible until modern sensor technologies became available and these sensors could be used in context with geodetic instruments. Hennes [1995] made practical investigations with a modified high-precision distance meter ME5000 by Leica, Switzerland. Casott [1999] and Deussen [2000] used a CCD area scan camera and Flach [2000] made investigations using a line scan camera. Furthermore, Hennes [1995], Deussen [2000], and Weiss [2001] also used a meteorological instrument, the small-aperture scintillometer SLS20 by Scin-tec, Germany, for this approach. The intention of these experiments was to demonstrate that the aforementioned turbulence parameters can be derived from the variations of the geodetic signal itself, i.e. from phase and intensity fluctuations.

Although these recent investigations were quite promising, presently the applicability of this approach suffers from several drawbacks. At presence, the atmospheric model is restricted
to a homogenous horizontal surface in open terrain. Investigations on the validity of the atmospheric model and evaluation of influence parameters will be addressed to in [Weiss, 2001]. Although the modeling of indoor and tunnel situations seems to be theoretically possible, it will take further considerable effort until a reliable correction method is available for measurements in these environments. Additionally, for the selection of the appropriate atmospheric model the sign of the temperature gradient must be known. Furthermore, also related to the atmospheric model, this approach solely derives correction values for the vertical angle. But as far as a number of applications is concerned correction of horizontal directions is highly expected and especially crucial.

Unlike all other approaches, as reported above, the dispersometer based on the dual-wavelength method constitutes a metrological solution to atmosphere-related effects. The dispersometer is potentially capable of refraction-corrected direction and angle measurements, the detrimental influence of atmospheric turbulence notwithstanding, by using the dual-wavelength method for dispersive air. Consequently, the correction of atmosphere-related effects works integrally and is available in real time. The challenge for realizing such a dispersometer is the development of a high-accuracy detection system and a stable dual-wavelength transmitter in connection with coaxial emission of monomode radiation at both wavelengths.

1.3 Motivation for the development of a novel dispersometer

The principle of the dual-wavelength method utilizes atmospheric dispersion, i.e. the wavelength dependence of the refractive index. Two beams at different wavelengths propagating from a common origin through the ambient atmosphere will be both affected by atmosphere-related effects, i.e. slightly bent in a refractive index gradient environment, and their optical paths will be separated due to dispersion. Consequently, one observes a difference angle between the two beams of different wavelengths at a receiver, e.g. a geodetic instrument. Fig. 1.1 shows the dispersometer principle for the vertical case. This measured difference angle which is called the dispersion angle $\Delta \beta$ is to good approximation proportional to the respective refraction angles $\beta_1$ and $\beta_2$. Thus, the determined refraction angles can be directly used for correcting the apparent direction. We note that the same relation can be found for the horizontal case.

![Fig. 1.1: Dispersometer principle](image-url)
Hertzsprung [1912] is credited for the original idea of utilizing atmospheric dispersion for the correction of atmosphere-related effects in astrometry. As far as terrestrial geodesy is concerned, the first discussion on the geodetic application seems to be originated by Näbauer [1924, 1929]. In these theoretical studies Näbauer gives a detailed analysis of optical path geometry and atmospheric composition.

Since these first theoretical proposals for utilizing the dual-wavelength method in geodetic metrology a larger number of instrumental approaches have been made. Reviews of these instrumental approaches, reflecting the respective state of their time, are given in [Prilepin, 1974; Glissmann, 1976; Prilepin and Golubev, 1979; Bahnert, 1983; Williams and Kahmen, 1982, 1984]. Startsev [1973] tried to compare dispersometer approaches using different techniques for determining the dispersion angle. In the following a short summary of the essential techniques applied in these realizations will be given and a review of approaches will be presented.

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**Fig. 1.2: Overview of the dispersometer approaches.**
Basically, as shown in Fig. 1.2, one can divide the dispersometer approaches into two groups by using different techniques at the receiving optics. This classification contains dispersometers with imaging optics in one class and dispersometers utilizing interferometry in the other class. The application of interferometry was considered to be less challenging in comparison with the application of imaging optics [Williams and Kahmen, 1984]. A review of the measurement techniques for small angles utilizing interferometry is given in [Bahnert, 1982a].

Khvostikov [1946, 1947] was the first to investigate a receiving system utilizing interferometry by placing a pair of slits in front of the objective of a precision leveling telescope. Thus, in the focal plane either a fringe system for each wavelength could be observed. The mutual displacement of these fringes is a measure for the dispersion angle. Although his laboratory experiments did not yield sufficient resolution and his subsequent proposals showed a lack of stability due to the large slit separation, these experiments can be regarded as the starting point for further approaches based on the interference phenomenon.

The idea of applying interferometric measurements for the dispersion angle was the central issue of research by Tengström [1967] starting in 1948. In the course of this work several so-called interfero-dispersometer (IDM) were built within three decades [Tengström, 1967, 1974a, 1977]. In order to obtain equal fringe spacing for both wavelengths which was considered to be a critical issue in the investigations by Khvostikov [Prilepin, 1970a] two adjustable pairs of slits, each pair with different filters for one wavelength, were introduced with the IDM 1. In contradiction to the IDM 1, Prilepin [1970b] suggested a method and an instrument based on a fixed slit separation, wherein the slits were spaced in a way that a vernier effect between the two sets of fringes occur. However, the lack of sensitivity motivated the development of a subsequent dispersometer (IDM 2) with optical magnification based on the Michelson principle [Bahnert, 1982b]. Additionally, a further approach used reflecting optics by means of a large spherical mirror (IDM 3). The measurements with these first IDMs were made visually by observing coincidence of both fringe systems. Although, laboratory tests were quite promising, these very sensitive instruments with large slit separations in combination with the visual observation method were not capable of conclusive measurements within the presence of atmospheric turbulence [Tengström, 1974a]. Consequently, Tengström applied smaller slit separations, longer focal lengths of the receiving optics and used photographic fringe recording [1974a, 1977]. In the final state of development the instrument consisted of a Cassegrain type telescope with a focal length of 6.26 m, a grating consisting of six slits mounted to the telescope objective, and a photographic camera installed in the focal plane of the telescope. In contradiction to the preceding experiments for the first time lasers, namely, HeNe, Ar\(^+\) ion and HeCd (ultraviolet) lasers, were used in the transmitter. As seen in the specifications of this system, the instrumental realization was very bulky. Furthermore, with the implementation of the long-focal-length Cassegrain telescope, the possibility for the implementation in a conventional geodetic instrument was prevented. Although a number of experiments were made [Tengström, 1974a, 1977; Martensson, 1978], these results are considered to be inconclusive due to the presence of atmospheric turbulence [Martensson, 1985]. The separation of the different laser sources induces a de-correlation of the beams with respect to refractive index fluctuations, therefore infringing the requirements for the dual-wavelength method. Additionally, it might be doubted that the proposal by Martensson [1985] which envisages a vertical separation of the laser sources can significantly reduce the influence of atmospheric turbulence on measurements using
the IDM. Any separation is especially detrimental considering sight lengths up to a few hundred meters which is envisaged for the most crucial applications, see section 1.2.

Unlike the approaches reported above, a compensation method by using the interferometric measurement method was reported in [Vshivkov, 1974; Vshivkov and Shilkin, 1975]. Herein an optical dispersion compensator was incorporated into the optical path of the receiving telescope. Although experiments were executed by correcting the readings of a second theodolite by the compensation technique, this applied visual observation method can be regarded as very time consuming and additionally lacks the possibility of automation.

A substantially different approach was introduced by Michajlov and Lazanov [1975]. In this instrument also coherent radiation at both wavelengths is generated by two gas laser sources, HeNe and HeCd laser. In contradiction to the aforementioned instruments, the interference effect is not accomplished by diffraction on the double slit, but by a birefringent bulky crystal, wherein the path difference between the ordinary and extraordinary beam leads to the interference fringes. A variation in beam angle causes a shift of the interference fringes. The mutual shifting which is equivalent to the dispersion angle, is then compensated by means of electro-optical intensity comparison in combination with a rotatable optical compensator plate. Reading of the angular plate position provides the dispersion angle. Additionally, both lasers are modulated and synchronous detection is employed in order to reduce external influences, e.g. background radiation. As a peculiarity of this approach, transmitter and receiver are incorporated in one instrument, whereas both beams are reflected by a corner cube. Although, the instrumental set-up was very advanced at that time, no investigations have been addressed to the mutual stability of both channels within the instrument and the application of a corner cube reflector which was rejected by Huiser and Gächter [1989] because a partial compensation of the dispersion effect might lead to inconclusive results.

As reported by Bahnert [1983] on a proposal by Jambaev, the methods using interference effects can be expanded by applying Fresnel zone plates in combination with coherent radiation in order to produce fringe pattern which can be utilized for the dual-wavelength method, as reported above. However, the interferometric method suffers from the severe disadvantage that the function of ordinary telescopes is limited by the implementation of the diffractive element. However, more straightforward approaches base on receiving telescopes with imaging optics, as it will be discussed in the following.

An early visual approach by application of imaging optics was introduced by Startsev and Tukh [1955]. Herein, Startsev and Tukh used a rotatable optical compensator to coincide the images at both wavelengths. A brief report of experiments using a refractive telescope is given by Startsev [1973]. However, these experiments showed a considerable lack of accuracy for this instrumental realization.

A proposal by Brein [1954] was the starting point of the development of several dispersometers based on reflecting imaging optics. Brein [1968] used in the first functional model a Cassegrain type telescope with a focal length of 6.25 m. The imaged light spots at both wavelengths were recorded on the same photographic plate, using different filters in separate exposures. Glissmann [1976] reported on severe drawbacks of the design by Brein. Consequently, Glissmann [1974] modified the Cassegrain type telescope by increasing the focal length to 13.7 m, using a rotating filter wheel in front of a position sensitive photodiode. Although this design enabled electro-optical measurements of the dispersion angle, it suffered from certain features which induce disturbances to the measurements, e.g. the presence of a rotating filter
wheel directly in front of the detector. The time consuming set-up of this instrument which was typically five hours [Glissmann, 1976] reflects the impracticability of this system. A continuation of this work was made by Kahmen [1983] which lead to an improved Cassegrain type telescope as reported in [Dallmann, 1986]. Although great effort was devoted to this project, a substantial breakthrough could not be achieved.

A rather different technique suggested by Dyson [1967] was to scan the composite images of both wavelengths by means of a rotating spiral grating. The instrument developed by Dyson [1974] and considerably improved by Williams [1974, 1977, 1979, 1981] consisted of a transmitter by coaxially combining the beams of a HeNe and a HeCd laser and a receiver, composed of a Cassegrain type telescope with the rotating spiral grating and a multi-channel phase detection system. After modulation by the rotating spiral grating the components of the composite beam at different wavelengths are separated by a wavelength selective beamsplitter system and fed to their respective phase detection system. By rotating a compensator plate in one optical channel, the dispersion induced phase difference between the beams of both wavelengths can be reduced to zero. In order to reduce the influence of atmospheric turbulence, either a non-modulated beam at each wavelength is extracted, fed to a separate detector and used as a scaling factor within the signal processing. Although still major difficulties limited this instrument, e.g. slip of the phase lock [Williams, 1977], crosstalk between both signals at the receiver, and errors due to drifting of the separated electronics for each channel, Kahmen [1983] considered this instrument as the most advanced system at that time.

A suggestion by Astheimer and McHenry [1969] also envisaged the use of a rotating spiral grating. Unlike the instrument developed by Dyson and Williams, this proposal envisioned the implementation of the grating at the transmitter. Both systems using a rotating spiral grating are very similar. However, by the suggested dimensions of the latter proposal, the measurement of the phase difference was considered to be more demanding [Williams and Kahmen, 1984].

Another approach very similar to the dispersometer by Dyson and Williams was presented by Fasching [1990, 1993]. Instead of the spiral grating, Fasching used a glass plate with opaque segments mounted to a piezoelectric vibration generator in order to diminish the moving mass. As a major drawback of this layout, this modulator showed a very high temperature dependence. Although time averages were tried, satisfying accuracy was not obtained [Fasching, 1993]. A subsequent work by Gaugitsch [1995] replaced the modulator based on the piezoelectric vibration generator by a magneto-optical modulator. However, in combination with the bulky set-up due to the long focal length of the receiving telescope, the separation of the optical and electronic channels for both wavelengths, and the implementation of bulky gas lasers at the transmitter, no improvements towards a practical, working dispersometer were made.

A very elegant method to obtain directly the refraction-free directions was suggested simultaneously by Glissmann [1977, 1978] and Williams [1978]. Herein, a coincidence procedure was proposed. The basic principle of this approach is to adapt the properties of the receiving telescope to the properties of the propagation medium, e.g. by effectuating a certain focal lengths ratio. Consequently, the optical axis of the receiving telescope points towards the refraction-free, i.e. true direction, if the spots at both wavelengths coincide in the focal plane. A brief review of the related theory is presented in section 2.3.1 and a realization of such a receiving telescope will be shown in section 3.2.1 of this thesis.

Considerable progress was made with a feasibility study of a dispersometer in connection with the rapid precision leveling system (RPLS) [Huff, 1984] as reported in [Wild Leitz, 1987;
Gächter and Huiser, 1987a; Huiser and Gächter, 1989; Ingensand, 1990]. For the first time a receiving short-focal-length telescope, comparable to ordinary telescopes of geodetic instruments, was implemented in a dispersometer. Consequently, in comparison with the preceding approaches a more compact set-up of the receiving unit was accomplished. As a consequence, very high requirements on the resolution capability of the detection system were postulated. Therefore, the gap-technology, as discussed in section 3.2.2, was applied. In the scope of this feasibility study, which was considered to be successful, experiments were made which revealed insights of the potential of the dual-wavelength method by spectral analysis of the measured and corrected angles, e.g. the spectral correlation of both wavelengths, and benefits for the instrumental design were derived [Gächter and Huiser, 1987b]. Due to difficulties within the experimental set-up, e.g. the performance of the dual-wavelength transmitter, and due to economic reasons, e.g. related to the operativeness of GPS, an industrial realization of this concept was not completed. As far as the overall concept was concerned this approach was the most advanced of all dispersometer function models reported in this section.

Within the scope of the RPLS-project, a different experimental set-up was built [Churnside et al., 1989]. The aim of this design was to analyze the influence of the turbulence on the dispersometer. Although considerable investigations were made, the results were partly inconclusive due to the peculiarities of the instrument.

Based on the instrumental approach reported in [Wild Leitz, 1987], a complete new development of a dispersometer was made in the course of this work. In order to overcome the difficulties within the experimental set-up a concept was created based on the further development and application of the most recent technologies. The basic principles of the instrumental development are summarized in chapter 3.

As addressed to within this chapter, the realization of this novel dispersometer is highly expected in geodetic sciences because at the moment neither an alternative metrological solution, nor the suitable modeling exist to achieve optical direction and angle measurements of high accuracy in the ambient air. The application of the dual-wavelength method is considered to possess the greatest potential of overcoming atmosphere-related effects. Though, a thorough demonstration of this potential was not succeeded so far, as reviewed in this section.

### 1.4 Outline of this thesis

In the introduction a short survey on the approaches for correction of atmosphere-related effects on geodetic direction and angle measurements is given. A subsequent analysis of the inherent advantages and disadvantages in combination with a comparison of these methods with the dual-wavelength method will highlight the motivation for developing the dispersometer. Based on the review of previous approaches in instrumental realization of a dispersometer, the key issues for the design of the novel dispersometer will be addressed to. Furthermore, the potential of a working dispersometer is presented and possible applications are mentioned.

Chapter 2 provides the theoretical background for the description of wave propagation through the atmosphere. By applying the dual-wavelength method the atmosphere-related effects are generally distinguished in systematic deviations and stochastic deviations, therefore it was necessary to treat both, geodetic refraction and optical turbulence. Based on the theory presented, the dual-wavelength method will be derived including a method for evaluating the dis-
persion of the complete system. Furthermore, the estimated orders of magnitude predicted for the instrumental realization will be given. In addition to the dispersometer principle, we will provide a statistics based validation of the dual-wavelength method.

In chapter 3 the instrumental realization of the novel dispersometer will be described. This chapter contains the technological key issues of the development process. Within this chapter either a complete section is devoted to the dual-wavelength transmitter and the dispersometer detection system. After a thorough evaluation of the dual-wavelength laser, we introduce a technique for effectuating coaxial single-mode propagation at both wavelengths by the application of optical fiber technology. For achieving position resolution in the order of several nanometers, as required for the dispersometer, we present the so-called gap-technology utilizing segmented photodiodes. In combination with this technology the design criteria and specifications of the analog and digital electronics will be discussed. In addition, the synchronization of the dual-wavelength transmitter with the detection system in combination with the demodulation scheme will be presented. Based on the experimental and theoretical analysis of the noise of the dispersometer, we provide an estimation of the envisaged performance of the dispersometer itself.

Chapter 4 is devoted to the performance tests of the realized dispersometer. Herein included, we provide a brief discussion of the experimental set-up. The main emphasis is on the experimental results, wherein we determine the difference position sensing accuracy of the detection system. Further, we demonstrate the existence of the detector inherent dispersion and evaluate the noise of the system.

In chapter 5 dispersometer measurements will be presented. We provide a detailed description of the signal processing flow. Besides the demonstration of the feasibility of applying the dual-wavelength method, a detailed study of the influence of the aperture diameter on the dispersometer measurements will be performed. Furthermore, experimental results for two basic atmospheric conditions will be presented. Additionally, we show the advantages of the dual-wavelength method by comparison with the conventional single-wavelength method.

In chapter 6 we summarize the main issues in instrumental development of the novel dispersometer. In addition, we discuss the feasibility of applying the dual-wavelength method and the benefits thereof. Finally, we give some conclusions.
2 Wave propagation through the atmosphere

In this chapter we discuss the theoretical basis for wave propagation through the atmosphere in connection with geodetic vertical angle measurements. The analog formalism for horizontal directions can also be found on the basis of the herein provided theoretical background. We restrict this chapter to the vertical component, because within this work we used a one-dimensional position sensitive detector (PSD).

As mentioned in the introduction, atmosphere-related effects occur within different time scales. The influences due to quasi-stationary refractive index gradients which are commonly referred to as refraction in geodetic context [Brunner, 1984a] will be addressed to in section 2.1. In order to interpret the dispersometer measurements, knowledge of the influence of turbulence on wave propagation is required, therefore section 2.2 provides this background with special emphasis on the optical phenomena related to the turbulent medium. Section 2.3 is devoted to the dual-wavelength method underlying the dispersometer measurements. Within this section also the impact of optical turbulence on the dispersometer measurements will be briefly discussed, the orders of magnitude in connection with dispersometer measurements will be estimated, and a statistics based validation of the dual-wavelength method will be given.

2.1 Geodetic refraction

A monochromatic wave can be mathematically described by a complex function of position \( r(x,y,z) \) and time \( t \), referred to as the complex wavefunction \( U(r,t) \) which satisfies the wave equation, Eq. (2.1)

\[
\nabla^2 U - \frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} = 0, \tag{2.1}
\]

where \( c = c_0/n \) is the speed of light in a medium with the refractive index \( n \). Eq. (2.1) can be directly derived from Maxwell’s equations. Furthermore, the complex wavefunction \( U(r,t) \) may be written in the form

\[
U(r,t) = U(r) \exp(i \omega t), \tag{2.2}
\]

wherein \( \omega \) is the angular frequency of the radiation and \( U(r) \) denotes the complex amplitude.

Inserting Eq. (2.2) into the wave equation (Eq. (2.1)), one obtains the Helmholtz equation, e.g. [Born and Wolf, 1993, p. 375]:

\[
(\nabla^2 + k_0^2 n^2(r)) U(r) = 0. \tag{2.3}
\]
In accordance with Eq. (2.2), in Eq. (2.3) $U(\mathbf{r})$ denotes the complex amplitude of a component of the electric field with the vacuum wavenumber $k_0$ and $n$ the local value of the refractive index of the air. We note that the refractive index $n = n(\mathbf{r})$ possesses a spatial dependence. Consequently, for the physical description of a monochromatic wave propagating within a non-uniform and non-isotropic medium as an approximation of the atmosphere, an exact solution of Eq. (2.3) is generally not possible, because the position dependent refractive index $n$ is a random variable and therefore not available along the propagation path. However, suitable approximate solutions for Eq. (2.3) can be found on the basis of elementary waves.

Under the assumption that the origin of wave propagation is a point source, the spherical wave case can be applied by using following ansatz [Saleh and Teich, 1991, p. 48]:

$$U(\mathbf{r}) = A(\mathbf{r})\exp(i k r) / r, \quad (2.4)$$

where $r = |\mathbf{r}|$ is the distance from the origin and $k = k_0 <n>$ denotes the wavenumber within the propagation medium. We note that the angular brackets denote an averaging process. Inserting this ansatz into Eq. (2.3) one yields

$$\left( \nabla^2 + 2\left(i k - \frac{1}{r}\right) \frac{r}{|\mathbf{r}|} \cdot \nabla + (k_0^2 n^2 - k^2) \right)A = 0. \quad (2.5)$$

Huiser and Gächter [1989] found a suitable solution of Eq. (2.5) by making following assumptions. First, as far as the orders of magnitudes are concerned, $1/r << k$ and therefore $1/r$ is negligible in Eq. (2.5), because the wavelength of the radiation is infinitesimally small compared with the distance to the source. Second, the second order derivatives may also be neglected. This can be physically interpreted as follows: Assuming smooth transitions between regions of different refractive indices, the variations in amplitude are small. Consequently, the changes of these variations in amplitude can be neglected. Additionally, the error resulting from this approximation was also estimated in [Huiser and Gächter, 1989]. As a third assumption Eq. (2.6) should be valid

$$|\Delta n(\mathbf{r})| = |n - <n>| << 1. \quad (2.6)$$

Consequently, one finds:

$$A(\mathbf{r}) = A_0(\mathbf{r}) = \exp(i k_0 \varphi(\mathbf{r})) \quad (2.7)$$

$$\varphi(\mathbf{r}) = \int_0^{\mathbf{r}} \Delta n(\mathbf{r}'/r) d\mathbf{r}'.$$

That means the phase of the wave $\varphi(\mathbf{r})$ in an arbitrary position is the integration of $\Delta n$ along the propagation path between the source and that point. An analog expression can be found for the plane wave case.
\[ U(r) = A(r) \exp(ikz)/r \]  

(2.8)

Herein the plane wave propagating from \( z = 0 \) in the \( z \)-direction is described.

\[ A(r) = A_0(r) = \exp(i\phi(r)) \]

(2.9)

\[ \phi(r) = \int_0^\infty \Delta n(x, y, z') dz' \]

In connection with geodetic measurements, the effects on the image formed by the receiving telescope of the geodetic instrument, e.g. the theodolite, are decisive. An analysis addressed to this topic can be found in [Huiser and Gächter, 1989], whereas in the following the essential results will be presented. We assume that the optical axis of the theodolite coincides with the \( z \)-axis, i.e. the theodolite points in the true direction towards the source. The intensity distribution in the focal plane of the receiving telescope \( I(u, v) \) in angular coordinates \( (u, v) \) is related to \( A(r) \) as written in Eq. (2.10)

\[ I(u, v) = \left| \int A(x, y, R) \exp(ik(xu + vy)) dx \right|^2. \]  

(2.10)

In Eq. (2.10) \( R \) denotes the sight length, i.e. the distance between the source and the receiving telescope. Under consideration of Eq. (2.10), one obtains for the measured apparent direction, e.g. the apparent vertical angle \( \beta \) due to the non-uniform and non-isotropic refractive index field

\[ \beta = \frac{\int \int |A(x, y, R)|^2 \frac{\partial \phi(x, y, R)}{\partial x} dx dy}{\int \int |A(x, y, R)|^2 dx dy}. \]  

(2.11)

In Eq. (2.11) the integration is performed over the complete aperture, leading to the center of gravity of the imaged intensity distribution as a measure of the apparent vertical angle. The absolute value of the amplitude is to first approximation \( |A| = 1 \) in accordance with Eq. (2.7). Consequently, the apparent vertical angle is the aperture average of the wavefront tilt \( \partial \phi/\partial x \). By pointing in the true direction \( \beta \) denotes the vertical refraction angle in Eq. (2.11).

A more commonly used description for the refraction angles in geodetic context, which is equivalent to the procedure described above, is based on the geometrical optics approach. By applying the geometrical optics approach by neglecting the finiteness of the wavelength, one can derive the eikonal equation [Born and Wolf, 1993, p. 112], a relation between the refractive index in a point, e.g. the geodetic instrument position, and the optical path length \( S \) between the origin, i.e. the source, and that point

\[ (\nabla S)^2 = n^2(x, y, z), \]  

(2.12)
where $S$ is a scalar function of position. Moritz [1961, 1967] introduced a representation of the refraction angles on the basis of a solution of the eikonal equation by series expansion. From this analysis follows that the refraction angle can be described to good approximation as a function of the refractive index and its partial derivatives.

$$\beta = -\frac{1}{R} \int_n^R \frac{\partial n(x, y, z)}{\partial y} \, dz$$  \hspace{1cm} (2.13)

Eq. (2.13) is the expression for the vertical refraction angle $\beta$ taking solely first order terms into account. In contradiction to a similar formula derived by Moritz [1962], the value for the refractive index was set to unity without loss of accuracy. The $z$-axis of the coordinate system, introduced at the position of the source, is orientated in direction source to receiving telescope, whereas the vertical refraction angle $\beta$ is measured at the receiving telescope. One observes in Eq. (2.13) that the refractive index gradients perpendicular to the optical axis are the decisive parameters. Furthermore, it is also important to note that the refractive index gradients are weighted in proportion to the distance from the source. Thus, refractive index gradients which occur near the receiving telescope contribute with larger portions to the total refraction angle than refractive index gradients close to the source. Consequently, the basic statement of Eq. (2.13) is that due to this position dependent weighting, the refractive index gradients have to be known along the complete propagation path. As observed in connection with Eq. (2.3) the refractive index and consequently, the refractive index gradient are local random variables and therefore not available along the propagation path.

In order to evaluate numerically the influence of certain refractive index gradient environments for coarse approximations, the refractive index gradient is assumed to be constant along the propagation path, therefore it follows from Eq. (2.13)

$$\beta = -\frac{1}{2} \frac{\partial n}{\partial y} \, R$$ \hspace{1cm} \text{for } \frac{\partial n}{\partial y} = \text{const.} \hspace{1cm} (2.14)$$

We note the truly approximate character of Eq. (2.14). Although coarse estimations can be made on this basis, this approach is not suitable for correcting optical high-accuracy measurements. Thus, a method is required to determine the vertical refraction angle with sufficient accuracy for all practical cases, wherein the refractive index gradient is not constant along the propagation path. The dual-wavelength method, derived in section 2.3, constitutes this required method.
2.2 Optical turbulence

Besides atmosphere-related effects due to quasi-stationary refractive index gradients, as described above, the influences of refractive index fluctuations resulting from atmospheric turbulence on the optical wave propagation have to be taken into account. In a similar way, as introduced within this work, Tatarkii [1993] distinguishes between regular and random inhomogeneities and Lawrence and Strohbehn [1970] speak of large- and small-scale variations. Optical turbulence can be defined as the fluctuations in the refractive index resulting from atmospheric turbulence [Beland, 1993]. The effects caused by optical turbulence include temporal intensity fluctuations (scintillation), image motion, and image blurring. Although the magnitudes of the individual fluctuations are very small, the cumulative effect in propagation through the atmosphere results in significant deviations from the ideal behavior. Consequently, for high-accuracy optical measurements, as it is the case for the dispersometer, knowledge of the turbulence induced influence is important for the design of such a system and the interpretation of the measurements. The relevant frequency ranges associated with optical turbulence reported in literature are very heterogeneous. Because the maximum frequency involved is decisive for the data acquisition system in terms of the sampling rate according to the Nyquist theorem, therefore we refer to the highest values in order to assure optimum system performance. Lawrence and Strohbehn [1970] specified 100 Hz as the maximum frequency of intensity fluctuations, wherein they note that the corresponding frequency of the phase fluctuations is significantly lower. The scintillation based value is in accordance with Gaugitsch [1995]. Lohmann [1999] gives generally 10 ms for the smaller time scale involved.

In general for the analysis of turbulence induced influences on optical measurements one divides the approach of the solution into three segments. The first segment involves the statistics of the random inhomogeneities of the refractive index along the propagation path. Second, a determination of the wave propagation statistics is required, wherein the statistics found in the first step for the propagation path are inserted. In a third step the obtained statistics of the propagated radiation have to be translated into statistics describing overall system performance, such as e.g. angle of arrival fluctuations.

In section 2.2.1 the first two steps are briefly discussed, wherein we highlight the essential influences of optical turbulence on the optical angular measurements in the two subsequent sections.

2.2.1 Modeling optical turbulence

Turbulence theory is based on the works of Kolmogorov [1941]. In contradiction to the stratified layer model which induces the aforementioned quasi-stationary refractive index gradient environment, the refractive index fluctuations are conceptionalized in the form of a spectrum of eddies. In the turbulent model each eddy possesses a constant refractive index. Key to this model is the hypothesis that the turbulent kinetic energy produced in the large scale eddies (energy containing range) is redistributed without loss to successively smaller and smaller eddies (inertial subrange) until finally dissipated by molecular viscosity into heat (dissipation range). The transfer of turbulent kinetic energy from larger to smaller scale eddies is known as the en-
eergy cascade. Propagation through a turbulent medium is affected by these eddies, whereas especially the small scale eddies are the optically most active ones in the inertial subrange.

Because most optical phenomena of interest depend on differential rather than absolute path lengths the spatial statistics of random refractive index variations and of random wavefronts can be described in terms of structure functions. The structure function of the refractive index under assumption of an isotropic medium is written as follows:

$$D_n(r) = C_n^2 r^{2/3}$$

for $$l_0 \leq r \leq L_0$$. (2.15)

In Eq. (2.15) $$C_n^2$$ denotes the refractive index structure parameter which is the important parameter for describing optical turbulence. $$C_n^2$$ is often referred to as the refractive index structure constant although underlying spatial and temporal variations exist. As the characteristic parameter of optical turbulence, $$C_n^2$$ is a measure for the turbulence strength varying from $$10^{-17} \text{m}^{-2/3}$$ for extremely weak turbulence to $$10^{-13} \text{m}^{-2/3}$$ or larger in the presence of strong turbulence, e.g. generated near the ground. Eq. (2.15) is only valid in the inertial subrange given with $$l_0 \leq r \leq L_0$$, wherein $$l_0$$ is the inner and $$L_0$$ the outer scale of turbulence. For $$l_0$$ which characterizes the transition to the dissipation subrange one finds typical magnitudes in the order of a few millimeters. For eddy sizes less than $$l_0$$ the turbulent kinetic energy is dissipated into heat due to viscosity effects. The outer scale of turbulence $$L_0$$ near the ground is approximated by $$L_0 = h/2$$, where $$h$$ is the distance from the ground [Hufnagel, 1989]. Brunner [1984b] gives an approximation by $$L_0 = 2h^{1/2}$$. The corresponding spectrum to the structure function in Eq. (2.15) is [Kolmogorov, 1941]

$$\Phi(\kappa) = 0.033 C_n^2 \kappa^{-11/3}$$

for $$2\pi/L_0 < \kappa < 2\pi/l_0$$. (2.16)

In Eq. (2.16) $$\kappa = 2\pi/l$$ denotes the spatial wavenumber, wherein $$l$$ is the scale size of the eddy. Thus Eq. (2.15) and Eq. (2.16) are equally valid and equivalent descriptions of the turbulent refractive index field in the inertial subrange. However, the Kolmogorov formulation of the refractive index spectrum is based on assumptions [Lawrence and Strohbehn, 1970] which are only valid in the inertial subrange, i.e. for infinite outer scale $$L_0$$ and an infinitesimally small inner scale $$l_0$$. In a number of experiments these assumptions are not applicable. Therefore the range of validity of Eq. (2.16) was extended by various authors in order to regard the behavior outside the inertial subrange. For the case wherein the inner scale $$l_0$$ is not negligible small, Tatarki [1971] provided within his improved form a cut-off for the dissipation range whereby the inner scale $$l_0$$ was introduced in the spectrum

$$\Phi(\kappa) = 0.033 C_n^2 \kappa^{-11/3} \exp \left( - \frac{\kappa^2}{\kappa_m^2} \right)$$

for $$2\pi/L_0 < \kappa < \kappa_m$$. (2.17)

where $$\kappa_m = 5.92/l_0$$. Further approaches of extending the range of validity of Eq. (2.16) can be found in, e.g. [Clifford, 1978; Reinhardt and Collins, 1972; Hill, 1978].
With Eq. (2.16) and Eq. (2.17) a statistical description of the random inhomogeneities of the refractive index can be made. Additionally, one has to determine the statistics involved in wave propagation through the atmosphere. Though a large number of approaches exist for this description, we base on results by the Rytov method of smooth perturbations (MSP) considered as the standard method [Tatarskii, 1993]. A review of alternative methods is given in e.g. [Strohbehn, 1968, 1978; Tatarskii, 1993; Beland, 1993]. The key question in connection with wave propagation statistics are turbulence-related effects on the amplitude and phase. Applying the MSP to the wave equation in scalar form, i.e. under the assumption that the turbulence regime is weak and backscatter effects are negligible [Strohbehn, 1978], the MSP provides solutions for the required fluctuations of the log-amplitude \( \chi = \log A \) and the phase \( \phi \). The log-amplitude fluctuations \( \sigma_\chi^2 \) can be expressed by integral representations according to [Lawrence and Strohbehn, 1970], e.g.:

\[
\sigma_\chi^2 = 4\pi^2 k^2 \int_0^R \int \sin^2 \left( \frac{\kappa^2 (R-r)}{2k} \right) \Phi(\kappa) \kappa d\kappa dr \\
\text{for plane waves,} \\
\sigma_\phi^2 = 4\pi^2 k^2 \int_0^R \int \sin^2 \left( \frac{\kappa^2 (R-r)}{2kR} \right) \Phi(\kappa) \kappa d\kappa dr \\
\text{for spherical waves.}
\]

For a locally isotropic random medium using Eq. (2.17) and under the consideration that \( C_n^2 = \text{const.} \) for the homogenous medium for the plane wave case following expression can be found:

\[
\sigma_\chi^2 = 0.31 C_n^2 k^{1/6} R^{1/6} \\
\text{for } l_0 \ll (\lambda R)^{1/2}. \tag{2.19}
\]

The relations for the amplitude fluctuations in Eq. (2.18) and Eq. (2.19) are only strictly valid for \( \sigma_\chi^2 < 0.64 \). Furthermore, aperture averaging reduces intensity fluctuations [Fried, 1966]. Additionally, saturation effects occur which have to be taken into account when information is deduced from the intensity fluctuations, e.g. scintillometry. For angle measurements intensity fluctuations do not have a direct detrimental influence in contradiction to phase fluctuations. However, for the design of the electro-optical receiver the magnitude of the intensity fluctuations contributes to the determination of the dynamic range. Furthermore, when focussing primarily on phase related information, e.g. angle measurements, the intensity related influences on the phase related information by applying electro-optical sensors have to be cancelled out, e.g. by scaling the acquired signal.

While the intensity fluctuations are caused by turbulence with an eddy size in the order of the Fresnel zone size \((\lambda R)^{1/2}\), for phase fluctuations larger scales are predominantly important, although, as we will show in section 2.3.3, small scale variations cannot be neglected. However, this poses the problem that the large scale portion of the refractive index spectrum does not have a universally valid shape. For this reason one uses the phase structure function \( D_\phi(d) \) which is less sensitive to large scales. One obtains for \( D_\phi(d) \) for a random homogenous medium according to [Lawrence and Strohbehn, 1970].
for plane waves and

\[ D_p(d) = 8 \pi^2 k^2 \int_{\kappa=0}^{\kappa=} \Phi(\kappa) \kappa d\kappa \left( 1 - J_0 \left( \frac{\kappa r d}{R} \right) \right) \cos^2 \left( \frac{\kappa^2 r(R-r)}{2 \kappa R} \right) dr \]

for spherical waves.

Using Eq. (2.17) for a locally isotropic random medium and under the consideration that \( C''_\varphi = \text{const.} \) for the homogenous medium one yields for the plane wave case:

\[ D_p(d) = 2.92 C^3 k^3 R d^{5/3} \]

for \( d \gg (\lambda R)^{1/2} \). (2.21)

In Eq. (2.21) one recognizes that the mean square difference of the phase observed at two points separated by the distance \( d \) parallel to the wavefront itself varies with \( d^{5/3} \) and increases linearly with the path length \( R \) and the refractive index structure parameter \( C''_\varphi \). Consequently, at this point we are in position to estimate the turbulence influences on the receiving system.

2.2.2 Turbulence induced angle of arrival fluctuations

In order to describe the effects induced by turbulence on the angular measurements, the phase structure function \( D_\phi(d) \) has to be related to angle fluctuations, i.e. angle of arrival fluctuations, observed in a receiving telescope. For the purpose of displaying the interaction between the turbulent eddies, Fig. 2.1 shows simplified the unperturbed and perturbed state of the wavefront propagating through turbulent medium as far as angle of arrival fluctuations are concerned.

![Diagram](image)

Fig. 2.1: Angle of arrival fluctuations induced by the turbulent medium
Tatarskii [1971] has shown a relation between the phase structure function $D_\phi(d)$ and the variance of the angle of arrival $\sigma_\beta^2$ for an interferometer with separation $d$:

$$\sigma_\beta^2 = \frac{D_\phi(d)}{k^2d^2}. \quad (2.22)$$

In Eq. (2.22) one observes that the variance of the angle of arrival $\sigma_\beta^2$ is a function of the separation $d$, therefore strongly related to the type of receiver. If a telescope is used instead of the introduced interferometer, the angle of arrival is closely related to the average tilt of the wavefront, as already observed for the quasi-stationary case in Eq. (2.11). In contradiction to the stricter interferometer case, in the telescope case spatial variations in the phase tend to average out if they are smaller than the aperture diameter $d$ [Churnside et al., 1989]. The difference between interferometer and telescope case is 3% [Lawrence and Strohbehn, 1970]. Inserting Eq. (2.21) into Eq. (2.22) yields for the plane wave case:

$$\sigma_\beta^2 = 2.92C_n^2d^{-1/3}R, \quad (2.23)$$

wherein the wavelength dependency is mutually cancelled out to this degree of approximation. For the spherical wave case the pre-factor is approximately three times smaller. Gurvich et al. [1968] reported on a pre-factor of $1.05$. Depending on the refractive index spectrum used in Eq. (2.20) slightly different pre-factors occur. However, in case of the dispersometer development where the magnitude of the angle of arrival fluctuations is required in order to optimize the design, the plane wave case, leading to higher values, was preferred. The procedure presented in [Hufnagel, 1989] is equivalent to the approach presented in this work.

### 2.2.3 Turbulence induced blur

A second important aspect of turbulence related effects is spot blurring in the focal plane of the receiving telescope. Because in the dispersometer we use a position sensing technique which induces a restriction towards the spotsize in the focal plane, see section 3.2.2, the maximum blurred spot has to be predicted. Because this requirement is crucial for the performance of the dispersometer, we exclusively use long exposure calculations in the subsequent derivation, although the turbulence blur can be significantly reduced using short exposures. Fig. 2.2 illustrates the influence of the turbulent eddies on the wavefront with respect to the turbulence blur.

In analog form to the approach presented in section 2.2.2, the phase structure function $D_\phi(d)$ has to be related to the turbulence blur. This is accomplished by introducing the coherence diameter $d_0$. To good approximation the coherence diameter $d_0$ for small intensity fluctuations can be written as follows:

$$d_0 = \left( \frac{D_\phi(d)}{6.884d^{5/3}} \right)^{-3/5}. \quad (2.24)$$
Inserting Eq. (2.21) into Eq. (2.24) yields for the plane wave case:

\[ d_0 = \left(0.4236C_n^2k^2R\right)^{3/5}. \quad (2.25) \]

Under the assumption of an ideal telescope with an aperture diameter \(d\) and a focal length \(f_{\text{opt}}\) we obtain in the absence of turbulence for the diffraction limited spot size \(2w\) in the focal plane:

\[ 2w = 2.44 \frac{\lambda}{d} f_{\text{opt}}. \quad (2.26) \]

In accordance to Fried [1966] we replace the aperture diameter \(d\) by the coherence diameter \(d_0\) because in the turbulent case the resolution is turbulence limited. Consequently, one obtains for the turbulence blur equivalent angle \(d\beta\):

\[ d\beta = 13.22 \lambda^{3/5} \left(C_n^2\right)^{3/5} R^{3/5}. \quad (2.27) \]

As described above, the turbulence blur equivalent angle \(d\beta\) can be significantly reduced by using short exposure times which is realized in the data acquisition scheme.

![Turbulent eddies](image)

**Fig. 2.2:** Wavefront distortion by the influence of the turbulent eddies leading to turbulence induced beam spreading.
2.3 Dispersometer principle

In the previous sections we presented the mathematical descriptions for the atmosphere-related effects. As reported above, we distinguish between influences due to refraction which cause systematic deviations and due to turbulence which are substantially random. Consequently, it is one important aim of the subsequent analysis to derive a purely metrological method to correct atmosphere-related effects. This method should eliminate the systematic effects due to refraction and should be applicable within the presence of optical turbulence. The dual-wavelength method, the underlying principle of the dispersometer, embodies this required method. In section 2.3.1 we derive the model equation for the dual-wavelength method based on the appropriate representation of the refractive index of the air. Furthermore, we will briefly highlight a special measurement procedure which is possible when certain requirements of the receiving optics can be met. However, in applying the dual-wavelength method one assumes that the detection system is dispersion-free or the dispersion of the detection system can be matched to the dispersion of the air, respectively. In practice, these both cases are generally not fulfilled. Therefore, we present in section 2.3.2 a self-calibration procedure wherein the measured quantities can be rigorously corrected for the dispersion of the detection system, so that the initial model equation is still strictly valid. In contradiction to the approaches reported in section 1.3, besides the RPLS-dispersometer, this procedure utilizes optical turbulence. In addition, we briefly discuss the impact of optical turbulence on the dual-wavelength method and present some estimated orders of magnitude whereon we base the instrumental design of the dispersometer, as it will be described in chapter 3. And finally, we present the theoretical background for evaluating the dual-wavelength method.

2.3.1 Correction for refraction

The dispersometer principle is based on dispersion, i.e. the wavelength dependence of the refractive index

\[ n = n(\lambda). \]  

(2.28)

The refractivity \( n - 1 \) for a gas can be expressed to a high degree of approximation as the product of the dispersion factor, depending solely on the wavelength \( \lambda \) and a density factor which is independent of the wavelength \( \lambda \) [Edlén, 1966]. Consequently, the dispersion factor remains constant for a selected source radiating on a constant wavelength. Such a source can be technologically realized in the form of a single-mode laser. The aforementioned density factor can be written in terms of the atmospheric states. Thus, the refractivity \( n - 1 \) of air can be expressed as follows:

\[ n - 1 = \mu(\lambda) F_1(p,T) + \mu'(\lambda) F_2(p_{\text{wv}},T). \]  

(2.29)
In Eq. (2.29) $\mu$ and $\mu'$ are the dispersion terms and $F_1$ and $F_2$ denote the spatial dependence in terms of the atmospheric states, i.e. the local temperature $T$, the pressure of air $p$, and the water vapor pressure $p_{wv}$. For dry air ($p_{wv} = 0$) the second term in Eq. (2.29) vanishes. A generally accepted formula for the refractivity of dry air is given by Edlén [1966]. This so-called 1965-dispersion formula is based on a standard air at 15 °C, 760 torr (101.3250 kPa) and a carbon dioxide (CO₂) content of 0.033 %.

\[
(n(\lambda) - 1) \cdot 10^8 = 8342.13 + 2406030 \left(130 - \frac{1}{\lambda^2}\right)^{-1} + 15997 \left(38.9 - \frac{1}{\lambda^2}\right)^{-1}
\]  

(2.30)

In Eq. (2.30) the wavelength $\lambda$ has to be inserted with the dimension $\mu m$. Edlén [1966] reported that the dispersion factor can by determined by relative measurements without accurate knowledge of temperature and pressure. Furthermore, because of the comparatively close similarity of the wavelength dependence for various gases, the dispersion factor is less sensitive to deviations from the correct composition of the gas sample than the absolute refractivity. Consequently, the dispersion of air can be determined with a much higher accuracy than its absolute refractivity. However, the high accuracy of the dispersion formula is eventually lost when extrapolated beyond the wavelength range where they are matched to data [Hill, 1996]. The wavelength range of the matched data for the 1965-dispersion formula spans from 230 to 2059 nm. Thus, this formula is applicable for the dual-wavelength method using one wavelength in the deep blue and one wavelength in the near infrared spectral range.

A further development of the 1965-dispersion formula, i.e. refinement of this formula in the infrared range, was made by Peck and Reeder [1972]. These authors included new measurements on eight infrared wavelengths from 724.7 to 1530 nm and made a new analysis on some parts of the data used by Edlén. In doing so, Peck and Reeder improved the formula in the infrared range, but obtained a slightly worse accuracy in the visible range compared to Edlén [1966]. Simulations have shown that the deviations between Edlén [1996] and Peck and Reeder [1972] for the present wavelengths utilized in the dispersometer, which maximum values are < 3·10⁻⁹, are negligible. A more detailed analysis of the refractive index formulae is beyond the scope of this work, especially when considering that besides the dispersion of air, the dispersion of the detection system which is the combined dispersion of the receiving telescope and the position sensitive detector exists. Detailed reviews of different approaches of refractive index formulae are given in [Deichl, 1984; Hübner, 1985; Hill, 1996; Rüeger, 1998].

As the starting point for the derivation of the dispersometer model equation we use the refraction angle in the representation given in Eq. (2.13), wherein we insert the form given with Eq. (2.29) for the dry air refractive index. Thus, the refraction angles $\beta_1$ for $\lambda_1$ and $\beta_2$ for $\lambda_2$ can be written as follows

\[
\beta_1 = -\frac{1}{R} \int_0^h \frac{\partial (\mu(\lambda_1) \cdot F_1)}{\partial y} \, dz
\]

(2.31)

\[
\beta_2 = -\frac{1}{R} \int_0^h \frac{\partial (\mu(\lambda_2) \cdot F_1)}{\partial y} \, dz.
\]
Regarding that $\mu$ denotes a wavelength dependent constant, the difference angle, i.e. the dispersion angle, can be written as in Eq. (2.32)

$$\beta_1 - \beta_2 = -\frac{1}{R} \mu(\lambda_1) \frac{\partial F}{\partial y} z dz + \frac{1}{R} \mu(\lambda_2) \frac{\partial F}{\partial y} z dz.$$  \hspace{1cm} (2.32)

Furthermore, one can write for the wavelength independent refraction angle:

$$\frac{\beta_1}{\mu(\lambda_1)} = \frac{\beta_2}{\mu(\lambda_2)} = -\frac{1}{R} \int \frac{\partial F}{\partial y} z dz.$$  \hspace{1cm} (2.33)

Inserting the expression given in Eq. (2.33), for e.g. $\lambda_1$, into Eq. (2.32), one obtains the dispersometer model equation (Eq. (2.34)), a relation wherein the refraction angle $\beta_1$ is proportional to the dispersion angle $\Delta \beta = \beta_1 - \beta_2$. Herein, the factor of proportion can be written in terms of the dry air refractivity given in Eq. (2.30)

$$\beta_i = \frac{\mu(\lambda_i)}{\mu(\lambda_1) - \mu(\lambda_2)} (\beta_1 - \beta_2) = \frac{n(\lambda_i) - 1}{n(\lambda_1) - n(\lambda_2)} \Delta \beta.$$  \hspace{1cm} (2.34)

In Eq. (2.34) the factor of proportion, consequently a wavelength dependent constant as well, is an analog form to the Abbé factor, the reciprocal dispersive power, conventionally defined for glass, e.g. [Schröder, 1990]. Based on Eq. (2.34) which constitutes the model equation for the dual-wavelength method, the choice of the wavelength pair plays a crucial role for the instrumental design of the dispersometer. Directly related to the choice of the wavelength pair is the resolution capability of the required refraction angle by a given focal length of the receiving telescope in combination with the resolution capability of the position sensitive detector. According to Eq. (2.30), inserted into Eq. (2.34), the theory predicts that the separation in wavelength should be maximal, whereby the shorter wavelength should be placed as far as possible into the deep blue spectral region. Furthermore, one observes in the run of the curve given by Eq. (2.30) that the advantages gained by placing the longer wavelength into the far infrared spectral range are not substantial compared with the choice of a near infrared wavelength. As far as the instrumental design and the design related simulations show, see e.g. section 3.2.5 and section 3.2.6 as far as the optical power is concerned, the radiation source should be a laser source. Herein, a limitation is given by the available laser sources in the deep blue spectral region as presented in section 3.1.2. Furthermore, as the basic principles underlying the design of the novel dispersometer postulate, as given in chapter 3, an overall monolithic system is the target issue of the instrumental realization. Consequently, besides the requirements given by the theory, the common optical and the common electronic channel should be able to deal with both wavelengths, as exemplified in terms of the spectral responsivity given by the position sensitive detector, see section 3.2.2. Taking both theory and design induced requirements into account, we will present a dual-wavelength laser in section 3.1.3 emitting on the 860 nm wavelength ($\lambda_1$) and the 430 nm wavelength ($\lambda_2$). The detailed wavelengths are given in section 3.1.3.
Consequently, the value of the reciprocal dispersive power for the given exact wavelengths in accordance with Eq. (2.30) and Eq. (2.34) amounts to

$$\frac{n(\lambda_1) - 1}{n(\lambda_1) - n(\lambda_2)} = -41.51.$$  \hspace{1cm} (2.35)

Thus, the accuracy of the dispersion angle must be at least 42-times better than the desired accuracy of the refraction angle. Simultaneously, the dispersion angle will be 42-times smaller than the required refraction angle. This implies the major challenge for metrologically utilizing the dispersion of air for the refraction-correction. The sign of the reciprocal dispersive power denotes that the beam at the shorter wavelength will be slightly more refracted than the beam at the longer wavelength.

We note that the model equation for the dual-wavelength method given in Eq. (2.34) is only strictly valid if the second term in Eq. (2.29) related to the water vapour pressure $p_{wv}$ can be neglected. Consequently, for deviating ambient conditions, i.e. within the presence of water vapor pressure gradients, a small correction term has to be introduced. Considerations of this small correction term are subject to the analysis by de Munck [1970], Prilepin [1973], Tengström [1974b], Glissmann [1976, 1978], and Brunner and Williams [1982]. Theoretically, the deviation from the dry air case due to the presence of water vapor pressure gradients could be cancelled out by introducing a third wavelength [Prilepin, 1973; Tengström, 1974b; Glissmann, 1976]. From a technological point of view this approach is not feasible because the resolution of the detection system had to be increased at least by a factor in the order of $10^2$ when introducing a third wavelength which meets the requirements postulated for the instrumental design. This makes the triple-wavelength method unrealistic. An alternative approach for introducing a correction due to water vapor pressure gradients was suggested by Brunner and Williams [1982]. These authors showed that the correction can be obtained from the knowledge of the Bowen ratio, a well-known ratio in meteorology in which the water vapor pressure gradient is related to the temperature gradient. Both authors conclude that in contradiction to measurements of the water vapor pressure gradients, the Bowen ratio can often be assumed to be constant and therefore derived from point measurements. However, the determination of the Bowen ratio is restricted to certain ambient conditions also discussed in [Brunner and Williams, 1982].

For the experiments within this work, as presented in chapter 5, the influence of the water vapor pressure gradients is negligible. First, the experiments take place in a laboratory environment, wherein the air is dried by air conditioning which yields a small value for the relative humidity and in combination with average room temperature a small value of the absolute water vapor pressure. Second, the laboratory environment can be assumed as a virtually source-free environment for the water vapor pressure, so that no significant gradients occur.

Furthermore, based on Eq. (2.33) an analog expression can be found which will be used below.

$$\beta_i = \frac{\mu(\lambda_i)}{\mu(\lambda_2)} \beta_2 = \frac{n(\lambda_1) - 1}{n(\lambda_1) - 1} \beta_2 = g_{aw} \beta_2.$$  \hspace{1cm} (2.36)
wherein $g_{\text{air}}$ denotes the dispersion of the air. Analog to Eq. (2.35) the dispersion of the air amounts to $g_{\text{air}} = 0.9765$ for the present wavelengths and standard dry air.

For a perfectly achromatic optical system of the receiving telescope, i.e. the focal length ratio for both wavelengths is unity, the required dispersion angle can be directly obtained from the displacement measured with the position sensitive detector. An alternative approach based on a special focal length ratio was proposed by Glissmann [1977, 1978] and Williams [1978].

For the case that the optical axis of the receiving telescope points correctly to the refraction-free direction towards an infinitely distant target we demand that coincidence occurs, i.e.

$$f_1 \beta_1 = f_2 \beta_2,$$  \hspace{1cm} (2.37)

wherein $f_1$ and $f_2$ are the focal lengths related to the wavelengths $\lambda_1$ and $\lambda_2$. We note that in the following, for the sake of simplicity, the indices 1 and 2 refer to $\lambda_1$ and $\lambda_2$, respectively. Consequently, when combining Eq. (2.37) and Eq. (2.36) following condition is required for the focal length ratio of the receiving telescope:

$$\frac{f_2}{f_1} = \frac{n(\lambda_1) - 1}{n(\lambda_2) - 1} = g_{\text{air}}.$$  \hspace{1cm} (2.38)

Thus, a dispersometer detection system consisting of a telescope with the focal length ratio given in Eq. (2.38) will directly point to the refraction-free direction if coincidence of the measured quantities for both wavelengths occurs. Introducing $\beta_0$ as the angle between the optical axis and the refraction-free direction, one can show in accordance with [Glissmann, 1978], that to good approximation the measured position displacement depends solely on $\beta_0$.

$$\bar{y}_1 - \bar{y}_2 = \beta_0 \frac{f_1}{(1 - g_{\text{air}})}.$$  \hspace{1cm} (2.39)

Consequently, coincidence occurs for $\beta_0 = 0$.

Based on this proposal for this coincidence method a so-called dispersion telescope was built in the course of the RPLS-project, e.g. cf. [Wild Leitz, 1987] which is described in detail in section 3.2.1. Because the generally assumed case which bases on an infinitely distant target is not given, the ratio given with Eq. (2.38) should be maintained for significantly shorter sight lengths $R$. To obtain this demanded ratio for different sight lengths, deviating wavelengths, and deviating atmospheric states which require an adaptation of Eq. (2.30), the so-called $\lambda$-tuning was built in the dispersion telescope as shown more detailed in section 3.2.1.

Besides the severe experimental difficulties in $\lambda$-tuning of the optics related to the smallness of the dispersion angle and the measurement range, as presented in section 3.2.2, which practically prevents the correct tuning, the assumption underlying the telescope construction was that the dispersion of the position sensitive detector itself is unity. In chapter 4 we will demonstrate that this assumption is not valid. Based on so-called scanning experiments executed within the course of the dispersometer performance tests the dispersion of the position sensitive
detector was accurately determined, as presented in section 4.2.1. Therefore, the coincidence method was not applicable with the dispersion telescope used. Furthermore, the combined dispersion of the detection system is not equal to one. Hence, a method had to be developed in order to determine the dispersion of the telescope and the position sensitive detector for all practical purposes, i.e. for all focal lengths and pointing directions.

For simplicity we assume that the dispersion of the position sensitive detector is inherent to the dispersion of the receiving optics. Hence, the dispersion of the detection system $g_0$ can be depicted in terms of an unknown focal length ratio in contradiction to the coincidence method. A first approach which is theoretically feasible is to determine separately the effective focal lengths by means of a scanning procedure. Herein, the telescope is tilted resulting in different values for $\beta_0$. Consequently, the quantities measured by means of the position sensitive detector can be directly related to angular quantities and thus, the required ratio $g_0$ can be found. Practically, as far as the attempt of a direct determination for both wavelengths is concerned, the tilting of the telescope with respect to an arbitrary vertical axis which does not coincide with the entrance pupil might introduce longitudinal spherical aberration which decreases accuracy. Therefore, a method will be introduced for the determination of $g_0$ which is possible with an arbitrary fixed pointing, as it will be shown in the subsequent section.

### 2.3.2 Determination of the combined dispersion by utilizing optical turbulence

In the following we present the essential theory related to the dispersometer principle utilizing optical turbulence for the determination of the combined dispersion of the atmosphere and the detection system $g_{\text{comb}}$. We will show that based on this theory a thorough self-calibration to correct the dispersion of the complete system is possible which is solely based on the acquired data. The procedure derived below is rigorous, i.e. the underlying principle of dispersometry is not infringed by introducing simplifying assumptions. As reported above, the dispersion of the detection system $g_0$ which is the combined dispersion of the receiving optics and the position sensitive detector is unknown. Because the combined dispersion of the atmosphere and the detection system $g_{\text{comb}}$ is the product of these dispersion factors one can write Eq. (2.40)

$$g_{\text{comb}} = g_0 \cdot g_{\text{air}}.$$  \hspace{1cm} (2.40)

Hence, determining the combined dispersion $g_{\text{comb}}$ leads to the dispersion of the detection system $g_0$ which is the correction for the position difference measured with the position sensitive detector as shown below. We now focus on the general case with the receiving telescope pointing towards an arbitrary direction as shown in Fig. 2.3.
Fig. 2.3: Schematic drawing of the general case of pointing with the receiving system to an arbitrary position within the presence of refraction.

Distinguishing between quantities related to the arbitrary pointing of the receiving telescope and the refraction induced quantities, one obtains according to Fig. 2.3 a physically exact expression related to the vertical angle $\beta_0$ between the refraction-free direction and the optical axis

$$y_{0i} = g_o \cdot y_{02}$$  \hspace{1cm} (2.41)

and for the refraction induced quantities

$$y_i = g_o \cdot g_{aw} \cdot y_2.$$  \hspace{1cm} (2.42)

Combining Eq. (2.41) and Eq. (2.42) leads to

$$\bar{y}_i = y_{0i} + y_i = g_o \cdot y_{02} + g_o \cdot g_{aw} \cdot y_2.$$  \hspace{1cm} (2.43)

The bar indicates the directly measurable quantities. For the following analysis the representation given in Eq. (2.44) is preferred:

$$0 = g_o \cdot y_{02} + g_o \cdot g_{aw} \cdot y_2 - \bar{y}_i.$$  \hspace{1cm} (2.44)

In practice Eq. (2.44) is not strictly valid, therefore one writes in terms of a model equation:

$$\varepsilon = g_o \cdot y_{02} + g_o \cdot g_{aw} \cdot y_2 - \bar{y}_i.$$  \hspace{1cm} (2.45)

In Eq. (2.45) $\varepsilon$ denotes the experimental error. We are now in position to determine the combined dispersion $g_{comb}$. Herein, we utilize turbulence induced fluctuations resulting in variances of the quantities given in Eq. (2.45). Hence, we apply to Eq. (2.45) the Gaussian law of variance propagation in its general form, e.g. [Koch, 1987, p. 113].
\[ D(y) = \begin{bmatrix} \sigma_y^2 \end{bmatrix} = A D(x) A' \] (2.46)

\( D(y) \) denotes the unknown covariance matrix consisting of a single element, the variance of the left hand side of Eq. (2.45). For the matrix \( A \) of the coefficients one finds:

\[ A = \begin{bmatrix} \frac{\partial \varepsilon}{\partial y_1} & \frac{\partial \varepsilon}{\partial y_{y_2}} & \frac{\partial \varepsilon}{\partial y_2} \end{bmatrix} = \begin{bmatrix} 1 & g_v & g_w \end{bmatrix}. \] (2.47)

For the covariance matrix \( D(x) \) of the right hand side of Eq. (2.45) one finds

\[ D(x) = \begin{bmatrix} \sigma_{y_1}^2 & \sigma_{y_1 y_2} & \sigma_{y_2}^2 \\ \sigma_{y_1 y_2} & \sigma_{y_1 y_2}^2 & \sigma_{y_2 y_2} \\ \sigma_{y_2}^2 & \sigma_{y_2 y_2} & \sigma_{y_2}^2 \end{bmatrix}, \] (2.48)

wherein we used the symmetry of the covariance matrix \( D(x) \). Executing the computation according to Eq. (2.46) one obtains:

\[ \sigma_z^2 = \sigma_{y_1}^2 - 2 g_v \sigma_{y_1 y_2} - 2 g_w \sigma_{y_1 y_2} + 2 \rho g_{wv} \sigma_{y_1 y_2} + g_v^2 \sigma_{y_1}^2 + g_w^2 \sigma_{y_2}^2. \] (2.49)

In Eq. (2.41) \( y_{02} \) denotes the displacement from the optical axis related to the refraction-free direction. If the receiving telescope is mechanically very stable positioned, no variations in \( y_{02} \) occur. Thus, \( y_{02} \) is a constant and its variance vanishes, \( \sigma_{y_{02}}^2 = 0 \). Consequently, the covariances related to the constant \( y_{02} \) and a random variable equal zero, therefore one can write

\[ \sigma_{y_1 y_2} = \sigma_{y_2 y_2} = 0. \] (2.50)

Furthermore, Eq. (2.51) is valid, because the variance of a random variable equals the variance of the sum of this random variable and a constant.

\[ \sigma_z^2 = \sigma_{y_1}^2. \] (2.51)

Additionally, from the same considerations as given above follows also Eq. (2.52)

\[ \sigma_{y_1}^2 = \sigma_{y_2}^2. \] (2.52)

Eq. (2.49) can now be simplified and written in terms of the measured quantities.
In Eq. (2.53) one observes that the variance of the experimental error $\varepsilon$ can be expressed in terms of the variances and covariances of the measured quantities. That means we are in position to calculate the combined dispersion $g_{\text{comb}}$ and thereof the dispersion of the detection system $g_0$ with the acquired data. It is now the demand that the experimental error $\varepsilon$, i.e. the fluctuation of the difference detected on the sensor, becomes minimal by computationally tuning the combined dispersion $g_{\text{comb}}$. This leads to the following ansatz:

$$\frac{\partial \sigma^2_{\Delta \beta}}{\partial g_{\text{comb}}} = 0.$$  \hspace{1cm} (2.54)

This leads to Eq. (2.55)

$$0 = -2\sigma^2_{\Delta \beta} + 2g_{\text{comb}}\sigma^2_{\Delta \beta}.$$ \hspace{1cm} (2.55)

Additionally, one can write for the covariance in Eq. (2.55) by applying the Gaussian law of variance propagation in its general form Eq. (2.46).

$$2\sigma^2_{\Delta \beta} = \sigma^2_{\Delta \beta} + \sigma^2_{\Delta \beta} - \sigma^2_{\Delta \beta}.$$ \hspace{1cm} (2.56)

Inserting Eq. (2.56) into Eq. (2.55) one finds for the combined dispersion of the atmosphere and the detection system $g_{\text{comb}}$ according to Eq. (2.57)

$$g_{\text{comb}} = \frac{\sigma^2_{\Delta \beta} + \sigma^2_{\Delta \beta} - \sigma^2_{\Delta \beta}}{2\sigma^2_{\Delta \beta}}.$$ \hspace{1cm} (2.57)

Using the combined dispersion of the atmosphere and the detection system $g_{\text{comb}}$ in Eq. (2.57) one obtains for the required dispersion of the detection system $g_0$ from Eq. (2.40). Hence, one can write for the angles depicted in Fig. 2.3:

$$\bar{\beta}_1 = \beta_1 + \beta_0 = \frac{1}{f_1} (y_{01} + y_1) = \frac{1}{f_1} \bar{y}_1$$ \hspace{1cm} (2.58)

$$\bar{\beta}_2 = \beta_2 + \beta_0 = \frac{1}{f_1} g_0 (y_{02} + y_2) = \frac{1}{f_1} g_0 \bar{y}_2.$$

For the dispersion angle corrected for the dispersion of the detection system using Eq. (2.58) following expression is valid:
\[ \beta_1 - \beta_2 = \frac{1}{f_1} \left( \overline{y}_1 - g_0 \overline{y}_2 \right). \quad (2.59) \]

And finally using Eq. (2.34) one obtains the required refraction angle \( \beta_i \) related to the radiation at \( \lambda_i \).

\[ \beta_i = \frac{n(\lambda_i) - 1}{n(\lambda_1) - n(\lambda_2)} \frac{1}{f_1} \left( \overline{y}_1 - g_0 \overline{y}_2 \right). \quad (2.60) \]

Eq. (2.60) constitutes the dispersometer model equation, wherein \( \beta_i \) is the desired quantity. Finally, one obtains from the measured and determined quantities:

\[ \beta_o = \overline{\beta}_1 - \beta_1. \quad (2.61) \]

This leaves the angle between the optical axis and the vertical as the only quantity which remains unknown. However, the determination of this quantity is initially covered by the theodolite system, i.e. it can be obtained by the vertical angle encoder reading in combination with theodolite observations. By the standard measurement procedure in both telescope faces, the residual errors due to an eventual off-axis position of the detector and errors due to asymmetries of the position sensitive detector are cancelled out, notwithstanding further errors in theodolite direction and angle measurements which are beyond the scope of this work. We emphasize that executing the standard measurement procedure in both telescope faces for the calibration due to the aforementioned residual errors is the decisive prerequisite for the determination of the refraction angle and the refraction corrected angle as well.

Summarizing the central issues of the dual-wavelength method in combination with the self-calibration of the system utilizing optical turbulence, we draw the following conclusions: A first conclusion is that the determination of the dispersion of the detection system is independent of the actual pointing of the optical axis. The crucial requirement for the pointing arises solely from the mechanical stability of the optical axis. Unlike dispersometer approaches reviewed in section 1.3, this procedure utilizes optical turbulence. However, as it will be shown in chapter 5, the presented self-calibration also works within a weakly turbulent regime. Furthermore, we demonstrated theoretically within the derivation of Eq. (2.57) that the combined dispersion of the atmosphere and the detection system can be directly obtained from the acquired data. Based on the theory proposed we conclude that the achromatic correction of the telescope is not a critical issue because it can be calibrated within the measurement process.

In addition to the dispersion angle corrected for the dispersion of the detection system, the completely dispersion corrected difference angle, i.e. corrected for the combined dispersion, can be written as follows:

\[ \Delta \bar{\beta} = \frac{1}{f_1} \left( \overline{y}_1 - g_0 \overline{g_{air}} \overline{y}_2 \right). \quad (2.62) \]
In order to evaluate the dual-wavelength method it is important to analyze the frequency behavior of the measured and corrected quantities. A brief discussion of the method used is given in section 2.3.5. As addressed to in this section, the power spectrum calculated from Eq. (2.62) is used to show the temporal behavior of the errors by applying dispersometry analog to [Huiser and Gächter, 1989]. The temporal integration of the power spectrum calculated according to Eq. (2.60) is used to show the variance as a function of the integration time.

2.3.3 The impact of optical turbulence on the dispersometer

In the previous section we derived a method for the determination of the dispersion of the detection system \( g_0 \) by utilizing the optical turbulence induced fluctuations in terms of variances of the measurable quantities. Unlike this procedure, a number of dispersometer approaches, as reviewed in section 1.3, were hampered within the presence of optical turbulence. Consequently, one has to introduce a method in order to validate the influence of optical turbulence on the dispersometer measurements. As discussed in section 2.2, optical turbulence occurs in a wide range of time scales, therefore the validation will be based on spectral analysis which will be discussed in section 2.3.5. In this section we focus on the physical explanation of the turbulent influences on the dual-wavelength method using a dispersometer.

The topic of turbulence influences on the dispersometer was extensively addressed to by Churnside et al. [1989] based on the power spectrum of the angle of arrival as derived by Clifford [1971]. Herein, we essentially follow the basic insights by Churnside et al. [1989]. However, although in principle the influence of turbulence could be predicted on the basis of this theory proposed, the associated experiments appear to be partly inconclusive because of the high noise content in the experimental data and the peculiarities of the set-up, e.g. a rotating filter wheel directly in front of the position sensitive detector.

In order to explain the effects related to the dispersometer method, one has to divide the turbulent regime into different regions according to the respective scale sizes, hence, a dominantly refractive regime and a dominantly diffractive regime can be found [Churnside et al., 1989].

At low frequencies angular fluctuations are caused by the largest turbulent eddies. The corresponding optical regime is related to the geometrical optics, hence, beam deflection and refraction occurs. Consequently, effects on both wavelengths \( \lambda_1 \) and \( \lambda_2 \) are expected to be highly correlated, i.e. the correlation of the angular fluctuations is \( \rho_{12} = 1 \). For this reason, the random low-frequency differences in angular deviation of both wavelengths are very similar to a mean refraction effect and thus, can be corrected using the dual-wavelength method.

In contradiction to the turbulent influences at low frequencies, angular fluctuations at high frequencies are caused by small eddies along the path. Consequently, both beams are diffracted by these eddies, rather than merely deflected and refracted. This regime is best described in terms of Fourier optics, e.g. [Saleh and Teich, 1991]. Since diffraction is strongly wavelength dependent the high-frequency angular fluctuations on the beams are uncorrelated to a large extent, i.e. \( \rho_{12} \) approaches zero. Thus, these fluctuations cannot be corrected using the dual-wavelength method. Though, the influences of these fluctuations are substantially random, see section 2.2, and can be averaged out by extending integration time. However, Churnside et al. [1989] also state that the extended observation period need not to be long compared with the
low-frequency fluctuations. The aforementioned partition of the turbulent regime into the re- 
fractive and the diffractive part can be shown by using a model power spectrum $W_\beta(f)$ of the 
age of arrival for a spherical wave, e.g. by Clifford [1971].

$$W_\beta(f) = 0.033 C_n^2 R w_\perp^{5/3} d^{-2} \left(1 - \sin(2\pi d f w_\perp^{-1})(2\pi d f w_\perp^{-1})^{-1}\right) f^{-8/3}, d \ll \sqrt{\lambda R} \quad (2.63)$$

$$W_\beta(f) = 0.066 C_n^2 R w_\perp^{5/3} d^{-2} \left(1 - \sin(2\pi d f w_\perp^{-1})(2\pi d f w_\perp^{-1})^{-1}\right) f^{-8/3}, d \gg \sqrt{\lambda R}$$

In Eq. (2.63) $f$ denotes the frequency and $w_\perp$ is the transverse wind speed. Due to Eq. (2.16) 
which is used as the refractive index spectrum herein, Eq. (2.63) is subject to the following re-
strictions $l_0 \ll d$, $(\lambda R)^{1/2} \ll L_0$, and $w_\perp/L_0 << f \ll w_\perp/l_0$. For the second case in Eq. (2.63) the 
model power spectrum is displayed in Fig. 2.4.

![Figure 2.4: Model power spectrum of the angle of arrival induced by the turbulent medium ac-

Fig. 2.4: Model power spectrum of the angle of arrival induced by the turbulent medium ac-

Fig. 2.4: Model power spectrum of the angle of arrival induced by the turbulent medium ac-

The x-axis is scaled by the normalized frequency $f_{norm} = f f_d$ with $f_d = w_\perp/d$. In Fig. 2.4 one 
observes the different frequency dependencies. For the displayed model power spectrum the low-
frequency part behaves as $f^{-2/3}$, whereas the high-frequency part behaves as $f^{-8/3}$. However, we 
note that apart from the different scaling, these absolute values of the frequency behaviors de-
pend strongly on the momentarily state of the atmosphere and on the parameters describing the 
instrument, wherein the size of the aperture is the decisive quantity. The absolute values of the 
exponents are immaterial for the subsequent analysis. Herein, the relative values of the expon-
ents are decisive, i.e. the relation of the frequency dependency in comparison between the sin-
gle wavelength angle of arrival fluctuations power spectrum and the power spectrum of the dis-
persion corrected difference angle as introduced in section 2.3.2. Furthermore, relative changes 
of the frequency dependencies in variation of the instrumental features, e.g. different aperture 
 sizes, by comparable turbulent conditions are important for the improvement of the instrumental 
design of the dispersometer.
On the basis of theoretical calculations Huiser and Gächter [1989] predicted that the turbulence compensation mechanism of the dispersometer, as explained above, should be very effective. As far as the low-frequency part of the power spectrum is concerned, the power spectrum of the dispersion corrected difference angle appears to be white, whereas the power spectra of both uncorrected angles of arrival show a strong frequency dependence, e.g. for the idealized case in a natural environment the frequency dependency can be estimated by $f^{-2/3}$. For the indoor scenario, wherein the experiments presented in chapter 5 take place, one expects that due to the very low transverse wind speeds $w_1$ indoors, caused e.g. by the air circulation generated by the air conditioning, the entire power spectrum is shifted towards the lower frequency regime. According to Huiser and Gächter [1989] the power spectra of both uncorrected angles of arrival will decrease more rapidly than $f^{-2/3}$ with increasing frequency. As a different approach of explanation for the deviating frequency behavior between model and indoor power spectra, we note that indoors the assumption of isotropic turbulence might not be valid. However, the model spectrum is based on this assumption. The power spectrum of the dispersion corrected difference angle should prevail to appear to be white. As far as the correlation of the angles of arrival at both wavelengths are concerned which characterizes the smooth transition between the refractive and the diffractive part of the power spectrum this correlation will also be shifted towards the lower frequency regime. Based on the explanation of the different optical regimes involved we conclude that the spectral analysis in combination with the analysis of the correlation $\rho_{12}$ is the suitable method for the validation of the dual-wavelength method within the presence of optical turbulence and for the determination of the required integration time $t_{\text{int}}$.

### 2.3.4 Estimated orders of magnitude

In the previous sections we derived equations and descriptions for the various parameters and magnitudes involved in the measurement process of the dispersometer. In order to specify instrumental parameters for the dispersometer, it is essential to know the magnitudes of the atmosphere induced angular influence and the limiting parameters of the receiving optical system. Therefore, a number of simulations were made. In this section, two characteristic cases will be presented, wherein we display the absolute values of the estimated orders of magnitude.

In Fig. 2.5 the atmosphere related influences for sight lengths up to $R = 250$ m are summarized for typical turbulent conditions, wherein the refractive index structure parameter was assumed to be $C_n^2 = 5 \times 10^{-14}$ m$^{-2/3}$. For the calculation of the refraction angle $\beta$ a constant transverse temperature gradient of $dT/dy = 0.1$ K/m was introduced. The resulting dispersion angle was calculated using Eq. (2.34) in combination with Eq. (2.30) for standard dry air. For the parameters of the receiving optical system, we chose for the aperture diameter $d = 42$ mm and for the focal length $f_1 = f_2 = 300$ mm. Furthermore we introduce for the wavelengths $\lambda_1 = 860$ nm and $\lambda_2 = 430$ nm. Consequently, for the angular value of the diffraction limit of the receiving telescope Eq. (2.26) was divided by the focal length. For the turbulence blur Eq. (2.27) was used. For the standard deviation of the angle of arrival induced by optical turbulence we calculated the square root of Eq. (2.23).
Fig. 2.5: Estimated orders of magnitude for sight lengths $R = 10$ m to 250 m.

In Fig. 2.5 one observes the dominating turbulence induced quantities. However, as reported above, these influences are substantially random and can be averaged out by extending integration time. Furthermore, one recognizes that for the strength of turbulence given, the turbulence blur is the dominating magnitude, already for a sight length $R > 44$ m for $\lambda_2$. Consequently, this issue has to be considered by the design of the detection system, see section 3.2.2. We note that the relation between refraction angle and the angle of arrival fluctuations in the simulation characterizes a special case. Normally, for the magnitude of the temperature gradient as assumed, the turbulence appears to be stronger.

Fig. 2.6: Estimated orders of magnitude for sight lengths $R = 1$ m to 50 m.

In addition to the first case we present an estimation for an indoor scenario characterized by the turbulence strength of $C_n^2 = 1 \cdot 10^{-15}$ m$^{-2/3}$. Consequently, the turbulence related quantities are not dominant in this case, but cannot be neglected. In Fig. 2.6 the refraction angle was assumed to be $\beta = 1$ μrad, whereas the major difficulty in applying the dual-wavelength method due to the resulting absolute value of the dispersion angle of $\Delta \beta = 0.024$ μrad becomes apparent.
2.3.5 Statistics based validation of the dual-wavelength method

As briefly mentioned in section 2.3.3, the spectral analysis, i.e. calculation of the power spectra $W(f)$ of the measured and determined quantities, provides an important signal processing tool for the dual-wavelength method. Herein, we focus on the important aspects derived from the analysis of the calculated power spectra $W(f)$. Additionally, we note that due to the significance of the low-frequency part of the power spectra $W(f)$, it is important to include the maximum sequence length possible into the computation process. As far as windowing is used, one has to ensure that the low-frequency part of the power spectrum is preserved.

The first aim of the calculation of the power spectra $W(f)$ of the measured and determined quantities is to derive the general frequency behavior. Herein, the relative frequency behavior of the power spectrum of the angle of arrival at $\lambda_1 W_1(f)$ which, as we will show in chapter 5, is very similar to the power spectrum of the combined dispersion corrected, i.e. corrected by $g_{\text{comb}}$, angle of arrival at $\lambda_2 W_2(f)$, and of the power spectrum of the dispersion corrected difference angle $W_3(f)$ is important. The difference angle corrected for the combined dispersion is determined by Eq. (2.62). Furthermore, by variation of the aperture size $d$, the optimum set-up for the turbulence compensation can be found by minimizing the power in $W_3(f)$, i.e. an aperture size $d$ has to be found, wherein $W_3(f)$ becomes white. For deriving quantitative values describing the relative frequency dependence within parts of the power spectra, appropriate exponents are calculated by least squares fits.

Furthermore, with the analysis of the power spectra a tool for distinguishing between atmosphere-related influences and influences related to the instrumental set-up is given, as it will be introduced for the instrumental set-up in chapter 4 and the atmosphere-related quantities in chapter 5. As far as it will be shown in section 3.2.6, the noise floor of the detection system depends on the total amount of optical power incident to the sensor. Consequently, the noise floor for different aperture sizes $d$ can be determined in order to evaluate the turbulence compensation of the dispersometer.

As briefly discussed in section 2.3.3, the analysis of the power spectra $W_1(f)$ and $W_2(f)$ in terms of their correlation $\rho_{12}$ gives the possibility to quantify the range of validity for the dual-wavelength method, i.e. the frequency regime wherein Eq. (2.34) holds, and the regime wherein both beams are substantially uncorrelated.

Moreover, knowledge of the power spectra enables the calculation of accuracy as a function of integration time $t_{\text{int}}$ when time averaging is involved. In the following we are focussing on the variance of the dispersion angle in accordance with Eq. (2.59) when time averaging is involved.

For temporal averaging processes low-pass filters are used [Czichos, 1989]. Consequently, the variance of the dispersion angle for a given integration time constant $\tau$ can be calculated as follows:

$$
\sigma_{\delta \tau}^2 = \int_0^\infty W_{\delta \tau}(f) |U(f)|^2 \, df.
$$

(2.64)
In Eq. (2.64) \( U_t(f) \) denotes the transfer function of the low-pass filter. For a low-pass filter of first order the square of the absolute value of the transfer function is

\[
|U_t(f)|^2 = \frac{1}{1+(2\pi f \tau)^2}.
\]  

(2.65)

Inserting Eq. (2.65) into Eq. (2.64) and for \( t_{mt} = \tau \), one derives finally in accordance with Eq. (2.60) for the variance of the refraction angle integrated over the time \( t_{mt} \):

\[
\sigma_{\beta,t_{mt}}^2 = \left( \frac{n(\lambda^2) - 1}{n(\lambda^2) - n(\lambda_0^2)} \right)^2 \int_{0}^{\infty} W_{\Delta \phi}(f) \frac{1}{1+(2\pi f \tau)^2} df.
\]

(2.66)

In Eq. (2.66) we assumed that the variance of the wavelength dependent constant and the variance of the focal length \( f_1 \) are negligible. However, within the analysis of the experiments, as presented in chapter 5, the influences of these quantities will be discussed. Within this model, the variance of the refraction corrected vertical angle \( \beta_{corr} \) can be written as follows:

\[
\sigma_{\beta_{corr}}^2 = \sigma_{\text{angle}}^2 + \sigma_{\beta_{t_{mt}}}^2,
\]

(2.67)

wherein \( \sigma_{\text{angle}} \) characterizes the accuracy of the vertical angle measurement system including the uncertainty within the two faces measurement procedure. Hence, with the knowledge of the power spectrum of the dispersion angle, it is possible to estimate the accuracy of the refraction corrected vertical angle as a function of the integration time online, if the aforementioned additional contributions to the overall accuracy, e.g. influence of the variance of the focal length \( f_1 \), were previously determined. Online estimation of measurement accuracy is highly desirable for a complete automation of the angular measurement system by using the dispersometer for obtaining refraction-corrected direction and angle observations.
3 Dispersometer

According to the dual-wavelength method the dispersometer consists of two modules: the dual-wavelength transmitter and the detection system being basically composed of a dispersion telescope and a position sensitive detector. Hereby, both modules have to be stable and compact devices. Consequently, one main focus of the instrumental realization was to develop the dispersometer with a monolithic structure. Both beams propagating from the laser light source to the detection system should be rigorously affected by the same influences. Solely the utilized physical effect, i.e. atmospheric dispersion, displaces both beams. This implies that both beams have to be strictly coaxial-guided. The dual-wavelength transmitter generates the spatially coincident origins of spherical wave propagation for both wavelengths. Furthermore, the detection system is also one monolithic system. This means that, with the exception of the wavelength dependent effects, e.g. the wavelength dependent absorption depth on the sensor, which have to calibrated, all impacts will affect both signals. In doing so, both beams are highly correlated with respect to the refractive index fluctuations of the ambient atmosphere as postulated in section 2.3.1 of this thesis.

One of the basic principles established by the tight specifications which are inherent in utilizing atmospheric dispersion was to apply, adapt, and advance recent technologies. The metrological application of the dual-wavelength method requires one operating wavelength in the blue spectral range. This spectral range is covered by only a few laser sources, see section 3.1.2. This induces, due to the lack of preceding investigations, that the behavior of passive (e.g. optical fibers) and active (e.g. sensors) optical devices cannot be very well predicted. Beyond that, the large span in operating wavelengths induces further difficulties into the design and realization process. Derived from the magnitudes involved in the measurement process of the dispersometer, see section 2.3.4, and under the consideration of the noise model presented in section 3.2.6, one of the most rigid demands leads to the application of advanced technologies to ensure that side effects can be minimized or well calibrated and therefore compensated leading to conclusive results which are not masked by system noise.

Another goal of the development was that a future integration into existing geodetic high-end total stations will be possible. Modern geodetic total stations can be characterized as multi-laser and multi-sensor devices. With the implementation of the dispersometer their accuracy potential should be fully exploitable, even when measuring in adverse meteorological conditions. Technologies had to be developed and applied which offer the potential towards manufacturing a compact dispersometer. Guidelines for the development should meet the established standards, e.g. the derived requirements prescribe receiving optics with similar specifications as standard theodolite telescopes. In combination with a monolithic design and the application of the most recent technologies it should be possible to incorporate the dispersometer into existing geodetic systems.
3.1 Dual-wavelength transmitter

3.1.1 Requirements and basic conception

In order to exploit the small effect of atmospheric dispersion, laser radiation at two different wavelengths which have to be optimized in spectral separation, see section 2.3.1, is required. The output power level, dependent on the efficiency of the laser, the transmission of the optical elements, and the geometry of radiating, should enable position detection above the system noise floor. Due to the detection scheme, as discussed in section 3.2, it is required to apply a temporal intensity modulation with suppression of the alternate wavelength. As presented in section 2.2, atmospheric turbulence arises in time scales down to 0.01 s. Regarding the Nyquist sampling theorem, a signal modulation frequency $f_{\text{mod}} = 250$ Hz is envisaged. To prevent crosstalk between the signals at both wavelengths induced by the dual-wavelength transmitter, modulation with a very high extinction ratio is demanded. One of the most important postulates of the detection scheme is to assure phase-locked sampling of the received signal. In order to synchronize modulation and sampling, i.e. $m f_{\text{mod}} \equiv f_{\text{samples}}$, wherein $m$ is a fixed integer, a synchronization signal derived from the modulator is necessary. Beyond this, the dual-wavelength transmitter has to generate a wavelength-coded reference signal for de-multiplexing the received signal on the detector side for the extraction of wavelength information.

As introduced in section 2.3.1 and section 2.3.2, the observation method of the dispersometer is characterized by measuring the dispersion angle between the two beams. This is realized by detecting the centers of gravity of the imaged intensity distributions in the focal plane of the receiving telescope with the accuracy requirement given in section 2.3.1 and section 2.3.4. As a consequence, either a so-called Gaussian TEM$_{00}$ beam for both wavelengths is required, as it will be discussed in more detail in section 3.1.5. In addition, coaxial emission of both beams with spatial coincidence < 0.5 μm assuming the shortest experimental sight length of $R = 17$ m, as reported in section 5.1, is postulated.

Fig. 3.1: Schematic drawing of the dual-wavelength transmitter. I: Electronics for the reference photodiode, II: DC-motor-driver, III: Signal processing for magnet encoder
The configuration of the dual-wavelength transmitter is depicted in Fig. 3.1. The core of the transmitter is the dual-wavelength laser generating blue light and IR radiation. Both beams will be modulated externally by passing a rotating filter disc. At this point a reference beam is extracted by an implemented two mirror system and lead on to the reference photodiode. Mutual power adjustment is enabled by a rotatable polarizer. Both alternately transmitted beams are coupled into an optical fiber by a microscope objective.

3.1.2 Laser sources

Laser diodes (LDs) are implemented in many modern geodetic instruments because of their advantages of high efficiency, compact size, reliability, long lifetime, and low cost. Reliable laser diode sources are currently available over a wide wavelength range from 630 nm to 2000 nm. However, for an optimized performance of the dispersometer, according to the dual-wavelength method presented in section 2.3.1, one wavelength has to be placed outside this aforementioned wavelength range. For this second laser source a deep blue light emitting laser is required.

Presently, mainly gas lasers cover the blue spectral range. These are in particular Ar⁺ ion laser emitting typically @458 nm and @488 nm and HeCd laser which lase typically on the 325 nm (ultraviolet) and the 442 nm line. Although gas laser technology is well established since many years, gas lasers possess a number of disadvantages. Their lifetime is limited by the lifetime of their laser tubes which amounts typically to only a few thousand hours. Furthermore, the electrical-power-to-light conversion efficiency is very small, e.g. for an air-cooled Ar⁺ ion laser model 2011-10SL, JDS Uniphase, USA, the electrical-power-to-light conversion efficiency amounts to 0.005 %, whereby the total power consumption is specified to 2 kW. Additionally, the device volume of gas lasers is larger than 10,000 cm³. These dimensions do not permit a direct incorporation into a theodolite as it is proposed for the dispersometer. Although several set-ups of dispersometer experiments, e.g. [Williams, 1979; Churnside et al., 1989] utilized HeCd laser as second laser sources, these devices are not suitable for modern geodetic "field"-instruments. These disadvantages of laser sources available in the blue spectral range have stimulated extensive research in the field of blue light emitting all solid state lasers.

In order to transfer the advantages of laser diode sources towards shorter wavelengths great effort has been devoted to the realization of blue semiconductor laser diodes. The first research was focussed on wide-bandgap II-VI semiconductors, leading to a first demonstration of a laser operating @490 nm [Haase et al., 1991]. Due to the remaining problems of II-VI devices, the first report on a III-V laser diode emitting @417 nm was considered to be a major breakthrough for blue semiconductor lasers using a semiconductor material which was believed to be less challenging [Nakamura et al., 1996; Nakamura et al., 2000]. Although first devices were commercially available most recently, it seems to be still a long way for these blue semiconductor diode lasers to become reliably operating devices. Especially, for the implementation in a dispersometer it will take further considerable effort in research and development until these devices meet the standards of nowadays IR laser diodes.

An alternative compact laser source which allows dual-wavelength operation with a spectral separation required for the dispersometer principle is based on frequency conversion. Frequency doubled laser diodes have the potential of compact and robust devices with high
efficiency [Fluck, 1995]. Utilizing high standard near IR (NIR) laser diodes as the source of the 
fundamental wavelength, blue light can be generated by frequency doubling of this wavelength. 
Application of this technology became possible both by advances in NIR semiconductor lasers 
and in nonlinear materials in conjunction with nonlinear conversion techniques. In the following 
section the implemented dual-wavelength laser based on frequency doubling of the wavelength 
of a semiconductor NIR laser diode in a bulk potassium niobate (KNbO₃) crystal will be pre-

sentent and its performance will be discussed.

3.1.3 Compact dual-wavelength laser by frequency conversion

KNbO₃ crystals are very attractive for frequency doubling the wavelength of near infrared laser 
diodes into the blue spectral range due to their high nonlinear coefficients. KNbO₃ crystals offer 
the favorable noncritical phase-matching (PM) possibility by temperature tuning between 
850 nm and 960 nm in crystals cut along their crystallographic axis [Fluck et al., 1996]. The 
corresponding frequency doubled wavelength range spans from 425 nm to 480 nm, respectively. 
Noncritical phase-matching, i.e. frequency conversion along the crystallographic axis, offers 
the advantages of high conversion efficiency and a large angular acceptance of the beam focussed 
into the crystal. By meeting the phase-matching condition the KNbO₃ crystal behaves as a spa-
tial filter, because the higher order modal content of the fundamental wavelength exceeds the 
angular acceptance of the KNbO₃ and consequently, does not contribute to the second harmonic 
generation (SHG). This is favorable for the beam quality of the second harmonic (SH) wave. 
However, temperature tuning in this configuration is more demanding because of the large in-
teraction length within the crystal.

The experimentally least challenging configuration is frequency doubling in a bulk crystal 
in single-pass geometry. As a drawback SH output power from single-pass bulk crystal schemes 
is intrinsically limited to a few percent conversion efficiency. Pliska [1997] reported normalized 
conversion efficiencies of typically 1 to 1.5 %/W cm⁻¹ from single frequency SHG measure-
ments on about 50 samples of KNbO₃ crystals. However, the development of high power CW 
NIR diode lasers enabled blue light power levels of several milliwatts.

During the course of this research work two lasers based on frequency doubling of the 
wavelength of a NIR laser diode in a bulk KNbO₃ crystal have been incorporated in the trans-
mitter set-up. Due to the experiences gained with the first model of the dual-wavelength laser a 
succeeding version was designed with improved specifications towards mechanical stability, 
power stability, and signal noise. Whereas the development of the first model was mainly fo-
cussed on high SH output power, for the second dual-wavelength laser source the design issues 
were concentrated on minimizing power fluctuations. Furthermore, it has to be pointed out that 
the technologies of the set-up developed with the first laser model are valid without any restric-
tions for the follow-up model. Therefore, this section is restricted to the second laser model.
In Fig. 3.2 the schematic drawing of the laser set-up is depicted. Here, the radiation at the fundamental wavelength is generated by a NIR semiconductor laser. This multi-longitudinal-mode CW laser diode itself offers a wavelength range of typically 10 to 15 nm, whereby the acceptance wavelength range for the SHG amounts to $\Delta \lambda_{\text{accept}} = 0.1$ nm. By introducing a cavity one obtains a discretization of the emitted frequency spectrum of the LD. From this discretization of the frequency spectrum the incorporated grating effectuates a selection of 4 to 5 longitudinal modes within $\Delta \lambda_{\text{accept}}$. The presence of these 4 to 5 longitudinal modes is substantial for the overall conversion efficiency. Each frequency generates its second harmonic, whereas all mutually available frequency combinations contribute to sum frequency generation (SFG). Here, in comparison to SHG solely, overall efficiency is increased by a factor 1.75 to 1.8 [Ducuing and Bloembergen, 1964].

In a first step maximum power output of the NIR-LD ($P_o = 0.5$ W) in conjunction with a low noise level at a wavelength which enables SHG at $\sim 25$ °C is achieved by optimizing the temperature of the NIR-LD in combination with tuning the current of the NIR-LD. In order to achieve frequency doubling the emitted radiation is then focussed into the bulk KNbO$_3$ crystal by the depicted lens system. For meeting the phase-matching conditions by temperature tuning of the KNbO$_3$ crystal, the KNbO$_3$ crystal is installed in an oven. In an iterative process the adjustments of the temperature of the KNbO$_3$ crystal, the temperature of the NIR-LD and the current of the NIR-LD are refined to effectuate optimum performance. Both emitted beams, at the fundamental and SH wavelength as well, are collimated by use of an achromatic collimation lens. Considering the efficiency of the frequency conversion, the power at the fundamental wavelength is approximately adjusted to the power of the SH by using an appropriate dichroic beam-splitter. Incorporation of the dichroic beam-splitter into the optical path was necessary, because the specifications of the detection scheme require balancing of both power levels ($P_o$ and $P_{2o}$) according to the spectral responsivity of the sensor. Due to the critical temperature conditions for second harmonic generation two temperature control loops have been incorporated into the laser system. In Fig. 3.3 the SH power dependence on the crystal temperature is shown.
Fig. 3.3: Dependence of the generated SH power on the temperature of the KNbO₃ crystal

For the high power NIR laser diode emitting at λₑ = 859.2 nm the optimal crystal temperature (KNbO₃, crystal length L = 10 mm) for the phase-matching condition amounts to 27.5 °C, see Fig. 3.3. One observes the typical tuning curve characterized by a best Gaussian fit calculated on the measured data. The experimentally determined full width at half maximum (FWHM) is 0.4 °C. One critical issue in temperature tuning is to avoid crystal temperatures on the flanks of the Gaussian curve. Here, in conjunction with the power fluctuations signal noise increases rapidly. For a peak to peak (p-p) SH power variation ΔP₂ω ≤ 3% which is equivalent to the SH power decrease caused by the drop out of one longitudinal mode, as reported below, the range of the controlled temperature has to be limited to ±0.07 °C centered on the optimum crystal temperature. Besides a direct effect on the SHG, temperature variations lead to variations of the laser cavity which is detrimental for the performance of the dual-wavelength laser as well. This temperature requirement in combination with the stability of the fundamental which is also temperature dependent is one of the most critical issues in instrumental realization.

During operation mutual interaction of the control loops assures an uniform performance. Unlike most systems utilizing semiconductor diode lasers, power monitoring by a so-called monitor photodiode in a feedback loop does not improve performance of the dual-wavelength laser, i.e. power stability and noise reduction. Monitoring SH output power of the laser is not sufficient due to the complexity of the applied nonlinear effects, as reported above. A possible solution would envisage spectral analysis of both beams which would increase the cost of the dual-wavelength laser.

In order to investigate the behavior of the complete dual-wavelength laser system measurements of the SH power stability were made with an optical power meter model 835 in combination with a 818-SL sensor, Newport, USA. In Fig. 3.4 the SH power behavior in the first 40 s after laser on from standby is shown. Here, SH power increased rapidly. The maximum value was reached within the first 30 s. Fig. 3.5 shows the typical power stability of the SH @429.6 nm within 70 minutes. The power drift of the generated SH light can be specified ≤ 1.4 %/h based on measurements for an ambient temperature of 22.7 °C. Long-term power stability variations are mainly caused by ambient temperature variations.
We note that the value determined for the SH power drift is valid as long as the same number of longitudinal modes is present. If the number of longitudinal modes changes from 5 to 4 then SH power decreases step-like by 3%. In addition to the long-term stability of SH power, power noise measurements were performed at @429.6 nm as shown in Fig. 3.6.

Fig. 3.6 shows power fluctuations in the time domain measured with a photodiode model 1801 by Thorlabs, USA, connected with an oscilloscope model 9410 by LeCroy, USA, over a period of 9 ms. The upper limiting frequency of the photodiode (125 MHz) determined the measurement bandwidth. For SH power noise the assumption of white noise is valid due to the random contribution of different modes within the frequency conversion process. Hence, the root mean square (rms) value of the SH power fluctuations for a limited bandwidth of 10 MHz was 1.1% of the average SH power. The noise of the dual-wavelength laser at the fundamental is at least a factor 2 lower. The factor 2 is solely valid for a single frequency laser, for any other laser this value is higher. Hence, SH power noise is the limiting noise factor for the performance of the dual-wavelength laser system.
For a complete summary of the measured and derived specifications for the dual-wavelength laser, the IR output power $P_{out} = 4.64$ mW @859.6 nm and the SH power $P_{2\omega} = 3.92$ mW @429.2 nm were measured after transmitting the dichroic beam-splitter for an optimal tuned laser using a Newport model 840 optical power meter in combination with a 818-SL sensor. Due to the nonlinear frequency conversion both beams are linearly polarized (@859.2 nm horizontally, @429.6 nm vertically). The actual polarization ratio for each wavelength is larger than 100:1. The beam pointing stability over a period of 2 hours is specified smaller than 50 $\mu$rad. However, due to the developed and applied technology reported in section 3.1.5, latter specification does not have a detrimental influence on the dispersometer measurements. A detailed study of the beam profiles of the dual-wavelength laser is presented in section 3.1.5.

One of the most decisive parameters derived from the dual-wavelength laser is the wavelength dependent constant which amounts to -41.51 for the present exact wavelengths by using Eq. (2.35) for standard dry air.

### 3.1.4 Modulation and synchronization

Due to the basic principle of a monolithic receiver there is one common optical and one common electronic channel for both wavelengths. For crosstalk-free detection and allocation of the correct wavelength a separation was introduced. The requirements induced by the dispersometer related theory demand a separation rather in time than in space. It is decisive for the application of the dispersion technique that this condition can be maintained even in presence of strong turbulence. Furthermore, the present layout of the receiver electronics, as presented in section 3.2.3, envisions a temporal intensity modulation. As a conclusion both beams will be transmitted alternately. Summarizing these criteria a modulation scheme for both wavelengths with a very high extinction ratio enabling a modulation frequency $f_{mod} = 250$ Hz for each wavelength is required. Due to the subsequent coupling into an optical fiber the mutual displacement of both beams should be minimized.

Laser diodes provide the inherent possibility of direct modulation as a function of the applied operation voltage up to several GHz. In contradiction to single-wavelength diode lasers, experiments showed that with a dual-wavelength laser the situation is much more complicated. The effect of the applied modulation on the dual-wavelengths laser is roughly analog to the laser start-up from stand-by, see Fig. 3.4. Here, with the variation of current of the NIR-LD, thermal effects occur, the frequency spectrum of the LD is slightly shifted resulting in a different phase-matching condition for the SHG, etc. It was investigated experimentally that for a temporal intensity modulation around the NIR-LD threshold, 50% SH power fluctuations occur [Fluck, 2000]. Furthermore, with the direct modulation scheme alternate emission of both wavelengths is not possible.

A theoretical evaluation of the possibilities and feasibility of external modulation schemes under the consideration of the factors given above was executed. This study resulted that configurations based on the different polarization states of the beams are inapplicable. The claim of a modulation frequency $f_{mod} = 250$ Hz for each wavelength and the demand for a very high extinction ratio lead to mutual exclusion. In addition, the polarization ratio (~100:1) of the dual-wavelength laser is not sufficient. Alternative solutions based either on the implementation of a second NIR diode laser, whereas the fundamental wavelength of the frequency doubled...
laser is totally blocked or the spatial separation of both beams, whereby in both schemes each wavelength is individually modulated, were rejected. Although basically feasible, two laser solutions possess the disadvantage of a more expendable set-up in conjunction with an experimentally more challenging system for beam guidance and combining. Furthermore, with the application of a second NIR laser the advantages of the dual-wavelength laser, e.g. compactness, correlation etc., are eventually lost.

As a conclusion a mechanical modulator was realized. This design provides intensity modulation and wavelength selection as well. Wavelength selection is achieved by optical Schott glasses BG39 and RG780. With a thickness of the absorption filter glasses of 3.0 mm mutual suppression of more than $10^5$ (50 dB) results which was demonstrated using a Lambda 9 spectrometer by Perkin Elmer, USA. This very high extinction ratio assures that evidently no crosstalk between blue light and IR on the receiver occurs which is induced by the dual-wavelength transmitter.

Within the fabrication process both filter glasses were cut into half discs and cemented forming a filter disc with a diameter of 15 mm and a centric bore hole with a diameter of 2.5 mm, see Fig. 3.7a and Fig. 3.7b. For an optimum performance of the NIR-LD backreflections are very crucial. It was demonstrated experimentally by inserting the anti-reflection (AR) coated KNbO$_3$ crystal into the optical path that the specified back reflection $< 0.1\%$ of the AR coating was not sufficient. That means a degradation of power output in addition with increasing noise if the back reflection ratio is larger than $-30\,\text{dB}$. To ensure suppression of backreflection, a $7^\circ$ angular deviation of the rotational axis of the filter disc from the propagation axis was introduced. From a thorough ray tracing calculation for this angular deviation under consideration of the tight requirements for the dual-wave coupling into one optical fiber, see section 3.1.5, the front and back surface of the filter disc were specified to be parallel within 1 arc second and to $\lambda/10$ peak to peak (p-p) of optical flatness. To prevent transmitter induced crosstalk at the transition of the beam over the joint filter edges two diametrical quadrants of the filter disc were opaquely coated. For the coating a 150 nm thick layer of aluminum was chosen. In a first step shielding masks were applied onto two diametrical quadrants. These applied gallium-arsenide wafers were used to produce very sharp edges. The shielding masks were aligned under a microscope in the way that the overlapping zones for assuring no filter-to-filter transitions are $\sim 1\,\mu\text{m}$ wide. In a second step the aluminum coating was evaporated on the blank quadrants.

Subsequently, the filter disc was mounted to the axis ($d_{\text{axis}} = 1\,\text{mm}$) of a small DC-motor with an outer diameter of 10 mm utilizing an especially manufactured device for force centering with respect to the outer filter disc diameter. The size of the motor was optimized with respect to the beam diameters in the set-up. From measurements of the beam propagation a maximum $1/e^2$-diameter of 0.7 mm resulted in the plane of the filter disc. That means the space available for both beams transmitting the filter glass is approximately 3 times the full beam diameter, see Fig. 3.7b for details.
For hardware-tuning of the phase relation between the dual-wavelength signal and the synchronization signal, as described below, the angular position of the filter disc with respect to the motor axis can be adjusted. In this scheme the filter disc rotating with constant angular velocity generates a temporal intensity modulation with a frequency of $f_{\text{mod}} = 250$ Hz (15000 rpm) for each wavelength. The layout of the filter disc leads to a duty cycle of $\frac{1}{4}$. Due to the finite size of the beam diameters, e.g. $1/e^2$-diameter, on the filter disc surface, the step rise time of the modulated intensity is non-zero. Considering the propagation of both beams and the specifications given above the respective rise time is $t_{\text{rise}} = 70 \mu s$ for the present set-up.

Beyond temporal intensity modulation and wavelength selection, a wavelength-coded reference signal for digital demodulation of the received signal on the detector side is generated using an extended set-up. The reference signal is directly extracted from the modulated dual-wave signal. The benefit gained is that a fixed phase relation between the dual-wave signal received on the sensor and the wavelength reference signal is given, even when frequency jittering occurs. Because two diametrical quadrants of the filter disc are coated by aluminum on the backside which functions as the first mirror plane, for each coated quadrant only one beam is reflected. Due to the $7^\circ$ angular deviation of the rotational axis of the filter disc from the propagation axis the beams are reflected on to the UDT-20D (UDT Sensors Inc., USA) reference photodiode via the second mirror (Ar-coating on a glass substrate) positioned next to the laser beam exit. In front of the chip surface of the UDT-20D a second RG780 absorption filter with a thickness of 3.0 mm is positioned. This filter totally blocks the from the coated BG39 filter reflected beam, see Fig. 3.1. After adjusting the gain and offset of the photodiode amplifier in combination with the tuning of the threshold of the applied Schmitt-trigger, the influence of the ambient light level and spurious Fresnel reflection were eliminated. Consequently, the electronics in connection with the reference photodiode provides the IR reference TTL-signal which is transmitted to the detection system. On the basis of the scheme presented, the IR reference TTL-signal is shifted in phase by $\pi/2$ of the dual-wave signal period, see Fig. 3.8.

Experiments with the complete dispersometer showed that phase-locking between modulation and sampling is crucial for the required accuracy and the validity of the subsequent spectral analysis on the measured and corrected data. We investigated that a trigger derived from the IR reference signal initiating the sampling process is not sufficient, even for sampling short
periods of several milliseconds, due to the drifting of the modulation frequency. The implementa-
tion of the pulse-width-modulation (PWM) technique for the motor-driver in combination
with multiple re-triggering did not yield significant improvements. First progress was made with
the application of a virtual sampling clock derived from the IR reference signal which enabled
continuously re-triggered acquisition. Due to the remaining problems in connection with the
frequency stabilization of the chopper an alternative solution was effectuated. The DC-motor
described above was replaced by a similar model with a built-in digital magnet encoder. This
magnet encoder possesses the same diameter ($d_{\text{encoder}} = 10 \text{ mm}$) as the DC-motor and it is fixed
on the rotational axis positioned opposite to the filter disc. One rotation of the filter disc is
equivalent to 12 pulses generated by the magnet encoder. Consequently, a strictly phase-locked
signal with a frequency $f_{\text{sync}} = 3 \text{ kHz}$ (in case $f_{\text{mod}} = 250 \text{ Hz}$) is obtained. To refine the resolution
in the time domain the frequency for the sampling process of the synchronization signal was
electronically doubled, $2f_{\text{sync}} = f_{\text{sample}}$. In Fig. 3.8 the signals derived from the modulator are
shown. The topic of timing and synchronization on the receiver is addressed to in section 3.2.4
more detailed.

3.1.5 Application of optical fiber technology

For a predominant number of applications it is favorable to generate a so-called Gaussian
TEM$_{0,0}$ beam, see Fig. 3.9. Lowest-order Gaussian beams offer the advantage of a smooth radial
symmetrical intensity distribution centered on the propagation axis. This quality is highly desir-
able in applications where the center of gravity of the intensity distribution is the quantity of
observation. For higher order beams the intensity distribution becomes potentially asymmetri-
cal, especially when energy is transferred between different transversal modes. For position
detecting experiments a systematic deviation occurs eventually which might include a temporal
dependence. Consequently, an apparent shift of the target occurs. Due to the existence of only
the lowest-order transversal mode the intensity distribution is invariant along the propagation
axis, see Fig. 3.9. Within a Gaussian TEM$_{0,0}$ beam time depending interference effects are
eliminated. Furthermore, Gaussian TEM$_{0,0}$ beams are diffraction limited and possess minimal
phase space volume [Siegman, 1990]. Resulting from the latter mentioned qualities the diffrac-
tion induced beam spreading, leading to the far-field divergence angle, becomes minimal for a Gaussian TEM\(_{0,0}\) beam. This is favorable for the radiometric optimization of opto-electronic systems utilizing laser radiation.

In optical applications in order to obtain Gaussian beam quality with a smooth intensity distribution one uses the techniques known as spatial filtering because in general the real laser beam is a superposition of Gaussian-Hermite waves (TEM\(_{pq}\) with \(p, q = 0, 1, \ldots\)). To achieve the desired Gaussian TEM\(_{0,0}\) beam one has to eliminate all the TEM\(_{pq}\)-waves with \(p, q \neq 0\). In Fig. 3.10 the intensity distributions of low-order TEM\(_{pq}\)-waves are depicted. The case \(p = q = 0\) is the wanted case.

With a dispersometer however, the situation is much more complicated, because at both present wavelengths a Gaussian TEM\(_{0,0}\) beam has to be generated. This requirement is especially crucial for the experiments presented in chapter 4. A mutual relocation of the centers of gravity of the intensity distribution would lead to an apparent variation of the dispersion induced displacement. In addition, due the experimental set-up this effect might be magnified by approximately a factor of 10. For dispersometer experiments with longer sight lengths an allocation of atmospherically induced effects on both beams is possible by ensuring Gaussian beam propagation. A further important requirement, especially for the feasibility of short distance \((R < 20 \text{ m})\) experiments, is coaxial emission of both beams with spatial coincidence better than 0.5 \(\mu\)m. Within this tolerance mutual angular beam stability has to be guaranteed. We investigated that the only existing technology on which this part of the development of the dual-wavelength transmitter can base is the application of optical fiber technology.
To meet the requirements of coaxial dual-wavelength monomode propagation as postulated above both beams have to be coupled into the same optical fiber with the quality that solely the fundamental mode can propagate within this fiber. Herein, a mode is characterized as a time independent intensity distribution. A conventional optical fiber is a radial symmetrical cylindrical waveguide. The simplest form of a single-mode fiber (SMF) is a step-index fiber with a central core in which the light is guided [Yeh, 1990], see Fig. 3.11. The core is embedded in an outer cladding with a lower refractive index $n_2$ than the refractive index of the core $n_1$. Under consideration of $n_1$ and $n_2$ a new parameter $\Delta$ can be defined such that

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \equiv \frac{n_1 - n_2}{n_1},$$

where $\Delta$ is a measure of the relative difference between the core refractive index and its cladding value [Jeunhomme, 1990; Yeh, 1990]. For an optical fiber with an effective guidance behavior the relative refractive index difference becomes $\Delta \ll 1$, e.g. for a FS-VS-2614 step-index fiber by 3M, USA, with $n_1 = 1.4585$ and $n_2 = 1.4530$ the relative refractive index difference amounts to $\Delta = 0.0038$, consequently.

To first approximation the simple model of internal reflection at the core-cladding boundary can be applied. Hence, rays with an incident angle smaller than the critical angle $\theta_c$ of total internal reflection are not guided and dissipate power into the cladding material (see Fig. 3.11).

![Fig. 3.11: Ideal step-index fiber with the geometrical optics propagation model.](image)

The angle of acceptance $\theta_c$ for the guiding of light is defined by the numerical aperture (NA) of the fiber. Whereas, for NA one finds:

$$\text{NA} = n_1 \sqrt{2\Delta}.$$  \hspace{1cm} (3.2)

Unlike this approximation, the exact theory, by applying Maxwell’s equations for the dielectric cylinder, shows merely different discrete modes propagating under different reflection angles, which is equivalent to different propagation velocities, see Fig. 3.13a. For theoretical predictions of monochromacity and guidance behavior within the fiber the normalized frequency $V$ is used:
Here, \( a_i \) is the radius of the core and \( \lambda \) is the free-space wavelength of the light source. For a normalized frequency of the fiber in the range of \( 0 \leq V < 2.4048 \) the fiber can support only the fundamental mode. This fundamental mode is linearly polarized transversely to the direction of propagation and hence, was named the LP\(_{01}\) mode by Gloge [1971]. Accordingly, this range defines the so-called "single-mode" regime. Calculating the solution of the wave equation derived from Maxwell's equations for the fundamental mode in a cylindrical waveguide, the value for the normalized cut-off frequency \( V_c = 2.4048 \) is the first zero of the zeroth-order Bessel function. A thorough analysis can be found in Gloge [1971]. Corresponding to the cut-off frequency \( V_c \), a theoretical cut-off wavelength \( \lambda_c \) exists beyond that propagation will be monomode:

\[
\lambda_c = 3.6950 \cdot a_i \cdot n_i \cdot (\Delta)^{1/2}. \tag{3.4}
\]

One has to note that in practice an ideal fiber does not exist. Due to microbends, core diameter fluctuations, and additional effects the effective cut-off wavelength \( \lambda_e \) is slightly below the theoretical value.

Theoretically, due to a zero cut-off frequency for the fundamental mode, the lowest order mode does always propagate within a cylindrical waveguide [Ungar, 1990]. But practically, as depicted in Fig. 3.12 below, for an operational wavelength \( \lambda_o \) far above the cut-off wavelength \( \lambda_c \), as it is the case with the IR wavelength, mode confinement becomes looser, whereby bend sensitivity increases. Hence, for single-mode guiding both wavelengths within the same fiber certain tradeoffs have to be made.

![Fig. 3.12: Guidance behavior of SMFs with operating wavelengths above cut-off wavelength](adapted from [Waveoptics, 1998]).
Because of the requirement of monomode propagation it had to be demonstrated experimentally that for both wavelengths only the fundamental mode is transmitted.

The typical parameters for the step-index fibers, i.e. SMF, used in the experiments are $NA = 0.11$ to $0.13$, nominal core diameter $2a_1 = 2.5$ to $3.5 \mu m$ and the cladding diameter $2a_2 = 80$ to $125 \mu m$. The smallness of the core diameter implied the very tight specifications for the fiber disc given in section 3.1.4 considering dual-wave coupling. For the use of a standard mounting system in the set-up and for high reproducibility of the coupling process nearly all fibers were standard FC-connectorized with a convex polish. In contradiction to an angular polish (typically $8^\circ$ angular deviation from the plane orthogonal to the fiber axis) which is often applied for decreasing backreflection, convex-polished endface geometry leads to a beam output direction which is coaxial with the propagation direction of the beams within in the fiber related to Snell’s law. Furthermore, free-space coupling can be performed more efficiently.

Yet another viewpoint of light launching into fibers is the design of the coupling optics. Optimizing the parameter $NA$, for a $NA$-matching between fiber and optics, and spot sizes, for matching the resulting spot sizes and the mode field diameter (MFD) of the fiber, for both wavelengths, we achieved best coupling efficiency with an achromatic well corrected 60X microscope objective (MO). Maximum efficiency for free-space coupling @429.6 nm was 55 %. More elaborated schemes to increase coupling efficiency were extensively discussed by Kawano [1986] and Ladany [1993].

Eq. (3.4) states that the first important condition for single-mode operation for both wavelengths is that the cut-off wavelength $\lambda_c$ of the fiber has to be lower than the shorter operating wavelength, i.e. $\lambda_c < 430 \, nm$. However, on the presently growing market of fiber optical technology, manufactures focus on the “communication”-wavelengths in the ranges of 1300 nm and 1550 nm. Cut-off wavelengths in the visible range are specified with large tolerances spanning several ten nanometers in wavelength. Secondly, the $\lambda_2 = 430 \, nm$ wavelength generated by frequency doubling is not a common wavelength, generated by commercially available laser sources. Fibers tailored to Ar*-lasers, e.g. SM488 by Waveoptics, USA, tend to have a higher cut-off wavelength than $\sim 420 \, nm$ which would be needed. Fibers for HeCd-lasers are not available on the market. In the course of the development 18 different single-mode fibers were tested concerning transmission and bend sensitivity whereby the exhibiting beam profiles were analyzed. We note that the results presented below were achieved in experiments with the first laser model emitting at $\lambda_1 = 860.5 \, nm$ and $\lambda_2 = 430.25 \, nm$ and are valid without restrictions for the second laser model emitting at $\lambda_1 = 859.2 \, nm$ and $\lambda_2 = 429.6 \, nm$, see section 3.1.3. The substantial results of the experiments are summarized below.

We investigated experimentally that for single-mode fibers which guide solely the fundamental mode at both present wavelengths (FS-VS-2614, $\lambda_c = 400 \, nm$ and FS-VS-2614, $\lambda_c = 420 \, nm$, 3M, USA) the mode confinement @860.5 nm becomes so loose that the complete measurable portion of the IR radiation is absorbed in the cladding material after approximately 200 mm of fiber length. This could be demonstrated using a set of FS-VS-2614, $\lambda_c = 400 \, nm$, 3M, USA, with fiber lengths ranging from 200 to 1000 mm. A further possibility for the transmission loss @860.5 nm was reported by Jeunhomme [1990]. Here, the presence of a depressed index profile may introduce a finite cut-off wavelength for the fundamental mode LP01 @860.5 nm. Furthermore, bend sensitivity increased in such a way that handling became practically impossible (P661/21, Schott, Germany). In addition, the mode field of the IR beam propa-
gating will become so large that the impurities of the cladding material acting as scattering centers completely distorted the beam profile. Another source of disturbances is that force centering within the connectorizing process will partly induce birefringence which also has a detrimental effect on the beam profile. We observed this effect on a CTF500C by Spectran, USA, centered and connectorized by Diamond, Switzerland. Although this latter source of errors was remedied by using non-centered (CTF500C, Spectran, USA) and cleaved bare fibers (SMC-A0515B, Spectran, USA), no significant improvements were made. Here, a visual inspection of the endfaces by microscope observation with a 40X magnification did not lead to conclusive results. We note that this inspection method is not rigorous because cracks which could in addition to endface defects also be an explanation for the poor guidance behavior of the larger IR mode fields are not likely to appear solely on the endface. For the present fibers which are enclosed by a thick acrylate jacket (~3 mm) an alternative visual inspection was not feasible.

Due to the fact that none of the fibers tested showed monomode propagation for both wavelengths in combination with transmission of sufficient optical power, we introduced a new technique for the solution of the monomode problem in the dual-wavelength transmitter. This new method is closely related to the physical properties of the different modes propagating within the fiber. Because of the characteristics of mode confinement in an optical fiber higher order modes are more weakly guided than the fundamental LP_{01}-mode. Introducing a certain curvature radius, e.g. by bending the fiber, higher order modes tend to become leaky, i.e. energy is radiated away in the radial direction into the cladding region. In the cladding the power contributed by those leaky modes decreases rapidly. Therefore, guidance behavior within the fiber can be manipulated and monochromacity can be obtained by mechanically forcing the fiber to a certain curvature. It was the aim of the experiments to determine a curvature radius and specify the required bent fiber length L_{coil} for which solely propagation of the LP_{01}-mode occurs. Fig. 3.13a and Fig. 3.13b show the schematic drawings of this principle.

![Fig. 3.13: Schematic drawing of the introduced technique. Wave propagation within the waveguide is described in the ray-optics approach.
  a) Propagation of the LP_{01} and LP_{11}-mode in a straight fiber
  b) Propagation of the LP_{01}-mode and leakage of the LP_{11}-mode in a bent fiber](image)
In addition to the exact theory, a simplified explanation of the applied method based on the geometrical optics model is possible. Under the consideration of propagation of merely the fundamental mode LP01 and the second-lowest order mode, the LP11-mode, one observes that in Fig. 3.13a the angle of total reflection for the LP11-mode is slightly closer to the critical angle \( \theta_c \) than for the LP01-mode. Introducing a certain curvature to the fiber, see Fig. 3.13b, the angle of total reflection for the LP11-mode falls below the critical angle \( \theta_c \) and its power is dissipated in the cladding, whereas the LP01-mode is solely guided.

For the characterization of the initial beam-profiles of the dual-wavelength laser the \( M^2 \)-factor is used as a measure of beam quality. This factor \( M^2 \geq 1 \), introduced by Siegman [1990], measures the relative far-field beam spreading in comparison to an appertaining ideal Gaussian TEM\(_{00}\) beam. Only for the ideal Gaussian TEM\(_{00}\) beam \( M^2 \) becomes unity. For the real laser beams emitted by the dual-wavelength laser the beam quality factors are \( M^2 \leq 1.2 \) @430.25 nm and \( M^2 = 1.5 \) @860.5 nm. The respective beam profiles at an arbitrary position are shown in Fig. 3.14a @860.5 nm and in Fig. 3.14b @430.25 nm. One observes that the \( M^2 \)-value for the blue beam is closer to unity than the respective \( M^2 \)-value for the IR beam. This is related to the frequency conversion configuration using a bulk crystal. As reported in section 3.1.3, the K\( \text{NbO}_3 \)-crystal acts as a spatial filter within the frequency conversion process. Furthermore, due to the structure of the semiconductor IR laser diode the emitted beam is highly elliptic.

For the experimental demonstration of this technique we chose optical fibers of the type FS-VS-2614, 3M, USA, which exhibited single-mode propagation @860.5 nm with satisfying power transmission. The cut-off wavelength \( \lambda_c = 448 \) nm indicates theoretically the possibility of co-excitation of the LP01 and the LP11-mode. The existence of this bi-modal regime was also demonstrated experimentally, see Fig. 3.16a. For introducing a curvature radius by bending the fiber, we fabricated a fiber coil with an experimentally determined diameter of 42 mm. For the fiber length \( L = 1 \) m the thread of the coil could accept 5 full windings of the fiber. The combined experimental set-up for the beam diagnostics and the determination of the beam propagation, as described below, is depicted in Fig. 3.15.
Fig. 3.15: Schematic drawing of the experimental set-up. The filter disc was used in a static mode. The number of windings around the fiber coil was varied experimentally from 0 to 5. The exhibiting intensity profiles were recorded with a TM-6 CCD-camera by Pulnix, USA. The relative distances from CCD-camera to fiber output for the determination of the exhibiting far-field divergence angles were measured by using the interferometer system HP5519A, Hewlett Packard, USA.

To purge the propagation @430.25 nm to single-mode the fibers were wound tightly around the coil, so that a regime could be found where the LP\textsubscript{11}-mode became leaky and solely the LP\textsubscript{01}-mode corresponding to the fiber-out-coupled free-space Gaussian TEM\textsubscript{00} beam was guided. To illustrate the effectiveness of this method the measured intensity profiles are depicted below. Fig. 3.16a shows the bi-modal regime characterized by the co-excitement of the LP\textsubscript{01} and LP\textsubscript{11}-mode. In Fig. 3.16b the beam @430.25 nm does merely contain the LP\textsubscript{01}-mode.

In order to analyze the fiber-out-coupled Gaussian TEM\textsubscript{00} beams we recorded the far-field intensity distribution with a CCD-camera model TM-6 by Pulnix, USA, and processed the data using a LBA100A laser beam analyzing system, Spiricon Inc., USA. Utilizing the complete dynamic range of the CCD-camera we attenuated the transmitted power of the beams by lowering the coupling efficiency. Unlike using absorption filters the true beam profile remains. Changing the launching-in geometry by beam-steering with microscope objective was considered as favorable for this kind of analysis. Because the LP\textsubscript{11}-mode is characterized as an asymmetrical mode it will be easier excited by a non-optimized asymmetrical coupling. This implies...
on the other hand, when single mode propagation exhibits with this launching-in condition, single-mode propagation will appear for all possible incoupling cases. Fig. 3.17a and Fig. 3.17b show the recorded typical profiles of the fiber-out-coupled beams for both wavelengths at an arbitrary CCD-camera position.

By application of an elliptical beam model by calculating the ratio of the minor and major width of the $1/e^2$ intensity, a roundness value of 0.987 at 860.5 nm and a roundness value of 0.990 at 430.25 nm results. Both beam profiles show very good correlation with the theoretical beam profiles. The beam profile at 860.5 nm exhibited a correlation of $\rho = 0.96$ for both the major and minor axis. For the beam profile at 430.25 nm a correlation of $\rho = 0.93$ for both the major and minor axis resulted. These fittings indicate that solely the fundamental mode is guided and show the effectiveness of the presented technique. These results could be reproduced by experiments with four fibers of the type FS-VS-2614, $\lambda_c = 448$ nm, $L = 1$ m, 3M, USA. These fibers were fabricated within one common drawing process from the same preform. We also determined experimentally that the proposed method works for a minimum number of 2 fiber windings around the coil. For ensuring the single-mode regime we chose $L_{\text{coil}} = 660$ mm for the subse-
quent experiments. Very similar results were obtained for a fiber of the type FS-VS-2614, $\lambda_0 = 428 \text{ nm}$, $L = 0.5 \text{ m}$, 3M, USA, although the fiber length permitted only two full fiber windings around the coil. Based on these measurements we assumed diffraction limited beam quality for the fiber-out-coupled beams, therefore in excellent approximation $M^2 = 1$. The Gaussian $TEM_{00}$ beam can be described by:

\[
\begin{align*}
    w(z) &= w_0 \left(1 + \left( \frac{z \lambda}{\pi w_0^2} \right)^2 \right)^{1/2}, \\
    R_{wF}(z) &= z \left(1 + \left( \frac{\pi w_0^2}{z \lambda} \right)^2 \right).
\end{align*}
\]  

Here, $w_0$ is the radius of the beam waist, the smallest constriction of the beam possible at the position $z = 0$, $w(z)$ denotes the beam radius at a given distance $z$ relative to the beam waist and $R_{wF}(z)$ is the appertaining radius of wavefront curvature. Eq. (3.5) and Eq. (3.6) indicate that for a known wavelength $\lambda$ in the medium in which propagation occurs the beam can be characterized solely by the radius of the beam waist $w_0$.

\[
z = z_R = \frac{\pi w_0^2}{\lambda}
\]  

The distance that a beam propagates from the waist before the beam radius increases by $\sqrt{2}$ is called the “Rayleigh range” given by Eq. (3.7). Furthermore, Eq. (3.7) separates approximately the “near-field” where the beam remains collimated over a shorter distance and the “far-field” where it diverges at a larger angle. We note that all measurements presented below and in the chapters 4 and 5 were made in the far-field. For the characterization of both beams in order to derive the radiometry of the system, see section 3.2.5, and the beam parameter of the dispersometer experiments, see section 4.2 and section 5.3, it is substantial to determine the radius $w_0$ of the beam waist. For this determination we used the relation between the far-field half-angle divergence $\theta$ and the radius of the beam waist $w_0$ due to the diffraction induced beam spreading in the far-field given in Eq. (3.8) below.

\[
\theta = \lim_{z \to \infty} \frac{w(z)}{z} = \frac{\lambda}{\pi w_0}
\]  

Due to the tight beam confinement by the optical fiber we assumed that the beam waists for both wavelengths coincide with the fiber output. For the metrological determination of the far-field half-angle divergence $\theta$ we measured the beam widths $D$ as a function of relative position in the propagation direction. By calculating the linear fits on this data and under consideration of Eq. (3.8) the radii $w_0$ of the beam waists for both wavelengths were obtained.
In these experiments we utilized the complete set-up as shown in Fig. 3.15. As a slight difference we used this time a Pulnix TM-6CN CCD-camera and a Spiricon LBA-500PC laser beam analyzing system for the determination of the beam widths in combination with the knife-edge technique proposed by Siegman et al. [1991]. For the clip levels we applied the standard values of 10-90 %. The resulting beam widths $D$ are equivalent to $1/e^2$-diameters. Relative distance variations were measured with a HP5519A interferometer. In optimizing the beam diameter to the CCD-chip, the distance range was 4 mm for 430.25 nm and 5 mm for 860.5 nm, whereby the CCD-camera positions were equally spaced by 0.5 mm with a positioning accuracy of 1 µm. Shortly before each series of measurements the system was calibrated for the correction of the temporal variations of system gain and offset. The determination of the half-angle divergence $\theta$ and the beam waist radius $w_0$ was executed for the same four fibers of the type FS-VS-2614, $\lambda_c = 448$ nm, $L = 1$ m, 3M, USA, as reported above. In the course of the determination of the propagation parameters approximately $10^3$ beam profiles were analyzed. First of all, the results obtained by utilizing the Spiricon LBA-100A could be reproduced. In Fig. 3.18 for all fibers the measured beam diameters $D$ as functions of their relative positions are shown:

![Fig. 3.18: Beam diameters $D$ vs relative CCD-camera positions. The calculated linear fits resulted a correlation of $\rho > 0.99$. Each optical fiber was wound 5 times around the coil resulting $L_{col} = 660$ mm.](image)

We obtained for the deviation of back and forth measurements differences $< 0.1 \%$ of the beam diameter $D$ and for multiple determinations at different beam intensity levels deviations $< 1 \%$.

The final numerical results are shown in the Table 3.1 below. Multiple determinations of these parameters showed standard deviations $< 1 \%$. Table 3.1 shows that the beam waist @860.5 nm is a factor of 2.5 to 2.9 larger than the beam waist @430.25 nm. The magnitude of the mode field diameter (MFD = $2w_0$) @430.25 nm is as expected for this wavelength. As an empirical value the MFD is assumed to be 15 % larger than the core diameter [Waveoptics, 1998]. For the IR wavelength operating far above the cut-off wavelength $\lambda_c = 448$ nm it can be estimated that the MFD is 7 to 8 times the operating wavelength [Waveoptics, 1998]. Hence, a larger portion of IR is transmitted in the fiber cladding. As far as the excellent beam quality is concerned this should not lead to limitations and is also in accordance with the theory of waveguides.
Table 3.1: Comparison of the determined half-angle divergences $\theta$ and the resulting radii $w_0$ of the beam waists for a set of sample fibers.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\theta$ @860.5 nm (mrad)</th>
<th>$\theta$ @430.25 nm (mrad)</th>
<th>$w_0$ @860.5 nm ($\mu$m)</th>
<th>$w_0$ @430.25 nm ($\mu$m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS-VS-2614-1</td>
<td>75.0</td>
<td>97.0</td>
<td>3.65</td>
<td>1.41</td>
</tr>
<tr>
<td>FS-VS-2614-2</td>
<td>83.2</td>
<td>104.5</td>
<td>3.29</td>
<td>1.31</td>
</tr>
<tr>
<td>FS-VS-2614-3</td>
<td>74.4</td>
<td>105.6</td>
<td>3.68</td>
<td>1.29</td>
</tr>
<tr>
<td>FS-VS-2614-4</td>
<td>75.1</td>
<td>96.7</td>
<td>3.65</td>
<td>1.42</td>
</tr>
</tbody>
</table>

As one can see in Table 3.1 fiber no 1 and fiber no 4 exhibit almost identical propagation parameters. With the exception of fiber no 2, all fibers show a very similar behavior for IR. As a result of this analysis it was suggested that the fiber no 1 was implemented in the dual-wavelength transmitter. Using Eq. (3.5) and Eq. (3.6) a complete propagation model can be established which allows a prediction of the radius $R_w(z)$ of wavefront curvature and of the beam radius $w(z)$ for any given distance $z$ from the transmitter. We emphasize the validity of the results presented in this section for the improved model of the dual-wavelength laser emitting at $\lambda_1 = 859.2$ nm and $\lambda_2 = 429.6$ nm. For the fiber no 1 and the improved laser model we measured for a blue light optimized coupling geometry $P_{2,0} = 1.43$ mW as a maximum value and with the same configuration $P_{0} = 0.96$ mW at the fiber output. Measurements were performed with a Newport optical power meter, model 840, in combination with a 818-SL sensor. Inserting the polarizer for mutual power adjustment we yielded a maximum value of $P_{2,0} = 0.77$ mW at the fiber output. Most importantly, both centers of gravity at the fiber output are in coincidence. We note that for the experiments presented in chapter 4 and in chapter 5 the active dual-wavelength target is characterized by the bare fiber output.

Conclusively, we demonstrated with the set-up presented in the preceding sections 3.1.3, 3.1.4 and 3.1.5 the realization of the dual-wavelength transmitter as shown in Fig. 3.19.

Fig. 3.19: Photograph of the realized dual-wavelength transmitter.
3.1.6 Transmitter optics

With the determination of the exhibiting half-angle divergences $\theta$ and the radii $w_0$ of the beam waists for both wavelengths in combination with the wavelengths themselves, the initial parameters for the design of the transmitter optics were given. Up to this stage the dual-wavelength transmitter is represented by the fiber output of the two beams. Because of the magnitude of the resulting half-angle divergence $\theta$, see section 3.1.5, a large amount of optical power cannot be collected by the receiving optics. Considering the radiometry of the dispersometer for sight lengths of a few hundreds of meters under adverse ambient conditions, large angular divergences are not bearable. Hence, it is one design issue for the beam shaping optics to improve far-field geometry of both beams.

For simplifying the alignment procedure on the transmitter side we propose to couple the optical fiber into the optical system of a theodolite. The pointing of the theodolite towards the receiving unit should tolerate a small angular pointing deviation in the order of 0.3 mrad (\(\sim 1\)). This magnitude was assumed to be tolerable for this kind of alignment procedures in precise surveying tasks. For this pointing accuracy two basic requirements for the optical system have to be met. First, the virtual images of the fiber output for both wavelengths have to be produced in the same image plane. Second, this image plane has to be located in the tilting axis of the aforementioned theodolite. If the first condition is not met, an apparent dispersion angle will result on the sensor. Infringing the second requirement, a systematic displacement occurs. Both effects are functions of the angular pointing deviation. Meeting these requirements results that the origin of spherical wave propagation is identical for both wavelengths and rotation-invariant. For achieving the required optical properties a proposed coupling of the dual-wavelength transmitter into one optical channel of a modern geodetic total station, e.g. tracking tacheometer, is shown in Fig. 3.20.

![Fig. 3.20: Schematic drawing of the coupling of the dual-wavelength transmitter into one optical channel of a modern geodetic total station. The virtual images of the fiber output coincide at the position of the tilting axis.](image-url)
3.2 Detection system

As postulated above the dispersometer detection system has to be a stable, compact, and monolithic unit. This implies for the detection system one optical and one electronic channel for both wavelengths, e.g. both beams will be imaged by one telescope onto the same sensor in combination with the appropriate analog and digital electronics as reported in this section. Consequently, a focal length of the receiving optics < 500 mm was required to meet the established standards of modern geodetic telescope-optics. As addressed to in section 3.2.1 in more details, the focal length of the dispersion telescope amounts to ~300 mm. Under the assumption of the smallest refraction angle of interest $\beta = 1 \mu$rad and under the consideration of the magnitude of the wavelength dependent constant (~41.51) for both present wavelengths, an absolute value of the dispersion angle of $\Delta\beta = 0.024 \mu$rad results which is equivalent to a spot displacement of 7.2 nm in the focal plane of the receiving optics. Analog results are obtained for the required difference position sensing accuracy. Furthermore, based on the additional magnitudes presented in section 2.3.4, the spatial position measurement range in the focal plane is mainly determined by the angle of arrival fluctuations in combination with the turbulence induced blur. Consequently, special care was taken for the design of the detection system regarding the technologies presented in section 3.2.1 and in section 3.2.2.

The very high accuracy demand for the detection system was one of the most critical issues in realizing the dispersometer. Presently, CCD-based sensors in combination with powerful digital image processing techniques are very attractive for opto-electronic position detection. However, recent works in geodetic context using CCD-cameras did not fall below of 1/100 pixel of system accuracy, even under advantageous ambient conditions and by application of advanced algorithms [Casott and Prenting, 1999; Casott, 1999; Prenting, 2000; Flach 2000]. Considering pixel sizes of ~10 pm, the required accuracy is exceeded by more than a magnitude. Additionally, CCD-based systems are limited to a sampling frequency of several hundreds of Hz, e.g. Casott [1999] reported on an image frequency of up to 200 Hz. In view of the temporal signal separation approach, shown in Fig. 3.8, this sampling frequency is not sufficient.

As an alternative to CCD-based sensors position sensitive detectors (PSDs) in combination with the appropriate analog electronics offer the required frequency response of several kHz. As reported in [UDT, 1982a], typical silicon photodiodes exhibit rise times < 5 $\mu$s. Basically, one classifies between lateral effect PSDs and segmented PSDs. Lateral effect PSDs are continuous single element planar diffused photodiodes [UDT, 1982b], whereas segmented PSDs are common substrate photodiodes divided into segments (two segments for the bi-cell PSD and four segments for quad-cell types) separated by a small gap [UDT, 1999]. In general, segmented PSDs offer higher position resolution and accuracy than lateral effect PSDs due to superior responsivity match between the elements, but are intrinsically limited by the qualities of the incident light spots in contrast to lateral effect PSDs. Edwards [1988] reported on resolutions better than 0.1 pm using bi- and quadrant-cell diodes. To increase these resolution capabilities of segmented PSDs by more than one order of magnitude a method proposed by Gächter [1981, 1984] is introduced in section 3.2.2, in section 3.2.3 and in section 3.2.4 the appropriate analog and digital electronics are presented.
3.2.1 Dispersion telescope

For the telescope of the detection system we used the dispersion optics initially designed for the experiments reported in [Wild Leitz, 1987; Gächter and Huiser, 1987a; Huiser and Gächter, 1989; Ingensand, 1990]. The design conception of the dispersion telescope was based on the so-called coincidence telescope proposed synchronously by Glissmann [1977, 1978] and Williams [1978]. Whereas the concept by Glissmann which envisions a modified Cassegrain type telescope is intrinsically limited by very tight mechanical tolerances of the optical double system and by imperfections of the mirror surfaces, the proposal by Williams based on introducing an additional lens combination to the optical path of the refractive optics was adopted, see Fig. 3.21 for details.

![Fig. 3.21: Schematic cross-section of the dispersion optics. Lens combination I mainly contributes with \( f' = 350 \text{ mm} \) to the overall refractive power of the optical system. Focussing is effectuated with lens combination II by translating IIa along the optical axis. III generates the required focal lengths ratio of the telescope and enables \( \lambda \) tuning. The PSD (IV) is installed in the focal plane of the telescope.]

The underlying principle of receiving optics from the coincidence type considers an adaptation of the dispersion of the ambient air to the focal lengths relation for both wavelengths. Hence, if the focal lengths ratio of the coaxial optics meets the condition given in Eq. (2.38), coincidence of both centers of gravity of both wavelengths occurs in the focal plane by pointing of the optical axis into the refraction-free, i.e. true, direction.

Furthermore, the actual magnitudes of the focal lengths are very decisive. A predominant number of previous dispersometer experiments was based on large focal lengths in order to enhance the position resolution capabilities, e.g. Brein [1954], Glissmann [1976] and Gaugitsch [1995] reported on focal lengths up to several meters. Additionally, Dallmann [1986] presented a telescope body design by Serrurier in order to mechanically stabilize large telescope constructions. However, in accordance with the development guidelines for the dispersometer such dimensions of the receiving optics are hardly practical and not acceptable because the critical issue of mechanical stability has to be considered as a decisive factor deteriorating system accuracy. In contradiction to these approaches, we will show in section 3.2.6 that the ratio of the position sensing range of the detector and the focal length of the receiving telescope is decisive. This implies that if the position sensing range of the detector can be kept small, the application of a short-focal-length receiving telescope is possible.
Consequently, for the dispersion optics, as displayed in Fig. 3.21, \( f_1 = 300.000 \text{ mm} \) and \( f_2 = 292.999 \text{ mm} \), derived from the optics design calculation, were considered as an acceptable compromise in respect of mechanical stability, the position sensing range of the detector, position resolution, and imaging qualities [Wild Leitz, 1987]. Using Eq. (2.38) for the focal length ratio \( \frac{f_2}{f_1} = 0.9767 \) results, whereby the deviation from the theoretic value which amounts to \( \varepsilon_{\text{dev}} = 0.9765 \), see section 2.3.1, can be neglected. This is additionally confirmed by considering that theoretically the dispersion of the optics, i.e. the focal length ratio, is tunable in a range from 0.973 to 0.980 as depicted in Fig. 3.21.

Moreover, we note that by applying the coincidence method by solely tuning the dispersion of the optics, as presented within the theoretical framework of the dual-wavelength method, see section 2.3.1, the PSD inherent dispersion has to be taken into consideration as well. We will show in section 4.2.1 that due to the PSD inherent dispersion the combined dispersion of the detection system exceeds the possible so-called \( \lambda \)-tuning range of the dispersion telescope. Furthermore, the smallness of the dispersion angle and the measurement range practically prevents correct mechanical tuning. Therefore, we presented in section 2.3.2 a self-calibration method by utilizing optical turbulence. This method will be applied and discussed within the scope of the dispersometer measurements in chapter 5. However, the knowledge of the initial purpose and realization of the dispersion telescope including the \( \lambda \)-tuning was inevitable.

The maximum aperture diameter \( d = 75 \text{ mm} \) can be continuously reduced for experimental purpose, e.g. for analyzing the dependence of the sensitivity of the turbulence compensation effectuated by the dual-wavelength method on the aperture size. Another viewpoint for the selection of this telescope was the very high quality of the lens system, the appropriate coating, and the high quality mechanical set-up, e.g. a centering accuracy for the optical system < 5 \( \mu \text{m} \) was specified.

### 3.2.2 Gap-technology for segmented position sensitive detectors

Gap-technology, as the position detection in the gap between the initially active cells of a PSD will be referred to within this work, utilizes special segmented PSDs. To stress the substantial differences between gap-technology and the standard procedure, the basic principle of the latter mentioned technique will be briefly summarized in the following.

The standard method in conjunction with segmented PSDs requires that the spotsize of the incident radiation is much larger than the gap width \( b \). Segmented PSDs possess separate photodiode elements masked onto a common substrate in the way that their cathode is shared. Each element is individually contacted (anode) so that a light spot radiating solely on a single segment can be electrically allocated to the related element. As this spot is translated across the detector surface, its energy is divided between adjacent segments, whereby the difference in electrical contribution between adjacent elements defines the relative spot position with respect to the center of the device. In Fig. 3.22 a photograph of the implemented dual-detector is shown and Fig. 3.23 pictures the gap area between two adjacent elements of the same PSD.
Unlike the standard method, position detection in the gap requires that the spotsize is much smaller than the gap width, hence $2w_{gap} \ll b$. Selected segmented PSDs exhibit irradiation sensitivity in the gap area. Within the elements of the segmented PSD the gap area performs as a lateral effect diode [Gächter, 1984]. Consequently, position is derived by dividing photon generated electrons within the substrate rather than profiling intensity distributions on the surface. When a laser beam is focused into the gap of a dual-detector both channels exhibit a current level. This current level is proportional to the total amount of optical power $\phi_{tot}$ incident to the gap area and proportional to the distance of the center of gravity of the spot to the respective gap-element boundary. Presently, the physical processes underlying the gap-technology have not been addressed to in detail, yet. We note that in order to electrically separate adjacent electrodes the required resistance will be introduced either by doping or by etching. In doing so, the gap area exhibits very similar characteristics as a lateral effect diode.

The basic principle for a discussion of the gap-method is shown in Fig. 3.24. As reported above, the initial requirement for the feasibility of this technique is $2w_{gap} \ll b$. For the Gaussian TEM$_{0,0}$ beam for each wavelength, as generated at the dual-wavelength transmitter, see section 3.1.5, one obtains for the irradiance $I(X,Y)$:

$$I(X,Y) = I_0 \cdot \exp \left( -2 \frac{X^2 + Y^2}{w_{gap}^2} \right).$$  \hspace{1cm} (3.9)

In Eq. (3.9) $I_0$ is the center peak irradiance, $X$ and $Y$ denote the coordinates of the axes of the Cartesian coordinate system of the beam.
In Fig. 3.24 \(i(A)\) and \(i(B)\) are the signal currents of channel A and B, respectively. Due to the fact that a dual-detector is a single axis device, solely \(y\) denotes the axis of the detector coordinate system. This \(y\)-axis is defined to be perpendicular to the segment-gap boundaries. Assuming a linear model for the lateral effect within the gap, one obtains for the signal currents \(i(A)\) and \(i(B)\) [Gächter, 1981]:

\[
\begin{align*}
  i(A) &= \frac{e \eta}{h \nu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(X,Y) \left[ \frac{1}{2} \left( y + \frac{X}{b} \right) \right] dX dY \\
  i(B) &= \frac{e \eta}{h \nu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(X,Y) \left[ \frac{1}{2} \left( y + \frac{X}{b} \right) \right] dX dY,
\end{align*}
\]

wherein \(e = 1.6022 \times 10^{-19} \text{ C}\) is the electron charge, \(h = 6.6261 \times 10^{-34} \text{ Js}\) the Planck constant, \(\nu\) the frequency of the radiation in Hz, and \(\eta\) the quantum efficiency of the detector. Separating the integral term in Eq. (3.10) leads to Eq. (3.11).

\[
\begin{align*}
  i(A) &= \frac{e \eta}{h \nu} \left( \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(X,Y) dX dY + \frac{X}{b} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(X,Y) \cdot X dX dY \right) \\
  &\quad + \frac{e \eta}{h \nu} \left( \frac{X}{b} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(X,Y) \cdot X dX dY \right)
\end{align*}
\]

\[
\begin{align*}
  i(B) &= \frac{e \eta}{h \nu} \left( \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(X,Y) dX dY - \frac{X}{b} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(X,Y) \cdot X dX dY \right) \\
  &\quad - \frac{e \eta}{h \nu} \left( \frac{X}{b} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(X,Y) \cdot X dX dY \right)
\end{align*}
\]

In Eq (3.11) the respective third integral term vanishes due to the assumption that the origin of the spot coordinate system coincides with the center of gravity of the imaged intensity distribution considering Eq. (3.9). Consequently, one obtains for the position sensitivity of the detector using gap-technology:
\[
\frac{\partial (i(A) - i(B))}{\partial y} = r_s \cdot \frac{2\phi_{\text{in}}}{b}
\]  

(3.12)

with the total optical power \( \phi_{\text{in}} \) on the sensor:

\[
\phi_{\text{in}} = \int\int I(X,Y) dX dY.
\]

(3.13)

Furthermore, in Eq. (3.12) \( r_s = e\eta/h\nu \) denotes the spectral responsivity of the sensor with the dimension \( A/W \) and is a measure of the effectiveness of the conversion of the optical power into electrical current. This figure is substantial for the investigations reported in sections 3.2.3, 3.2.5, and 3.2.6.

By applying the technique for position detection in the gap of a segmented PSD, e.g. a dual-detector, the signals obtained at the electrodes are proportional to the position of the center of gravity of the beam, see Eq. (3.14), whereby as one observes in Eq. (3.12) independent of the spotsize \( 2w_{\text{gap}} \), if \( 2w_{\text{gap}} \ll b \) is valid. Furthermore, it can be stated that the position sensitivity is increased by diminishing the gap width \( b \). This result will be validated in section 3.2.6 by the expression of the position sensitivity by noise figures.

To eliminate the first order influences due to power variations on the detection of position, one introduces a normalization, yielding a dimensionless position according Eq. (3.14)

\[
y_{\text{gap}} = \frac{i(A) - i(B)}{i(A) + i(B)}.
\]

(3.14)

We note that in connection with the experiments presented in chapter 4 and 5 we used a slightly different approach within the digital signal processing scheme, as presented in section 4.2, because the signal currents \( i(A) \) and \( i(B) \) are not directly available from the dual-wavelength signal available at the detection system.

For the implementation of the gap-technology in the dispersometer detection system under considerations of the focal length of the dispersion optics of \( \sim 300 \) mm, see section 3.2.1, we transferred the estimated orders of magnitude derived in section 2.3.4 into the focal plane of the receiving optics, see Fig. 3.25.

As displayed in Fig. 3.25, taking the turbulence induced beam spreading into account results in spotsizes of \( 2w_{\text{gap}} = 20 \) \( \mu \)m. In addition, the turbulence induced angle of arrival fluctuations are assumed to cause image shifting typically in the range of \( 3 \) \( \mu \)m. These figures contributed to the specifications of segmented PSDs which exhibit irradiation sensitivity @859.2 nm and @429.6 nm in the gap area. Taking these orders of magnitude into account, the gap width was specified to \( 150 \) \( \mu \)m \( \geq b \geq 100 \) \( \mu \)m.
As the result of the sensor evaluation we have chosen modified SPOT-2D sensors, by UDT Inc., USA, see Fig. 3.22 and Fig. 3.23. The modification is such that the protective glass was removed. Although the sensor surface is subject to contamination by moisture and dust and has to be handled very delicately, the cover window would act like an additional dispersive element which might be disadvantageous within the detection system. The nominal gap width which is specified to \( b = 127 \, \mu m \) was preliminary to the implementation in the detection system measured optically in connection with the performance tests as presented in section 4.1.1. We note that for the following estimations we use \( b = 127 \, \mu m \).

Another specification, as introduced above, is the spectral responsivity \( r_s \), of the detector. For the case \( 2w_{gap} \gg b \) the typical spectral responsivities based on the properties of silicon amount to \( r_s = 0.6 \, \text{A/W @859.2 nm} \) and \( r_s = 0.1 \, \text{A/W @429.6 nm} \). In contrast, measurements using the gap-technology revealed for the gap area \( r_s = 0.39 \, \text{A/W @859.2 nm} \) and \( r_s = 0.28 \, \text{A/W @429.6 nm} \). These values are in very good agreement with the measurements presented in [Wild Leitz, 1987] for a similar PSD of the same type.

A further specification of interest, required for the noise model of the detection system, is the noise equivalent power of the detector \( \text{NEP}_{DL} \). Noise equivalent power is the amount of optical power falling on a photodiode which produces a signal equal to the noise generated internally by the photodiode [UDT, 1982c].

\[
\text{NEP}_{DL} = \frac{i_{eq}}{r_s} = \left( 2e I_d \Delta f + \frac{4k_B T \Delta f}{R_L} \right)^{1/2}
\]  

In Eq. (3.15) \( i_{eq} \) is the total equivalent noise current referred to photodetector input, \( I_d \) is the dark current of the detector, \( \Delta f \) denotes the noise measurement frequency bandwidth, \( k_B = 1.3807 \times 10^{-23} \, \text{J/K} \) is the Boltzmann constant, \( T \) the absolute temperature, and \( R_L \) the load resistance of the diode. Generally, shot noise and Johnson or thermal noise contribute to the total equivalent noise current \( i_{eq} \) within a photodiode denoted as the first and second term of the
square root in Eq. (3.15). For the UDT SPOT-2D sensor typical noise figures per root Hertz are \( \text{NEP}_{DL} = 2 \times 10^{-14} \text{ W/Hz}^{1/2} @ 859.2 \text{ nm} \) and \( \text{NEP}_{DL} = 3 \times 10^{-14} \text{ W/Hz}^{1/2} @ 429.6 \text{ nm} \).

3.2.3 Analog electronics

In combination with the PSD and the digital electronics, the analog electronics of the sensor plays a crucial role for the performance of the dispersometer detection system. For the basic conception of an appropriate PSD amplifier, specifications are determined by the dual-wavelength signal, the characteristics of the UDT SPOT-2D sensor, and the specifications given by the layout of the digital electronics, i.e. the analog to digital converter (ADC). Due to very high accuracy demand on position detection in the focal plane and the limited amount of optical power available on the sensor a very low-noise amplifier was required.

As it will be shown in section 3.2.5 considering the radiometry of the dispersometer, position detection should be possible for received optical power down to the nanowatt level. Therefore, high-gain amplification of the UDT SPOT-2D sensor signal is required. Additionally, the amplification scheme should be able to filter the modulated dual-wave signal within the frequency bandwidth given in Fig. 3.8. By an optimized matching of the amplifier frequency bandwidth to the dual-wave signal, the influence of noise on the measurements can be significantly reduced. Additionally, for the derivation of further requirements the adaptation of the analog electronics to the digital electronics has to be considered. At this point, the resolution of the ADC has to be taken into account. For the elimination of potential quantization noise within the analog to digital conversion process, it is one design issue to match the voltage noise at the amplifier output with the least significant bit (LSB) of the ADC. In this section the design issues for the required very low-noise high-gain transimpedance amplifier will be presented and the performance of the analog electronics will be evaluated.

In Fig. 3.26 the scheme of the analog electronics layout is displayed. In order to meet the requirements postulated above a dual-stage amplifier circuit with a very low electronic drift was designed. The sensor signal, induced by photons generating the signal currents \( i(A) \) and \( i(B) \), see Eq. (3.10), is fed to the first stage. Here, the transimpedance amplifier converts these signal currents to voltages by \( V_1(A) = i(A) \cdot R_1 \) and \( V_1(B) = i(B) \cdot R_1 \), respectively. In the second stage the alternate current (AC) part of the signal, induced by the modulation of the dual-wavelength transmitter, is filtered and amplified with the factor \( \frac{R_2}{R_1} \). This is substantially effectuated by introducing the first to second stage coupling capacity \( C_2 \) in the layout of the analog electronics circuit as displayed in Fig. 3.26. Consequently, the photodiode amplifier acts as a band-pass filter, whereas the overall transimpedance gain \( \gamma_{TA} \) (or current-to-voltage gain) of both stages can be calculated as follows:

\[
\gamma_{TA} = \frac{V_{out}(A)}{i(A)} = \frac{R_1 \cdot R_2}{R_2}.
\]

For the signal current \( i(B) \) an expression analog to Eq. (3.16) is valid. Unlike depicted in Fig. 3.26, we utilize the same operational amplifier chip for both channels A and B. In doing so, identical drift behavior for both channels can be obtained. Therefore, using Eq. (3.14) to this
degree of approximation the influence of electronic drifts on the position detection can be greatly reduced.

Fig. 3.26: Schematic drawing of the circuit of the dual-channel transimpedance amplifier

In the circuit layout of the dispersometer detection system we use for the first stage $R_1 = R_f = 1 \, \text{M} \Omega$ and $C_f = 33 \, \text{pF}$ as feedback parameters, for the first and second stage coupling $R_2 = 30.1 \, \text{k} \Omega$ and $C_2 = 220 \, \text{nF}$, and for the second stage $R_3 = 301 \, \text{k} \Omega$ and $C_3 = 100 \, \text{pF}$. Consequently, using Eq. (3.16) the transimpedance gain amounts to $\gamma_{TA} = 1 \times 10^7 \, \text{V/A}$, e.g. for $P_{2\omega} = 1 \, \text{nW}$ of optical power on the sensor the output voltage $V_{\text{out}} = 2.8 \, \text{mV}$ results to first approximation. From these parameters and under the consideration of Eq. (3.16) one observes that the feedback resistance for the first stage $R_f = 1 \, \text{M} \Omega$ is the most critical resistance within the circuit. In contradiction to the operational amplifier chips, here the common use for both channels is not possible. For that reason we took special care by mutually matching the feedback resistors $R_f$ of both channels. From resistance measurements in a temperature range from 15 to 30 °C we chose two samples out of 20 pre-selected resistors of the same type which exhibited an identical absolute value and an identical temperature behavior.

For the first stage operational amplifier which is the core of the transimpedance amplifier, the operational amplifier of the model OPA2111 by Burr-Brown, USA, was finally selected. In combination with the feedback resistance $R_f$ the selection of this operational amplifier is extremely crucial for the performance of the complete circuit as it will be shown in the following. Based on the experiences gained with a number of different operational amplifier alternatives, we found that basically an optimization of three specifications, namely the input noise current $i_{nA}$, the input noise voltage $e_{nA}$, and the open loop gain $A_o$, is most decisive. A detailed review along with the respective definitions of the noise quantities is given in [National Semiconductor, 1974]. We found experimentally, wherein the initial values were derived by numerical simulation that for the envisaged performance of the detection system the input noise current $i_{nA}$ should be in the order of fractions to a few $\text{fA/Hz}^{1/2}$ whereby the input noise voltage $e_{nA}$ must
not exceed 50 nV/Hz1/2. The open loop gain should be \( A_v > 1 \cdot 10^6 \). For the OPA2111 following typical specifications are given with \( i_{\text{sh}} = 0.8 \text{ fA/Hz}^{1/2} \), \( e_{\text{sh}} = 8 \text{ nV/Hz}^{1/2} \) @1 kHz, and for the open loop gain \( A_v = 125 \text{ dB} \) [Burr-Brown, 1993]. As far as the second amplifier stage is concerned, the requirements are considerably less challenging. For this second stage which contributes with a factor 10 \((R_3/R_2)\) to the amplifying process the selection of an operational amplifier of the type TL072 by Texas Instruments, USA, with slightly poorer performance, e.g. \( i_{\text{sh}} = 0.01 \text{ pA/Hz}^{1/2} \), \( e_{\text{sh}} = 18 \text{ nV/Hz}^{1/2} \) @1 kHz is sufficient.

Besides the selection of the sensor and the electronic circuit components, the mode of operation of the photodiode affects the performance and the layout of the amplifier circuit as well. Generally, one distinguishes between photoconductive and photovoltaic mode. Each mode has its benefits: photoconductive mode effectuated by the application of a reverse bias can improve the speed of response and linearity [UDT, 1999]. However, noise currents will be increased with the applied reverse bias. Unlike photoconductive operation of the PSD, the detector remains unbiased in the photovoltaic mode. Consequently, any additional noise current is eliminated. Due to the requirements on the detection system which demand low-noise amplification of low optical power levels within a frequency range up to several kHz, this latter mode of operation was preferred. Furthermore, we demonstrated experimentally the superior noise qualities of the detection scheme operating in photovoltaic mode by operating the same detector set-up in photoconductive and photovoltaic mode, alternately.

In addition to the layout of the amplifier circuit, the stability of the power supply of the operational amplifier in connection with the reduction of power supply noise coupling is substantial for the performance of the analog electronics. Noise induced by the power supply, due to the finite power supply rejection ratio of the operational amplifier, might couple into the signal path reproducing a portion of the noise signal at the operational amplifier. There, the induced noise combines with the present input noise and will be amplified [Graeme, 1997]. In order to prevent this coupling of power supply noise and to stabilize the supply voltage, we designed a two-stage power stabilization scheme. Herein, an external power supply provides a stabilization as the first stage, whereas the additional supply voltage stabilization circuit on a separate print which effectuates the refined stabilization is directly implemented in the detection unit as the second stage.

In the course of the amplifier development special care was also taken for reducing the influence of further external noise sources by shielding the complete analog electronics, minimizing the distance between the PSD electrodes and the first stage operational amplifier input, by separating amplifier and power stabilization print within the detection system housing, and by using coaxial cables connecting the analog and digital electronics. In order to minimize the influence of secondary effects, e.g. temperature effects, we physically arranged the circuit components symmetrically. The complete analog electronics in combination with the PSD is installed in a shielded housing which can be mounted to the dispersion telescope, see section 3.2.1, and installed in the nanopositioning system as presented in section 4.1.2. Furthermore, the detection system can be sealed with a black anodized cap in the way that external electromagnetic disturbances are substantially eliminated and irradiation of the PSD surface is prevented.

In order to evaluate the performance of the analog electronics and to predict the position sensitivity of the detection system we executed noise bandwidth measurements in combination with the analysis of the noise spectrum. Additionally, we determined the rms voltage noise of the detection system as presented in the following. We note that for all the noise measurements
presented within this section the UDT SPOT-2D and the analog electronics were installed in their housing shielded by the aforementioned black anodized cap. In addition, all measurements have been performed on a plate connected to ground potential.

The noise bandwidth measurements and the measurements for analyzing the noise spectrum were executed by using a HP3561 Dynamic Signal Analyzer by Hewlett Packard, USA. For the lower 3 dB point we found $f_1 = 34$ Hz and for the upper 3 dB point $f_2 = 3.0$ kHz was obtained. Consequently, the lower 3 dB point which was assumed to be critical when coinciding with the modulation frequency $f_{mod} = 250$ Hz exhibits a sufficient separation in frequency. The upper frequency point is at $f_{samp}/2$ which is also favorable considering the Nyquist theorem. From these measured quantities the effective noise measurement frequency bandwidth can be approximated by $\Delta f = 4.7$ kHz [Texas Instruments, 1999].

The noise spectra were analyzed utilizing the same measurement set-up. In Fig. 3.27 the typical noise spectrum is displayed. Here, the stop frequency of 6.25 kHz for the measurement was placed outside $\Delta f$ intentionally.

![Graph showing noise spectrum](image)

Fig. 3.27: Noise spectrum of the UDT SPOT-2D in conjunction with the amplifier for channel A. The complete detection system was shielded against radiation. The quantities related to the cursor position show the characteristic values @250 Hz. Channel B showed identical behavior.

One observes in Fig. 3.27 that the noise spectrum is predominantly smooth, whereby no resonant frequencies occur which might indicate the presence of external noise sources. Generally, the thoroughly flat run of the curve within the noise frequency bandwidth $\Delta f$ can be recognized. Derived from several noise spectra by varying the measurement bandwidth of the signal analyzer which also effectuates variation of the resolution bandwidth, we obtained for the rms noise voltage output density $e_{rms} = 1.29 \mu V/\text{Hz}^{1/2}$ as a mean value. Using Eq. (3.16) under consideration of the measured spectral responsivity of the PSD $r_s$ for the detector system limited noise equivalent power for $\lambda_1 = 859.2$ nm $\text{NEP}_{DL} = 3.31 \times 10^{-13}$ W/Hz$^{1/2}$ and for $\lambda_2 = 429.6$ nm $\text{NEP}_{DL} = 4.61 \times 10^{-13}$ W/Hz$^{1/2}$ were determined.
In addition to the noise spectrum analysis, the rms noise voltages at the amplifier output were measured by use of a HP3400A rms voltmeter by Hewlett Packard, USA, which integrates over a frequency bandwidth from 1 Hz to 1 MHz. In doing so, we obtained for the rms noise voltage $V_{\text{rms}(A)} = V_{\text{rms}(B)} = 90$ μV. For the conversion of the rms noise voltages at the amplifier output to the detector system limited noise equivalent power $\text{NEP}_{\text{DSL}}$ at the input Eq. (3.17) holds:

$$\text{NEP}_{\text{DSL}} = \frac{1}{r_i} \frac{V_{\text{rms}} \cdot R_3}{\sqrt{\Delta f \cdot R_f \cdot R_j}}.$$  

(3.17)

With Eq. (3.17) the respective values can be calculated to $\text{NEP}_{\text{DSL}} = 3.36 \times 10^{-13}$ W/Hz$^{1/2}$ for $\lambda_1 = 859.2$ nm and $\text{NEP}_{\text{DSL}} = 4.68 \times 10^{-13}$ W/Hz$^{1/2}$ for $\lambda_2 = 429.6$ nm. Regarding these results, a very good agreement between the different measurement methods is obvious. Deviations of the noise equivalent power $\text{NEP}_{\text{DSL}}$ within different measurements using the same method and by comparison to the alternate method are found to be $< 1 \times 10^{-14}$ W/Hz$^{1/2}$. Furthermore, by comparing detector system limited noise equivalent power $\text{NEP}_{\text{DSL}}$ with the noise equivalent power of the detector $\text{NEP}_{\text{DL}}$, see section 3.2.2, we can assume that the amplifier acts as the dominant noise source. In order to underlay this assumption we applied an amplifier noise model. The model parameters are shown in Fig. 3.28.

![Amplifier noise model](image)

Fig. 3.28: Characteristic parameters of the amplifier noise model

In accordance to Fig. 3.28 the influence parameters are the input noise current $i_{nA}$, the input noise voltage $e_{nA}$, the feedback resistance $R_f$ of the amplifier, and the shunt resistance $R_{sh}$ of the PSD. Therefore, the amplifier noise model can be written as follows:

$$\text{NEP}_{\text{DSL}} = \frac{1}{r_i} \left( \frac{e_{nA}}{R_{sh}} \right)^2 + \left( \frac{i_{nA}}{R_{sh}} \right)^2 + \frac{4k_B T}{R_{sh}} + \frac{4k_B T}{R_j} \right)^{1/2}.$$  

(3.18)

The shunt resistance $R_{sh}$ of the UDT SPOT-2D amounts typically to $R_{sh} = 100$ MΩ. Hence, one finds for the $\text{NEP}_{\text{DSL}} = 3.28 \times 10^{-13}$ W/Hz$^{1/2} @ 859.2$ nm and $\text{NEP}_{\text{DSL}} = 4.57 \times 10^{-13}$ W/Hz$^{1/2}$
@429.6 nm for \( T = 293 \, ^\circ\text{K} \). We note that in contradiction to amplifier noise models proposed in the references listed, e.g. [Graeme, 1997; Texas Instruments; 1999; National Semiconductor, 1974], Eq. (3.18) is related to the noise equivalent power. The representation was preferred, because the ratio of the NEP and the amount of optical power on the sensor is decisive for the detector performance, as it will be shown in section 3.2.6. Comparing the model calculations with the measurements above we conclude that the noise of the detection system is limited by the amplifier. Furthermore, for the parameters of the selected circuit components one observes that within the amplifier noise model Johnson or thermal noise of the feedback resistance \( R_f \) is the predominant noise effect. Under this assumption one can write for the rms noise voltage \( V_{\text{rms}} \)

\[
V_{\text{rms}} = \frac{R_3}{R_2} \cdot \left( 4k_B T \Delta f \cdot R_f \right)^{1/2}.
\]  

(3.19)

Using Eq. (3.19) one obtains \( V_{\text{rms}} = 87 \, \mu\text{V} \). Consequently, this noise source contributes with approximately 97 \% to the total measured rms noise voltage. Because here Johnson noise predominantly contributes to the noise budget of the analog electronics and this noise source is characterized by a Gaussian probability density function [Texas Instruments, 1999], we consider the amplifier output noise voltage as white noise. This introduces certain benefits for the further analysis.

In order to connect the analog electronics to the digital electronics noise considerations are also important. An optimized noise input level enables a resolution of the ADC beyond the theoretical resolution it provides [Loewenstein, 2000]. This increase is achieved by ensuring that the rms noise voltage \( V_{\text{rms}} \) falls into the range \( 0.5 \, \text{LSB} \leq V_{\text{rms}} \leq 1 \, \text{LSB} \), wherein the least significant bit (LSB) of the \( n \)-bit ADC is defined by Eq. (3.20).

\[
\text{LSB} = \frac{\text{FSR}}{2^n}.
\]  

(3.20)

In Eq. (3.20) \( \text{FSR} \) denotes the full scale range of the ADC. The condition given with the noise range above is equivalent to the principle of dither. For \( V_{\text{rms}} < 0.5 \, \text{LSB} \) the digital data may be fully affected by the quantization noise \( q_n \) varying from 0 to 0.5 \( \text{LSB} \), nonlinearities and other imperfections of the ADC notwithstanding. If noise voltage \( V_{\text{rms}} \) falls into the aforementioned range, the effective resolution of the ADC can be improved by several bits due to oversampling. However, for the adaptation of the amplifier to the ADC we used a slightly expanded range \( 0.29 \, \text{LSB} \leq V_{\text{rms}} \leq 1 \, \text{LSB} \) under the assumption that with present background radiation the overall noise level will be increased. The lower limit was approximated by the rms value of the quantization noise \( q_{\text{rms}} = \text{LSB}(12)^{1/2} \) which can be derived from the triangular waveshape of the ADC transfer function [Zuch, 1979]. For the selected 16-bit ADC, see section 3.2.4, and a \( \text{FSR} = 20 \, \text{V} \), a least significant bit of \( \text{LSB} = 305.18 \, \mu\text{V} \) results. One observes that \( V_{\text{rms}} \) falls solely into the expanded range. However, for optimum performance under exclusion of additional noise sources, either the ratio \( R_3/R_2 \) has to be enlarged, or the voltage input range of the ADC adjusted. Preventing a hardware modification we adjusted the programmable gain of the ADC to enable oversampling.
3.2.4 Digital electronics

The requirements for the digital electronics envision phase-locked analog to digital conversion in combination with demodulation of the dual-wavelength signal. Therefore, we stress that two data acquisition features are most decisive. First, this is the analog to digital conversion process itself. Second, in combination with the ADC an appropriate timing scheme is substantial for enabling phase-locked sampling and decoding of the acquired data.

Based on a sensitivity analysis we considered a 16-bit ADC implemented in the detection system as sufficient. Moreover, regarding a sampling frequency $f_{\text{sample}} = 6 \text{ kHz}$ the aforementioned resolution should be maintained. Consequently, the settling time of the ADC unit is an important specification as well.

In the course of evaluating the digital electronics we tested several 16-bit ADC boards in connection with the UDT SPOT-2D and the analog electronics considering resolution, stability, noise behavior, and settling time. We finally selected a PCI-6032E multifunction data acquisition board by National Instruments, USA. Besides the actual performance of the PCI-6032E, further benefits arise from the implementation of this National Instruments board, because the complete data acquisition and system control software of the dispersometer is based on LabVIEW, National Instruments, USA, which simplified implementation. Hence, development time was substantially reduced in comparison to register level programming.

The block diagram of the PCI-6032E is shown in Fig. 3.29 below:

![Block diagram of the PCI-6032E, based on [National Instruments, 1999]](image-url)
One observes in Fig. 3.29 that due to the architecture of the 16-bit ADC unit, the analog input signals are first routed from the appropriate channels via the multiplexer (mux) and are fed subsequently to the programmable gain instrumentation amplifier (PGIA). Here, the signals are amplified to assure analog to digital conversion with maximum resolution. Multiplexing is required, because solely one single PGIA and ADC, respectively, is used for both input channels. Unlike using a single PGIA and ADC combination for each channel, which is basically feasible in our case, e.g. by using two PCI-6032E boards synchronized via the displayed real-time system integration (RTSI) bus, both signals are effected by the same influences, e.g. drift, nonlinearities, etc., by passing the identical PGIA and ADC chip. Moreover, this is well in accordance with the respective basic design principle. However, a limitation of the depicted circuit design arises in the finite settling time of the instrumentation amplifier. The worst case settling time for all possible gain levels and consequently, for all voltage input ranges (maximum FSR = 20 V) is specified to \( t_{set} \leq 40 \mu s \) for 0.5 LSB accuracy [National Instruments, 1999]. However, due to applying the same gain for both channels and due to the magnitudes of the measurement range under consideration that the light spots are positioned around the middle of the gap, the voltage difference between both channels can be kept small which also leads to a small settling time. Furthermore, it could be demonstrated experimentally by executing the settling time tests proposed by Kasin [1993] that at \( f_{sample} = 6 \text{ kHz} \) and for an identical bipolar voltage range in the order of several volts and therefore common gain for both channels, the influence of the PGIA settling time is fully negligible.

In addition to the considerations in connection with multiplexing the analog input signals, the topic of calibrating the ADC unit has to be addressed here to ensure that the PCI-6032E works within the given specifications. As displayed in Fig. 3.29, the PCI-6032E board possesses so-called calibration DACs, wherein calibration constants are written in order to compensate for the pregain and postgain offset, the gain error, and linearity drifts. In a self-calibration procedure provided by the PCI-6032E board which can be initiated by software, the aforementioned error sources are calibrated without external signal connections under consideration of the onboard calibration reference. The remaining significant residual deviation after calibration is gain error due to temperature induced drifting of the reference voltage. The long-term stability of the onboard calibration reference is specified to 6 ppm/(1000 h)\(^{1/2}\). The calibration of this potential error source can be, at least theoretically, addressed by applying a very accurate (< 0.001%) external reference in the order of several volts. However, no external calibration was executed in connection with the dispersometer experiments based on the following considerations. First, the residual error which stems from the temperature drift of the reference voltage particularly due to warming-up of the device can be reduced by allowing substantially longer warm-up times than recommended for the PCI-6032E (\( t_{warm} \sim 15 \text{ min} \)) up to several hours for the experimental set-up. Second, due to the aforementioned signal characteristics this error term can be kept small. Moreover, as a third and most decisive consideration, the remaining uncertainty within the gain can be well calibrated within the overall calibration of the dispersometer. This global system calibration which encompasses the measurement procedure is the decisive calibration. The aforementioned self-calibration procedure of the PCI-6032E board is implemented in the control software of the dispersometer. After each calibration procedure which is frequently executed a new set of calibration constants is stored in the calibration DACs.

Furthermore, in Fig. 3.29 the timing unit of the PCI-6032E board is depicted, wherein its core is the data acquisition timing controller (DAQ-STC). Within the present application the
DAQ-STC controls timed analog-to-digital conversion, therefore enabling phase-locked sampling of the dual-wavelength signal at both channels. In order to initiate the externally timed sampling by the synchronization signal generated on the transmitter side, this signal is connected to the DAQ-STC. Herein, the synchronization signal acts as the external sampling clock, whereas in the applied edge detection mode each low to high transition of the TTL-signal generates a pulse which initiates the ADC unit to take one single sample of each channel. The inter-channel delay, mainly caused by the finite settling time of the PGIA, as reported above, is controlled by the DAQ-STC autonomously and therefore minimized. By applying this scheme a strictly phase-locked acquisition was assured, because modulation and synchronization are derived from the same rotating filter disc, as introduced in section 3.1.4. As a consequence, this scheme works, even when frequency jittering occurs.

To enable the digital demodulation of the dual-wavelength signal, in order to find the relation between the digitized signal parts and the respective wavelengths, the IR reference signal is fed to the DAQ-STC as a digital start trigger in the so-called post-triggered acquisition mode. That means, the data acquisition is initiated by the rising edge of the IR reference signal and timed by the synchronization signal. Because the relation of the occurrence between the start trigger and the first pulse is hardware dependent and therefore fixed, due to the fixed mounting of the filter disc, this scheme assures a robust demodulation of the digitized dual-wavelength signal for the further signal processing. The schematic drawing of the sample timing scheme is shown in Fig. 3.30.

![Schematic drawing of the data acquisition timing scheme using the DAQ-STC chip of the PCI-6032E board. The synchronization and IR reference signal generated by the dual-wavelength transmitter are used as the external sampling clock and the digital start trigger, respectively.](image)

Before working with the PCI-6032E board, the actual performance was thoroughly tested and evaluated considering the criteria reported in this section. For these performance tests the PCI-6032E was connected to the analog electronics of the detection system, whereby the sensor was shielded against irradiation as reported in section 3.2.3. For preventing temperature effects the board was warmed up 24 h which was also done in connection with the experiments presented...
in chapter 4 and 5. Fig. 3.31 shows a typical example of the performance, shortly after the self-calibration procedure was executed.

![Graph showing performance of the PCI-6032E board connected to the detection system (channel A). Both channels were sampled with $f_{\text{sample}} = 6.25$ kHz for 10k times. In the histogram each class marks 1 LSB for the 16-bit ADC and a FSR = 20 V. For channel B an identical behavior resulted.]

Derived from those measurements the total system noise including the quantization noise was found to be 0.57 LSB, therefore below the specified value. By frequent re-calibration of the board virtually no systematic deviations to preceding sampling series were recognized. Furthermore, we found by connecting the analog input with ground potential that both channels exhibit an offset which is induced by the analog electronics. Because these offsets remain stable during measurement time, they can be eliminated easily within the further signal processing.

### 3.2.5 Radiometry

As derived in section 3.2.2 and in section 3.2.3, the noise quantities of the detection system can be expressed in terms of equivalent power levels, e.g. $\text{NEP}_{\text{DSL}}$. Consequently, the performance of the detection system depends on the total amount of optical power on the sensor $\phi_{\text{tot}}$, as addressed to in Eq. (3.12), in combination with the integration time $t_{\text{int}}$. Thus, knowledge of the total amount of optical power on the sensor $\phi_{\text{tot}}$ enables to predict detector performance. In the following we provide a radiometric model to estimate the total amount of optical power on the sensor $\phi_{\text{tot}}$, in consideration of the envisaged parameters of the experiments presented in chapter 5. The model parameters, related to the geometrical properties of the set-up in combination with the emitted and received amounts of optical power, are displayed in Fig. 3.32.
In Fig. 3.32 $R$ denotes the distance between transmitter and receiver, $d$ the aperture diameter of the dispersion telescope and $\phi_{out}$ the optical output power at the transmitter. Under consideration of the half-angle divergence $\theta$ the radiant intensity $J_{out}$ out of the optical fiber is defined as in the following:

$$J_{out} = \frac{\phi_{out}}{\Omega} = \frac{\phi_{out}}{\pi \theta^2}. \quad (3.21)$$

In Eq. (3.21) $\Omega$ denotes the Gaussian beam solid angle. By maintaining the $1/e^2$ criterion for the half-angle divergence $\theta$, as reported in section 3.1.5, the equivalent circular cone contains 86% of total beam power in the far-field [Siegman, 1986]. Under consideration of Eq. (3.21) the total amount of optical power on the sensor $\phi_{out}$ can be obtained by

$$\phi_{out} = T_e \cdot \frac{\pi d^2}{4 R^2} \cdot J_{out} \cdot e^{-\kappa R} \cdot R^{-1}. \quad (3.22)$$

in accordance with [Gächter, 1981]. Eq. (3.22) is the ratio of the emitted Gaussian beam solid angle and the solid angle wherein power can be collected by the receiving optics. Furthermore, this ratio is corrected for propagation loss due to the atmosphere and the receiving optics, wherein $T_e = 0.5$ is the transmittance of the dispersion telescope at both wavelengths. The last term in Eq. (3.22) takes the extinction by the atmosphere into account. Within this estimation we used for the approximation of the extinction coefficient $\zeta = 0.2$ km$^{-1}$ for $\lambda_1$ and $\zeta = 0.32$ km$^{-1}$ for $\lambda_2$ for clear conditions and a visual sight length of $\sim 15$ km [Wolfe and Zissis, 1989; Elion and Elion, 1979; Gächter, 1986]. We note that these extinction figures are average values for the broadly defined conditions as given above. As far as this estimation is concerned their validity is well in accordance with the other magnitudes in Eq. (3.22). We also note that for a detailed analysis of the extinction coefficients for a specific atmospheric condition the use of the high-resolution transmission molecular absorption database (HITRAN) is recommended [Rothman et al., 1998].

Under the assumption of the power output $P_{tot} = \phi_{out,1} = 0.96$ mW and the half-angle divergence $\theta_1 = 75.0$ mrad for $\lambda_1 = 859.2$ nm and $P_{tot} = \phi_{out,2} = 1.43$ mW and $\theta_2 = 97.0$ mrad for $\lambda_2 = 429.6$ nm, we obtained for the radiant intensities by using Eq. (3.21) $J_{out,1} = 5.43 \cdot 10^2$ W/sr
and $J_{\text{out},2} = 4.84 \times 10^{-2}$ W/sr. One observes that relatively small radiant intensities result. These are caused by the large-diffraction induced beam spreading which originates from the small core diameter of the optical fiber, see section 3.1.5. The total amount of optical power on the sensor $\phi_{\text{tot}}$ for various sight lengths $R$ and aperture diameters $d$ is displayed in Fig. 3.33 for $\lambda_1$ and in Fig. 3.34 for $\lambda_2$.

Fig. 3.33: Simulation of the propagation of optical power @859.2 nm

Fig. 3.34: Simulation of the propagation of optical power @429.6 nm

Due to the similar radiant intensities for both wavelengths, both figures resemble each other as far as the orders of magnitudes are concerned. A more detailed analysis reveals that the received blue light power level is slightly lower than the received IR power level. Hence, in accordance with the higher signal noise level, see section 3.1.3, and the lower spectral sensitivity of the sensor, see section 3.2.2, the SH radiation is assumed to be the limiting factor of the system. Therefore, we will limit the further considerations to $\phi_{\text{tot},2}$, see section 3.2.6. One observes the rapid decrease of optical power, mainly contributed by the large angular beam spreading. Furthermore, by simulating optical power transfer for various aperture diameters, as used in experiments reported in chapter 5, the large influence of the aperture diameter of the receiving optics can be demonstrated.

Up to this point all considerations were based on the radiation emitted directly from the fiber output which characterizes the dual-wavelength transmitter. In order to demonstrate the potential of the developed dual-wavelength transmitter in combination with the transmitter optics, as proposed in section 3.1.6, for application with longer sight lengths, we assumed a smaller half-angle divergence of $\theta_t = 2.0$ mrad for the critical SH radiation. This order of magnitude is comparable with half-angle divergences used in commercially available electro-optical systems, e.g. in electro-optical distance meters. Thus, $\theta_t = 1.5$ mrad results for $\lambda_1$ assuming the same optical magnification for both wavelengths. Hence, the amount of optical power received on the sensor is $\phi_{\text{tot},1} = 3.06 \times 10^{-7}$ W for $\lambda_1$ and $\phi_{\text{tot},2} = 2.64 \times 10^{-7}$ W for $\lambda_2$ at a sight length of $R = 250$ m and for an aperture diameter of $d = 20$ mm. Consequently, we demonstrated that by reducing beam divergence to the proposed values, the received optical power levels can be increased by more than 3 orders of magnitude at a sight length of $R = 250$ m.
3.2.6 Influence of the noise sources on position detection

Within this section influence of noise on the measurements, i.e. the determination of position in the focal plane of the receiving optics, will be analyzed based on the predicted optical power levels on the detector, see section 3.2.5, in combination with the noise quantities of the detection system determined in section 3.2.3. Furthermore, we will expand the analysis towards additional noise sources, e.g. laser noise, etc., which were not regarded so far. Finally, conclusions will be drawn for the set-up of the dispersometer experiments.

In analog representation to the noise equivalent power NEP, the noise equivalent displacement NED in combination with the noise equivalent angle NEA have been introduced by Gächter [1981]. Regarding the gap-technology, as presented in section 3.2.2, the NED for the case $2w_{gap} \ll b$ can be written as follows:

$$NED = \frac{1}{2} \cdot \frac{b \cdot NEP}{\phi_{rot}}. \quad (3.23)$$

Herein, NEP denotes the noise equivalent power contributed by the respective noise source. Accordingly, one obtains for the NEA:

$$NEA = \frac{1}{2} \cdot \frac{b \cdot NEP}{f_{rot} \cdot \phi_{rot}}. \quad (3.24)$$

$NED$ and $NEA$, respectively, depend on the total amount of optical power on the sensor. Furthermore, in accordance with Eq. (3.12), the gap width $b$ is directly related to the position sensitivity. In order to describe generally the potential performance of a detection system, the detection system inherent characteristic noise equivalent displacement $CNED$ has been introduced in [Wild Leitz, 1987]:

$$CNED = \phi_{rot} \cdot NED. \quad (3.25)$$

Consequently, for the dispersometer detection system we obtain $CNED = 21.4 \cdot 10^{9} \text{nm-W/Hz}^{1/2}$ for $\lambda_1 = 859.2 \text{nm}$ and $CNED = 29.7 \cdot 10^{9} \text{nm-W/Hz}^{1/2}$ for $\lambda_2 = 429.6 \text{nm}$. Based on these figures we are now able to calculate the $NED$ as a function of the received power at the respective distance from the dual-wavelength transmitter. In Fig. 3.35 such a calculation was executed based on the received power levels simulated in Fig. 3.34. For reasons mentioned above, the calculation is consequentially restricted to the shorter wavelength. Based on considerations for the set-up of the forthcoming experiments, a maximum sight length of $R = 50 \text{ m}$ in a laboratory environment was available.
One observes the rapid increase of $NED$ for longer sight lengths, especially for smaller aperture diameters. As a noise figure of merit the $NED$ possesses the dimension $m/Hz^{1/2}$. Hence, the $NED$ has to be related to the integration time $t_{int}$ in order to achieve position information. As prescribed by the requirements for position detection in the gap, see section 3.2.2, analog requirements result for the $NED_{int}$ and the $NEA_{int}$. Hence, for a working dispersometer $NED_{int} \leq 7.2$ nm ($NEA_{int} \leq 0.024$ μrad) is postulated. Under consideration of an integration time $t_{int} = 1$ s, the $NED_{int}$ falls below the postulated value for all aperture diameters used in this simulation and for a separation of $R = 20$ m between transmitter and receiver, see Fig. 3.35. For a more detailed analysis of the $NED_{int}$ in dependence of the integration time, this relation is depicted in Fig. 3.36 for a sight length of $R = 20$ m.

Fig. 3.35: Simulation of $NED$ for various sight lengths $R$.

Fig. 3.36: $NED_{int}$ of the detector as a function of integration time $t_{int}$ at $R = 20$ m

Fig. 3.36 also confirms that the required $NED_{int}$ was reached for all aperture diameters within 1 s of integration time. The break-even point for the smallest aperture of $d = 20$ mm is at
As a result from this analysis, sight lengths of $R < 20$ m were taken into consideration for the experimental set-up.

Whereas the analysis above was limited to the impact of the inherent noise of the detection system on the position detection, the influence of external noise sources will be briefly discussed in the following.

Laser noise can be regarded as the initial external noise source. In section 3.1.3 we derived that the laser inherent SH noise is substantially white. Hence, for the integration time $t_{\text{int}} = 1$ s, the signal-to-noise ratio of the laser signal amounts to $\text{SNR} = 2.9 \times 10^5$. Thus, the SNR available from the laser source is more than one order of magnitude larger than required for the present realization of the dispersometer. As a consequence, this noise source is considered to be negligible. Quantum noise fluctuations within the laser [Siegman, 1986] contribute to the total laser noise, however as a small fraction. Thus, this source of error is already regarded and therefore is also negligible.

A further substantial noise source in connection with electro-optical systems is background radiation. We note that during the experiments presented in chapter 4 and 5, the laboratory was totally darkened. Hence, the influence of background radiation was minimized by the ambient conditions of the experimental set-up. Furthermore, the digital signal processing (DSP) scheme in combination with the signal structure, as introduced in 3.1.4, offers a possibility to eliminate the influence of background radiation on the acquired signal, i.e. digitized signal, see section 4.2.

Unlike laboratory conditions, the influence of background radiation has to be minimized within the design of the dispersometer under consideration of adverse conditions of background radiation. One key issue, previously considered, is an appropriate temporal modulation of the signal, see section 3.1.4, in connection with a suitable amplifier acting as a band-pass filter, see section 3.2.3. Furthermore, the amount of background radiation on the sensor is dependent on the field-of-view (FOV) of the receiving optics in combination with the albedo of the imaged background. Theoretically, the measurement range in the focal plane of the receiving optics is determined by the gap width $b = 127$ μm. By introducing an aperture with $d = b = 127$ μm directly in front of the PSD, the FOV of the dispersion telescope can be reduced to $\Omega_{\text{ele}} = 1.4 \times 10^{-7}$ sr. Consequently, for $R = 250$ m the imaged background spans an area with a diameter of ~106 mm. The albedo of this area has to be minimized. Another design possibility is to apply optical filter technique. As far as the analog electronics are considered to act as a band-pass filter, interference filters exhibit the analog performance for optical wavelengths. By applying interference filters the amount of background radiation and consequently, the influence of background noise can be substantially reduced. Because the wavelengths generated by the dual-wavelength laser are very well known, interference filters can be tailored with very small band-passes.

By realizing the aforementioned design issues, i.e. reducing the FOV, diminishing the albedo of the background by placing a black, non-reflecting shield around the dual-wavelength transmitter output, and by applying interference filters, the influence of background noise can be substantially reduced. In doing so, numerical simulations showed that even in adverse conditions background noise is not the dominant noise source. Furthermore, the noise of the detection system acts as the predominant noise source. Under these considerations, the required position accuracy can be obtained within $t_{\text{int}} = 1$ s of integration time for a sight length $R = 250$ m, regarding the reduced half-angle divergences $\theta$ as proposed in section 3.2.5. Consequently, from
these points of view the envisaged performance of the dispersometer, as far as the realized modules are concerned, has been shown. We conclude that from the instrumental point of view these realized dispersometer modules work. Considering the proposed transmitter optics, sight lengths of $R = 250 \text{ m}$ are feasible, even in adverse conditions. Furthermore, we have shown that the experiments in the laboratory environment are possible even solely with the fiber output characterizing the dual-wavelength transmitter. Here, sight lengths of $R \approx 20 \text{ m}$ were proposed in order to analyze the influence of various aperture diameters $d$, e.g. for analyzing the dependence of the sensitivity of the turbulence compensation. However, as far as beam quality is concerned these bare fiber experiments are advantageous.
4 Performance tests of the realized dispersometer

As a prerequisite for the understanding of the dispersometer experiments within the presence of atmosphere-related effects, the testing procedures of the realized dispersometer, as presented in this chapter, should indicate the actual performance of the dispersometer itself, i.e. without the disturbing influences of the ambient air. In order to analyze the operativeness and the system performance of the complete dispersometer both modules were involved in the test series.

Besides the analysis of the operativeness which includes the verification of the data acquisition timing scheme, the synchronization, and the sampling scheme, basically three different types of key experiments were addressed to within the course of the performance tests.

First, scanning experiments, see section 4.2.1, were executed. Within these experiments analysis of the position sensing accuracy and the linearity of position detection were the main issues. As far as the prescribed specifications in combination with the implementation of the short-focal-length telescope are concerned, as given in section 3.2.2, difference position sensing with the accuracy of 7.2 nm had to be demonstrated. Furthermore, homogeneity of position sensing within the gap should be analyzed. Here, potential defects or discontinuities in the characteristic curves of the detection system should be detected. Additionally, the appropriate type of the calibration function should be evaluated, i.e. the best linear fit suggested by the theory in section 3.2.2 had to be verified, e.g. in terms of the correlation coefficient. Moreover, a thorough analysis of the wavelength dependent effects was made in order to determine the PSD inherent dispersion.

The second type of experiments involved the so-called static experiments, as presented in section 4.2.2. Herein, the precision of the detection system was analyzed, whereby the dimensionless positions were transformed to metric dimensions by using the appropriate calibration function. Within these experiments several seconds of continuous data were acquired. Consequently, any kind of drift could be analyzed.

A third type of experiments, based on the static experiments, was motivated by the theoretical framework given in section 3.2.6. In order to evaluate the spectral noise portion related to the atmosphere and the respective noise portion related to the instrumental set-up, spectral analysis of the dispersometer noise floor was made, as presented in section 4.2.3. As shown in section 3.2.6, the noise floor of the detection system depends on the total amount of optical power onto the sensor. Consequently, the optical power incident to the sensor was carefully adjusted, simulating the different aperture diameters used within the experiments presented in chapter 5.

4.1 Set-up for performance tests

In order to obtain conclusive results in the course of the performance tests, the design of the experimental set-up was the most critical issue. Due to the smallness of the magnitudes in the range of a few nanometers special care was taken in realizing a stable and monolithic apparatus. To prevent temperature effects due to material expansion the complete experimental set-up was located in an actively temperature-stabilized laboratory environment. Additionally, to avoid long-term temperature changes which would cause the same detrimental effects as mentioned
above the complete testing procedure was fully automated in order to minimize the elapsing time during the individual experiments. Herein, the core of the experimental set-up was embodied by a nanopositioning system, as presented in section 4.1.2. In Fig. 4.1 the schematic drawing of the experimental set-up is displayed.

Fig. 4.1: Experimental set-up of the dispersometer performance tests

Fig. 4.1 shows both dispersometer modules implemented in the experimental set-up. Due to the experiences gained in connection with the analysis of the beam profiles, as reported in section 3.1.5, we did not use imaging optics within this set-up. This was motivated considering that any additional optics of standard quality might distort the ideal beam profiles. Furthermore, with the implementation of the dispersion telescope, see section 3.2.1, the distance between fiber output and telescope objective would have been substantially enlarged to several meters. Consequently, this distance would make separating between the instrument-related and atmosphere-related effects impossible. As it will be shown in the subsequent section, the set-up becomes potentially very compact by leaving out any imaging optics between fiber output and PSD surface.

Furthermore, one observes in Fig. 4.1 that the detection system, i.e. here the PSD in combination with the analog electronics, is directly incorporated into the nanopositioning system, as mentioned in section 3.2.3. This allows positioning along three axes, wherein the bold axis denotes the axis of positioning within the scanning experiments. For aligning the fiber output with respect to the PSD surface, the fiber connector is incorporated in a gimbal mount which is installed on a three axes translation stage. The respective degrees of freedom are displayed in Fig. 4.1. For assuring maximum stability the fiber connector and the detection system with the nanopositioning system are installed in one monolithic block.

As also schematized in Fig. 4.1 the analog signals from the detection system and the digital signals from the dual-wavelength transmitter are lead to the PCI-6032E board via shielded cables and the shielded connector box. The nanopositioning system is controlled by the personal computer (PC) via the digital controller. Hence, the complete data acquisition and nanopositioning of the PSD is controlled by one PC.
4.1.1 Preliminary measurements and determination of parameters

Preliminary to the incorporation into the experimental set-up, the different sample PSDs were subject to optical measurements in order to derive the key parameters for the appropriate spacing between the fiber output and the PSD chip surface, and for the correct positioning of the PSD chip surface in the focal plane of the dispersion telescope. The sensor dimensions were measured by means of a microscope, whereas the sample movements in the object plane were recorded by two orthogonally positioned linear encoders. The dimension along the z-axis of the PSD, i.e. the axis collinear with the normal to the PSD chip surface, which is critical dimension for the installation procedure, was measured by means of the depth of focus of the microscope, whereas the focus movements were read out by a third linear encoder. For the latter specification given above the distance from the chip surface to the bottom of the case was decisive. Besides the accurate determination of the required quantities, the optical method utilizing microscope observations offers intrinsically the additional benefit of direct visual inspection of the chip surface. Consequently, discontinuities and impurities can be directly identified within the measurement process. This is important, because due to the removal of the protective glass the surface of the PSD is exposed, as reported in section 3.2.2. In doing so, an example of the microscopic inspection is given in Fig. 3.23.

The optical measurements showed that the actual gap width of the UDT SPOT-2D sample used in the dispersometer detection system amounts to \( b = 125.9 \, \mu m \), therefore slightly smaller than the specified nominal value \( (b = 127 \, \mu m) \). The measurement accuracy was determined to \( \sigma = 0.3 \, \mu m \). Additionally, it was observed that all sample PSDs exhibit very sharp edges and well defined geometrical properties, e.g. parallelism, which is beneficial for the overall performance. Herein, deviations from the geometrical conditions were found to be significantly below measurement accuracy.

![Fig. 4.2: Propagation model for the dispersometer performance tests](image)

After installation in the detection system the position of the PSD chip surface along the z-axis could be determined with an accuracy of \( \sigma < 3 \, \mu m \). In combination with the finite depth of focus in connection with the model of Gaussian beams, as introduced in section 3.1.5, character-
ized by the Rayleigh range, the herein achieved accuracy is sufficient. Furthermore, the spot-sizes on the chip surface can be accurately controlled by the fiber output to PSD chip distance. Due to the fact that the beams coupled out from the fiber output directly radiate on the PSD chip surface, the Gaussian beam propagation model and the associating parameters presented in section 3.1.5 can be directly applied. Under the consideration of the estimated orders of magnitude, as presented in section 2.3.4 and in section 3.2.2, the turbulence induced blur has to be considered for the realistic simulation of spot sizes.

Fig. 4.2 shows the increase of the spot-sizes within a distance of 200 µm from the fiber output. This model was calculated for the fiber no 1 using Eq. (3.5) for both present wavelengths in the far-field characterized by the Rayleigh range \( z_R \). The respective values for the Rayleigh range \( z_R \) according to Eq. (3.7) are also displayed in Fig. 4.2. Within the experimental set-up the distance between fiber output and PSD chip surface was accurately adjusted to 147 µm by a very precise mechanical spacer. Consequently, considering the Rayleigh ranges given above, the far-field behavior was assured. The corresponding spot-sizes can be found to \( 2w = 23.2 \mu m @ 859.2 \text{ nm} \) and \( 2w = 28.7 \mu m @ 429.6 \text{ nm} \). In comparison with the estimated orders of magnitude given in section 2.3.4 and 3.2.2, these values are in good agreement with the moderately turbulent case. Therefore, as far as spot-sizes are concerned the presented experimental set-up gives the possibility of a realistic testing environment. Additionally, the set-up is ultra compact, external influences by the ambient air on both beams are totally eliminated. Disturbances induced by multiple reflections between fiber facet and PSD chip surface which would lead to enlarged spot-sizes are initially minimized by the AR-coating of the PSD chip surface. Furthermore, the actual spot-sizes can be determined within the measurement process by using a similar knife-edge method as reported in section 3.1.5. Herein, the sharp transition between initially active element and the gap functions as the knife-edge.

### 4.1.2 Nanopositioning system

Besides the overall stability of the experimental set-up and the correct adjustment between fiber output and PSD chip surface, mechanical positioning of the PSD down to the nanometer range was considered as one of the most challenging requirements in connection with the dispersometer performance tests.

Due to the requirements of position sensing accuracy of the dispersometer detection system the positioning of the PSD should enable small movements with a smallest step width in the order of the required difference position sensing accuracy. For the testing procedure it is important that the actual position can be detected including a feedback signal that the positioning was completed. Additionally, the step width should be variable in order to enable certain scan modes by a constant time elapsing and constant amount of acquired data. These experiments include the global characteristics of the sample PSD in terms of homogeneity tests. Step width must be minimized close to the center of the gap in order to derive the position sensing accuracy of the detection system. The specified measurement range depends on the dimensions of the PSD and the spot-sizes. For the characteristic behavior of the PSD using gap-technology the active element to gap transitions are decisive as well, also cf. [Gächter, 1984]. Hence, for the selected UDT SPOT-2D in combination with the resulting spot-sizes, see section 4.1.1, a maximum translation range of 200 µm was assumed to be sufficient. To enable scanning on adjoin-
ing lines with defined separation and for an envisaged expansion towards 2-dimensional PSDs of the same type, the maximum translation range was specified for two orthogonal axes. A thorough evaluation of different technologies for the realization of nanopositioning resulted that the envisaged specifications can be solely fulfilled by a nanopositioning system based on piezoelectric technology. Typical fields of application for these piezo flexure nanopositioners and -scanners include scanning microscopy, micro-manufacturing, and disk drive testing.

Such a piezo-based nanopositioning system was developed by Physik Instrumente, Germany. As one can see in Fig. 4.1, the dispersometer detection system is incorporated into the clear aperture of the piezo flexure stage, wherein the PSD chip surface coincides with the XY plane of the piezo flexure stage. The 3-axis system of the implemented version P-527.3CL offers linear translation ranges of $200 \times 200 \times 20 \mu m$. The working principle of the P-527.3CL is based on low voltage piezoelectric actuators (PZTs) and flexures employed as the drive and guiding system. By the integration of capacity position feedback sensors the hysteresis and drift (creep) effects contributed by the PZTs are eliminated, as well as very high linearity, reproducibility of positioning, and positioning accuracy can be obtained [Physik Instrumente, 1998]. The complete nanopositioning system P-527.3CL was calibrated by the manufacturer using a displacement measuring interferometer ZMI 1000 by Zygo, USA. The accuracy of the P-527.3CL, according to the calibration certificates [Physik Instrumente, 1999], can be specified to 2 to 3 nm under the consideration of the calibration function and under the strict requirement that the ambient conditions which prevailed for the calibration can be met. Consequently, it was required that after installation the parameters of the control loop were checked and the complete system was operated in an actively temperature-stabilized laboratory environment with respect to the ambient conditions at the calibration site.

### 4.2 Experiments

Unlike the simplified form of the calculation of position on the PSD given with Eq. (3.14), the correct demodulation of the sampled signal at both channels A and B requires a slightly more expendable digital signal processing scheme. Experiments in connection with the performance tests showed that the trigger position with respect to the sampled signal remains identical. Consequently, modifying the sampled signal by means of digital signal processing was possible autonomously. Due to the finite rise time $t_{rise} = 70 \mu s$ and the sampling frequency $f_{sample} = 6 \text{ kHz}$, it had to be guaranteed that no samples used within the signal processing coincide with the rising or falling signal edges. Therefore, the signals initially consisting of 24 samples for each period were subset and concatenated resulting in modified signals consisting of 16 samples.

<table>
<thead>
<tr>
<th>No of sample</th>
<th>Signal @ $\lambda_1$</th>
<th>Signal @ $\lambda_2$</th>
<th>Background</th>
<th>Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 4</td>
<td>-</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>5 to 8</td>
<td>-</td>
<td>-</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>9 to 12</td>
<td>×</td>
<td>-</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>13 to 16</td>
<td>-</td>
<td>-</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>
In Table 4.1 the first 16 samples (one period) of the resulting signal structure are given. This period can be repeated in order to characterize the complete signal. Consequently, demodulation with respect to the wavelength content of the signal can be made by splitting each period in halves. Additionally, one observes the presence of the background and offset portions superimposed on the acquired samples. However, by subtracting the part of the period wherein no measurement signal is transmitted from the part of the period in which the measurement signal is superimposed by the background and offset portion results in a background and offset corrected signal for either wavelength. It was demonstrated experimentally that preceding integration is favorable for this analysis. In accordance to Table 4.1 the corrected signal for the first period can be calculated for both channels A and B and both wavelengths $\lambda_1$ and $\lambda_2$, indicated as the subscript at the channel symbol, as follows:

$$
A_1 = (s_{A09} + s_{A10} + s_{A11} + s_{A12}) - (s_{A13} + s_{A14} + s_{A15} + s_{A16})
$$

$$
B_1 = (s_{B09} + s_{B10} + s_{B11} + s_{B12}) - (s_{B13} + s_{B14} + s_{B15} + s_{B16})
$$

$$
A_2 = (s_{A01} + s_{A02} + s_{A03} + s_{A04}) - (s_{A05} + s_{A06} + s_{A07} + s_{A08})
$$

$$
B_2 = (s_{B01} + s_{B02} + s_{B03} + s_{B04}) - (s_{B05} + s_{B06} + s_{B07} + s_{B08})
$$

wherein $s$ denotes the sample and the indices indicate the respective channel and number of sample within a period. This procedure is repeated throughout all periods. Hence, the positions on the PSD can be calculated as follows:

$$
\bar{y}_1 = c_1 \frac{A_1 - B_1}{A_1 + B_1}
$$

$$
\bar{y}_2 = c_2 \frac{A_2 - B_2}{A_2 + B_2}
$$

In Eq. (4.2) $c_1$ and $c_2$ denote constants introduced for the transformation to metric dimensions.

In preliminary experiments we observed a certain amount of crosstalk between the signals of both wavelengths. Based on a thorough analysis on this topic in combination with simulations on the electronic layout we found out that crosstalk is contributed by the first to second stage coupling capacitor $C_2$. Hence, we changed the capacity in a first series of experiments from 220 nF to 22 $\mu$F and in a second series of experiments we bridged over the first to second stage coupling capacitor $C_2$ by a wire connection. We demonstrated experimentally that for $C_2 = 22$ $\mu$F the apparent crosstalk could be virtually eliminated. However, to guarantee crosstalk-free detection the following experiments are executed with bridged over $C_2$. This was possible due to the laboratory setting of the experiments.

Prior to the installation of the detection system into the clear aperture, the fiber out-coupled beam profiles were thoroughly analyzed. In the course of the experiments presented in the following sections, both power levels were coarsely matched by means of mutually varying coupling efficiency. Fine tuning with respect to the spectral responsivities of the detection system was effectuated by means of the rotatable polarizer, see Fig. 4.1.
4.2.1 Scanning experiments

In order to enable a comparison between the individual scanning experiments and to utilize optimally the translation range of the P-527.3CL, the fiber output was approximately centered on position 100 µm in the P-527.3CL coordinate system. We note that due to the dismantling and resetting between the experiments there is no absolute position. Shortly before each experiment the modulation frequency was accurately tuned to \( f_{\text{mod}} = 250 \) Hz. In each scanning experiment 1536 samples were acquired for a single P-527.3CL position according to 64 positions on the PSD. The mean value calculated from these 64 positions is equivalent to a time averaged value over \( t_{\text{int}} = 256 \) ms.

![Graph of position sensing homogeneity](image)

**Fig. 4.3:** Evaluation of the position sensing homogeneity of the UDT SPOT-2D. The travel range of the P-527.3CL amounts to 100 µm. The step width is 0.5 µm.

In Fig. 4.3 a typical example for the evaluation of the position sensing homogeneity within the gap of the UDT SPOT-2D is displayed. In this experiment the measurement range was equivalent to about 80 % of the gap. One observes the very smooth run of the characteristic curve. Throughout the complete measurement range no discontinuities occur. Besides a linear range centered approximately on position 100 µm, one observes a slightly s-shaped curve over the complete measurement range. At the edges of the gap range, due to the finite spotsizes, which are in good agreement with the values predicted in Fig. 4.2, the laser spots do not fall completely into the gap region. Consequently, the characteristic curves show a significantly different run at the gap to active element boundaries. Within the scope of the homogeneity tests more than 70 % (1.9 mm) of the overall gap region was analyzed. The run of the characteristic curve in Fig. 4.3 could be reproduced very successfully. It was demonstrated that the UDT SPOT-2D offers excellent homogeneity within the gap for the present wavelengths.

As a subsequent type of experiments the measurement range and the step width was decreased to demonstrate the position sensing accuracy within the center of the PSD. In Fig. 4.4...
the results of a scanning experiment with a travel range of 10 μm and a step width of 50 nm are visualized.

Fig. 4.4: Scanning experiment with travel range 10 μm. The step width is 50 nm. The solid lines denote the best linear fits.

In order to increase the readability of the graph due to the large number of observations, Fig. 4.5 shows a more detailed view of this scanning experiment. We note that Fig. 4.5 is based on the same data as Fig. 4.4.

Fig. 4.5: Detailed view in a range of 2 μm. The step width amounts to 50 nm. The solid lines are the best linear fits.
One observes in Fig. 4.4 and in Fig. 4.5 the excellent linearity of the detection system. Based on the calculation of the best linear fits we obtained in this example for the position sensing accuracy $\sigma = 4.6 \text{ nm @} 859.2 \text{ nm}$ and $\sigma = 6.1 \text{ nm @} 429.6 \text{ nm}$. Consequently, for the difference position sensing accuracy we obtained $\sigma = 7.6 \text{ nm}$. We note that these results are also based on averages of 64 positions per P-527.3CL position. Furthermore, it has to be stated that the achieved accuracy specification also includes the positioning error of the P-527.3CL. The deviation of the correlation coefficient $\rho$ from unity is $< 1 \times 10^{-5}$. Furthermore, the magnitudes of the residuals nowhere exceed a maximum value of 16 nm. Equivalent results could be obtained, even after dismantling and re-building the experimental set-up. For the difference position sensing accuracy calculated from the total number of scanning experiments, including scanning experiments with a step width of 10 nm and 50 nm, we obtained $\sigma = 7.3 \text{ nm}$.

In order to demonstrate the reproducibility of the position sensing we compare back and forth measurements executed in two subsequent experiments. In Fig. 4.6 the position differences for the back and forth measurements are plotted as a function of P-527.3CL positions. To obtain metric dimensions the dimensionless positions on the PSD were transformed according to the parameters of the best linear fits. One observes in Fig. 4.6 that the maximum magnitude of the position differences amounts to 10.5 nm @ 859.2 nm and 10.3 nm @ 429.6 nm. Due to the fact that the positions for both wavelengths show a very similar behavior the position differences between back and forth positioning are assumed to be caused, at least partly, by the nanopositioning system.

![Fig. 4.6: Comparison between back and forth measurements. Measurement range is 10 \(\mu\text{m}\). The step width amounts to 50 nm. The difference in recording time between the measurements at position 95 \(\mu\text{m}\) for both experiments was about 10 minutes.](image-url)

One observes in Fig. 4.4 the different slopes of the best linear fits for both wavelengths. These different slopes indicate the presence of wavelength dependent effects of the detection system, i.e. detector inherent dispersion. For a thorough analysis of the dispersion $g_{\text{PSD}}$ of the PSD we adopted the following model:
\begin{equation}
\bar{y}_i = g_{PSD} \cdot \bar{y}_i + \delta,
\end{equation}

wherein \( \delta \) takes small asymmetries of the PSD into account. We note that unlike stated in Eq. (4.2) we solely used dimensionless positions on the PSD in order to assure a separation of parameters. This model was experimentally verified and the unknown parameters were determined.

Fig. 4.7 shows an example of the determination of the parameters of the PSD dispersion model according to Fig. 4.4. For this experiment presented we obtained for the dispersion of the PSD \( g_{PSD} = 0.9216 \) with an accuracy of \( \sigma = 2.2 \times 10^{-4} \). For the error term due to the asymmetries of the PSD we found \( \delta = -3.423 \times 10^{-3} \) with an accuracy of \( \sigma = 7.49 \times 10^{-6} \). The value of \( \delta \) corresponds to 0.325 \( \mu \)m. Furthermore, measurements have shown that the value \( \delta \) shows excellent stability. We note that this error term due to the asymmetries of the PSD for the present wavelengths can be cancelled out by the standard measurement procedure in both telescope faces.

In order to analyze the reproducibility of the dispersion value of the detector we executed a large number of experiments under varying conditions, e.g. varying incident optical power on to the PSD, travel range, step width, etc. As a mean value we obtained for the dispersion of the PSD \( g_{PSD} = 0.9234 \) with an accuracy of \( \sigma = 9.7 \times 10^{-4} \).
4.2.2 Static experiments

Besides the scanning experiments static experiments were also executed. Aim of this analysis was to demonstrate the behavior of a continuous series of positions in a fixed position of the P-527.3CL approximately centered on the middle of the gap.

![Residuals @859.2 nm in a fixed P-527.3CL position.](image)

**Fig. 4.8:** Residuals @859.2 nm in a fixed P-527.3CL position.

![Residuals @429.6 nm in a fixed P-527.3CL position.](image)

**Fig. 4.9:** Residuals @429.6 nm in a fixed P-527.3CL position.

The data acquisition period was chosen to be 10 s. For the modulation and consequently, for the sampling frequency we used identical values as in the preceding experiments. Hence, each ex-
experiment consisted of 2500 positions computed for each wavelength. Each position was calculated in analog form to the scheme presented in Table 4.1 according to Eq. (4.1) and Eq. (4.2). From these 2500 positions a mean value was computed, wherein the residuals @859.2 nm are plotted in Fig. 4.8 and the residuals @429.6 nm are plotted in Fig. 4.9.

Both aforementioned figures show that within the data acquisition time no drifting occurs, i.e. both graphs are centered around the zero position. This observation is very important for the envisaged integration and averaging periods for the dispersometer measurements. Hence, it was assumed that the modification of the detection system might lead to a degraded stability behavior. However, unlike hitherto assumed very high stability could be demonstrated.

In order to describe the measurement precision from repetitive measurements the standard deviation for a single position was calculated to $\sigma = 11.3 \text{ nm} @859.2 \text{ nm}$ and to $\sigma = 11.7 \text{ nm} @429.6 \text{ nm}$. These values could be confirmed in a number of subsequent experiments. Furthermore, a statistics based data analysis showed that by extending averaging time the respective standard deviations can be significantly decreased. Consequently, the accuracy of the relative position difference sensing between the spots at both wavelengths can be improved by increasing integration time as far as solely the performance of the dispersometer system is concerned.

Another positive aspect derived from the static experiments deals with the reliability of the system performance. First, within the performance tests, whereas the static experiments and the forthcoming spectral analysis of the system noise floor span the largest data acquisition period, no drop-outs or blunders were observed. We note that in connection with the static experiments the residuals of a single position nowhere exceed the maximum absolute magnitudes of 42.2 nm @859.2 nm and 36.2 nm @429.6 nm as displayed in Fig. 4.8 and in Fig. 4.9.

4.2.3 Spectral analysis of the system noise floor

Unlike the experiments presented in the two preceding sections 4.2.1 and 4.2.2, wherein the levels of optical power incident to the PSD were solely mutually adjusted, the spectral analysis of the system noise floor was additionally based on well defined absolute power levels. We showed with Eq. (3.22) that the total amount of optical power incident to the detector depends on the square of the aperture diameter. Hence, it was possible to simulate the different power levels due to the various effective aperture diameters used in the experiments presented in chapter 5. We note that in order to simulate the actual amounts of optical power for the sight length $R$ and the various aperture diameters $d$ used within the dispersometer measurement, the experiments for the analysis of the system noise floor were made after the dispersometer measurements. In doing so it was assured that the power levels incident to the detector were in good agreement for the experiments evaluating the system noise floor and the dispersometer measurements. In fact, care was taken that the power levels for the experiments presented within this section were always slightly lower than for the dispersometer measurements in order to derive the worst case values.

Besides equivalent levels of optical power, we chose the same sampling scheme, i.e. number samples of a single sequence and the total number of sequences, as for the experiments presented in chapter 5. In doing so a sequence consisted of 73728 samples leading to 3072 positions calculated for each wavelength. The equivalent acquisition duration for one period was 12.288 s. The frequency range corresponding to this sampling scheme is 0.08 Hz to 125 Hz. The
The total number of sequences was 50 with approximately 10 s between two consecutive sequences. The complete description of this sampling and data acquisition scheme is given in section 5.2.

Fig. 4.10: System noise power spectra corresponding to dispersometer measurements for the experiment sight length $R = 17$ m and $d = 75$ mm (fully-opened aperture).

Fig. 4.11: System noise power spectra corresponding to dispersometer measurements for the experiment sight length $R = 17$ m and $d = 42$ mm.
Fig. 4.12: System noise power spectra corresponding to dispersometer measurements for the experiment sight length $R = 17$ m and $d = 30$ mm.

Fig. 4.13: System noise power spectra corresponding to dispersometer measurements for the experiment sight length $R = 17$ m and $d = 20$ mm.
For each experiment three power spectra were calculated. Hence, each figure shown above includes one power spectrum for each wavelength and additionally the power spectrum for the dispersion uncorrected difference between both wavelengths. In order to enable a comparison between the power spectra presented in this section and in section 5.3 all power spectra are plotted with the same dimensions.

In Fig. 4.10 to Fig. 4.13 the measured and calculated system noise power spectra are visualized. In all plots above the light gray line denotes the power spectrum for $\lambda_1 = 859.2$ nm, the dark gray line denotes the power spectrum for $\lambda_2 = 429.6$ nm. The black line is the difference power spectrum.

First, it can be stated that with the system noise power spectra given in Fig. 4.10 to Fig. 4.13 the validity of Eq. (3.23) and Eq. (3.24), respectively, could be thoroughly demonstrated. One observes that by decreasing the aperture diameter $d$ and consequently, by decreasing the total amount of power $\phi_{tot}$ incident to the PSD, the noise floor of the system increases. A second conclusion on the basis of the system noise spectra is that the overall shape of these spectra appears to be flat over the entire frequency band. However, at certain frequencies peaks occur in the displayed power spectra. Due to the very low noise floor these resonant frequencies can be observed best in Fig. 4.10. In all graphs the largest power contribution is at 100 Hz. The corresponding external noise source which induces these noise spikes was assumed to be related to the external power supply. Additionally, a second spike appears at 50 Hz, see Fig. 4.10 and Fig. 4.11, which is also caused by the external power supply. Even though special care was taken in order to suppress these external noise sources, see section 3.2.3, these residual contributions remain in the data. Fortunately, this external influence is very stable and clearly defined, so it can be easily eliminated on the basis of the present analysis. A third broader irregularity was solely observed in a single experiment simulating the fully-opened aperture, see Fig. 4.10. The corresponding frequency is at 3 Hz. This contribution might be induced externally as well, but its origin cannot be precisely evaluated.

As far as the mutual relation of the system noise floor is concerned one observes the very similar behavior at both wavelengths. Furthermore, with the exception of the full aperture experiment, both wavelengths show the same absolute amount of power in the entire frequency band, whereas the noise floor for the difference angle is significantly higher. This can be understood in terms of correlation. Due to the exclusion of the atmosphere both beams are substantially uncorrelated. Hence, the noise at both wavelengths propagates as well, leading to a higher noise floor for the difference power spectrum. As an exception in Fig. 4.10 the noise floor @429.6 nm is significantly higher than the noise floor @859.2 nm. A possible explanation can be given on the basis of the technological principle of the dual-wavelength laser. The higher noise floor can be caused either by a change in the number of longitudinal modes contributing to the frequency conversion process leading to a significantly lower amount of optical power, or by an increase in laser noise due to the malfunction of the temperature stabilization of the KNbO$_3$-crystal.
4.3 Discussion of results

It was the global aim of all experiments presented in this chapter to thoroughly evaluate the system performance of the complete dispersometer, the dispersion telescope excluded. In the course of these performance tests the excellent degree of reliability for the synchronization and the very good stability of the reference signal was demonstrated. These properties enable very robust demodulation of the dual-wavelength signal and consequently, the demodulation can be executed autonomously. This is beneficial for the signal processing with respect to the dispersometer measurements. As mentioned before, no drop-outs and blunders occurred within the course of these experiments. These errors would have been caused, either by irregularities within the synchronization and wavelength referencing process, or by interrupts within the generation of the dual-wavelength radiation. Consequently, as far as these important aspects are concerned very good performance could be demonstrated.

As reported above the stability of the experimental set-up was a critical issue in connection with the performance tests. It was demonstrated in all three types of experiments that due to the very compact and monolithic design sufficient stability was achieved. Furthermore, the nanopositioning system plays an important role in the experimental set-up. With the incorporation of the P-527.3CL it was possible to position the PSD with a smallest step width of 10 nm in the course of the scanning experiments. Hence, the position sensing capability of the dispersometer detection system could by thoroughly analyzed. Due to the layout of the experiments it was not possible to strictly separate the positioning accuracy of the P-527.3CL and the position sensing accuracy of the dispersometer detection system. Although the P-527.3CL offers excellent positioning accuracy and linearity, Fig. 4.6 indicates that the position sensing accuracy of the dispersometer detection system is even higher.

It was one aim of the scanning experiments to analyze the position sensing homogeneity within the gap of the UDT SPOT-2D. In the course of these experiments, which covered the complete gap region in measurement direction and 70 % of the overall gap region, excellent position sensing homogeneity was observed.

In order to evaluate the difference position sensing accuracy we analyzed spot positions at both wavelengths derived from scanning experiments. Based on the total number of scanning experiments with step widths of 10 nm and 50 nm we obtained for the difference position sensing accuracy $\sigma = 7.3$ nm which is slightly worse than the required difference position sensing accuracy of $\sigma = 7.2$ nm. However, we note that this value is too pessimistic because it includes the positioning error of the P-527.3CL. Furthermore, each position was solely calculated as a mean value from 64 positions. As demonstrated within the course of the static experiments, by extending averaging time standard deviation for a single position and consequently, the difference position sensing accuracy can be increased. Hence, the required difference position sensing accuracy can be achieved.

A very important aspect of the performance of the dispersometer detection system is the PSD inherent dispersion $g_{PSD}$. As already mentioned in section 2.3, this dispersion value does not equal unity. Based on the scanning measurements a PSD dispersion model was adopted and the PSD inherent dispersion was determined. Additionally, the results of these measurements could be successfully reproduced. The quantitative determination of the PSD inherent dispersion $g_{PSD}$ motivated the self-calibration procedure as presented in section 2.3.2. The existence of
the PSD inherent dispersion $g_{PSD}$ is closely related to the fabrication process of the PSD and the properties of the material in the gap region of the PSD. In order to electrically separate both electrodes either by doping or etching the required resistance will be introduced which changes the material properties. Wherein the lateral extension of the introduced resistance exhibits a depth dependence which also induces a depth dependence of the electric field. Together with the wavelength dependent absorption depth which is also material related a wavelength dependent effective gap width occurs. Although the PSD inherent dispersion $g_{PSD}$ was precisely determined, its actual value is immaterial for the dispersometer experiments presented in chapter 5, because herein the self-calibration procedure was applied.

The final series of experiments in this chapter was devoted to the spectral analysis of the system noise floor related to several different dispersometer measurements with varying aperture diameter. It was demonstrated that by decreasing optical power incident to the PSD the spectral noise floor increases in the entire frequency band. Besides influences related to the external power supply at 50 Hz and 100 Hz which can be clearly defined, the general shape of the power spectra appears to be flat. For the experiments simulating the smaller apertures one observes that the external influences are masked by the increasing system noise floor. With the exception of the fully-opened aperture experiment noise floors for both wavelengths are virtually identical. This indicates a very similar noise behavior of the detection system for a finely tuned dual-wavelength transmitter. Possible explanations for the discrepancy in the first experiment are given in section 4.2.3. In all experiments the noise floor of the difference angle is higher. As already mentioned in section 4.2.3 the reason therefore is given by the analysis of the correlation of the positions at both wavelengths. This was quantitatively confirmed by a statistical analysis.

In addition to the scanning and static experiments the knowledge of the atmospherically undisturbed system noise floor for different aperture diameters completes the characterization of the dispersometer performance and makes a differentiation between dispersometer-related and atmosphere-related effects possible.
5 Dispersometer measurements

The main emphasis of this chapter is on the demonstration of dispersometer measurements, i.e. herein the feasibility of applying the dual-wavelength method in combination with the self-calibration procedure, as presented in section 2.3.2, should be shown experimentally. After a brief description of the experimental set-up, see section 5.1, and a summary of the signal processing flow, see section 5.2, section 5.3 is devoted to the dispersometer measurements.

Besides dispersometer measurements for a single atmospheric condition, the question of dispersometer performance in differing atmospheric conditions is raised. Although the dispersometer experiments take place in a laboratory environment which is normally air conditioned, variations of the laboratory atmosphere were effectuated by the operation modes of the air conditioner itself. The appertaining experimental results will be shown in section 5.3.2.

Another important issue in connection with the dispersometer measurements is the dependence of the dual-wavelength method, and herein the turbulence compensation technique, on the effective aperture diameter. Because initially the dispersion telescope possesses an aperture diameter of $d = 75$ mm, which is larger than the aperture diameter of standard geodetic instruments, it was the aim of the further experiments to investigate the behavior of preferably smaller apertures. Consequently, section 5.3.3 is devoted to experiments with smaller aperture diameters. A discussion of the achieved results given in section 5.4 will close this chapter.

We note that the dispersometer measurements shown within this chapter are solely a fraction of the number of executed experiments. However, they possess a truly representative character. Due to the relative small amount of experiments that can be presented within this work, section 5.4 will be expanded for a discussion of the complete experimental data, e.g. in terms of reproducibility.

5.1 Experimental set-up

Analog to the requirements for the performance tests presented in the preceding chapter, one of the most important requirements was the stability of the instrumental set-up. Based on the theoretical framework of the self-calibration procedure given in section 2.3.2, we derived that the stability of the optical axis of the receiving dispersion telescope was the most critical issue. In order to guarantee maximum stability for the complete detection system, the telescope was fixed on an optical bench, whereby this optical bench was mounted to a granite table. For the initial alignment of the optical axis towards the dual-wavelength transmitter the dispersion telescope can be tilted similar as a telescope installed in a theodolite axis system, however in a very limited angular range. To prevent longitudinal spherical aberration within the alignment process the rotational axes intersect in the entrance pupil of the dispersion telescope.

In Fig. 5.1 the experimental set-up is displayed. One observes that again solely the bare fiber output characterizes the active dual-wavelength target. The sight length for the dispersometer measurements was $R = 17$ m which is slightly smaller than estimated in section 3.2 in order to assure a sufficiently large SNR even for the smallest aperture diameter. Based on the Gaussian beam propagation model adopted in section 3.1.5 the magnitudes of the spotsizes at
the position of the telescope aperture position were calculated to $2w = 2.55 \text{ m} @ 859.2 \text{ nm}$ and $2w = 3.29 \text{ m} @ 429.6 \text{ nm}$ using Eq. (3.5).

As one can see in Fig. 5.1 the fine adjustment between the dual-wavelength transmitter and the detection system is executed by means of the combined rotation and translation stage which offers five degrees of freedom. Consequently, in a mutual collimating process the dual-wavelength transmitter and the detection system were aligned. For the final and most accurate adjustment of the pointing of the optical axis of the dispersion telescope towards the fiber output, we minimized the spot positions on the PSD calculated according to Eq. (4.2).

![Fig. 5.1: Experimental set-up for the dispersometer measurements](image)

In the course of these experiments the detection system was mounted to the dispersion telescope wherein due to the accurately referenced $z$-position of the PSD, the chip surface was brought to coincidence with the specified position of the focal plane.

Preliminary to the experiments we maximized and mutually matched the power levels incident to the PSD. Therefore, we optimized the coupling efficiency for $429.6 \text{ nm}$ to the maximum value given in section 3.1.5. In order to match the power levels incident to the PSD fine tuning was effectuated by rotating the polarizer, see Fig. 5.1, while monitoring the signal levels of samples 1 to 4 and 9 to 12 for each period. After this adjustment the beam profiles were thoroughly checked using the Pulnix TM-6CN CCD-camera in combination with the Spiricon LBA-500PC laser beam analyzing system. We demonstrated very good agreement with the measurements presented in section 3.1.5.

### 5.2 Signal processing

In the course of the dispersometer measurements we used the same data acquisition strategy as reported in chapter 4. One can generally distinguish between scanning experiments and the actual dispersometer measurements. We used the scanning experiments initially for optimally focussing of the receiving optical system within the adjustment process and preliminary to each dispersometer measurement for the derivation of a parameter for the transformation of the di-
Dimensionless positions on the PSD to angular quantities. For the latter scanning experiments we reduced the travel range and decreased the step-width.

Dual-wavelength signal (analog) → A/D-Conversion

73,728 samples per sequence
50 sequences

Calculation of \( y_1(t), y_2(t) \)

3072 positions @859.2 nm per sequence
3072 positions @429.6 nm per sequence
3072 difference positions per sequence

\( f_{\text{scan}} = 250 \) Hz

Synchronization signal

\( f_{\text{scan}} = 6 \) kHz (phase-locked)

Calculation of \( y_1(t), y_2(t) \)

Mean, variance @859.2 nm per sequence
Mean, variance @429.6 nm per sequence
Mean, variance of difference per sequence

Self-calibration procedure

Combined dispersion \( \rho_{\text{comb}} \)

Transformation to angular dimension (rad)

Dispersion correction

\@859.2 nm \@429.6 nm

\@429.6 nm

\@859.2 nm

Calculation of:

Angle of arrival
Dispersed corrected angle of arrival
Dispersion corrected difference angle

\( C_{\alpha} \)

Spectral analysis

\( W_1(f) \)
\( W_2(f) \)
\( W_3(f) \)

Parametric description by best linear fit from 0.08 to 1.00 Hz
Calculation of correlation \( \rho_{\alpha} \) as a function of \( f \)
Parametric description by best linear fit from 0.08 to 1.00 Hz
Parametric description by best linear fit from 0.08 to 1.00 Hz

Frequency behavior of \( W_1(f) \)
Frequency behavior of \( W_2(f) \)
Frequency behavior of \( W_3(f) \)

Fig. 5.2: Processing flow for the dispersometer measurements
As already introduced in the course of the performance tests, we used for the modulation frequency \( f_{\text{mod}} = 250 \, \text{Hz} \) resulting in \( f_{\text{sample}} = 6 \, \text{kHz} \) in the strictly phase-locked mode, as visualized in Fig. 3.30, for both types of experiments. Additionally, by using the same trigger and reference signal modes we obtained the identical signal structure as schematized in Table 4.1.

For the scanning experiments we acquired 1536 samples at both channels resulting in 64 spot positions for both wavelengths, whereas for the actual dispersometer measurements we acquired 73728 samples at both channels per sequence leading to 3072 positions for both wavelengths. In each experiment of the actual dispersometer measurements we recorded 50 sequences. In order to introduce a temporal spacing between the single sequences we introduced a 10 s pause by software. In combination with the data and file management each sequence took 24 s. Consequently, the duration of a single experiment was 20 minutes.

In Fig. 5.2 the processing flow for the dispersometer measurements is visualized. Starting from the acquired digital signal, in a first step the dimensionless positions on the PSD for each wavelength and the difference between the dimensionless positions for both wavelengths were computed resulting in 3072 values for each wavelength and for the difference per sequence. In a subsequent step we computed the mean value and the variance for each sequence, whereas the variance for each wavelength and the variance for the difference are the required values for the self-calibration procedure according to Eq. (2.57). Consequently, by using the calculated dispersion value and the transformation parameter derived from the scanning experiments for each sequence either 3072 angles of arrival at \( \lambda_1 \), dispersion corrected angles of arrival at \( \lambda_2 \), i.e. scaled in relation to the angle of arrival at \( \lambda_1 \), and dispersion corrected difference angles are computed. The subsequent spectral analysis bases on this data, whereby each power spectrum, analog to the spectral analysis presented in section 4.2.3, contains a frequency range from 0.08 Hz to 125 Hz corresponding to the sampling scheme presented above. Hence, from each single experiment we obtain the power spectrum \( W_1(f) \) of the angle of arrival @859.2 nm, the power spectrum \( W_2(f) \) of the combined dispersion corrected angle of arrival @429.6 nm, and the power spectrum \( W_3(f) \) for the combined dispersion corrected difference angle. We note that a very similar processing scheme holds for the power spectrum \( W_{\delta}(f) \) of the detection system dispersion corrected difference angle, i.e. the dispersion angle. Unlike the scheme presented in Fig. 5.2 the dispersion angle is corrected by \( g_0 \). This power spectrum is decisive for the computation of the accuracy of the refraction angle \( \beta_1 \) as a function of the integration time. Also from this data we calculated the refractive index structure parameter \( C_n^2 \).

Based on the computed power spectra following subsequent calculations were performed. In order to analyze the frequency behavior of the measured and dispersion corrected magnitudes, either a parametric description by best linear fit for the frequency range from 0.08 Hz to 1 Hz was computed. For the analysis of the correlation \( \rho_{12} \), as introduced in section 2.3.3, we calculated the correlation \( \rho_{12} \) as a function of the frequency from \( W_1(f) \) and \( W_2(f) \). And finally, for the analysis of the variance as a function of the integration time a computation according to Eq. (2.64) in combination with Eq. (2.65) was executed. We note that in accordance to Eq. (2.66) for the variance of the refraction angle the numerical integration of the low-pass filtered power spectrum has to be multiplied by the square of the wavelength dependent constant given in Eq. (2.35).
5.3 Experimental results

A basic prerequisite for utilizing the gap-technology by using telescope optics is to focus both spots completely into the gap. Although position sensing using gap-technology is independent of the spotsize $2w_{gap}$ if $2w_{gap} \ll b$, as derived in section 3.2.2, the measurement range is significantly decreased for an ill-focussed receiving optical system. As addressed to in section 3.2.1, besides focusing, the dispersion telescope possesses the $\lambda$-tuning function. Opposed to this possibility, we demonstrated in section 4.2.1 that the PSD inherent dispersion $g_{PSD}$ exceeds significantly the possible tuning range. Furthermore, due to the application of the self-calibration procedure mechanical $\lambda$-tuning becomes immaterial. Consequently, in combination with the focussing process we adjusted and fixed the $\lambda$-tuning to a practical position.

In order to focus the dispersion telescope correctly we applied a PSD based approach using a knife-edge method as already introduced in section 4.1.1. In a series of scanning experiments we minimized the apparent spotsizes observed on the gap to active element boundaries. In Fig. 5.3 the results of a single scanning experiment are displayed. It was demonstrated experimentally that by moving the fiber output vertically instead of tilting the dispersion telescope an improved angular resolution can be obtained. Furthermore, in combination with the higher degree of stability a better angular reproducibility was obtained. Hence, from this vertical positioning an effective tilt angle can be calculated.

For the line scan displayed in Fig. 5.3 we used a travel range of 1.17 mrad and step width of 30 µrad. As one can observe in Fig. 5.3, within this travel range the complete gap region is included.

Fig. 5.3: Line scan over the complete gap region. Travel range is 1.17 mrad, step width is 30 µrad.

Due to the line scan over the entire gap region and parts of the active elements, Fig. 5.3 shows explicitly the underlying principle of gap-technology. One observes that for both wavelengths
the regions of the graph where the spot is on either the right or left active element is characterized by the almost flat run of the curve. This behavior in combination with the steepness of the curve within the gap shows that the spotsizes are much smaller than the gap width.

In an idealized case the absolute values of the positions on the active elements of the PSD using gap-technology, characterized by the ratio given in Eq. (4.2), should equal unity for the dimensionless computation. In Fig. 5.3 one observes a slight deviation from this ideal behavior. Furthermore, the absolute values for the dimensionless spot positions on the active elements @429.6 nm are closer to unity than the equivalent values @859.2 nm. This can be understood in terms of radiation penetrating into the PSD material. Hence, a certain portion of photon generated electrons will generate a current on the opposite channel. The difference in the deviation from unity for both wavelengths is based on the different absorption depths for both wavelengths. Due to the deeper penetration of the IR radiation, e.g. cf. [UDT, 1999], a comparatively larger number of photon generated electrons will be detected at the opposite channel. As reported in section 4.3, the different absorption depths also contribute to the PSD inherent dispersion. In combination with the dispersion of the optics the combined dispersion of the detection system becomes apparent within the scanning experiments. We note that due to the relatively high dispersion of the PSD, the experimentally determined order of magnitude for the combined dispersion of the detection system exceeds most significantly the tuning range of the dispersion telescope. In this section we do not address to the determination of the dispersion of the detection system due to the intrinsic disadvantages of this method, as discussed in section 2.3.1. We also note that the angle measurements in all experiments presented in this section were made in one telescope face, because of the special mounting of the dispersion telescope. However, by implementing the dispersometer into modern geodetic total stations measurements in both telescope faces are made possible which is postulated in section 2.3.2 and in section 4.2.1.

5.3.1 Air conditioned laboratory atmosphere

In this first series of experiments we used the fully-opened aperture with \( d = 15 \) mm in an air conditioned laboratory atmosphere. Due to the ventilation a certain amount of transverse wind speed was introduced. The mean room temperature was determined to \( T = 20.3 \) °C. The scheme visualized in Fig. 5.2 was used for the data acquisition and processing. We found for the refractive index structure parameter \( C_{n}^{2} = 4 \times 10^{-15} \text{ m}^{2/3} \) related to IR radiation @859.2 nm. This value characterizes a moderately turbulent atmosphere, mainly caused by the aforementioned ventilation. Such ambient conditions prevail for a number of industrial measurement tasks. We note that in accordance with the previous chapters all measurements and computations are related to the IR radiation. Based on the properties of the instrumental realization, as shown in chapter 3, this method was advantageous due to the superior properties of the dispersometer inherent technologies in the IR spectral region.

From the self-calibration procedure the combined dispersion \( g_{\text{comb}} \) was determined. We found for the combined dispersion for a complete day of experiments including 8 complete single experiments, i.e. either a number of \( -1.3 \times 10^{5} \) angles of arrival for both wavelengths and difference angles, \( g_{\text{comb}} = 0.8959 \). In the course of these experiments the complete detection system, after focussing, was solely touched in order to change the aperture diameter. The accuracy of the combined dispersion was determined taking all 8 single experiments into account, in-
including experiments with two different aperture diameters. The typical time gap between two experiments was 1 h. We found for \( \sigma_{\text{comb}} = 6.4 \times 10^{-4} \) for all experiments, taking solely experiments with an aperture diameter of \( d = 42 \) mm into the calculation we obtained an improved accuracy of \( \sigma_{\text{comb}} = 4.4 \times 10^{-4} \).

After the self-calibration procedure which was individually applied for each experiment the aforementioned power spectra \( W_1(f) \) of the angle of arrival \( @859.2 \text{ nm} \), \( W_2(f) \) of the dispersion corrected angle of arrival \( @429.6 \text{ nm} \), and \( W_3(f) \) of the dispersion corrected difference angle were calculated. In Fig. 5.4 a log-log plot of the aforementioned power spectra is displayed.

![Fig. 5.4: Power spectra of the observed and dispersion corrected angular quantities. The upper curves are the power spectra of the angle of arrival (IR and blue) and dispersion corrected (IR and blue) angle of arrival. The lower curve denotes the power spectrum of the dispersion corrected difference angle. The effective aperture of the dispersion telescope was \( d = 75 \) mm. The dashed lines are the respective best linear fits for a frequency range of 0.08 to 1 Hz.](image)

One observes in Fig. 5.4 that the power spectra of the apparent target observations, i.e. the power spectra of the angle of arrival for both wavelengths, are virtually indistinguishable when corrected for dispersion. As already discussed in section 4.2.3, the disturbances around 50 Hz are externally induced and are assumed to be caused by the external power supply. Unlike the power spectra of the system noise floor, virtually no peaks occur @100 Hz. This may be the case due to the different position of the detection system itself in the laboratory.

For the interpretation of the power spectra as shown in Fig. 5.4 the knowledge of the low-frequency behavior is substantial. Consequently, for the parametric description of the low-frequency part we calculated best linear fits in a frequency range of 0.08 to 1 Hz. The low-frequency part of the power spectra can be well approximated by:
\[
\begin{align*}
\log(W_1(f)) &= -1.69 \log(f) - 13.4 \\
\log(W_2(f)) &= -1.66 \log(f) - 13.3 \\
\log(W_3(f)) &= -0.87 \log(f) - 14.7
\end{align*}
\] (5.1)

One observes that the power spectrum of the dispersion corrected difference angle \(W_3(f)\) decreases not as fast with increasing frequency as the power spectra of the angle of arrival of both wavelengths \(W_1(f)\) and \(W_2(f)\). But, in contradiction to the predictions summarized in section 2.3.3, the power spectrum of the dispersion corrected difference angle \(W_3(f)\) still shows a considerable frequency dependence.

In addition to the spectral analysis the correlation \(\rho_{12}\) between the angles of arrival at both wavelengths can be used to compare the effectiveness of the present instrumental set-up, e.g. the aperture diameter, for certain ambient conditions. Furthermore, the correlation \(\rho_{12}\) characterizes the frequency range where the dual-wavelength method holds, as discussed in section 2.3.3.

![Fig. 5.5](image.png)

**Fig. 5.5:** Correlation \(\rho_{12}\) between the angles of arrival @859.2 nm and @429.6 nm as a function of frequency for \(d = 75\) mm.

In Fig. 5.5 one observes that the correlation \(\rho_{12}\) between the angles of arrival at both wavelengths decreases with increasing frequency. This frequency behavior characterizes the transition from the dominantly refractive part to the dominantly diffractive part of the appertaining power spectrum as described in section 2.3.3. At low frequencies one observes the very high correlation between the angles of arrival at both wavelengths due to refractive effects and deflection of both beams caused by the larger turbulent eddies. At high frequencies, corresponding to the influences of very small turbulent eddies, the angles of arrival at both wavelengths are substantially uncorrelated. As characterized by the energy cascade involved, the transition from the refractive part to the diffractive part of the power spectrum is smooth. As an aspect of the instrumental realization of the dispersometer, Fig. 5.5 also demonstrates the effectiveness of the
monolithic design, see chapter 3. One observes in Fig. 5.5 that the disturbances around 50 Hz are well correlated. That means both wavelengths are affected by the same influences.

In section 2.3.5 a method for the statistics based validation of the dual-wavelength method is given. Consequently, the variance of the refraction angle $\sigma_{\beta^2}$ can be calculated as a function of the integration time $t_m$. We note that this method involves the integration of the low-pass filtered power spectrum of the dispersion angle $W_{\Delta f}(f)$ according to Eq. (2.66).

![Graph showing the variance of the refraction angle $\sigma_{\beta^2}$ as a function of the integration time $t_m$ for $d = 75$ mm.]

In Fig. 5.6 the variance of refraction angle $\sigma_{\beta^2}$ as a function of the integration time $t_m$ is depicted. Accuracy is increasing by extending integration time. The accuracy of the refraction angle for single-face telescope observation amounts to $\sigma_{\beta} = 0.5 \mu$rad after an integration time of 12 s.

In the subsequent experiments we used an aperture diameter of $d = 42$ mm in an air conditioned laboratory atmosphere. The aim of these experiments was to analyze the dispersometer measurements with a set-up for the receiving optics equivalent to the dimensions of standard theodolite telescopes. The mean room temperature for the selected experiment was $T = 20.6^\circ$C. In executing this experiment directly after the first experiment presented in this section, we can imply very similar ambient conditions. This assumption is supported by the value found for the refractive index structure parameter $C_n^2 = 4 \times 10^{-15}$ m$^{-2/3}$ which is identical as in the first experiment. Hence, the ambient atmosphere prevailing can be characterized as moderately turbulent.
Fig. 5.7: Power spectra of the observed and dispersion corrected angular quantities. The upper curves are the power spectra of the angle of arrival (@859.2 nm, light gray line) and dispersion corrected (@429.6 nm, dark gray line) angle of arrival. The lower curve denotes the power spectrum of the dispersion corrected difference angle. The effective aperture of the dispersion telescope was $d = 42$ mm. The dashed lines are the respective best linear fits for a frequency range of 0.08 to 1 Hz.

Fig. 5.8: Correlation $p_{12}$ between the angles of arrival @859.2 nm and @429.6 nm as a function of frequency for $d = 42$ mm.
In Fig. 5.7 a log-log plot of the three characteristic power spectra is displayed. For the parametric description in a frequency range of 0.08 to 1 Hz these power spectra can be well approximated by:

\[
\begin{align*}
\log(W_1(f)) &= -1.22 \log(f) - 13.0 \\
\log(W_2(f)) &= -1.23 \log(f) - 13.0 \\
\log(W_3(f)) &= -0.12 \log(f) - 15.1
\end{align*}
\]  

(5.2)

One important conclusion on the basis of Fig. 5.7 in combination with the parametric description in the low-frequency regime is that with the aperture diameter reduced to \(d = 42\) mm the frequency dependence of the power spectrum of the dispersion corrected difference angle \(W_2(f)\) was significantly reduced. Because we observed at very similar ambient conditions for both selected experiments presented to this point, we conclude that the aperture diameter is a very sensitive parameter for the dispersometer performance.

Similar observations can be made in Fig. 5.8. The correlation \(\rho_{12}\) between the angles of arrival at both wavelengths is much higher in the low-frequency regime compared to the preceding experiment. However, correlation decreases faster for \(d = 42\) mm than for \(d = 75\) mm. Besides the influence of the turbulent eddies, for the aperture diameter \(d = 42\) mm the system noise floor will be approached earlier, see section 4.2.3, which is substantially uncorrelated. However, the benefits gained by using the smaller aperture diameter will be preserved.

Moreover, the variance of the refraction angle as a function of the integration time is decisive for evaluating dispersometer performance.

![Fig. 5.9: Variance of the refraction angle \(\sigma_{\beta_1}^2\) as a function of the integration time \(t_{\text{int}}\) for \(d = 42\) mm.](image-url)
In Fig. 5.9 the variance of refraction angle $\sigma_{\beta}^2$ as a function of the integration time $t_{int}$ is depicted. Accuracy increases much faster by extending integration time than for the $d = 75$ mm case. After an integration time of 12 s the accuracy of the refraction angle for single-face telescope observation amounts to $\sigma_{\beta} = 0.3 \mu$rad.

In order to highlight the influence of the smaller aperture on the accuracy when time averaging is involved, we compare the temporal behavior of the accuracy of the dual-wavelength method with the temporal behavior of the precision of the uncorrected angles of arrival. Therefore, in Fig. 5.10 the variance of the angle of arrival @859.2 nm is plotted as a function of the integration time $t_{int}$.

![Graph showing variance of angle of arrival as a function of integration time](image)

**Fig. 5.10:** Variance of the angle of arrival @859.2 nm as a function of the integration time $t_{int}$ for $d = 42$ mm.

We note that Fig. 5.10 does not consider any systematic errors due to atmosphere-related influence on the uncorrected angle of arrival fluctuations @859.2 nm. Consequently, the lower values of the variances in Fig. 5.10 are immaterial. Comparing Fig. 5.9 and Fig. 5.10 one observes that the accuracy of the refraction angle increases much faster than the precision of the angle of arrival for the present aperture diameter and for the present ambient conditions.

### 5.3.2 Variation in laboratory atmosphere conditions

In addition to the experiments in very similar laboratory atmosphere conditions, wherein the aperture of the receiving optics was changed, experiments for different atmospheric conditions within the laboratory were envisaged. The most efficient way to effectuate this kind of variation was to shut down the complete air conditioner. Consequently, for the selected experiment pre-
presented in this section we assume a stratified atmosphere. Furthermore, the influence of the air circulation by the ventilation was eliminated, i.e. the wind speed was minimized. The mean room temperature for the selected experiment was \( T = 20.9 \, ^\circ\text{C} \). The refractive index structure parameter was determined to \( C_n^2 = 4 \cdot 10^{-16} \, \text{m}^{2/3} \) which is more than a magnitude smaller than for the experiments presented in the preceding section. Hence, the ambient atmosphere can be characterized as very weakly turbulent. We note that such ambient conditions are prevalent in closed, non-air-conditioned corridors which also can be found in connection with industrial measurement tasks.

Again, in Fig. 5.11 a log-log plot of the three characteristic power spectra is displayed.

Fig. 5.11: Power spectra of the observed and dispersion corrected angular quantities. The upper curves are the power spectra of the angle of arrival (@859.2 nm, light gray line) and dispersion corrected (@429.6 nm, dark gray line) angle of arrival. The lower curve denotes the power spectrum of the dispersion corrected difference angle. The effective aperture of the dispersion telescope was \( d = 42 \, \text{mm} \). The dashed lines are the respective best linear fits for a frequency range of 0.08 to 1 Hz.

For the parametric description in a frequency range of 0.08 to 1 Hz the power spectra displayed in Fig. 5.11 can be well approximated by:

\[
\begin{align*}
\log(W_1(f)) &= -2.05 \log(f) - 15.2 \\
\log(W_2(f)) &= -2.04 \log(f) - 15.1 \\
\log(W_3(f)) &= -0.17 \log(f) - 15.2
\end{align*}
\]  

(5.3)

From Fig. 5.11 and the appertaining parametric descriptions of the low-frequency range one observes the sensitivity of the power spectra of the angles of arrival for both wavelengths to-
wards the turbulence strength. In this very weakly turbulent regime, $C_n^2 = 4 \cdot 10^{-16} \text{ m}^{2/3}$, the power spectra of the angles of arrival for both wavelengths $W_1(f)$ and $W_2(f)$ decrease faster than $1/f^2$ with increasing frequency. In connection with the analysis of the measurement precision as the function of the integration time we will show that this frequency behavior is very detrimental. Moreover, one recognizes that for the power spectrum of the dispersion corrected difference angle the results of the second experiment presented in section 5.3.1, wherein also the aperture diameter of $d = 42 \text{ mm}$ was used, could be very well reproduced. As far as these ambient conditions are concerned which are realistic cases for indoor measurements the dual-wavelength method is to a high degree insensitive to the presently prevailing turbulent conditions.

Analyzing the correlation $\rho_{12}$ between the angles of arrival at both wavelengths one observes in Fig. 5.12 that for this very weakly turbulent case the correlation is significantly lower compared with the second experiment presented in section 5.3.1. This is mainly caused by the very weakly turbulent regime prevailing for this experiment. As predicted in section 2.3.3 for lower transverse wind speeds which are effectuated by the shut down of the air conditioner the entire power spectrum of the angle of arrival is shifted to the lower frequency regime. Within the theoretical framework presented in section 2.3.3, the significantly higher frequency dependence of the power spectra of the angles of arrival for both wavelengths $W_1(f)$ and $W_2(f)$ in Fig. 5.11 can be explained. Consequently, the correlation $\rho_{12}$ between the angles of arrival at both wavelengths is also shifted to the lower frequency part leading to an overall lower correlation as displayed in Fig. 5.12 above.

In order to demonstrate the efficiency of the dual-wavelength method when time averaging is involved in Fig. 5.13 the variance of the refraction angle and the variance of the angle of arrival @859.2 nm is plotted as a function of the integration time. We note that the variance of
the refraction angle is characterized by the solid line, whereas the dashed line denotes the variance of the uncorrected pointing towards the apparent target positions @859.2 nm.

One observes in Fig. 5.13 the variances decrease with increasing integration time. However, both variances improve very differently. Whereas the accuracy of the refraction angle improves with the square root of the integration time \((t_{\text{int}})^{1/2}\), the precision of the uncorrected apparent target direction improves only with \((t_{\text{int}})^{1/20}\). The latter behavior is catastrophic for the conventional, i.e. single-wavelength, method. This implies that even by extending integration time, the pointing precision cannot be significantly improved. On the other hand Fig. 5.13 shows that time averaging leads to a significant improvement of accuracy for the dual-wavelength method.

![Fig. 5.13: Variance of the refraction angle and variance of the angle of arrival @859.2 nm as a function of the integration time](image)

Although the accuracy of the refraction angle is initially worse than the precision of the apparent target direction, due to the faster improvement of \(\sigma_{\text{int}}^2\) with increasing integration time there is a break-even point approximately at \(t_{\text{int}} = 10\) s. After an integration time of 12 s the accuracy of the refraction angle for single-face telescope observation amounts to \(\sigma_{\text{int}} = 0.1\) μrad.

### 5.3.3 Variation of the aperture diameter

Based on the experience gained in the course of the experiments presented in section 5.3.1 and due to the investigations by Gächter and Huiser [1987b], we recognized that the effective aperture diameter of the dispersion telescope is a very sensitive parameter. Motivated by the improvement of the experimental results as far as the dual-wavelength method was concerned by
using a smaller aperture diameter, the experiments presented in this chapter focus on the question of system performance for even smaller aperture diameters.

Consequently, in the first experiments presented in this section we used an aperture diameter of \( d = 30 \) mm in an air conditioned laboratory atmosphere. Due to the operating air conditioner we assume very similar ambient conditions as prevailing for the experiments presented in section 5.3.1. The measured mean room temperature for this experiment was \( T = 20.7 \) °C. Due to the operating air conditioner we assume similar ambient conditions including a refractive index structure parameter \( C_n^2 = 6 \cdot 10^{-15} \text{ m}^{-2/3} \) which is slightly larger than for the experiments shown in section 5.3.1. Hence, the ambient atmosphere can also be characterized as moderately turbulent. Prior to this experiment we executed the same experiments as shown in the selection of experiments presented in section 5.3.1. Herein, we demonstrated very good reproducibility, although the time elapsed between these two experimental series was 2.5 months.

![Power spectra graph](image)

**Fig. 5.14:** Power spectra of the observed and dispersion corrected angular quantities. The upper curves are the power spectra of the angle of arrival (\( @859.2 \) nm, light gray line) and dispersion corrected (\( @429.6 \) nm, dark gray line) angle of arrival. The lower curve denotes the power spectrum of the dispersion corrected difference angle. The effective aperture of the dispersion telescope was \( d = 30 \) mm. The dashed lines are the respective best linear fits for a frequency range of 0.08 to 1 Hz.

Analog to the preceding experiments the power spectra displayed in Fig. 5.14 can be well approximated in a frequency range of 0.08 to 1 Hz by:

\[
\log(W_1(f)) = -1.29 \log(f) - 12.8 \\
\log(W_2(f)) = -1.29 \log(f) - 12.8 \\
\log(W_3(f)) = -0.01 \log(f) - 14.9
\]
The most important conclusion derived from Fig. 5.14 is that the power spectrum of the dispersion corrected difference angle $W(f)$ appears to be white as predicted by the theory presented in section 2.3.3. We note that the same approximation also holds for the entire frequency range. Hence, within the ambient conditions given, we found an instrumental set-up, wherein the dispersometer is insensitive towards the impact of optical turbulence. The frequency behavior of the apparent target direction at both wavelengths is comparable to the second experiment presented in section 5.3.1.

![Graph](image)

**Fig. 5.15:** Correlation $\rho_{12}$ between the angles of arrival @859.2 nm and @429.6 nm as a function of frequency for $d = 30$ mm.

Furthermore, as far as the correlation $\rho_{12}$ between the angles of arrival at both wavelengths is regarded, as visualized in Fig. 5.15, the highest correlation so far for the low-frequency regime results for this presently used aperture diameter. The decrease of the correlation with increasing frequency is very similar to the second experiments in section 5.3.1, using a 42 mm aperture and similar ambient conditions.

In Fig. 5.16 the variance of the refraction angle and the variance of the angle of arrival @859.2 nm is plotted as a function of the integration time. As already observed in the previous sections, both variances improve differently with increasing integration time. As also demonstrated with the previous experiment, the accuracy of the refraction angle improves with the square root of the integration time $(t_{int})^{1/2}$. This time, the precision of the uncorrected apparent target direction improves with $(t_{int})^{1/20}$ for an integration time of up to $t_{int} = 0.4$ s. By further extending integration time the uncorrected pointing precision improves with $(t_{int})^{1/2}$.

Furthermore, one observes in Fig. 5.16 as well that the variance of the refraction angle starts initially worse than the uncorrected pointing precision, but due to the faster improvement of $\sigma_{\theta}^2$ with increasing integration time the break-even point is reached at $t_{int} = 7$ s. After an integration time of 12 s the accuracy of the refraction angle for single-face telescope observation
amounts to $\sigma_{\beta} = 0.2 \mu\text{rad}$. Taking the measured turbulence strength into account, this experiment has given the best results.

![Graph showing variance of refraction angle and angle of arrival](image)

**Fig. 5.16:** Variance of the refraction angle (solid line) and variance of the angle of arrival $\theta_{859.2}$ nm (dashed line) as a function of the integration time $t_{in}$ for $d = 30$ mm

The experimental results presented in section 5.3.1 and in this section seem to imply that the performance of the dispersometer can be improved by decreasing the effective aperture diameter. However, in opposition to this dependence found for the previously shown results, we note, as shown in section 3.2.5, that the received amount of optical power depends on the square of the aperture diameter and, as addressed to in section 3.2.6, the NEA depends on the amount of received optical power. That means by taking radiometric and noise considerations into account, the size of the optimal aperture diameter is also determined by the system noise floor. In order to experimentally analyze the system performance using a very small aperture diameter, we made dispersometer measurements with $d = 20$ mm. We executed the experiments presented in this section immediately successively, so that we imply that the same laboratory atmosphere prevails for both experiments.

In Fig. 5.17 the characteristic power spectra resulting from this experiment are displayed. Analog to the preceding experiments within this chapter, the displayed power spectra are parametrically described in a frequency range of 0.08 to 1 Hz:

\[
\begin{align*}
\log(W_{1}(f)) &= -1.14\log(f) - 12.7 \\
\log(W_{2}(f)) &= -1.13\log(f) - 12.7 \\
\log(W_{3}(f)) &= -0.13\log(f) - 14.1
\end{align*}
\]
Fig. 5.17: Upper curves are the power spectra of the angle of arrival (@859.2 nm, light gray line) and dispersion corrected (@429.6 nm, dark gray line) angle of arrival. Lower curve denotes the power spectrum of the dispersion corrected difference angle. The effective aperture of the dispersion telescope was \( d = 20 \) mm. The dashed lines are the respective best linear fits for a frequency range of 0.08 to 1 Hz.

For the correlation \( \rho_{12} \) between the angles of arrival at both wavelengths one finds:

Fig. 5.18: Correlation \( \rho_{12} \) between the angles of arrival @859.2 nm and @429.6 nm as a function of frequency for \( d = 20 \) mm.
By comparing the results of this experiment, see Fig. 5.17 and Fig. 5.18, with the results of the preceding experiment in this section, see Fig. 5.14 and Fig. 5.15, one observes deteriorated results. As previously predicted, the system noise seems to be no longer negligible with respect to the impact of optical turbulence. Due to the significantly higher system noise floor, see Fig. 4.13, this noise floor will be earlier approached, as one can see in Fig. 5.17. However, the general shape of the dispersion corrected difference angle power spectrum for low frequencies is preserved.

As far as the correlation $\rho_{12}$ between the angles of arrival at both wavelengths is concerned, one observes a lower correlation at lower frequencies in Fig. 5.18 compared to Fig. 5.15. Here, the system noise might have a detrimental influence on the dispersometer performance as well. Due to the lower SNR for the 20 mm aperture diameter, the angles of arrival at both wavelengths will be partly de-correlated. The impact of the lower SNR also applies for the accuracy of the refraction angle as a function of the integration time $t_{int}$. Although accuracy improves with extending integration time similarly to the 30 mm case, after an integration time of 12 s the accuracy of the refraction angle for single-face telescope observation is $\sigma_\theta = 0.3 \mu$rad.

### 5.4 Discussion of results

It was the global aim of this chapter to demonstrate that the dual-wavelength method in combination with the dispersometer realized within the frame of this thesis works. In a large number of experiments, wherein this chapter contains only a few characteristic experiments, we demonstrated that applying the dual-wavelengths is feasible and furthermore, by using the realized dispersometer certain benefits in contradiction to the conventional, i.e. single-wavelength, measurements can be gained. Additionally, as derived from the various experiments, the instrumental set-up of the dispersometer was modified in order to optimally apply the dual-wavelength method.

Preliminary to the dispersometer measurements we showed that the receiving optics of the detection system can be focussed by utilizing the gap to active element transitions in a knife-edge method. Furthermore, it was experimentally demonstrated that by vertically moving the fiber output instead of tilting the complete detection system, a very accurate transformation of the dimensionless positions on the PSD to angular dimensions can be achieved. Herein, the maximum uncertainty in scaling is 2%. We note that by extending the integration time within the scanning experiments an improved accuracy is expected. A further advantage of this method was that due to the fixed mounting of the detection system, the very high stability of the optical axis of the receiving optics required for the self-calibration method could be easily obtained.

In the course of the dispersometer measurements we executed more than 26 individual experiments resulting in a total of 8.7 h of data. This data was processed in accordance with Fig. 5.2. Each series of dispersometer measurements using a specific experimental set-up, i.e. either the variation of the ambient atmosphere conditions or the variation of the aperture diameter, was reproduced on at least two different days. We obtained very good reproducibility of the experimental results. Vice versa, based on the shapes of the resulting power spectra the initial experimental set-up could be concluded.

The same high degree of reproducibility was obtained in connection with the applied self-calibration method. Herein, we obtained for the accuracy of the combined dispersion, e.g. for
one focus position including 8 complete individual experiments throughout a complete day, \( \sigma_{\text{comb}} = 6.4 \times 10^{-4} \). This accuracy implies, e.g. for a refraction angle in the order of 10 \( \mu \text{rad} \) an uncertainty of < 4 \%e, whereby the receiving telescope points approximately towards the apparent direction. By taking solely experiments with an aperture diameter of \( d = 42 \text{ mm} \) into the calculation, we obtained an improved accuracy of \( \sigma_{\text{comb}} = 4.4 \times 10^{-4} \) for the same span of time. Herein, one observes that the smaller aperture, due to the higher correlation between the angles of arrival at both wavelengths, gives better results. Similar results were obtained for the other experiments using different experimental settings. However, all determined quantities are well in agreement with dispersion values predicted on the basis of individual dispersion measurements. Consequently, the self-calibration procedure by utilizing optical turbulence provides a very elegant method for the correction of the dispersion of the detection system. We note that due to the weakly and moderately turbulent conditions, which prevail in a laboratory environment, the accuracy potential of the self-calibration method might be underestimated. We expect that higher turbulence strengths lead to even better results.

We also confirmed experimentally that the power spectrum of the dispersion corrected difference angle becomes white by using an optimal aperture diameter as shown in section 5.3.3, whereas the power spectra of the angles of arrival at both wavelengths exhibit a large frequency dependence. The characteristics of the dispersion corrected difference angle power spectrum show that for the present instrumental set-up the dual-wavelength method is rather insensitive towards the influences of optical turbulence. Furthermore, due to the equivalent characteristics of the power spectrum of the dispersion angle \( W_\alpha(f) \), the accuracy of the refraction angle improves faster than the precision of the angles of arrival at both wavelengths. Experiments showed that by using smaller aperture diameters the accuracy of the refraction angle improves with the square root of the integration time \( (t_{\text{int}})^{1/2} \).

As far as the laboratory atmosphere is concerned, we performed dispersometer measurements in basically two different ambient conditions which can be regarded as typical for industrial measurements indoors. In both cases the application of the dual-wavelength method was successful. Derived from the experimental results for the higher turbulence strength prevailing, the characteristics of the power spectrum of the dispersion corrected difference angle indicate that applying the dual-wavelength method in outdoor field experiments is potentially promising.

A detailed study of the performance of the dispersometer for different aperture diameters was addressed to in the course of the dispersometer measurements. It was confirmed that by using smaller apertures improved results can be obtained. This is well in accordance with the theoretical predictions and experimental investigations by [Gächter and Huiser, 1987b; Huiser and Gächter, 1989; Chumside et al., 1989]. For the present instrumental layout and under the prevailing ambient conditions the optimal aperture diameter was determined to \( d = 30 \text{ mm} \). We demonstrated experimentally that for this aperture diameter in a moderately turbulent laboratory atmosphere the accuracy of the refraction angle for single-face telescope observation amounts to \( \sigma_\beta = 0.2 \mu \text{rad} \) after an integration time of 12 s. We note that by increasing SNR, e.g. by reducing the divergence angles for both wavelengths at the dual-wavelength transmitter, the optimal aperture diameter can even be smaller than specified above. This confirms the presumption that the dispersometer can be implemented into modern geodetic total stations.
6 Summary and conclusions

6.1 Development of the dispersometer

Within this work we demonstrated the feasibility of realizing a dispersometer which is capable of utilizing atmospheric dispersion for the correction of atmosphere-related influences, in order to obtain unprecedented accuracy in optical direction and angle measurements. The major challenges for realizing the dispersometer were the generation of coaxial emission of monomode radiation at two spectrally optimized wavelengths and the development of the high-accuracy detection system. The development of the dispersometer was principally enabled by focusing on three key technologies: dual-wavelength generation by frequency conversion, optical fiber technology, and gap-technology. Within this work detailed studies of these three key technologies were performed leading to an optimized implementation of the dual-wavelength laser, the introduction of a novel technique for achieving coaxial monomode propagation at two spectrally wide-separated wavelengths by one single-mode fiber and optical position sensing accuracy of a few nanometers.

Based on a thorough evaluation of laser sources emitting in the deep blue spectral range we demonstrated that a dual-wavelength laser by frequency conversion, i.e. second harmonic generation and sum frequency generation, is clearly suited for the implementation in the dual-wavelength transmitter. The presented compact laser source intrinsically provides several advantages for the overall conception of the dual-wavelength transmitter. Due to the underlying principle of this laser we simultaneously obtain radiation at two accurately specified wavelengths with very narrow and stable spectral linewidths. Furthermore, the low second harmonic power noise level was experimentally demonstrated. Based on noise model calculations we predicted that the influence of the laser inherent second harmonic noise which appears to be white on the position sensing is negligible. An intrinsic disadvantage of the dual-wavelength laser is that direct individual modulation of the radiation at both wavelengths individually is not possible. As a consequence, we developed an external modulator. We demonstrated temporal intensity modulation with $f_{\text{mod}} = 250 \text{ Hz}$ for each wavelength with mutual suppression of the alternate wavelength by more than 50 dB. Furthermore, by extracting either an optical and electronic signal at the modulator, strictly phase-locked and robust wavelength referenced data acquisition was made possible. It was demonstrated experimentally that this also applies if jittering of the modulation frequency occurs.

Due to the very high accuracy requirements in difference position sensing induced by the smallness of the measurable quantities, coaxial monomode propagation at both wavelengths, i.e. generation of either a Gaussian $\text{TEM}_{0,0}$ beam of both wavelengths, was postulated. A novel technique based on the application of optical fiber technology was established within this work, wherein this novel technique was described by the ray-optics approach within a waveguide. Measured beam profiles were found to agree excellently with Gaussian $\text{TEM}_{0,0}$ beam profiles expected from theory. Correlation between measurement and theory was found to be significantly larger than 0.9. Additionally, the measured magnitudes of the mode field diameters were well in agreement with the magnitudes predicted by the theory of cylindrical waveguides for monomode guidance. Beyond this, we adopted a propagation model, wherein the decisive
model parameter was determined by beam profile measurements as well. Based on this model, we predicted the quantities required for the design of the detection system and for the various experimental set-ups. Due to the application of optical fiber technology it is now possible to couple both beams into one optical channel of a modern geodetic total station. A first proposal was presented within the scope of this dissertation.

In contradiction to a large number of dispersometer approaches which focussed on the enlargement of the focal length of the receiving telescope in order to increase the position resolution capability, it was verified within this work that besides a sufficient signal-to-noise ratio, the ratio of the position sensing range of the detector and the focal length is decisive. In order to achieve optical position sensing with the accuracy of a few nanometers by using a short-focal-length receiving telescope, we applied the so-called gap-technology utilizing special segmented position sensitive detectors. This technology offers intrinsically a very small position sensing range. A detailed theoretical study on gap-technology showed the underlying principle. In combination with the thorough experimental investigations on gap-technology, as summarized below, this thesis contains a complete treatment addressed to this technology. For the performance of the dispersometer detection system, the analog electronics plays a crucial role. Consequently, a very low-noise high-gain amplifier was required. We demonstrated that the analog electronics works very close to the theoretical limits. A detailed theoretical and experimental analysis of the noise of the detection system showed the excellent performance which was summarized using the characteristic noise equivalent displacement as a figure of merit. For the present detection system we determined \( \text{CNED} < 30 \times 10^{-9} \text{nm-W/Hz}^{1/2} \). In connection with the digital electronics special emphasis was on the data acquisition timing scheme in order to guarantee the aforementioned strictly phase-locked and robust wavelength referenced sampling of the dual-wavelength signal.

As a prerequisite for the interpretation of the dispersometer measurements, an experimental investigation was devoted to the actual performance of the dispersometer itself. Because the basic potential of the difference position sensing using gap-technology had to be evaluated, the development of a special experimental set-up based on a nanopositioning system was required. As far as position sensing homogeneity within the gap of the selected position sensitive detector is concerned, measurements within an area of 70% of the overall gap region showed excellent position sensing homogeneity. We also observed excellent linearity of position sensing, indicated by a deviation of the correlation from unity which amounted to \( < 1 \times 10^{-5} \). Furthermore, we obtained for the difference position sensing accuracy \( \sigma = 7.3 \text{ nm} \) using an integration time of only \( \tau_{\text{int}} = 256 \text{ ms} \). Because of the short integration time and the positioning error of the nanopositioning system included, this result implies an actually higher accuracy, especially for longer integration times. One very important aspect of the performance of the detection system that was addressed in detail is the inherent dispersion of the position sensitive detector. In order to quantitatively determine the position sensitive detector inherent dispersion, an appropriate dispersion model was adopted. Motivated by the existence of the position sensitive detector inherent dispersion we presented a self-calibration procedure which corrects the dispersion of the complete detection system, as discussed below. A final series of experiments was devoted to the atmospherically undisturbed system noise floor for various virtual aperture diameters. Consequently, it is now possible to differentiate between dispersometer-related and atmosphere-related effects.
6.2 Dual-wavelength method

As a prerequisite for the theoretical framework whereupon the dual-wavelength method is based, we provided the mathematical descriptions of atmosphere-related effects. Because atmosphere-related effects arise in a large span of time scales, we treated both, systematic deviations caused by a quasi-stationary refractive index gradient environment, generally referred to as refraction in geodetic context, and stochastic deviations resulting from optical turbulence. For the latter source of influences we outlined the derivation of the related optical phenomena, namely, angle of arrival fluctuations and turbulence induced beam spreading.

A detailed theoretical study was devoted to the dual-wavelength method itself. On the basis of the experimental investigations addressed to the position sensitive detector inherent dispersion, we concluded that one basic prerequisite for the application of the dual-wavelength method, assuming a dispersion-free or precisely dispersion-tuned detection system, does not apply. Furthermore, we investigated that dispersion measurements for calibration on the basis of scanning procedures do not deliver satisfying results. As a solution we presented a self-calibration procedure by utilizing the impact of optical turbulence. We showed both theoretically and experimentally that by executing the self-calibration procedure, dispersometer measurements are rigorously corrected for the dispersion of the detection system. This self-calibration procedure, which has not been reported in literature so far, possesses the decisive advantages that it obviates the need of additional measurements and the dispersion correction can be computed and applied in real time. Furthermore, the important aspect of the impact of optical turbulence on the dispersometer measurements was addressed in detail. Herein, we discussed the influence of the dominantly refractive and the dominantly diffractive part of a model angle of arrival power spectrum on the dual-wavelength method. Theoretical calculations underlined the major challenges in the instrumental design of the dispersometer. Additionally, we presented a method for a statistics based validation of the dispersometer measurements.

A substantial part of this thesis focussed on actual dispersometer measurements. A large number of experiments was executed for a sight length of 17 m in a laboratory environment. In this laboratory environment we simulated two basic atmospheric conditions which are typical for industrial measurement tasks indoors. Additionally, a detailed study of the influence of the effective aperture diameter on the dispersometer measurements was performed.

Dispersometer measurements were found to agree very well with the theoretically predicted performance. We confirmed experimentally that the power spectrum of the dispersion corrected difference angle becomes white by using an optimal aperture diameter, whereas the power spectra of the apparent, i.e. non-refraction-corrected, target directions at both wavelengths exhibit a large frequency dependence. Based on these characteristics we conclude that the dual-wavelength method is rather insensitive towards the influences of optical turbulence. This was demonstrated for very weakly turbulent and for moderately turbulent ambient conditions. Derived from the correlation of the angles of arrival at both wavelengths, we experimentally confirmed the transition from the dominantly refractive to the dominantly diffractive regime of the calculated power spectra. We also investigated experimentally that the aperture diameter of the receiving optics is a key parameter for the dispersometer performance. The optimal aperture diameter for the present instrumental layout and the prevailing ambient conditions was 30 mm. For theodolite-like and smaller apertures it was demonstrated that the accu-
racy of the refraction angle improves with the square root of the integration time. Opposed to this characteristic, the precision of the apparent target directions at both wavelengths improves significantly slower. Moreover, this latter behavior is strongly dependent on the ambient conditions prevailing. There are certain ambient conditions, wherein the precision of the apparent target direction cannot be improved by reasonably extending integration time. However, this does not affect the dual-wavelength method. In a moderately turbulent atmosphere the accuracy of the refraction angle for single-face telescope observation was found to be 0.2 μrad (0.01 mgon) after an integration time of 12 s.

### 6.3 Conclusions and outlook

Summarizing the theoretical investigations, the key technologies involved in the instrumental development, and the experimental results presented in this dissertation, it can be concluded that the realized dispersometer in combination with a theodolite is capable of refraction corrected angle measurements, the influences of optical turbulence notwithstanding. It was demonstrated that the dual-wavelength method embodies a purely metrological solution for correcting atmosphere-related influences on high-accuracy direction and angle measurements.

We have shown that with the presented self-calibration procedure online correction for the dispersion of the detection system is possible. Unlike hitherto assumed, this calibration method uses optical turbulence which can be understood in terms of optical turbulence acting as a stochastic carrier for angular dispersion. We suggest that instead of the special dispersion telescope the same type of an achromatic well-corrected telescope as presently used in modern geodetic instruments can be implemented. We also confirmed that small apertures give better results. The correlation of the angles of arrival at both wavelengths is a decisive indicator for the efficiency of the dual-wavelength method. By using smaller apertures, smaller parts of the incoming wavefronts will be imaged on the position sensitive detector. As far as refractive index fluctuations are concerned, the wavefront tilts at both wavelengths are higher correlated within a smaller range characterized by the smaller aperture. Furthermore, aperture averaging at both wavelengths individually which might lead to a de-correlation is reduced by using smaller apertures. In addition to this behavior, we have shown that the signal-to-noise ratio of the dispersometer is also a decisive parameter. It might be the case that by increasing the signal-to-noise ratio even smaller apertures, as suggested above, lead to optimum performance. The application of optical fiber technology and the envisaged implementation of a standard telescope confirm the presumption that the dispersometer can be implemented into modern geodetic total stations. This implementation in combination with the standard measurement procedure in both telescope faces in order to eliminate residual errors is one decisive prerequisite for the determination of the refraction angles and the refraction corrected angles. However, if the residual errors by measurement in both telescope faces were determined and their temporal stability was assured, single-face telescope observations lead to the refraction corrected angles as well. Hence, in combination with the realized dispersometer real-time determination of refraction corrected angles seems to be possible. Furthermore, concluded from the self-calibration method and from the experimental results for the prevailing higher turbulence strength, applying the dual-wavelength method in outdoor field experiments is potentially promising.
Improvements are expected by an industrial realization of the dispersometer and also by implementing the dispersometer into modern geodetic total stations, e.g. with respect to field-operativeness. Herein, especially the reduction of the divergence angles at the dual-wavelength transmitter will make longer sight lengths in combination with smaller aperture diameters possible. Furthermore, the implementation of blue laser diodes, when meeting the standards of nowadays infrared laser diodes, would significantly enhance efficiency and reduce overall costs. By direct modulation of the blue laser diodes more sophisticated modulation schemes seem to be on hand. We note that the beam quality of these blue laser diodes is not a critical issue because of the availability of the novel technique for achieving coaxial monomode propagation which was established within this work.
References

Abbreviations

AO Applied Optics
AP Applied Physics
AVN Allgemeine Vermessungs-Nachrichten
DGK Deutsche Geodätische Kommission
IEEE Institute of Electrical and Electronics Engineers, New York
JOSA Journal of the Optical Society of America
ÖZfV Österreichische Zeitschrift für Vermessungswesen
SPIE Society of Photo-Optical Instrumentation Engineers
VPK Vermessung, Photogrammetrie. Kulturtechnik
VR Vermessungswesen und Raumordnung
VT Vermessungstechnik
ZfV Zeitschrift für Vermessungswesen


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Curriculum Vitae

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