Departure time choice equilibrium problem with partial implementation of congestion pricing

Tokyo Institute of Technology
Postdoctoral researcher
Katsuya Sakai

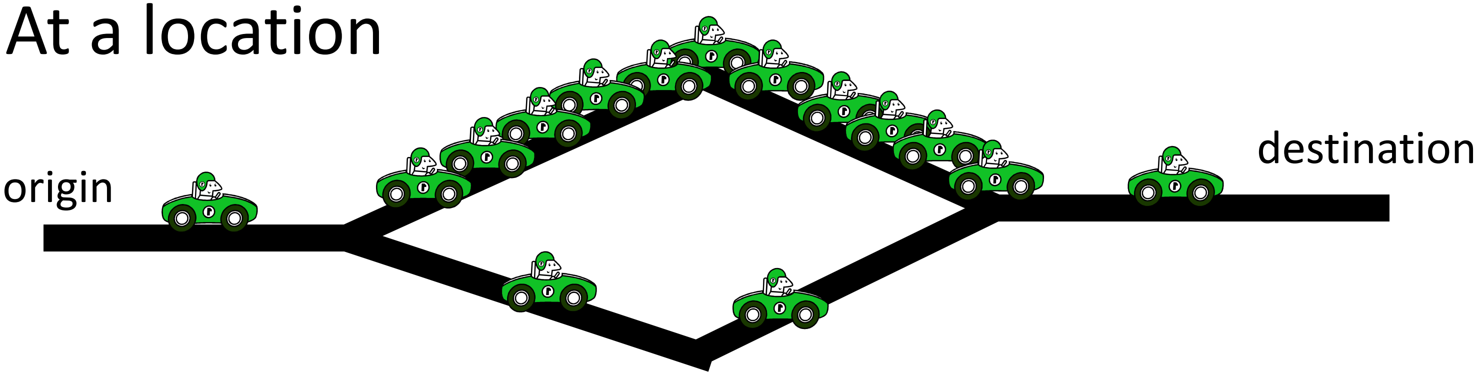
Contents

Problems of congestion

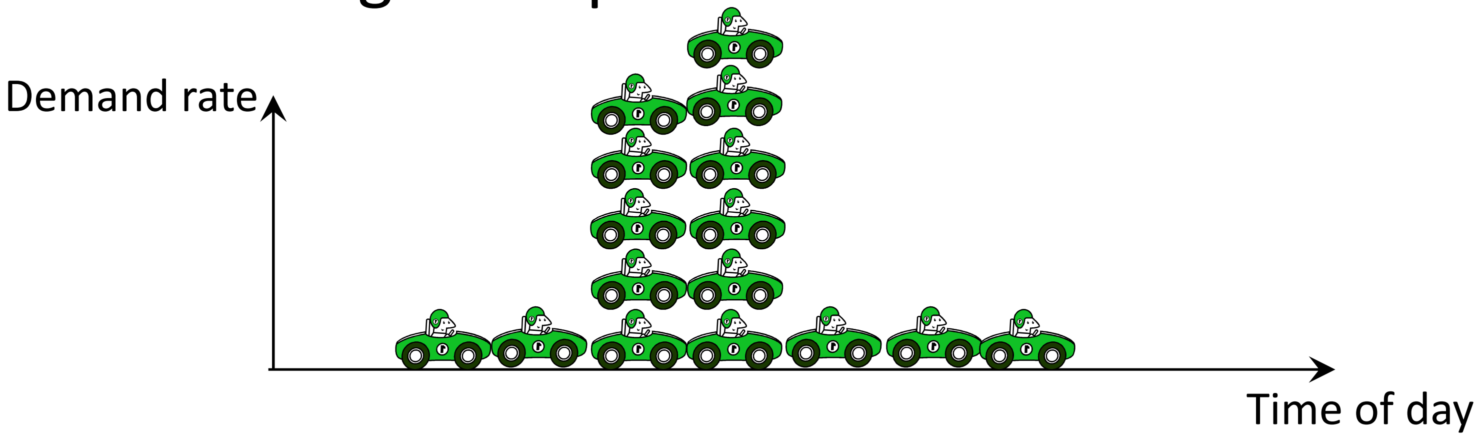
- wasting a lot of resources
  - Time
  - Energy
  - Mental health
  - …

How congestion occurs?

- caused by a concentration of demand
  - At a location

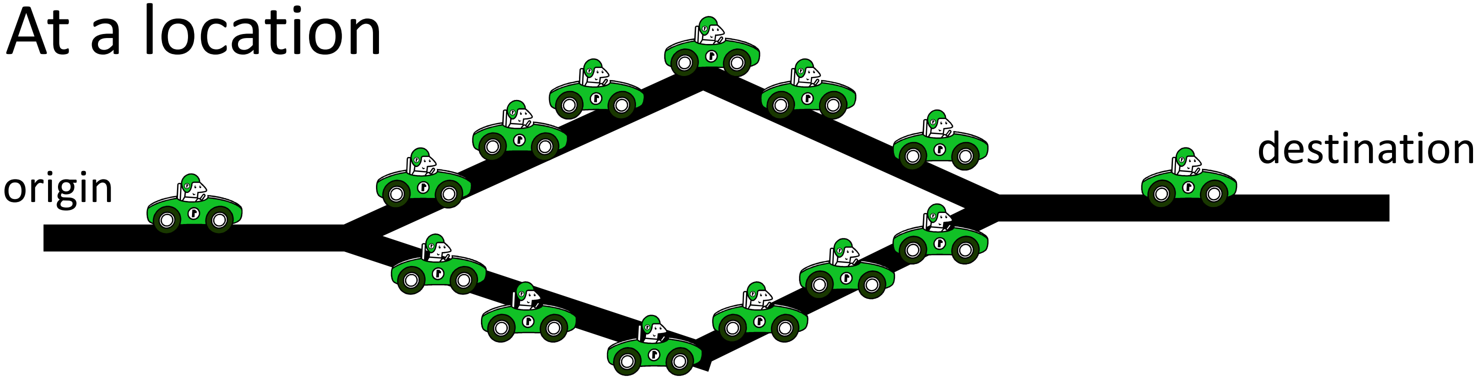


  - During short period

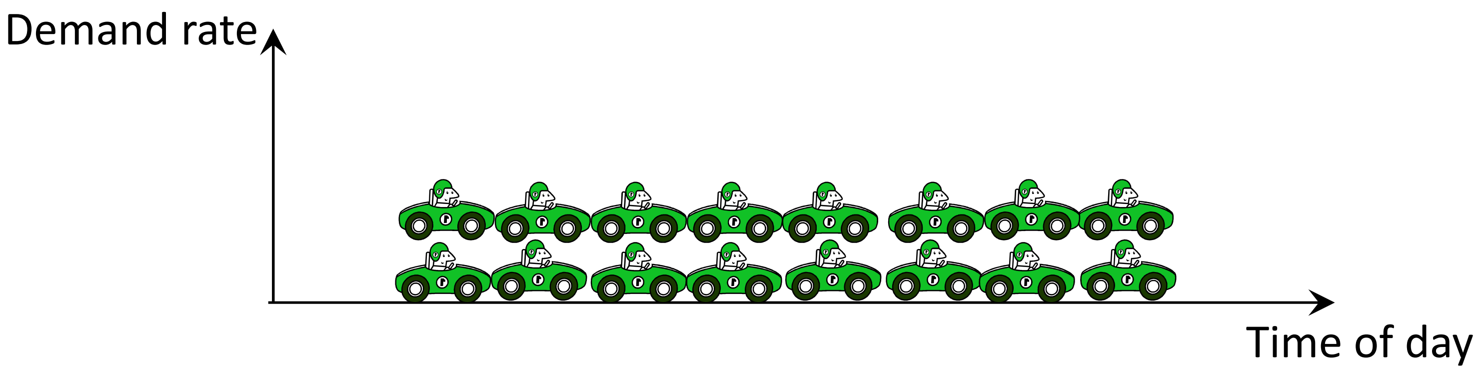


How to eliminate congestion?

- distributing the concentrated demand
  - At a location



  - During short period



Congestion management

- Information provision
  - Congestion length
  - Travel time

- Physical control (quantitative control)
  - Ramp metering
  - Advance booking

- Economic approach (pricing control)
  - Congestion charging
  - Tradable bottleneck permits

TSU Tokyo Tech Transport Studies Unit

Congestion management

Approach	Advantages	Limitations
Information provision	Travel time can be reduced	Total travel time cannot be minimized
Quantitative control	Total travel time can be minimized	Social cost cannot be minimized
Pricing control	Social cost can be minimized	Equity problem; poor people may suffer a loss

- Focusing on pricing control (economic approach)
  - Tradable bottleneck permits (TBP) scheme

Tradable bottleneck permits scheme (Akamatsu et al. 2006)

1. Road administrators issue a right that allows a permit holder to pass through the bottleneck at a pre-specified time period ("bottleneck permits").
2. A new auction market is established for bottleneck permits differentiated by a pre-specified time.

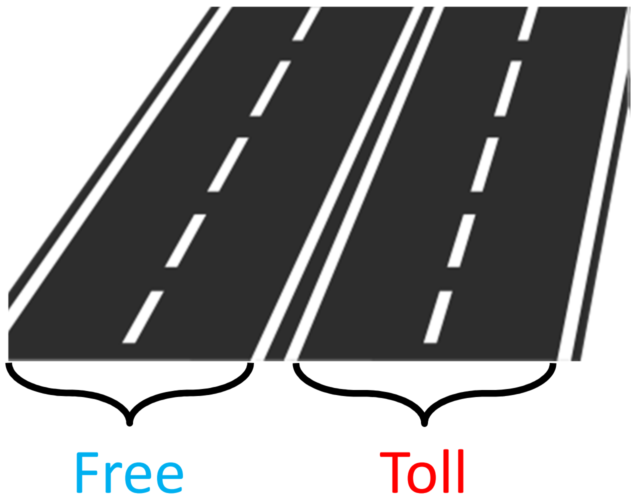
– As a result of auction, toll is determined by each commuter's willingness to pay (value-pricing)
– Under the toll, social cost is minimized

Equity problem of congestion charging

- The Welfare Effects of Congestion Tolls with Heterogeneous Commuters (Arnott et al., 1994)
  - The poor suffer a loss
  - The rich get a benefit

Suggestion

- Applying congestion pricing only a portion of road (lanes).
  - Drivers can choose
    paying or not paying



- How much portion should be charging lanes?
- Don’t the poor suffer a loss?

How to evaluate pricing scheme?

- Comparing equilibrium states before/after pricing applied, we can know the effect of congestion pricing.

Equilibrium
BEFORE pricing
- Commuter's travel cost
- Social cost

applying pricing

Equilibrium
AFTER pricing
- Commuter's travel cost
- Social cost

Need to calculate equilibrium

Objective

- To formulate the departure time choice equilibrium problem and to solve it when the TBP scheme is partially applied.
- To examine the welfare effect of partial congestion pricing with heterogeneous commuters.

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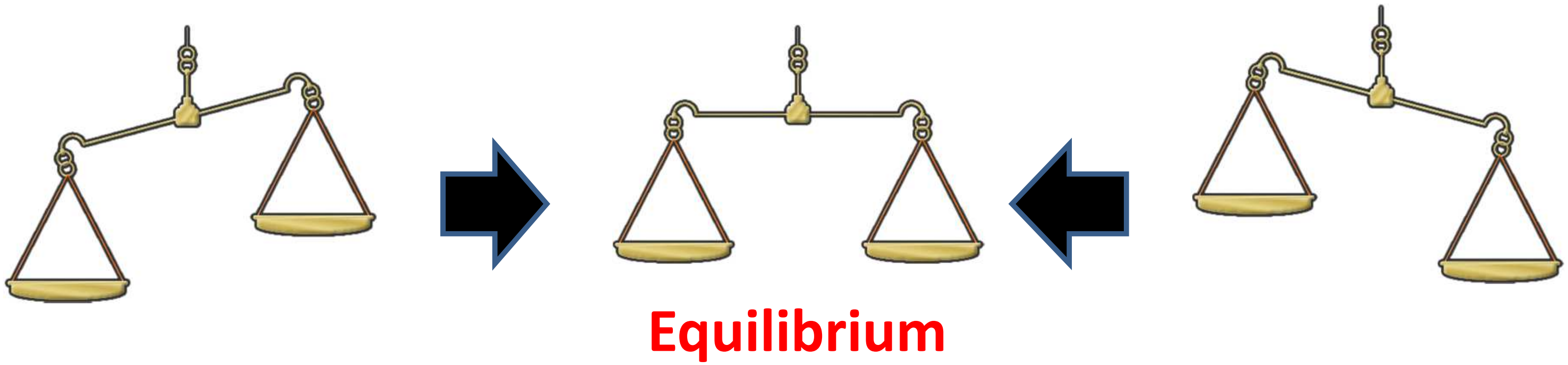
What is equilibrium?

- The most realizable state.
- At equilibrium, nobody can improve their travel cost by unilaterally changing their behavior.



What is equilibrium?

- Route choice equilibrium (Wardrop, 1952)
  - The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.
- Departure time choice equilibrium (Vickrey, 1969)
  - Times => costs
  - Routes => departure time

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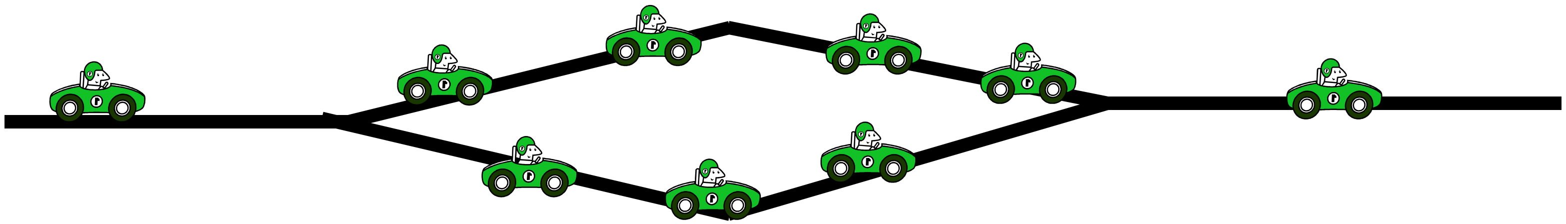
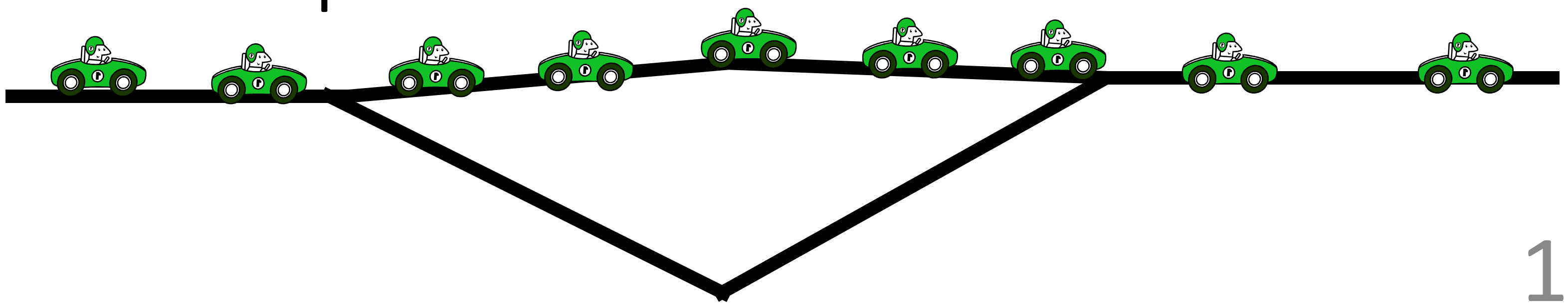
Route choice equilibrium (Wardrop, 1952)

- Both routes are used
  - Travel time of two routes must be same



- Either of routes is used
  - Travel time of used road must be smaller or at most equal to that of the unused road.



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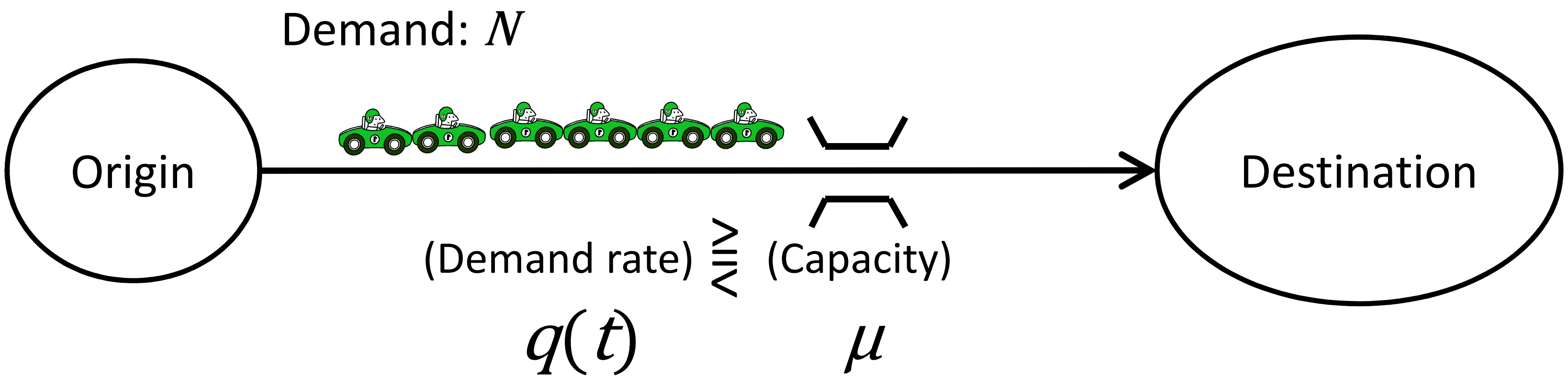
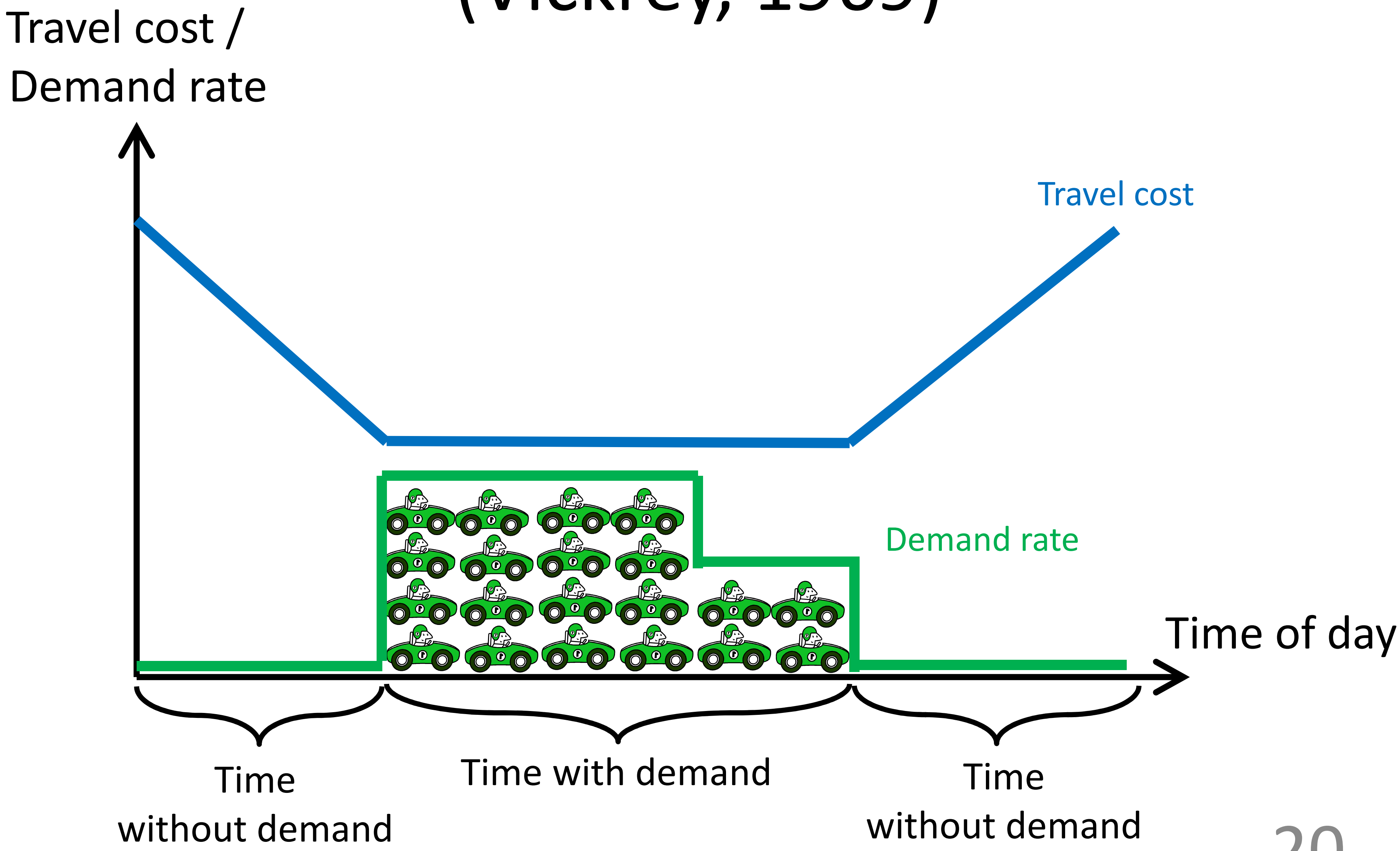
Departure time choice problem (Vickrey, 1969)

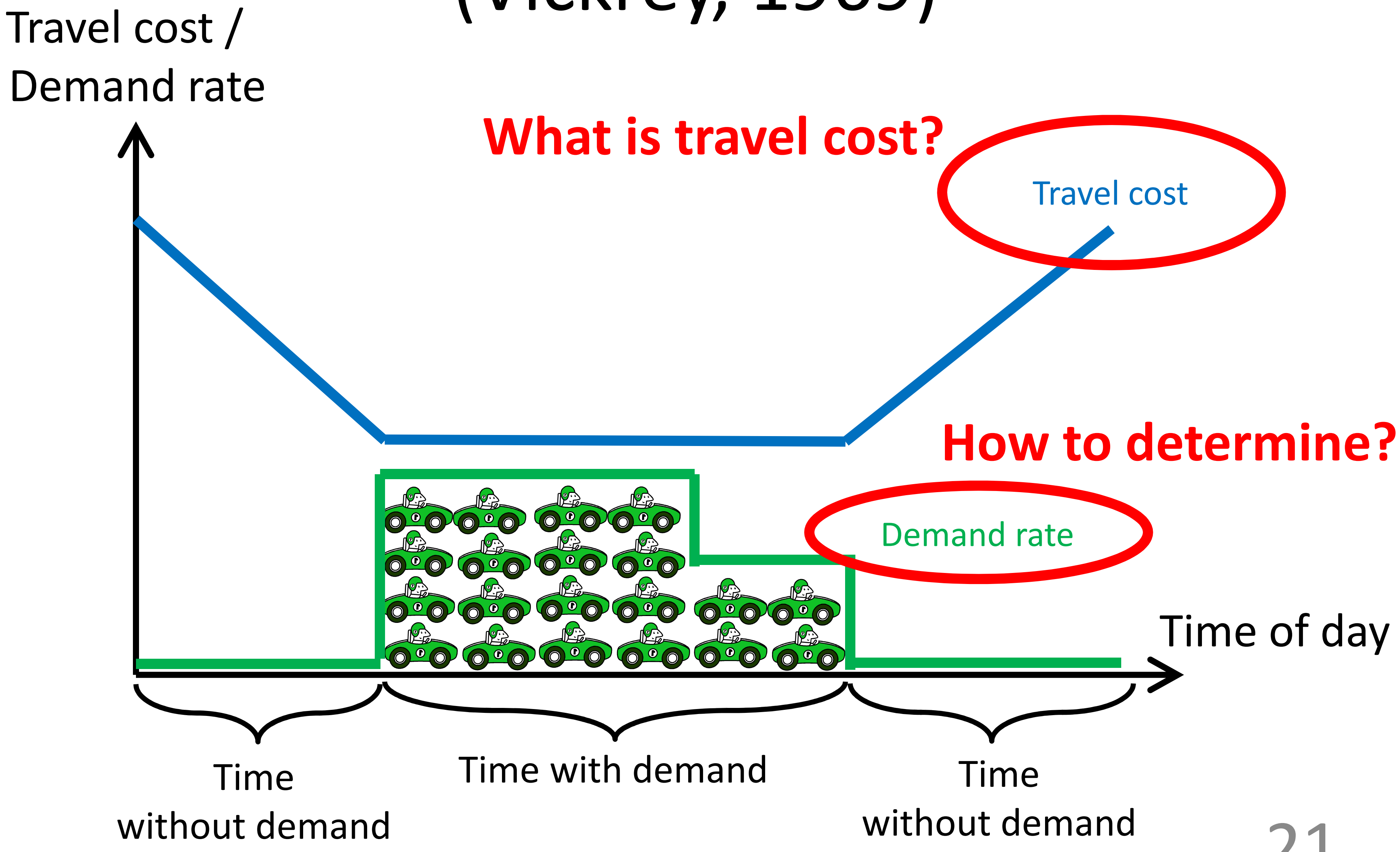
- Focusing on morning peak hour (commute) problem
- Employing bottleneck model, we simply describe traffic congestion one-to-one network with a single bottleneck



Departure time choice equilibrium (Vickrey, 1969)



Departure time choice equilibrium (Vickrey, 1969)

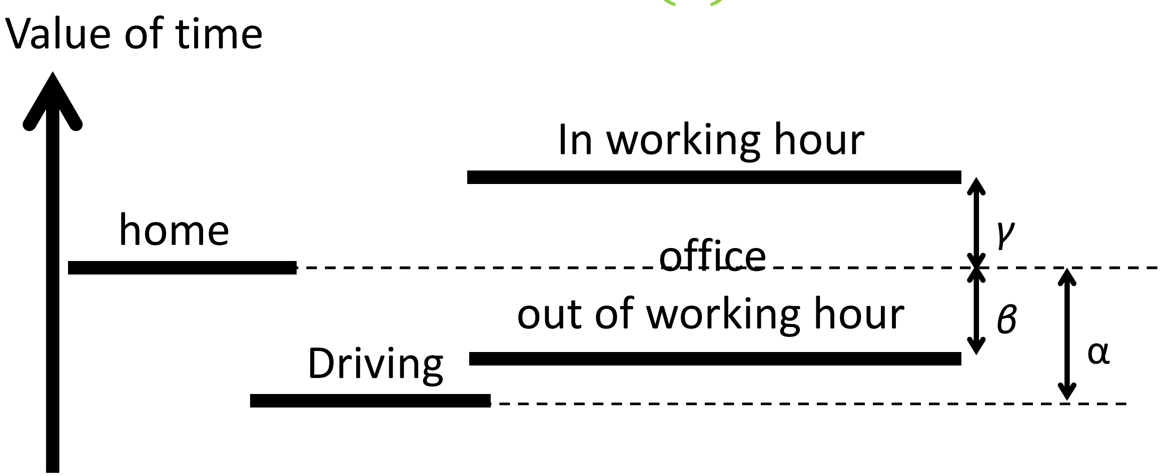


Departure time choice equilibrium (Vickrey, 1969)

- Defining travel cost with tradeoff between waiting and schedule delays.

$$TC = \begin{cases} \underbrace{\alpha w}_{\text{Waiting cost}} - \beta \cdot (t - t_0) + p + const & \text{if early arrival} \\ \alpha w + \underbrace{\gamma \cdot (t - t_0)}_{\text{Schedule cost } c(t)} + \underbrace{p}_{\text{Toll cost}} + const & \text{if late arrival} \end{cases}$$

Waiting cost — Schedule cost $\boldsymbol{c(t)}$ — Toll cost



TC: travel cost
w: waiting delay
α: marginal cost of waiting delay
β: marginal cost of early arrival
γ: marginal cost of late arrival
t_0: desired arrival time
t: actual arrival time

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    1. Solution procedure
    2. Numerical example
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Equilibrium conditions without toll

- Departure (arrival) time choice equilibrium condition;

$$\begin{cases} TC(t) = \alpha \cdot w(t) + c(t) = \overline{TC} & \text{if} \quad q(t) > 0 \\ TC(t) = \alpha \cdot w(t) + c(t) \geq \overline{TC} & \text{if} \quad q(t) = 0 \end{cases} \quad \forall t$$

- Demand-supply equilibrium condition;

$$\begin{cases} q(t) = \mu & \text{if} \quad w(t) > 0 \\ q(t) \leq \mu & \text{if} \quad w(t) = 0 \end{cases} \quad \forall t$$

- Flow conservation

$$\sum_t q(t) = N$$

- Non-negativity constraint

$$q(t) \geq 0 \quad \forall t$$

TC: travel cost
$\overline{TC}$: equilibrium travel cost
q : demand rate
μ : bottleneck capacity
w: waiting delay
N : total demand

How to solve equilibrium problem?

- Solving the equivalent optimization problem (Iryo and Yoshii, 2007)
- Minimizing total schedule cost (waiting-time base)

$$min_{\{q(t)\}} \sum_t \frac{c(t)}{\alpha} \cdot q(t)$$

$$\text{s.t.} \quad q(t) \leq \mu$$

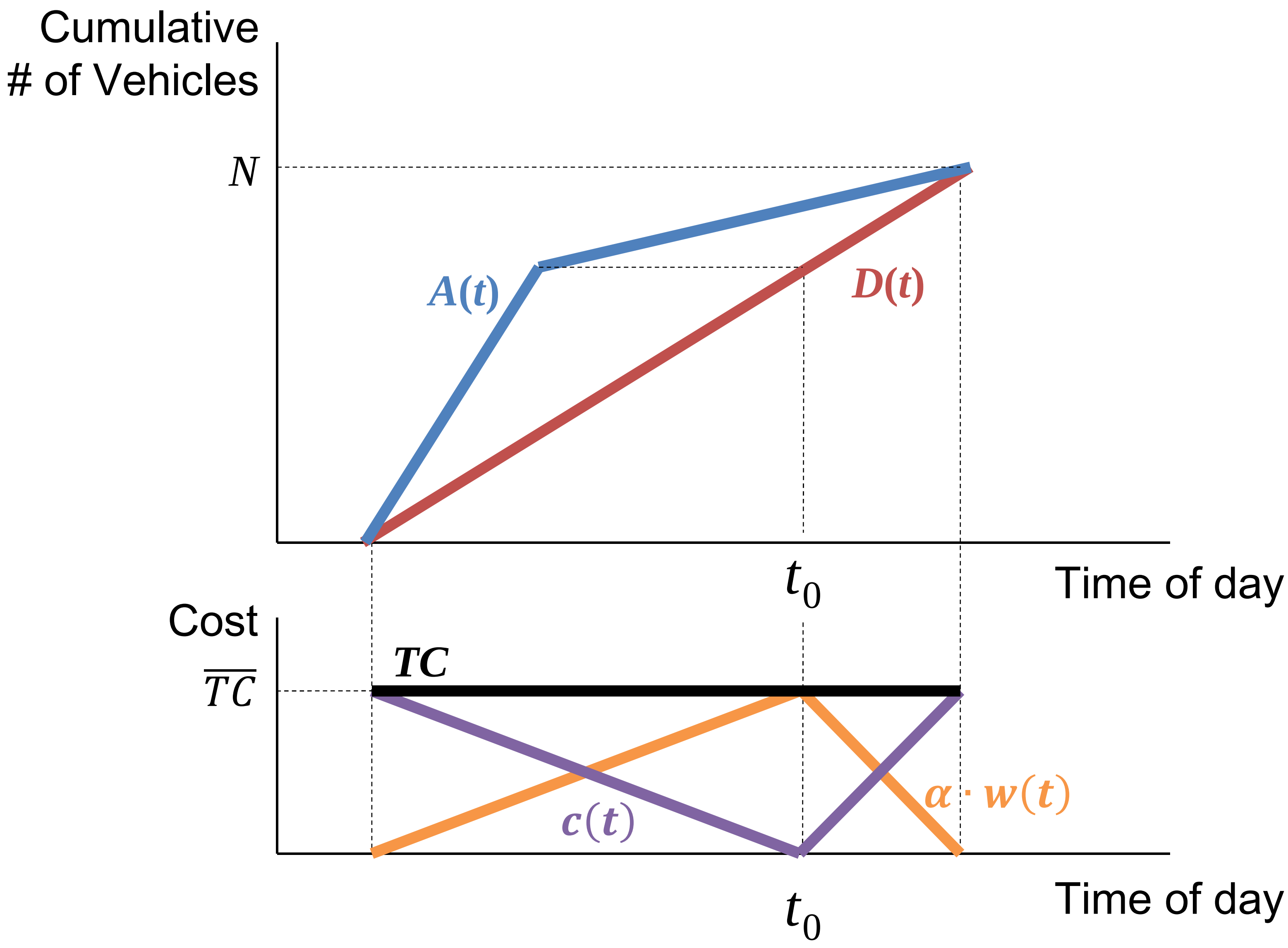
$$\sum_t q(t) = N$$

$$q(t) \geq 0$$

  – KKT conditions are equal to the original problem

*It is also possible to analytically solve the equilibrium problem.

Equilibrium solution without toll



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Equilibrium conditions with toll (TBP)

- Departure (arrival) time choice equilibrium condition;

$$\begin{cases} TC(t) = c(t) + p(t) = \overline{TC} & \text{if } \ q(t) > 0 \\ TC(t) = c(t) + p(t) \geq \overline{TC} & \text{if } \ q(t) = 0 \end{cases} \quad \forall t$$

- Demand-supply equilibrium condition;

$$\begin{cases} q(t) = \mu & \text{if } \ p(t) > 0 \\ q(t) \leq \mu & \text{if } \ p(t) = 0 \end{cases} \quad \forall t$$

- Flow conservation

$$\sum_t q(t) = N$$

- Non-negativity constraint

$$q(t) \geq 0 \quad \forall t$$

TC: travel cost
$\overline{TC}$: equilibrium travel cost
p: price of bottleneck permits
q: demand rate
μ: bottleneck capacity
p: price of TBP
N: total demand

Equivalent optimization problem

– Minimizing total schedule cost (monetary base)

$$min_{\{q(t)\}} \sum_t c(t) \cdot q(t)$$

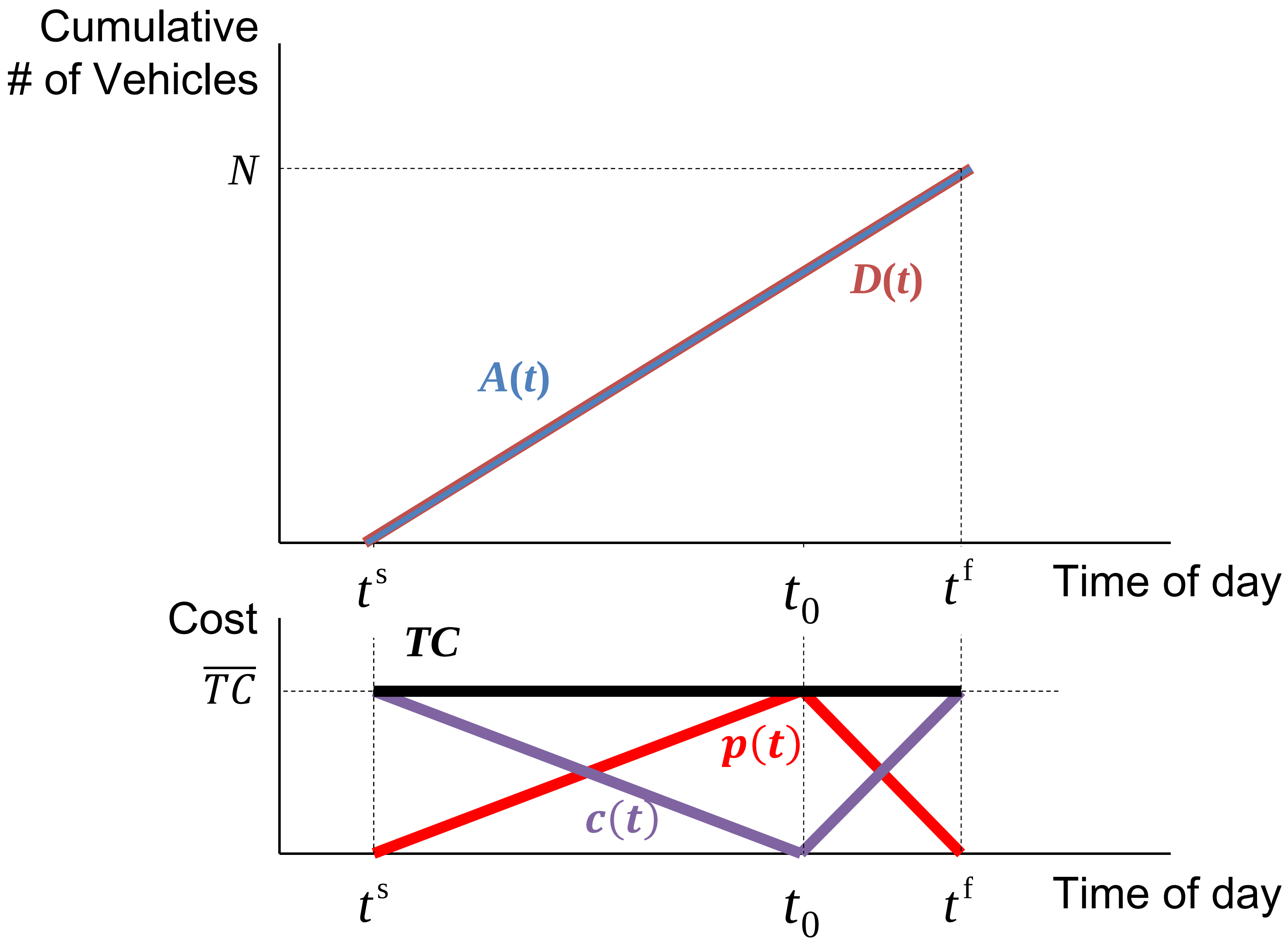
$$\text{s.t.} \quad q(t) \leq \mu$$

$$\sum_t q(t) = N$$

$$q(t) \geq 0$$

*It is also possible to analytically solve the equilibrium problem.

Equilibrium solution with TBP



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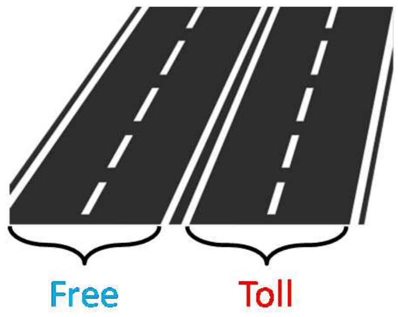
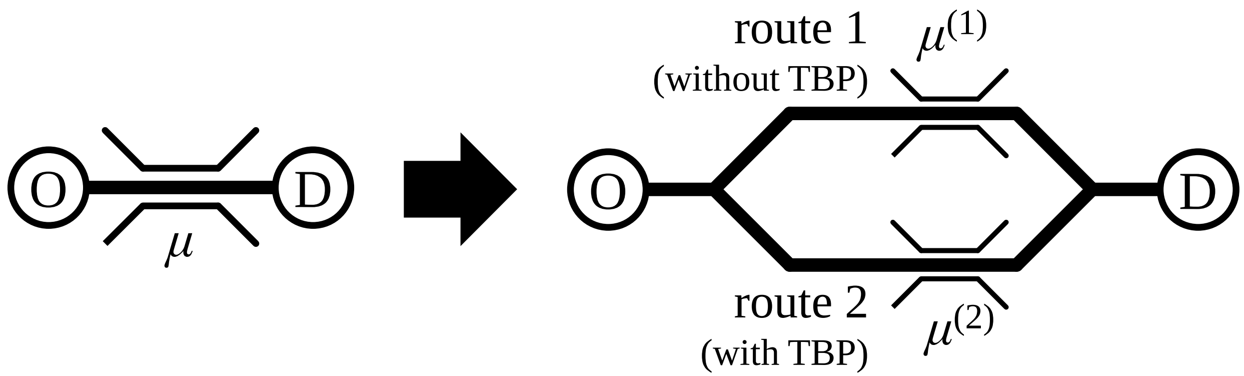
Departure time choice equilibrium problem with partial implementation of tradable bottleneck permits

- Problem including departure time choice and lane (route) choice problem
- Commuter' heterogeneity
  - Value of time (waiting delay cost)
  - Flexibility of job (schedule delay cost)





Equilibrium conditions

- Departure (arrival) time and route choice equilibrium condition;

$$\begin{cases} TC^{(r)}(t) = \overline{TC} & \text{if} \;\; q^{(r)}(t) > 0 \\ TC^{(r)}(t) \geq \overline{TC} & \text{if} \;\; q^{(r)}(t) = 0 \end{cases} \quad \forall r, t$$

- Demand-supply equilibrium condition in route 1;

$$\begin{cases} q^{(1)}(t) = \mu^{(1)} & \text{if} \;\; w(t) > 0 \\ q^{(1)}(t) \leq \mu^{(1)} & \text{if} \;\; w(t) = 0 \end{cases} \quad \forall t$$

- Demand-supply equilibrium condition in route 2;

$$\begin{cases} q^{(2)}(t) = \mu^{(2)} & \text{if} \;\; p(t) > 0 \\ q^{(2)}(t) \leq \mu^{(2)} & \text{if} \;\; p(t) = 0 \end{cases} \quad \forall t$$

- Flow conservation

$$\sum_t \sum_r q^{(r)}(t) = N$$

- Non-negativity constraint

$$q^{(r)}(t) \geq 0 \quad \forall r, t$$

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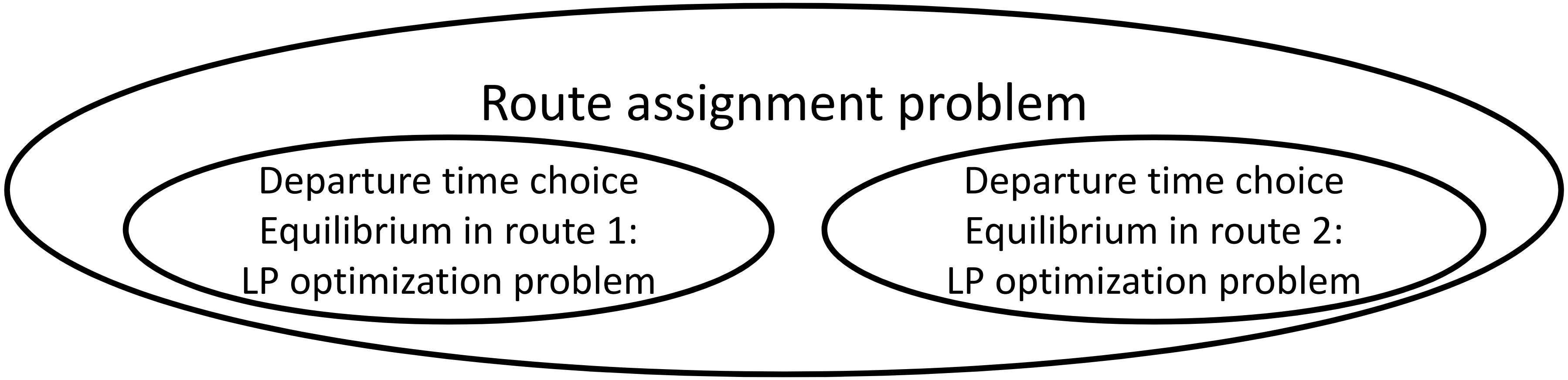
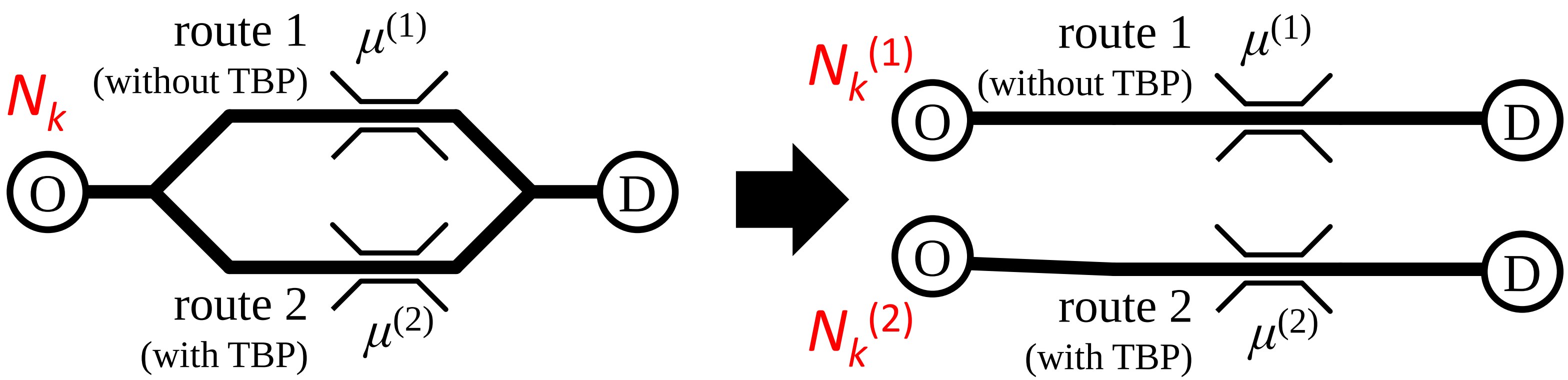
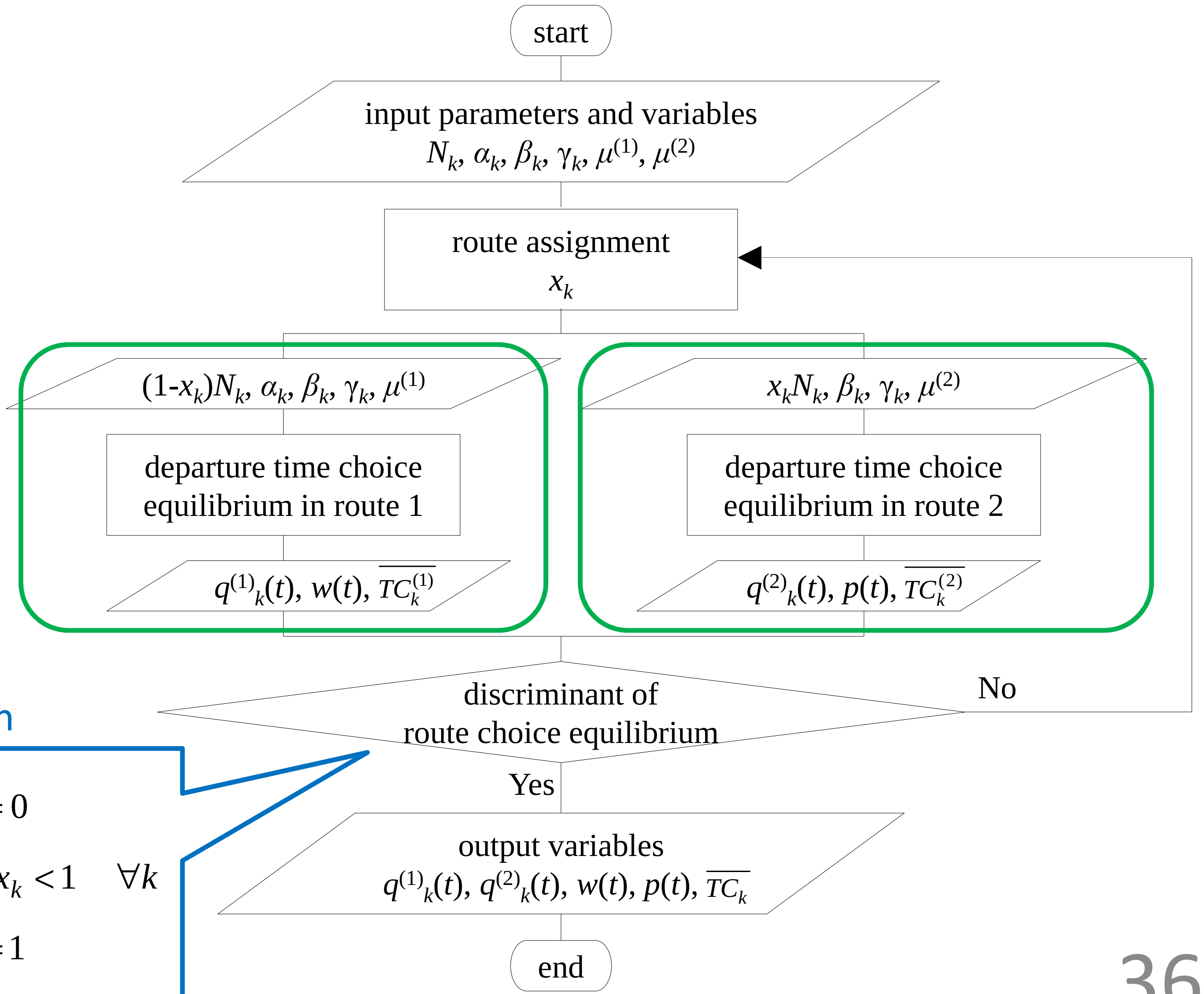
How to solve?

- No equivalent optimization problem
- Dividing the problem into 3 parts





Solution algorithm



Solving departure time choice equilibrium in each route

Route choice equilibrium condition

$$\begin{cases} \overline{TC}_k^{(1)} < \overline{TC}_k^{(2)} & \text{if} \quad x_k = 0 \\ \overline{TC}_k^{(1)} = \overline{TC}_k^{(2)} & \text{if} \quad 0 < x_k < 1 \\ \overline{TC}_k^{(1)} > \overline{TC}_k^{(2)} & \text{if} \quad x_k = 1 \end{cases} \quad \forall k$$

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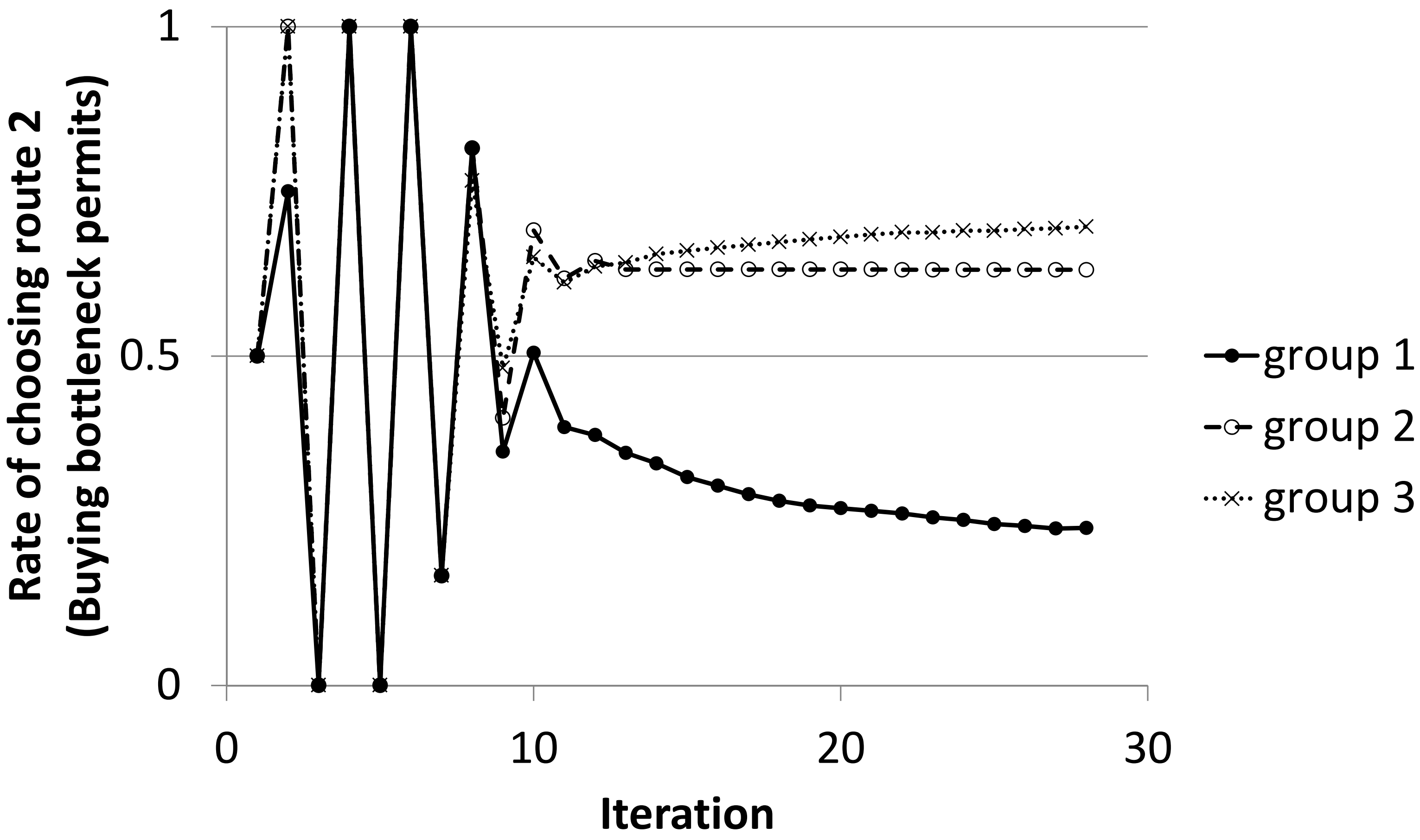
Input

- Bottleneck capacity: μ=100 veh/min
- Capacity without toll: $\mu^{(1)}$=40 veh/min
- Capacity with toll: $\mu^{(2)}$=60 veh/min

Model inputs

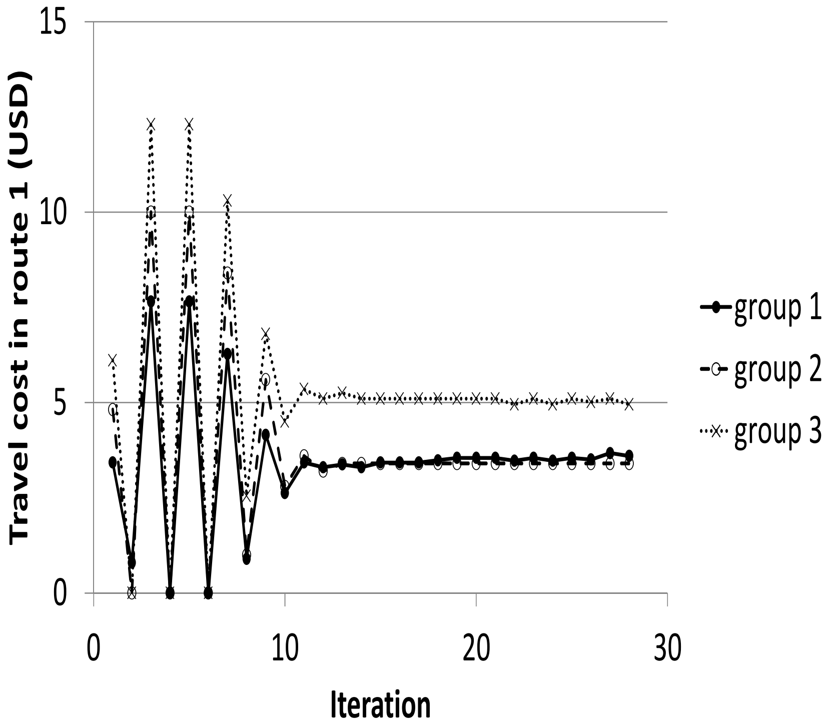
	N_k (veh)	t_0	α_k (USD/min)	β_k (USD/min)	γ_k (USD/min)
k=1	1000	8:50	0.25	0.20	0.30
k=2	2000	9:10	1.00	0.10	0.20
k=3	3000	9:00	0.50	0.15	0.40

Convergence to equilibrium

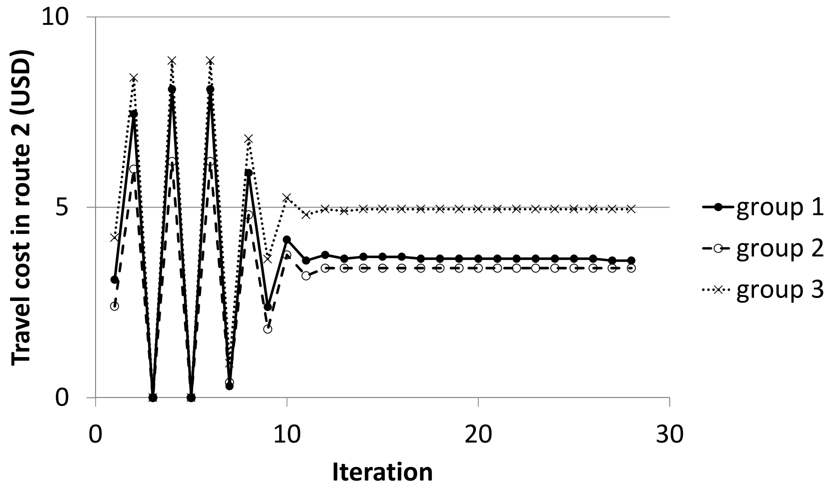


Convergence to equilibrium

Route 1 (without toll)

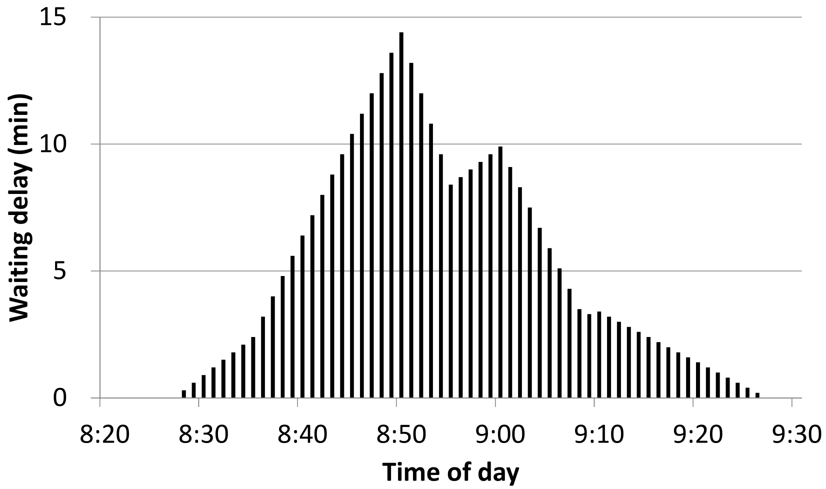


Route 2 (with toll)

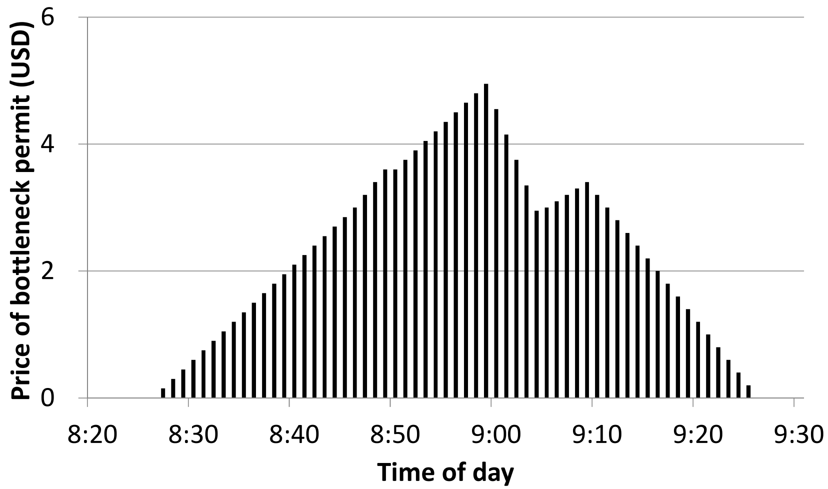


Equilibrium state

Waiting delay



Price of TBP



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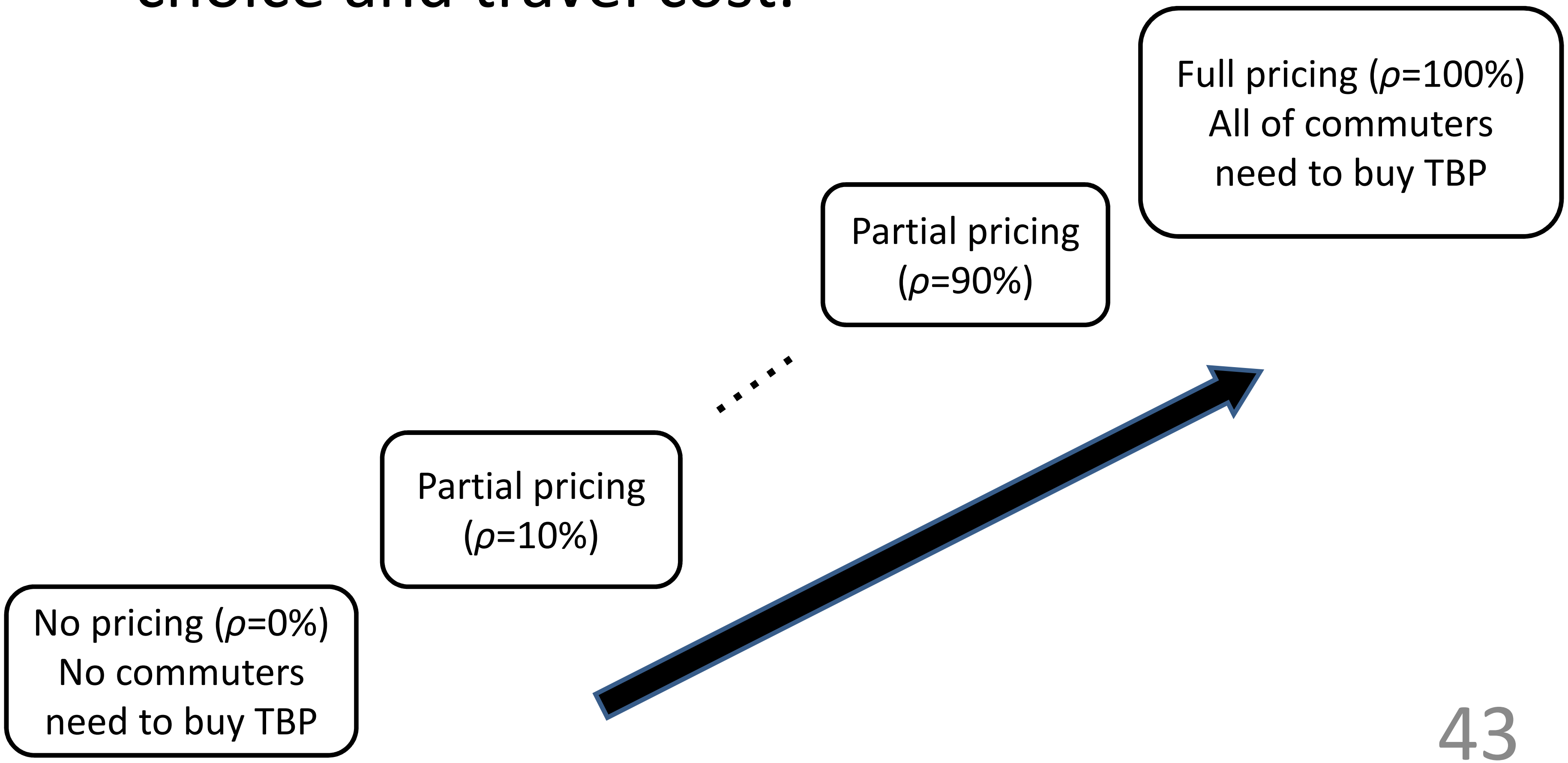
Welfare effect analysis

- Examining changes in commuter's route choice and travel cost.



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Welfare effect analysis
Input

- Commuter's heterogeneity in value of time and flexibility of job

(value of time) \ (Job flexibility)		Flexible δ=0.20	Inflexible δ=0.80
Poor	α=0.20	[Group 1]	[Group 2]
Rich	α=1.00	[Group 3]	[Group 4]

δ: flexibility of job ($\delta=\beta/\alpha$)
β/γ assumed constant

	q_k (veh)	t_0	α_k (USD/min)	δ	β_k (USD/min)	γ_k (USD/min)
k=1	1000	9:00	0.20	0.20	0.04	0.08
k=2	1000	9:00	0.20	0.80	0.16	0.32
k=3	1000	9:00	1.00	0.20	0.20	0.40
k=4	1000	9:00	1.00	0.80	0.80	1.60

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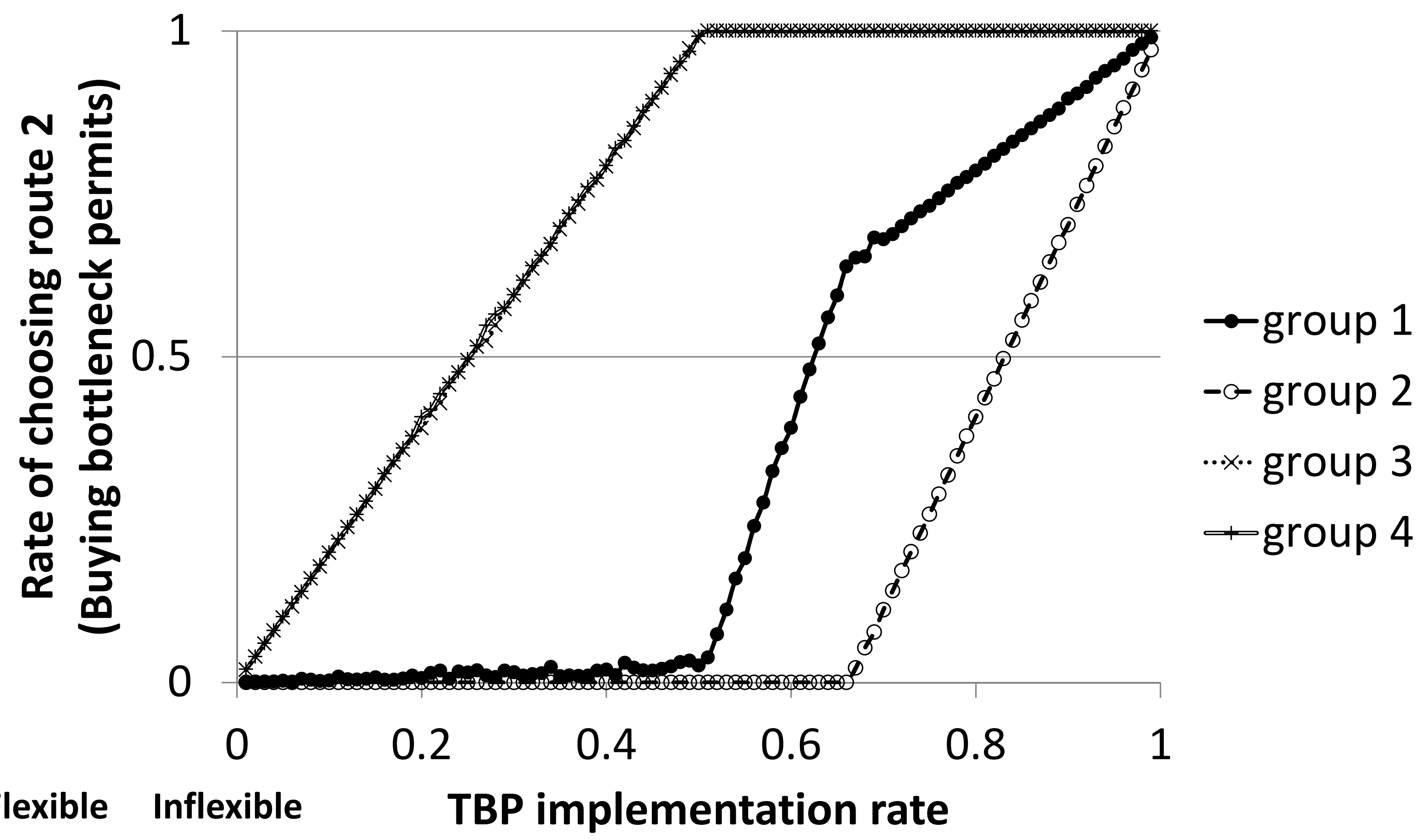
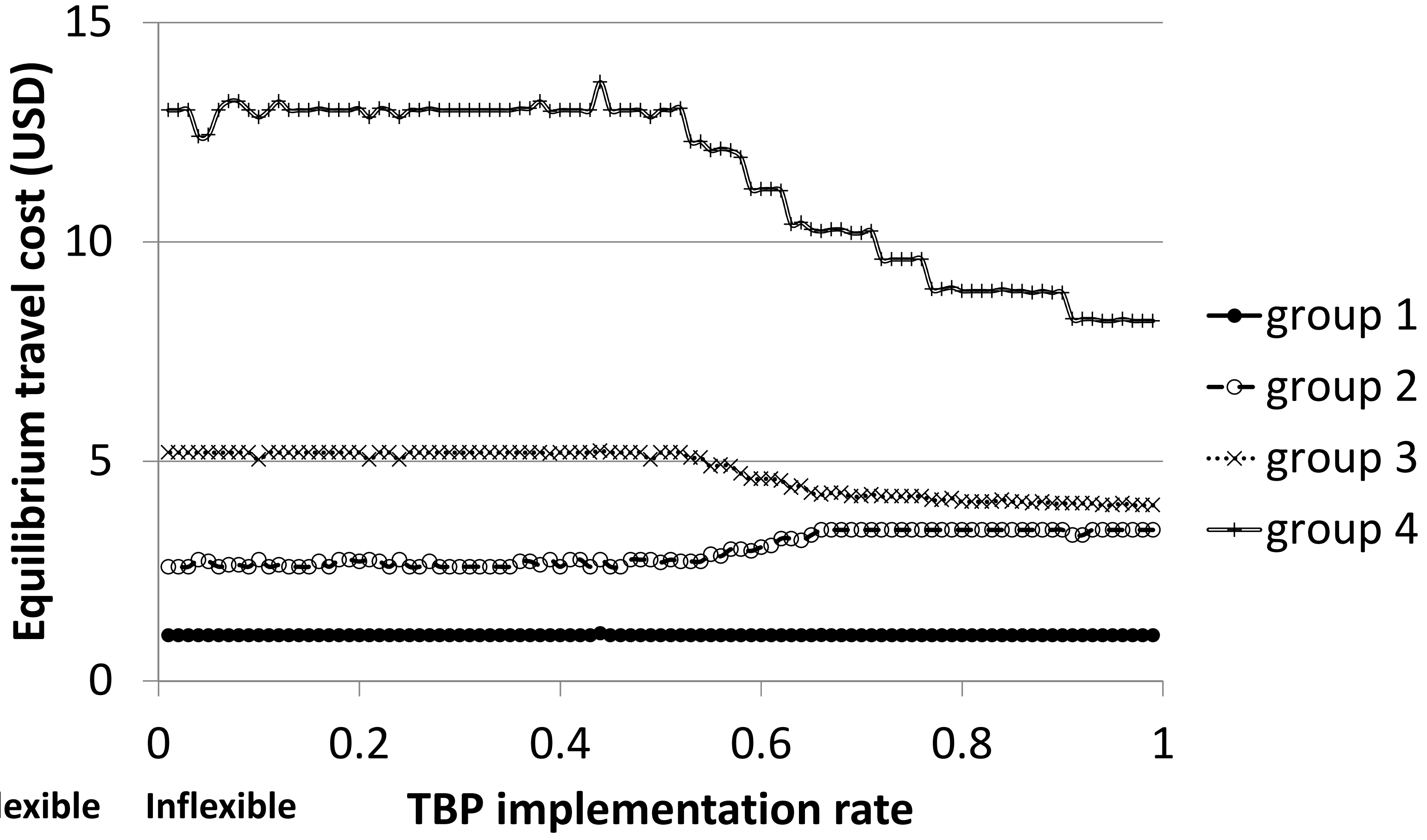
Change in route choice



	Flexible	Inflexible
Poor	group 1	group 2
Rich	group 3	group 4

Change in travel cost



	Flexible	Inflexible
Poor	group 1	group 2
Rich	group 3	group 4

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Conclusions

- Equilibrium problem under partial TBP implementation
- Welfare effect of partial congestion pricing scheme
- Possibility of congestion pricing scheme harming nobody even if the toll revenues are not refunded
- Future works
  - To prove uniqueness of equilibrium point

References

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