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Spatial fluctuation on speed-density relationship of pedestrian dynamics

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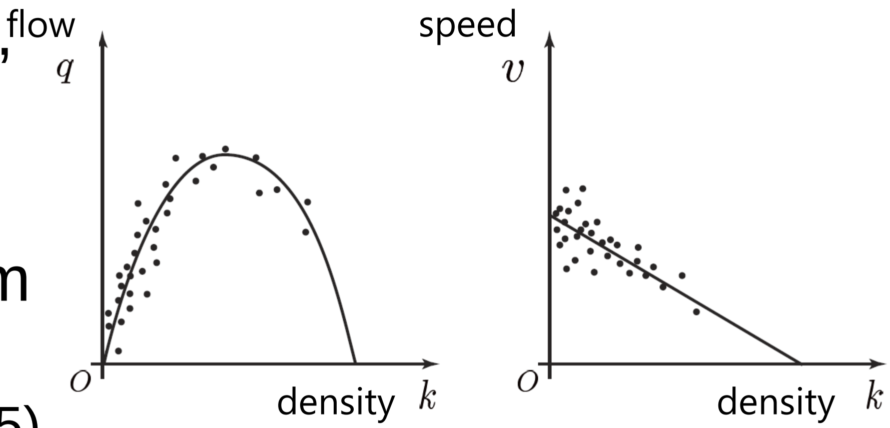
Background – bottleneck detection

- Bottleneck determines the maximum flow in an area
- Therefore, bottleneck detection is important in evaluating the LOS of pedestrian facilities
- Existing method: Fruin (1971) needs a priori knowledge regarding the position of bottlenecks
  - Area to be evaluated should be decided in advance
- Studies on simulation and modelling (e.g., Kirchner+ 2003; Schadschneider+ 2009; Hoogendoorn+ 2005; Zhang+ 2014) have also dealt with situations with obvious bottleneck
- However, in the real situation,
  - there is no a priori knowledge
  - there is no obvious bottleneck
  - our intuitive understandings often fail

Background – pedestrian fundamental diagram

- Speed-density-flow relationship, namely fundamental diagram, is useful in vehicular flow



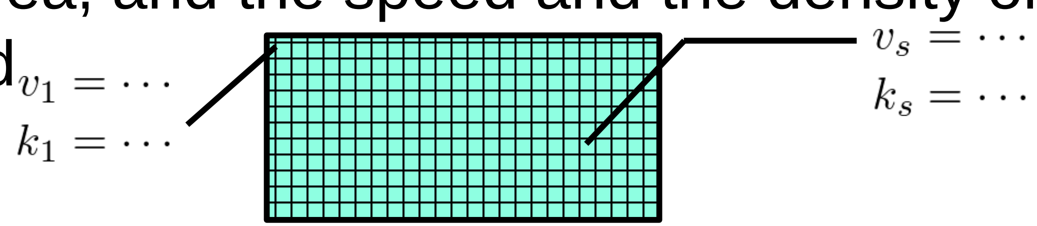
- Pedestrian fundamental diagram is analysed for many years (e.g., Seyfried+ 2010; Flötterrod+ 2015)
- Generally this fundamental relationship depends on time *t*, position *x* and individual *n*; e.g., $v=V(k,t,x,n)$
- In particular, in case of pedestrian flow, the effect of position may be significant even in small area
- From this viewpoint, if there is no obvious bottleneck in the area, the bottleneck detection seems to be almost equivalent to finding the fundamental relationship with minimum value of maximum flow in the area

Objective

- To detect <u>unobvious</u> bottlenecks in pedestrian flow

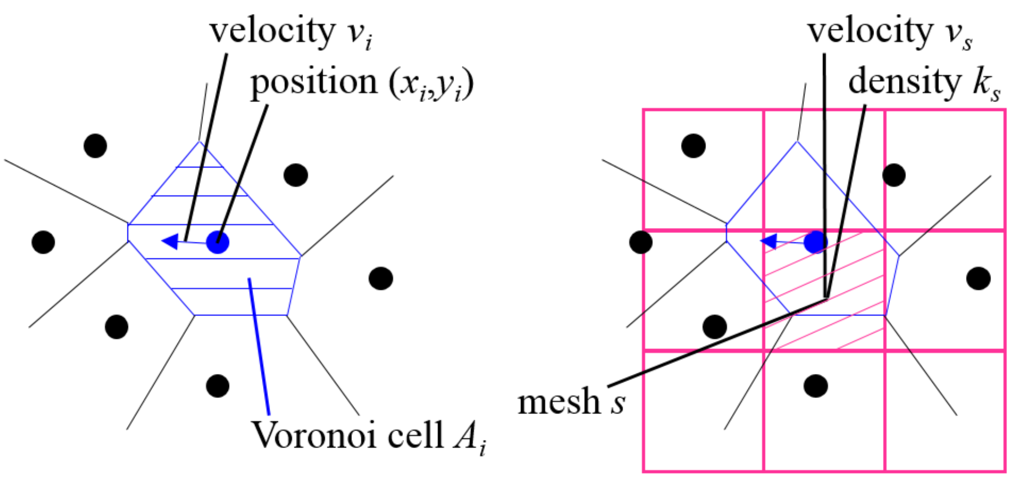
- Two main ideas:
  - If the facility does not have an obvious bottleneck, then the speed-density relationship (=fundamental diagram) at arbitrary positions of the facility are not the same, but only slightly different from each other
  - This difference can be modelled as spatial dependence;
    if the positions are close to each other, then the FDs are more similar to each other

- To model this dependency, mesh *s*={1,2,…,S} is generated over an experimental area, and the speed and the density on each mesh is calculated



Methodology – fundamental diagram

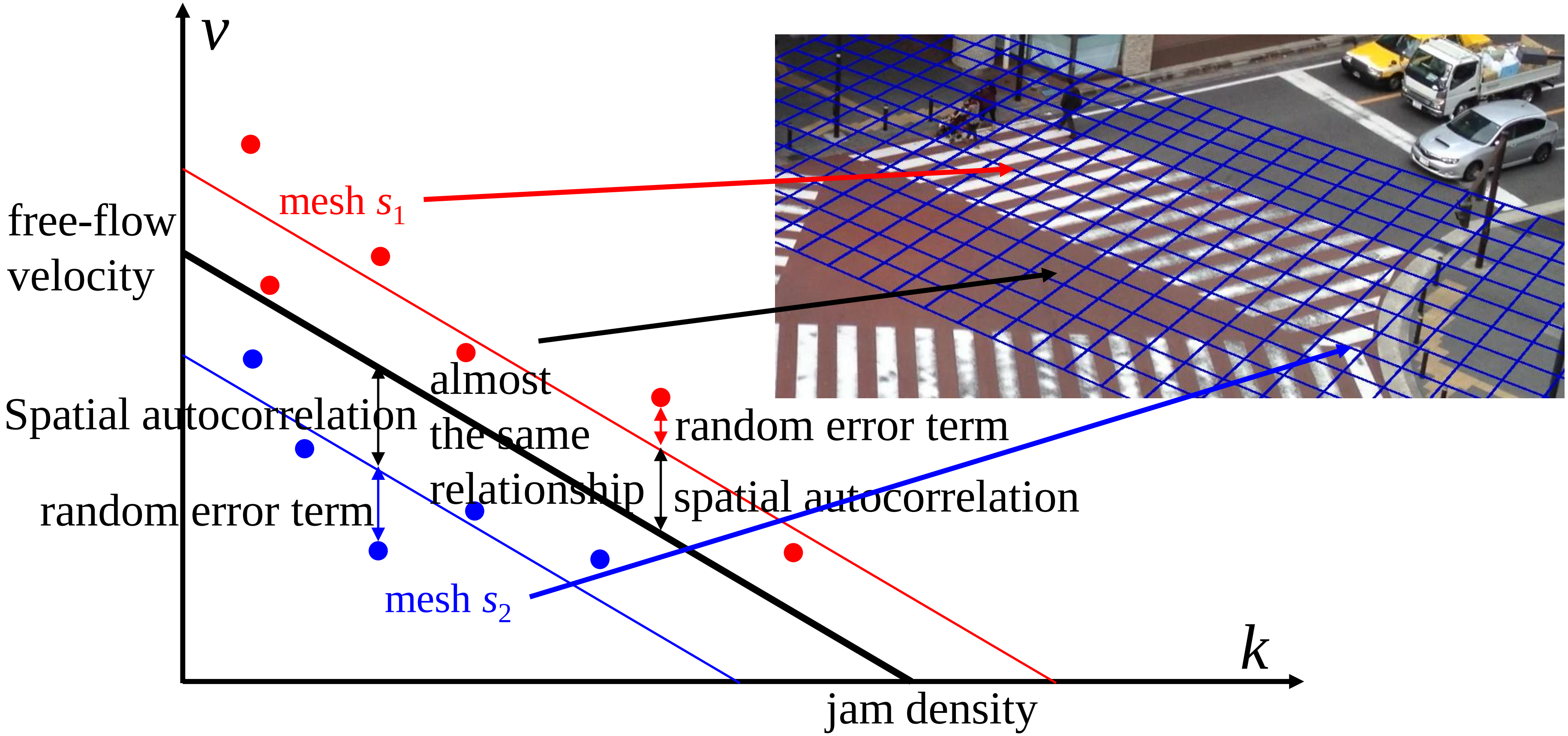
- The Voronoi diagram method (Steffen+ 2010; Zhang 2011) is employed to define flow indicators (speed and density)
- At each time instance, Voronoi diagram for pedestrians is drawn, and indicators for each mesh *s* are calculated as
  - $v_{xy}(t) = v_i(t) \text{ iff } (x,y) \in A_i(t)$
  - $k_{xy}(t) = 1/V_i(t)$
  - $\begin{cases} v_s(t) = \dfrac{\iint_s v_{xy}(t)dxdy}{d^2} \\ k_s(t) = \dfrac{\iint_s k_{xy}(t)dxdy}{d^2} \end{cases}$
  - where V_i is volume of A_i and *d* is mesh size



- Parametric form is assumed as Greenberg $v = \beta_0 + \beta_1 \ln k$ or Greenshields $v = \beta_0 + \beta_1 k$

Methodology – spatial dependence

- We assume that
  - almost the same FDs are drawn in the area
  - but they are slightly different
  - and the difference is caused by spatial dependence



Methodology – eigenvector spatial filtering

- Eigenvector Spatial Filtering (Griffith, 2003)
  - a method to model spatial dependence
  - easier parameter estimation thanks to linear regression form
  - estimates can be understood as "spatial map pattern"

- ESF regression

$$y_s = \boxed{\beta_0 + \sum_i x_{i,s}\beta_i} + \boxed{\sum_j E_{j,s}\gamma_j} + \boxed{\varepsilon_s}$$

explained variable y_s at mesh s

Non-spatial linear regression
- explanatory vector at mesh s: $\mathbf{x}_s$
- random error term at mesh s: ε_s
- parameter vector: $\boldsymbol{\beta}$

Spatial dependence term
- linearly independent vector
- spatial dependence variable(s) at mesh s: $\mathbf{E}_s$
- parameter vector: $\boldsymbol{\gamma}$

- $\mathbf{E}_s$ is derived from the Moran's I coefficient, which is the statistic of spatial dependence

Methodology – eigenvector spatial filtering

- Moran's I coefficient shows spatial dependence
- Spatial autocorrelation

Positive dependence
– Land price

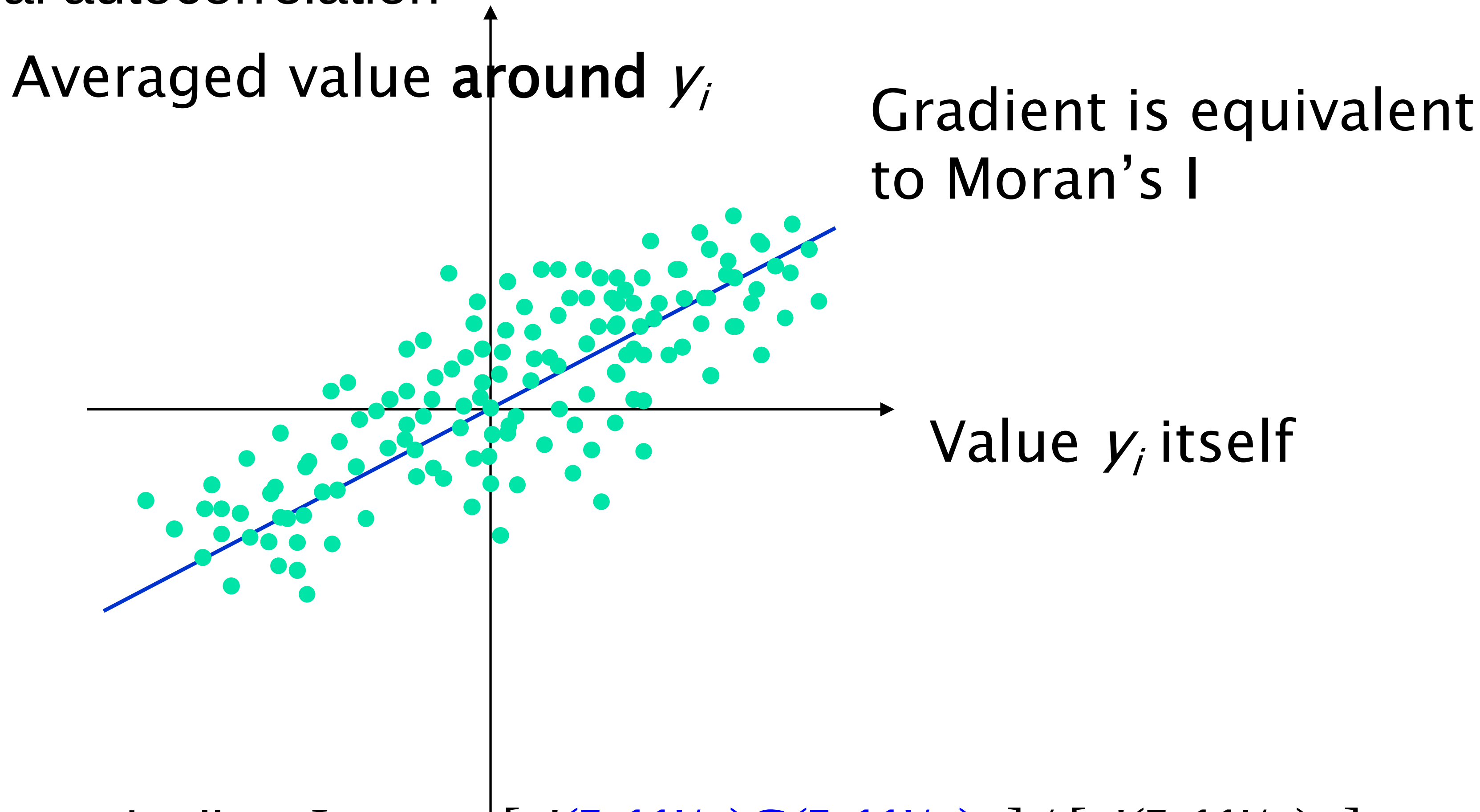
Random

Negative dependence
– Store location

Moran's I ~ 1 Moran's I ~ 0 Moran's I ~ −1

Methodology – eigenvector spatial filtering

- Moran's I coefficient shows spatial dependence
- Spatial autocorrelation



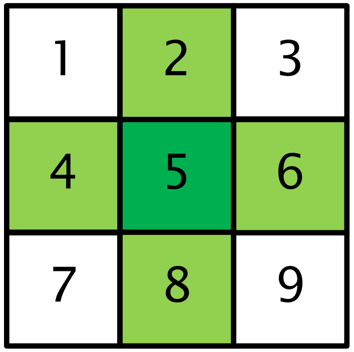
- Mathematically, $I_{moran} = [\mathbf{y}'(\mathbf{I}-\mathbf{1}\mathbf{1}'/n)\mathbf{C}(\mathbf{I}-\mathbf{1}\mathbf{1}'/n)\mathbf{y}] \,/\, [\mathbf{y}'(\mathbf{I}-\mathbf{1}\mathbf{1}'/n)\mathbf{y}]$

Methodology – eigenvector spatial filtering

- ESF regression $$y_s = \boxed{\beta_0 + \sum_i x_{i,s}\beta_i} + \boxed{\sum_j E_{j,s}\gamma_j} + \boxed{\varepsilon_s}$$
- $\mathbf{E}_s$ is derived from the Moran's I coefficient

$$I_{moran} = [\mathbf{y}'(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{C}(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{y}] \,/\, [\mathbf{y}'(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{y}]$$

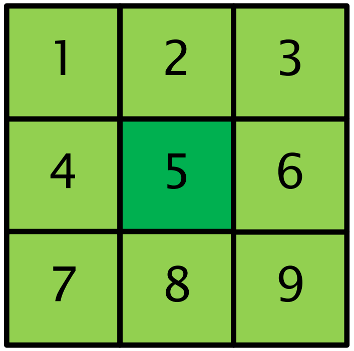
- Contiguity matrix **C**: $n \times n$ "distance" matrix
  - Rook matrix



$C_{i,j} = 1$ iff
i and *j* share a common **boundary**

$$\begin{bmatrix} 0&1&0&1&0&0&0&0&0\\ 1&0&1&0&1&0&0&0&0\\ 0&1&0&0&0&1&0&0&0\\ 1&0&0&0&1&0&1&0&0\\ 0&1&0&1&0&1&0&1&0\\ 0&0&1&0&1&0&0&0&1\\ 0&0&0&1&0&0&0&1&0\\ 0&0&0&0&1&0&1&0&1\\ 0&0&0&0&0&1&0&1&0 \end{bmatrix}$$

  - Queen matrix



$C_{i,j} = 1$ iff
i and *j* share a common **vertex**

$$\begin{bmatrix} 0&1&0&1&1&0&0&0&0\\ 1&0&1&1&1&1&0&0&0\\ 0&1&0&0&1&1&0&0&0\\ 1&1&0&0&1&0&1&1&0\\ 1&1&1&1&1&1&1&1&1\\ 0&1&1&0&1&0&0&1&1\\ 0&0&0&1&1&0&0&1&0\\ 0&0&0&1&1&1&1&0&1\\ 0&0&0&0&1&1&0&1&0 \end{bmatrix}$$

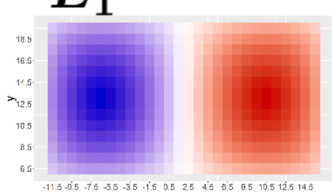
Methodology – eigenvector spatial filtering

- ESF regression $$y_s = \boxed{\beta_0 + \sum_i x_{i,s}\beta_i} + \boxed{\sum_j E_{j,s}\gamma_j} + \boxed{\varepsilon_s}$$
- $\mathbf{E}_s$ is derived from the Moran's I coefficient

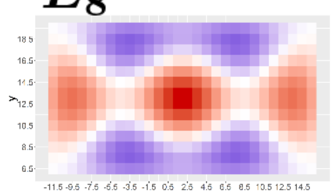
$$I_{moran} = [\mathbf{y}'(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{C}(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{y}] \,/\, [\mathbf{y}'(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{y}]$$

- Contiguity matrix **C**: $n \times n$ "distance" matrix
- Additional explanatory variables **E** is generated by the **eigenvectors** of $(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{C}(\mathbf{I}\text{-}\mathbf{11}'/n)$
- Spatial term $\Sigma_j(\mathbf{E}_j\gamma_j)$ shows "spatial map pattern"
- **Multiscale** dependence pattern is generated by $\mathbf{E}_j\gamma_j$
  - Eigenvector with larger eigenvalue shows more global map pattern, and smaller shows more local pattern

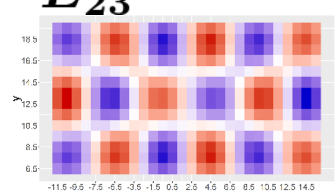
E_1



E_8

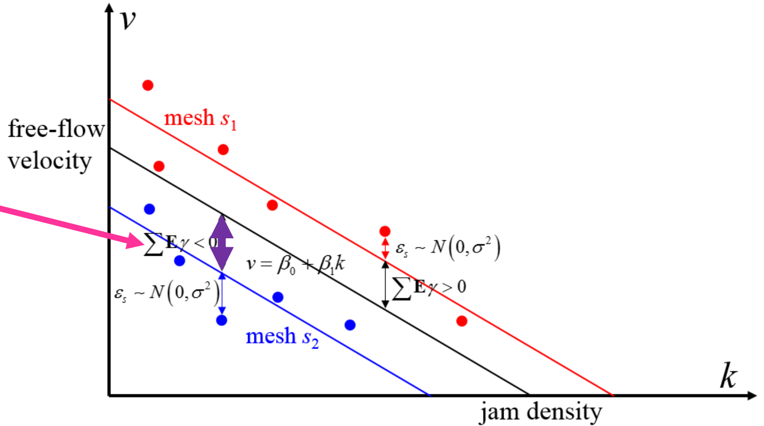


E_{23}



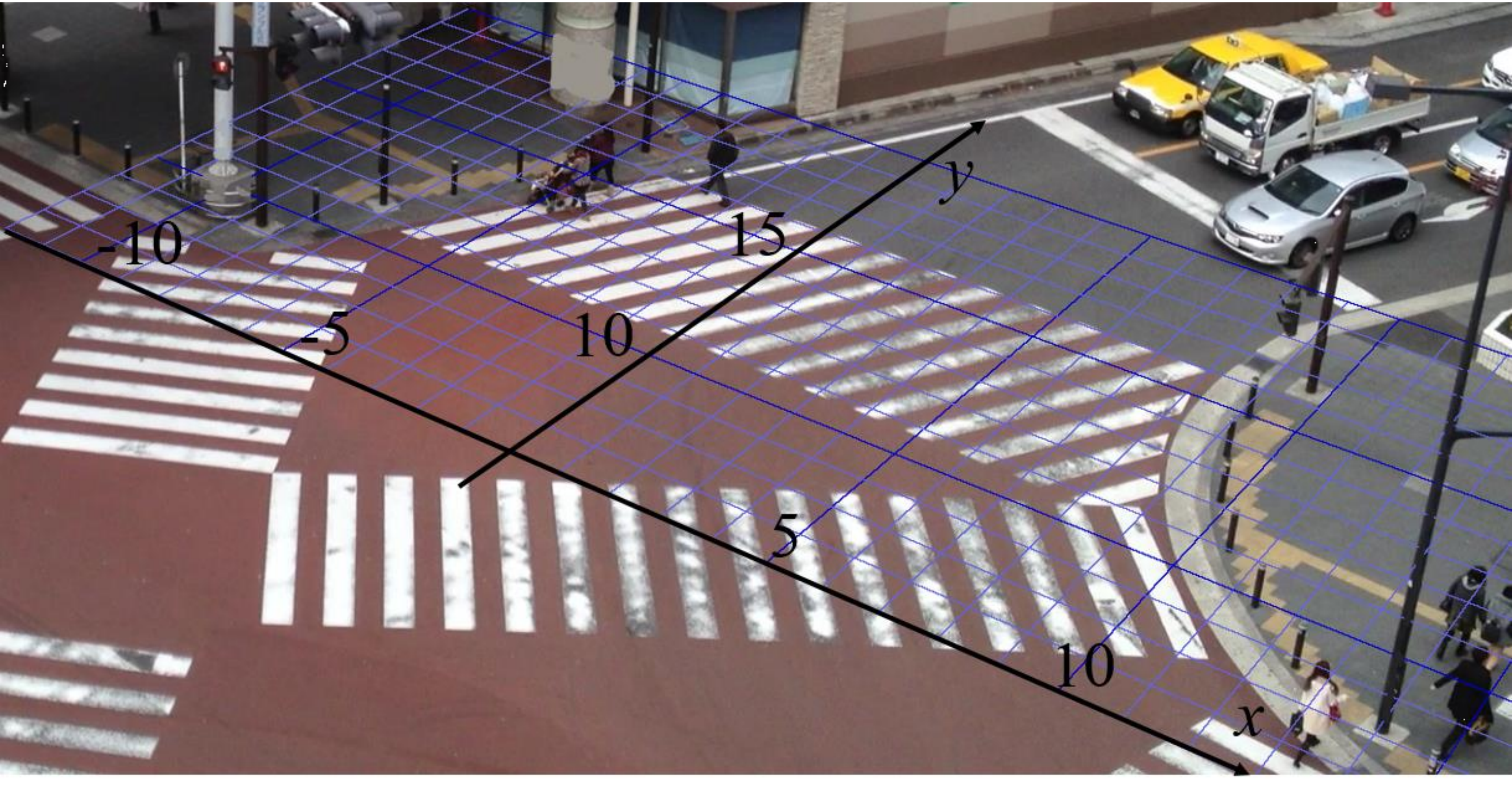
Methodology – eigenvector spatial filtering

- ESF regression $$y_s = \boxed{\beta_0 + \sum_i x_{i,s}\beta_i} + \boxed{\sum_j E_{j,s}\gamma_j} + \boxed{\varepsilon_s}$$
- $\mathbf{E}_s$ is derived from the Moran's I coefficient
  $$I_{moran} = [\mathbf{y}'(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{C}(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{y}] \,/\, [\mathbf{y}'(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{y}]$$
- Contiguity matrix **C**: $n \times n$ "distance" matrix
- Additional explanatory variables **E** is generated by the eigenvectors of $(\mathbf{I}\text{-}\mathbf{11}'/n)\mathbf{C}(\mathbf{I}\text{-}\mathbf{11}'/n)$
- Spatial term $\Sigma_j(\mathbf{E}_j\gamma_j)$ shows "multiscale spatial map pattern"
- $\mathbf{E}_j$ are selected by some criteria as LASSO and t-value of γ
- At last, if the density is the same, then the estimated velocity is $\Sigma_j(\mathbf{E}_j\gamma_j)$ different from the area average
- The mesh with the smallest $\Sigma_j(\mathbf{E}_j\gamma_j)$ implies the bottleneck



Application – data and coordinates

- Pedestrian crossing in front of Kawasaki (川崎) station
- 18424 samples for 1[m] mesh, 11466 samples for 2[m] mesh with time step 1[s]



Application – setup comparison

- Apply 8 patterns
  - Mesh size: 1m / 2m, Contiguity matrix: Rook / Queen, Regression: Greenberg / Greenshields
- AICs of ESF regression are always smaller than that of conventional non-spatial regression

Mesh 1m	Greenshields	Greenberg
Non-spatial	15799.15	15855.30
Rook	15164.03	15260.99
Queen	15164.06	15261.04

Mesh 2m	Greenshields	Greenberg
Non-spatial	12029.24	12077.30
Rook	11958.48	12019.14
Queen	11960.93	12020.60

- Obtained spatial patterns are similar irrespective of the setup

Application – estimated parameters

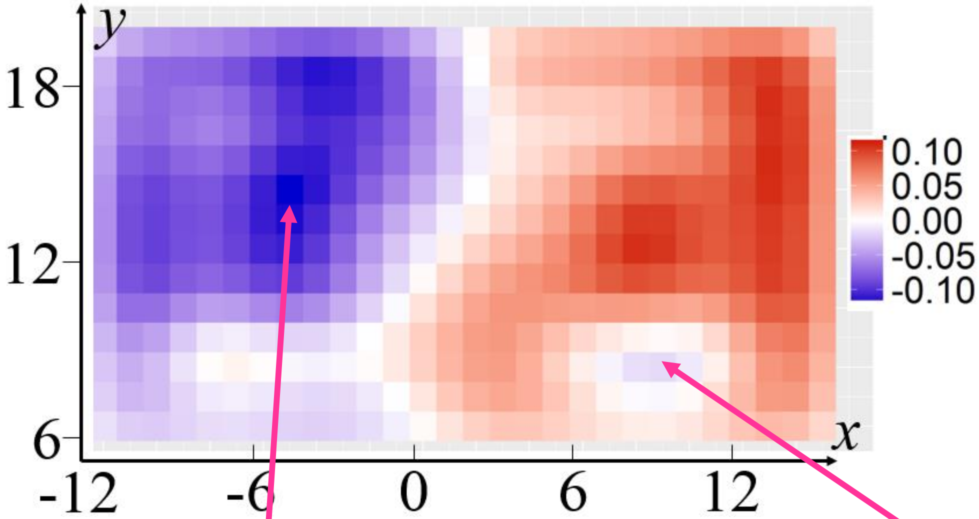
- 1[m] mesh, Rook, Greenshields
- 13 eigenvectors were selected to explain spatial dependence

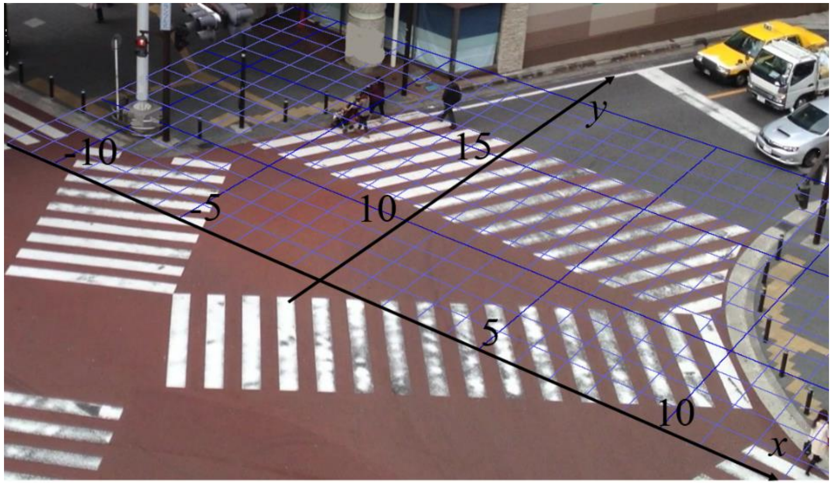
Parameters		Proposed method $v_s = \beta_0 + \beta_1 k_s + \sum_j E_{j,s}\gamma_j + \varepsilon_s$ Estimate	t-value	Conventional (non-spatial) method $v_s = \beta_0 + \beta_1 k_s + \varepsilon_s$ Estimate	t-value
Constant	β_0	1.53	494.39 **	1.53	491.30 **
Gradient	β_1	-0.57	-11.92 **	-0.61	-12.90 **
Eigenvectors	γ_1	-7.07	-19.38 **		
	γ_3	1.65	4.45 **		
	γ_4	2.14	5.85 **		
	γ_5	-1.28	-3.51 **		
	γ_6	0.80	2.19 *		
	γ_7	-0.92	-2.50 *		
	γ_{10}	-2.47	-6.77 **		
	γ_{11}	-1.53	-4.19 **		
	γ_{19}	1.02	2.79 **		
	γ_{21}	1.17	3.22 **		
	γ_{23}	0.93	2.55 *		
	γ_{30}	1.29	3.55 **		
	γ_{61}	-0.76	-2.08 *		
AIC		15164.0		15691.5	
RMSE		0.133		0.137	

**: 1% significant, *: 5% significant

Application – estimated bottleneck

- Whole spatial pattern: $\Sigma_j(\mathbf{E}_j\gamma_j)$





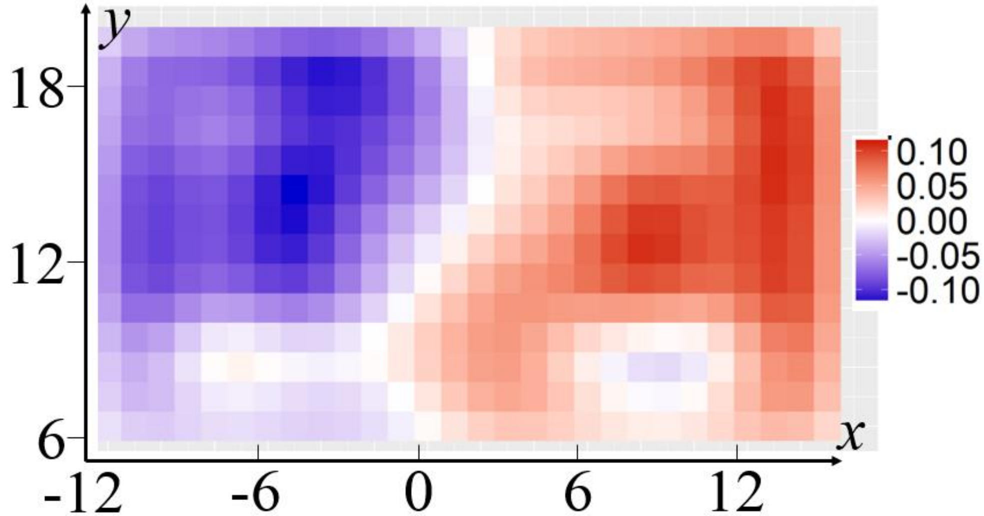
- Red mesh shows positive $\Sigma_j(\mathbf{E}_j\gamma_j)$ and blue shows negative
- Two bottlenecks were detected

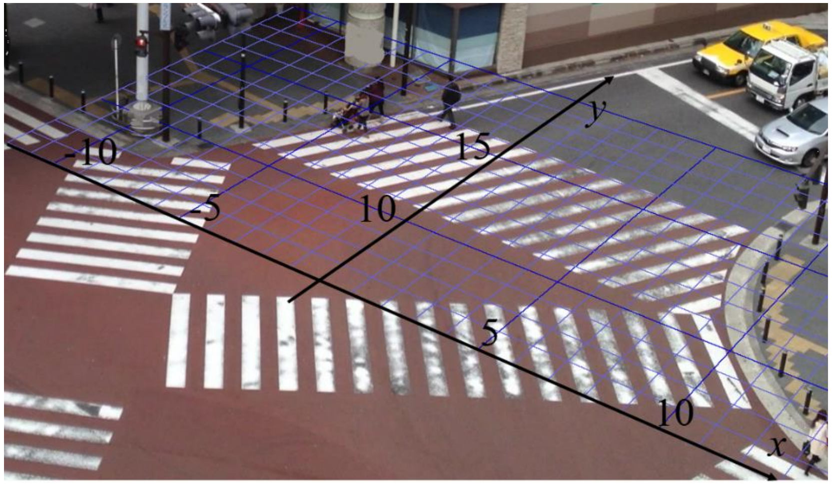
Global bottleneck
- Capacity in left-hand side is lower
- Destination is limited to the station in the left-hand side, where there is many stores in the right-hand side

Local bottleneck
- The corner of this intersection, and pedestrian flow may intersect there

Application – estimated spatial pattern

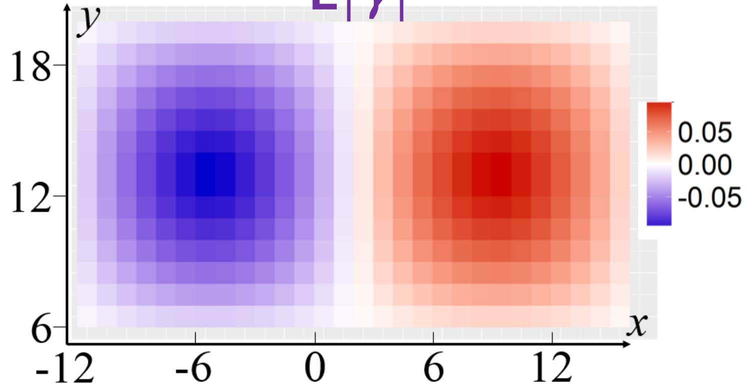
- Whole spatial pattern: $\Sigma_i(\mathbf{E}_i \gamma_i)$



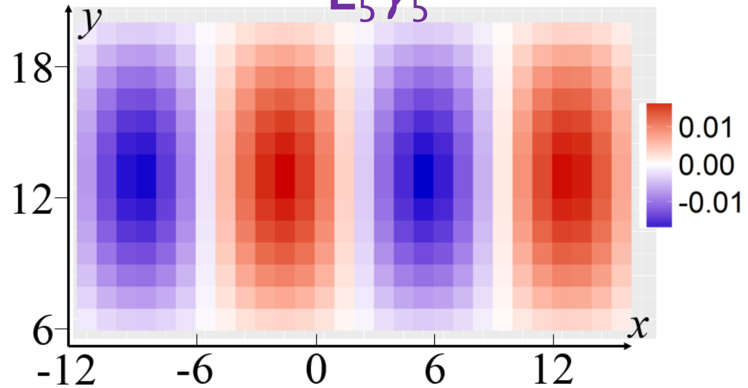


- Multiscale map pattern decomposition also shows the characteristics of this crossing: left-hand side has lower capacity

$\mathbf{E}_1 \gamma_1$

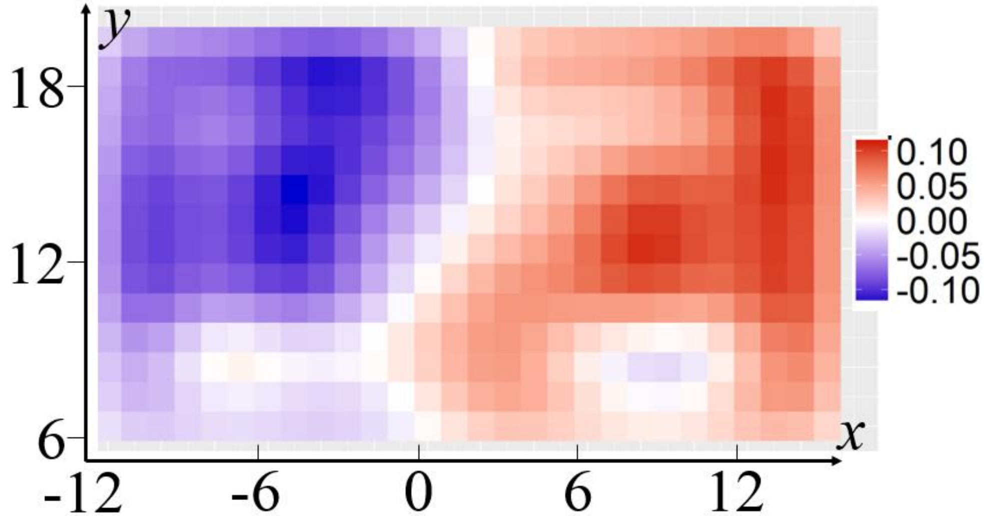


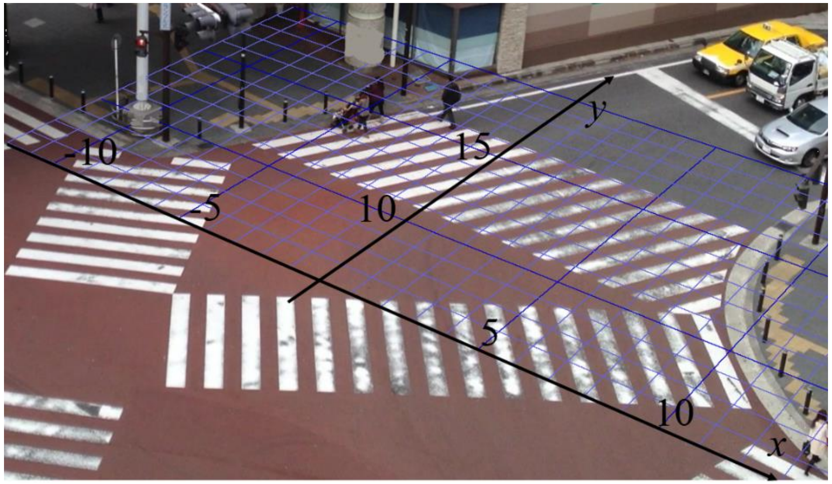
$\mathbf{E}_5 \gamma_5$



Discussion – practical meaning

- Whole spatial pattern: $\Sigma_i(\mathbf{E}_i\gamma_i)$

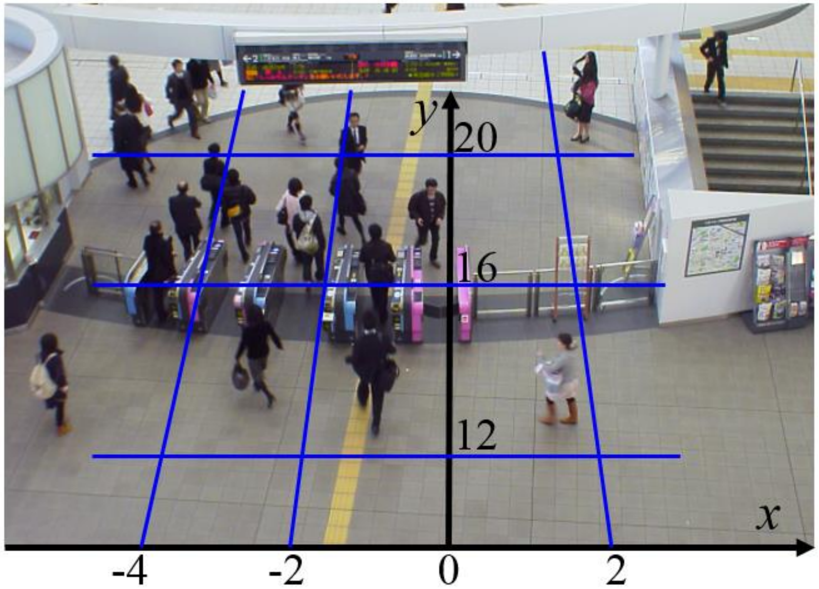


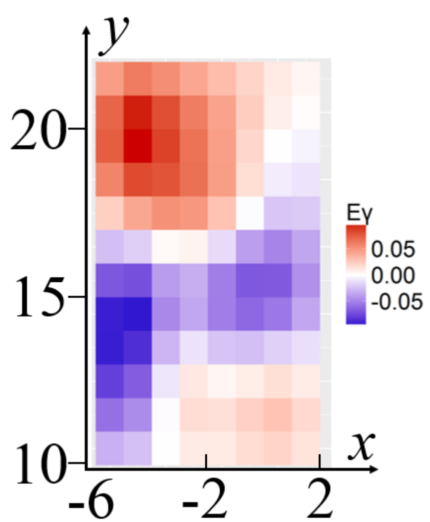


- If we measure the capacity of this crossing **at only right-hand side edge**, it might be overestimated around 0.2 [m/s], which is over 10% of the pedestrian speed

Another application to railway station

- Coordinates and results





		Proposed method $v_s = \beta_0 + \beta_1 k_s + \Sigma_j(\gamma_j \mathbf{E}_{j,s}) + \varepsilon_s$		Conventional non-spatial method $v_s = \beta_0 + \beta_1 k_s + \varepsilon_s$	
		Estimate	t-value	Estimate	t-value
Intercept	β_0	1.31	479.89[a]	1.32	488.52[a]
Gradient	β_1	−0.53	−21.08[a]	−0.55	−23.16[a]
Eigenvectors	γ_1	6.67	19.84[a]		
	γ_2	−1.46	−4.28[a]		
	γ_3	5.17	14.98[a]		
	γ_4	5.59	16.62[a]		
	γ_6	1.76	5.26[a]		
	γ_7	−3.58	−10.55[a]		
	γ_8	2.66	7.95[a]		
	γ_{11}	1.19	3.55[a]		
	γ_{12}	1.44	4.30[a]		
	γ_{15}	−1.10	−3.28[a]		
	γ_{16}	−2.28	−6.81[a]		
	γ_{21}	−0.68	−2.03[b]		
	γ_{23}	−0.88	−2.63[a]		
AIC		−4716.1		−3526.0	

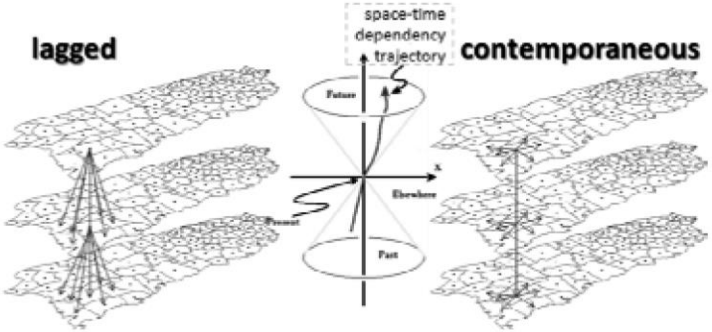
[a] 1% significant,
[b] 5% significant, respectively.

Conclusions and future directions

- Summary
  - Eigenvector spatial filtering is introduced to model the spatial dependence on fundamental diagrams in a small area
  - In this modelling, smaller speed with the same dencity implies bottlenecks
  - Two bottlenecks are detected on pedestrian crossing data
- Future directions
  - Extension of the model to spatio-temporal type, using three-dimensional indicators
  - Exploration of consistency of the spatial dependence pattern with shockwave (kinematic wave theory)
  - Application to congested flow
  - Application of nonlinear fundamental relationship by MLE estimation
- This work has been published as J. Stat. Mech. 2017 033402

Expansion to 3-dimensional model

- It seems to be easy
  - 3D-Voronoi indicator: Nikolić & Bierlaire, 2015
  - 3D-ESF: Griffith 2010, 2012
- However there are some difficulties
  - Calculation cost
  - Setting time interval is rather difficult
    - If an interval is not large enough, 3D-Voronoi indicator itself has an effect like filtering and we cannot distinguish it from the spatio-temporal dependence
  - Structure of spatio-temporal dependence is difficult to determine
    - Griffith (2012) proposed 2 types:



- So far, we cannot obtain any good results…