Departure time choice equilibrium problem with partial implementation of congestion pricing

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Problems of congestion

• wasting a lot of resources
  – Time
  – Energy
  – Mental health
  – …
How congestion occurs?

• caused by a concentration of demand
  – At a location
  – During short period
How to eliminate congestion?

• distributing the concentrated demand
  – At a location
  – During short period
Congestion management

• Information provision
  – Congestion length
  – Travel time

• Physical control (quantitative control)
  – Ramp metering
  – Advance booking

• Economic approach (pricing control)
  – Congestion charging
  – Tradable bottleneck permits
## Congestion management

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<th>Approach</th>
<th>Advantages</th>
<th>Limitations</th>
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<td>Information provision</td>
<td>Travel time can be reduced</td>
<td>Total travel time cannot be minimized</td>
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<tr>
<td>Quantitative control</td>
<td>Total travel time can be minimized</td>
<td>Social cost cannot be minimized</td>
</tr>
<tr>
<td>Pricing control</td>
<td>Social cost can be minimized</td>
<td>Equity problem; poor people may suffer a loss</td>
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</table>

- Focusing on pricing control (economic approach)
  - Tradable bottleneck permits (TBP) scheme
Tradable bottleneck permits scheme (Akamatsu et al. 2006)

1. Road administrators issue a right that allows a permit holder to pass through the bottleneck at a pre-specified time period (“bottleneck permits”).

2. A new auction market is established for bottleneck permits differentiated by a pre-specified time.
   - As a result of auction, toll is determined by each commuter’s willingness to pay (value-pricing)
   - Under the toll, social cost is minimized
Equity problem of congestion charging

- The Welfare Effects of Congestion Tolls with Heterogeneous Commuters (Arnott et al., 1994)
  - The poor suffer a loss
  - The rich get a benefit
Suggestion

• Applying congestion pricing only a portion of road (lanes).
  – Drivers can choose paying or not paying

• How much portion should be charging lanes?
• Don’t the poor suffer a loss?
How to evaluate pricing scheme?

• Comparing equilibrium states before/after pricing applied, we can know the effect of congestion pricing.
Objective

• To formulate the departure time choice equilibrium problem and to solve it when the TBP scheme is partially applied.
• To examine the welfare effect of partial congestion pricing with heterogeneous commuters.
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What is equilibrium?

- The most realizable state.
- At equilibrium, nobody can improve their travel cost by unilaterally changing their behavior.
What is equilibrium?

• Route choice equilibrium (Wardrop, 1952)
  – The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.

• Departure time choice equilibrium (Vickrey, 1969)
  – Times => costs
  – Routes => departure time
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Route choice equilibrium (Wardrop, 1952)

• Both routes are used
  – Travel time of two routes must be same

• Either of routes is used
  – Travel time of used road must be smaller or at most equal to that of the unused road.
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Departure time choice problem (Vickrey, 1969)

- Focusing on morning peak hour (commute) problem
- Employing bottleneck model, we simply describe traffic congestion one-to-one network with a single bottleneck

![Diagram of one-to-one network with a single bottleneck]
Departure time choice equilibrium
(Vickrey, 1969)
Departure time choice equilibrium
(Vickrey, 1969)

Travel cost / Demand rate

What is travel cost?

How to determine?

Time of day

Time without demand

Time with demand

Time without demand

Travel cost

Demand rate
Departure time choice equilibrium (Vickrey, 1969)

- Defining travel cost with tradeoff between waiting and schedule delays.

\[ TC = \begin{cases} 
  \alpha w - \beta (t - t_0) + \rho + \text{const} & \text{if early arrival} \\
  \alpha w + \gamma (t - t_0) + \rho + \text{const} & \text{if late arrival} 
\end{cases} \]

- **Value of time**
  - *TC*: travel cost
  - *w*: waiting delay
  - *α*: marginal cost of waiting delay
  - *ρ*: marginal cost of early arrival
  - *β*: marginal cost of late arrival
  - *γ*: marginal cost of late arrival
  - *t₀*: desired arrival time
  - *t*: actual arrival time

- **Waiting cost**
  - **Schedule cost**
  - **Toll cost**
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Equilibrium conditions without toll

- Departure (arrival) time choice equilibrium condition:
  \[
  \begin{cases}
  TC(t) = \alpha \cdot w(t) + c(t) = \overline{TC} & \text{if } q(t) > 0 \\
  TC(t) = \alpha \cdot w(t) + c(t) \geq \overline{TC} & \text{if } q(t) = 0
  \end{cases}
  \forall t
  \]

- Demand-supply equilibrium condition:
  \[
  \begin{cases}
  q(t) = \mu & \text{if } w(t) > 0 \\
  q(t) \leq \mu & \text{if } w(t) = 0
  \end{cases}
  \forall t
  \]

- Flow conservation
  \[
  \sum_t q(t) = N
  \]

- Non-negativity constraint
  \[
  q(t) \geq 0 \quad \forall t
  \]

\[TC\]: travel cost  \\
\[\overline{TC}\]: equilibrium travel cost  \\
\[q\]: demand rate  \\
\[\mu\]: bottleneck capacity  \\
\[w\]: waiting delay  \\
\[N\]: total demand
How to solve equilibrium problem?

• Solving the equivalent optimization problem (Iryo and Yoshii, 2007)

• Minimizing total schedule cost (waiting-time base)

\[
\min_{\{q(t)\}} \sum_t \frac{c(t)}{\alpha} \cdot q(t)
\]

s.t. \( q(t) \leq \mu \)

\[
\sum_t q(t) = N
\]

\( q(t) \geq 0 \)

– KKT conditions are equal to the original problem

*It is also possible to analytically solve the equilibrium problem.*
Equilibrium solution without toll

Cumulative 
# of Vehicles

Time of day

Cost

$\bar{TC}$

$T(t)$

$D(t)$

$A(t)$

$N$

$T_0$

$\alpha \cdot w(t)$

$c(t)$

Time of day

$T_0$
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Equilibrium conditions with toll (TBP)

• Departure (arrival) time choice equilibrium condition;
  \[
  \begin{aligned}
  T_C(t) &= c(t) + p(t) = \overline{T_C} & \text{if } q(t) > 0 \\
  T_C(t) &= c(t) + p(t) \geq \overline{T_C} & \text{if } q(t) = 0
  \end{aligned}
  \forall t
  \]

• Demand-supply equilibrium condition;
  \[
  \begin{aligned}
  q(t) &= \mu & \text{if } p(t) > 0 \\
  q(t) &\leq \mu & \text{if } p(t) = 0
  \end{aligned}
  \forall t
  \]

• Flow conservation
  \[
  \sum_t q(t) = N
  \]

• Non-negativity constraint
  \[
  q(t) \geq 0 \quad \forall t
  \]

\(T_C\): travel cost  
\(\overline{T_C}\): equilibrium travel cost  
\(p\): price of bottleneck permits  
\(q\): demand rate  
\(\mu\): bottleneck capacity  
\(p\): price of TBP  
\(N\): total demand
Equivalent optimization problem

- Minimizing total schedule cost (monetary base)

\[ \min_{\{q(t)\}} \sum_t c(t) \cdot q(t) \]

subject to

\[ q(t) \leq \mu \]
\[ \sum_t q(t) = N \]
\[ q(t) \geq 0 \]

*It is also possible to analytically solve the equilibrium problem.*
Equilibrium solution with TBP

Cumulative # of Vehicles

Cost

$\frac{TC}{N}$

Time of day

$t^s$

$t_0$

$t^f$

Time of day

$t^s$

$t_0$

$t^f$
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Departure time choice equilibrium problem with partial implementation of tradable bottleneck permits

• Problem including departure time choice and lane (route) choice problem

• Commuter’ heterogeneity
  - Value of time (waiting delay cost)
  - Flexibility of job (schedule delay cost)
Equilibrium conditions

- Departure (arrival) time and route choice equilibrium condition:
  \[\begin{align*}
  TC_r(t) = TC & \quad \text{if } q^r(t) > 0 \\
  TC_r(t) \geq TC & \quad \text{if } q^r(t) = 0
  \end{align*}\] \(\forall r, t\)

- Demand-supply equilibrium condition in route 1:
  \[\begin{align*}
  q^{(1)}(t) = \mu^{(1)} & \quad \text{if } w(t) > 0 \\
  q^{(1)}(t) \leq \mu^{(1)} & \quad \text{if } w(t) = 0
  \end{align*}\] \(\forall t\)

- Demand-supply equilibrium condition in route 2:
  \[\begin{align*}
  q^{(2)}(t) = \mu^{(2)} & \quad \text{if } p(t) > 0 \\
  q^{(2)}(t) \leq \mu^{(2)} & \quad \text{if } p(t) = 0
  \end{align*}\] \(\forall t\)

- Flow conservation
  \[\sum_t \sum_r q^{(r)}(t) = N\]

- Non-negativity constraint
  \[q^{(r)}(t) \geq 0 \quad \forall r, t\]
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How to solve?

- No equivalent optimization problem
- Dividing the problem into 3 parts

**Route assignment problem**

- Departure time choice
- Equilibrium in route 1: LP optimization problem
- Equilibrium in route 2: LP optimization problem
Solution algorithm

start

input parameters and variables
\( N_k, \alpha_k, \beta_k, \gamma_k, \mu^{(1)}, \mu^{(2)} \)

route assignment
\( x_k \)

Solving departure time choice equilibrium in each route

\[(1-x_k)N_k, \alpha_k, \beta_k, \gamma_k, \mu^{(1)}\]

departure time choice equilibrium in route 1

\( q^{(1)}_k(t), w(t), TC^{(1)}_k \)

\( x_kN_k, \beta_k, \gamma_k, \mu^{(2)}\)

departure time choice equilibrium in route 2

\( q^{(2)}_k(t), p(t), TC^{(2)}_k \)

discriminant of route choice equilibrium

\[
\begin{cases}
TC^{(1)}_k < TC^{(2)}_k & \text{if } x_k = 0 \\
TC^{(1)}_k = TC^{(2)}_k & \text{if } 0 < x_k < 1 \quad \forall k \\
TC^{(1)}_k > TC^{(2)}_k & \text{if } x_k = 1
\end{cases}
\]

output variables
\( q^{(1)}_k(t), q^{(2)}_k(t), w(t), p(t), TC_k \)

No

Yes

end
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Input

- Bottleneck capacity: $\mu=100$ veh/min
- Capacity without toll: $\mu^{(1)}=40$ veh/min
- Capacity with toll: $\mu^{(2)}=60$ veh/min

Model inputs

<table>
<thead>
<tr>
<th>$k$</th>
<th>$N_k$ (veh)</th>
<th>$t_0$</th>
<th>$\alpha_k$ (USD/min)</th>
<th>$\beta_k$ (USD/min)</th>
<th>$\gamma_k$ (USD/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>8:50</td>
<td>0.25</td>
<td>0.20</td>
<td>0.30</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>9:10</td>
<td>1.00</td>
<td>0.10</td>
<td>0.20</td>
</tr>
<tr>
<td>3</td>
<td>3000</td>
<td>9:00</td>
<td>0.50</td>
<td>0.15</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Convergence to equilibrium

Rate of choosing route 2
(Buying bottleneck permits)

Iteration

- group 1
- group 2
- group 3
Convergence to equilibrium

Route 1 (without toll)  Route 2 (with toll)
Equilibrium state

Waiting delay

Price of TBP
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Welfare effect analysis

- Examining changes in commuter’s route choice and travel cost.

No pricing ($\rho=0\%$)
No commuters need to buy TBP

Partial pricing ($\rho=10\%$)

Partial pricing ($\rho=90\%$)

Full pricing ($\rho=100\%$
All of commuters need to buy TBP
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Welfare effect analysis
Input

- Commuter’s heterogeneity in value of time and flexibility of job

<table>
<thead>
<tr>
<th>(value of time)</th>
<th>(Job flexibility)</th>
<th>Flexible</th>
<th>Inflexible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor</td>
<td>$\alpha=0.20$</td>
<td>[Group 1]</td>
<td>[Group 2]</td>
</tr>
<tr>
<td>Rich</td>
<td>$\alpha=1.00$</td>
<td>[Group 3]</td>
<td>[Group 4]</td>
</tr>
</tbody>
</table>

$\delta$: flexibility of job ($\delta = \frac{\beta}{\alpha}$)

$\beta/\gamma$ assumed constant

<table>
<thead>
<tr>
<th>$q_k$ (veh)</th>
<th>$t_0$</th>
<th>$\alpha_k$ (USD/min)</th>
<th>$\delta$</th>
<th>$\beta_k$ (USD/min)</th>
<th>$\gamma_k$ (USD/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k=1$</td>
<td>1000</td>
<td>9:00</td>
<td>0.20</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>$k=2$</td>
<td>1000</td>
<td>9:00</td>
<td>0.80</td>
<td>0.16</td>
<td>0.32</td>
</tr>
<tr>
<td>$k=3$</td>
<td>1000</td>
<td>9:00</td>
<td>1.00</td>
<td>0.20</td>
<td>0.40</td>
</tr>
<tr>
<td>$k=4$</td>
<td>1000</td>
<td>9:00</td>
<td>0.80</td>
<td>0.80</td>
<td>1.60</td>
</tr>
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Change in route choice

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<td>Poor</td>
<td>group 1</td>
</tr>
<tr>
<td></td>
<td>group 2</td>
</tr>
<tr>
<td>Rich</td>
<td>group 3</td>
</tr>
<tr>
<td></td>
<td>group 4</td>
</tr>
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### Change in travel cost

**Equilibrium travel cost (USD)**

- **Flexible:**
  - Poor: group 1
  - Rich: group 3

- **Inflexible:**
  - Poor: group 2
  - Rich: group 4

**TBP implementation rate**

![Graph showing change in travel cost for different flexible and inflexible groups with TBP implementation rate.]
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Conclusions

- Equilibrium problem under partial TBP implementation
- Welfare effect of partial congestion pricing scheme
- Possibility of congestion pricing scheme harming nobody even if the toll revenues are not refunded
- Future works
  - To prove uniqueness of equilibrium point
References


