Homo economicus vs the reptilian brain: bridging the gap between choice modelling, mathematical psychology and neuro-science

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Main contributors

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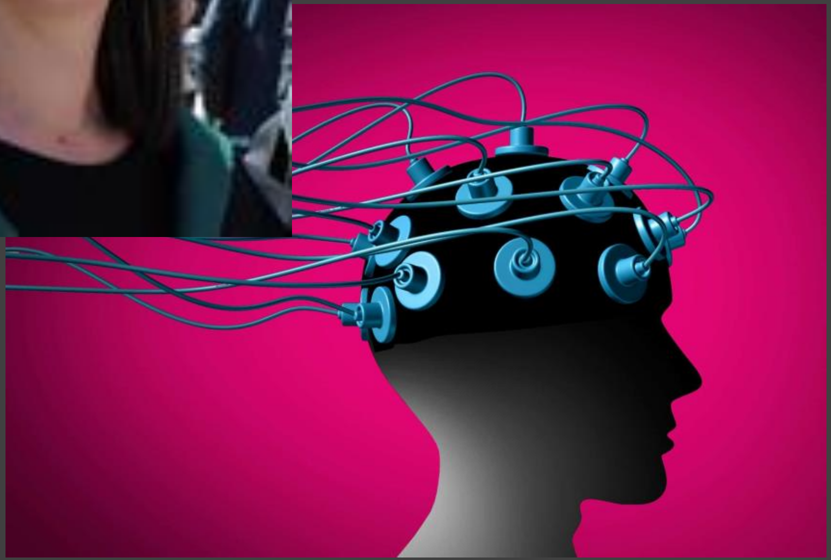
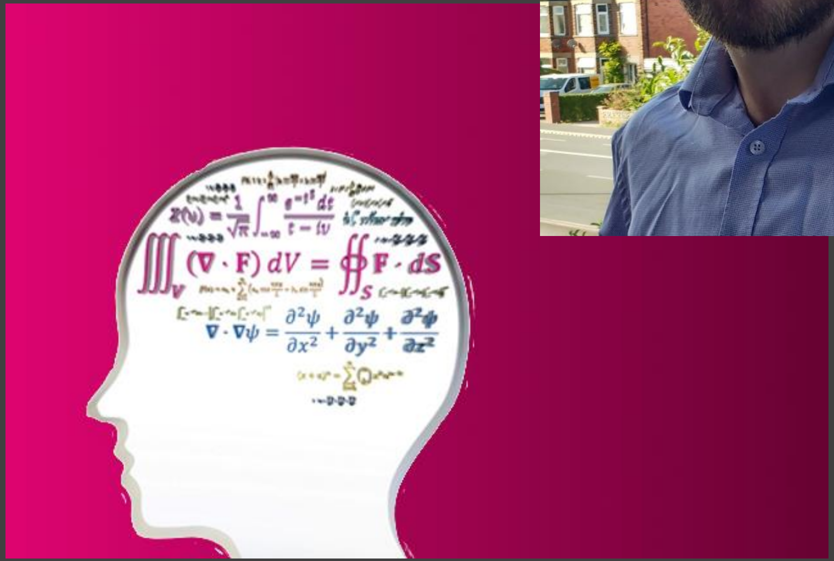
Brave new departures: neuroscience and mathematical psychology





Topics

- Insights from neuro-science
- Development of ideas from mathematical psychology
- Bringing the two together

Part 1: introduction and motivation

Which wine do you prefer?

	Hess Select	Cabernet Sauvignon	USA	2014	£14
	Campo Viejo	Rioja	Spain	2011	£10
	Santa Rita	Cabernet Sauvignon	Chile	2014	£10
	Tesco Australian	Shiraz	Australia	2017	£6

Systematic approach at the alternative level

	Hess Select	3	Cabernet Sauvignon	3	USA	3	2014	2	£14	-9	2
	Campo Viejo	2	Rioja	1	Spain	1	2011	6	£10	-6	4
	Santa Rita	1	Cabernet Sauvignon	3	Chile	2	2014	2	£10	-6	2
	Tesco Australian	0	Shiraz	2	Australia	0	2017	0	£6	-3	-1

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Penalise worst alternative at attribute level

	Hess Select	0	Cabernet Sauvignon	0	USA	0	2014	0	£14	-1	-1
	Campo Viejo	0	Rioja	-1	Spain	0	2011	0	£10	0	-1
	Santa Rita	0	Cabernet Sauvignon	0	Chile	0	2014	0	£10	0	0
	Tesco Australian	-1	Shiraz	0	Australia	-1	2017	-1	£6	0	-3

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Reward best alternative at attribute level

	Hess Select	1	Cabernet Sauvignon	1	USA	1	2014	0	£14	0	3
	Campo Viejo	0	Rioja	0	Spain	0	2011	1	£10	0	1
	Santa Rita	0	Cabernet Sauvignon	1	Chile	0	2014	0	£10	0	1
	Tesco Australian	0	Shiraz	0	Australia	0	2017	0	£6	1	1

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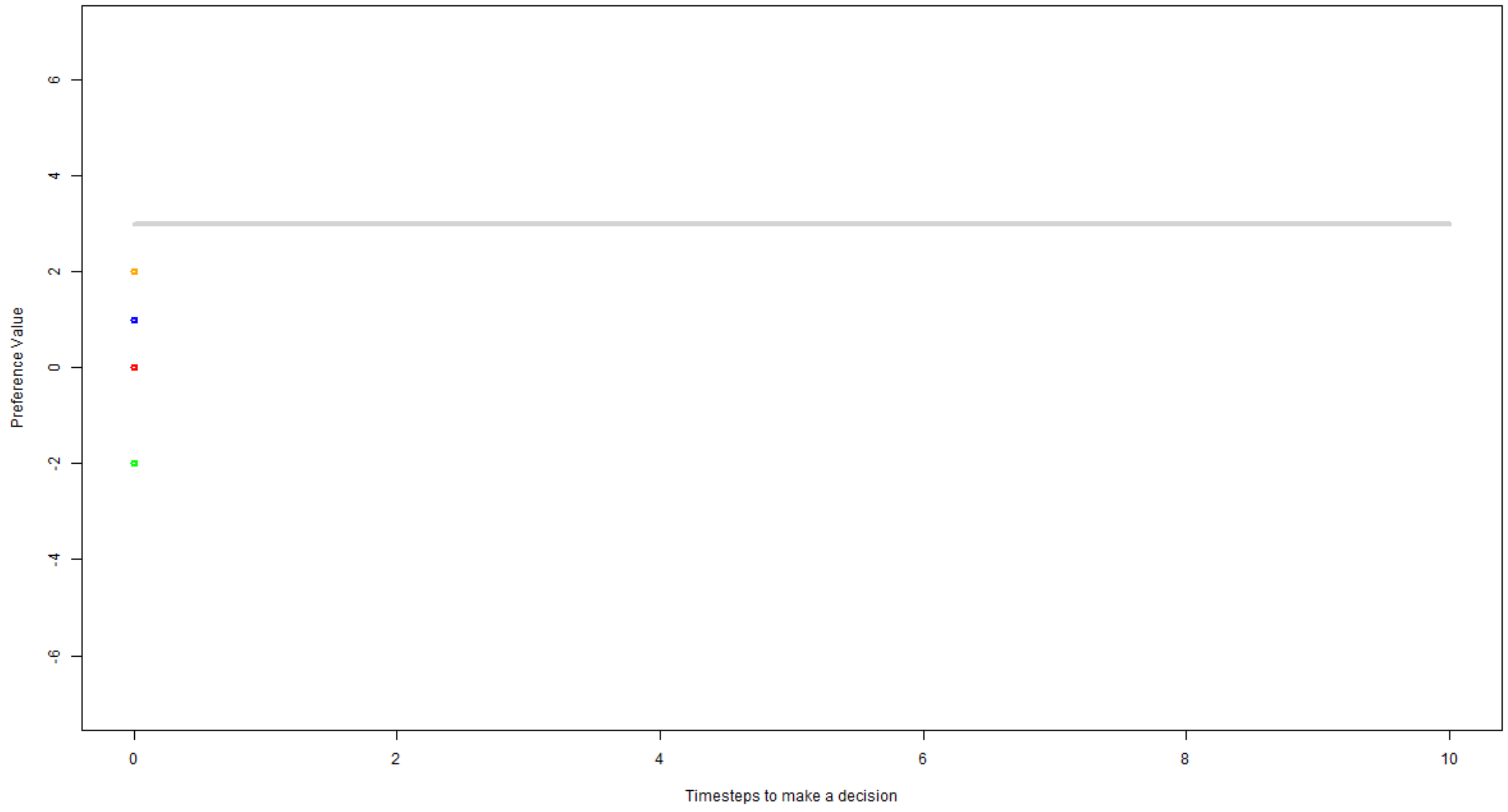
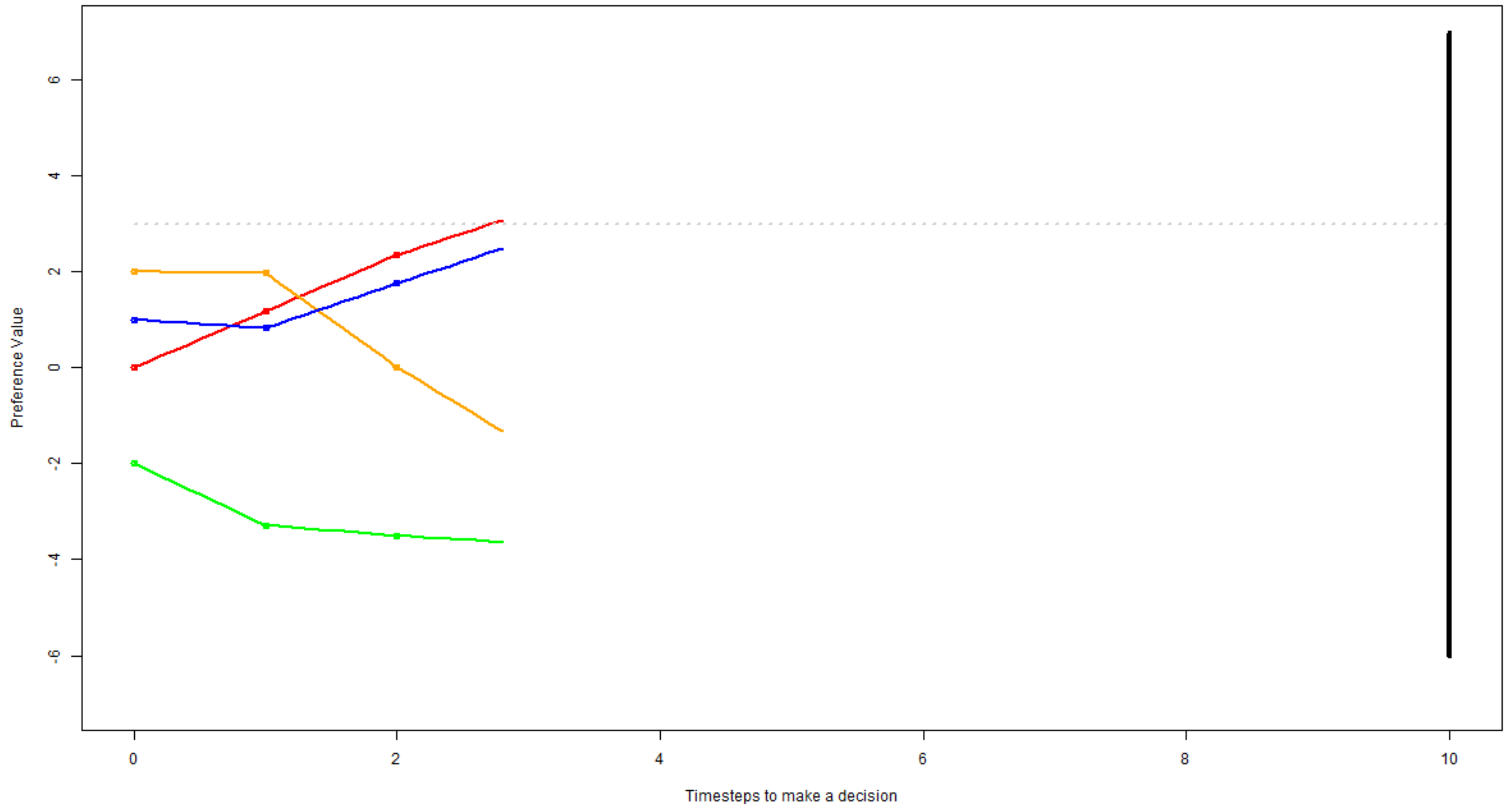
Assumptions made here about the choice process

- Order in which alternatives and attributes are evaluated does not influence outcome
  - contradicts ideas from quantum theory
- Constant “value” for alternatives
  - our models are “single shot” approaches
- Information considered in systematic way
  - randomness in the evaluation process is not modelled explicitly
- Very different in mathematical psychology
- Our viewpoint: if you’re willing to let go of RUM, you should consider “bigger” departures than e.g. RRM



Hess Select
Campo Viejo
Santa Rita
Tesco Australian Red

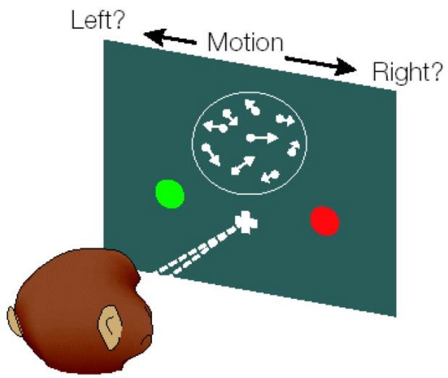
A dynamic approach with a preference threshold

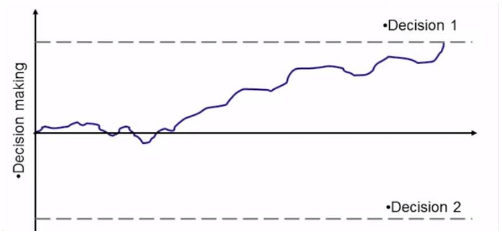


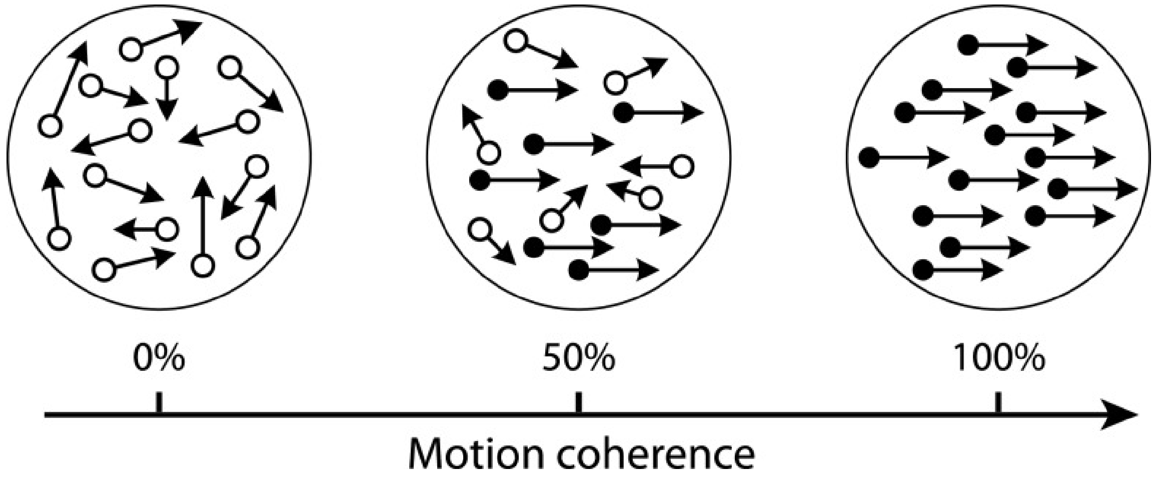
... but if we are given more time

Accumulation in the brain

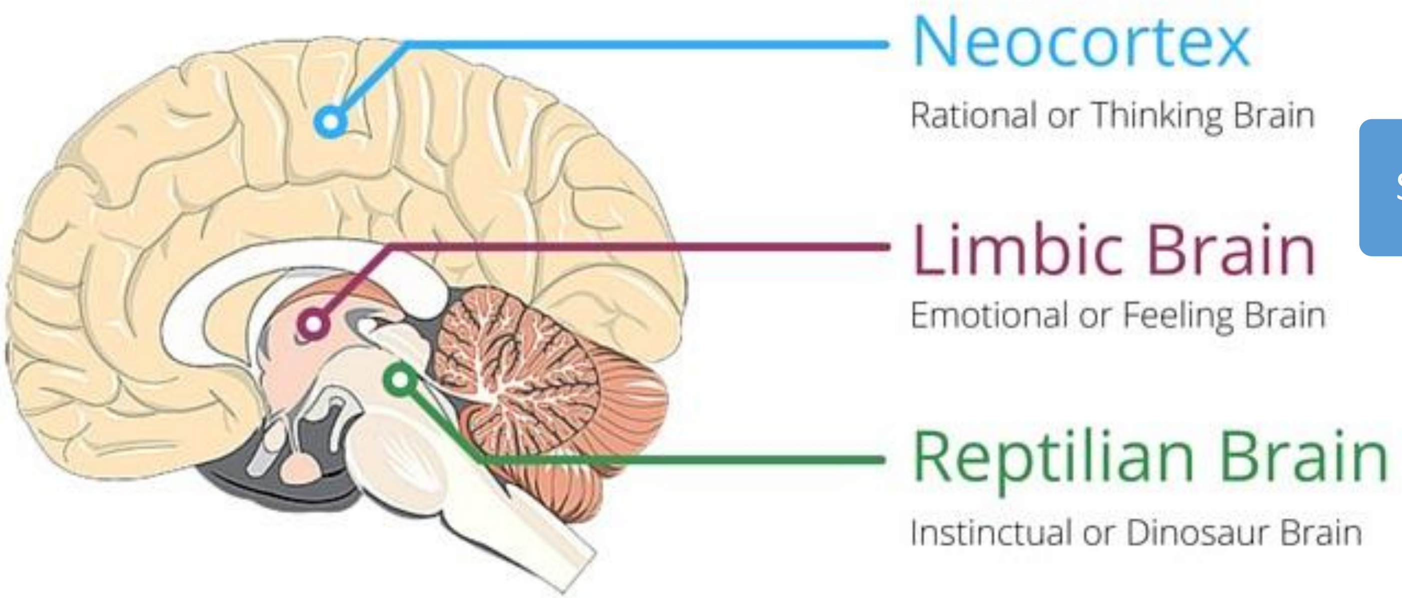
- Study by Britten et al. (1992)
- Monkeys required to indicate direction of the movement of dots on the screen by looking either to the left or right

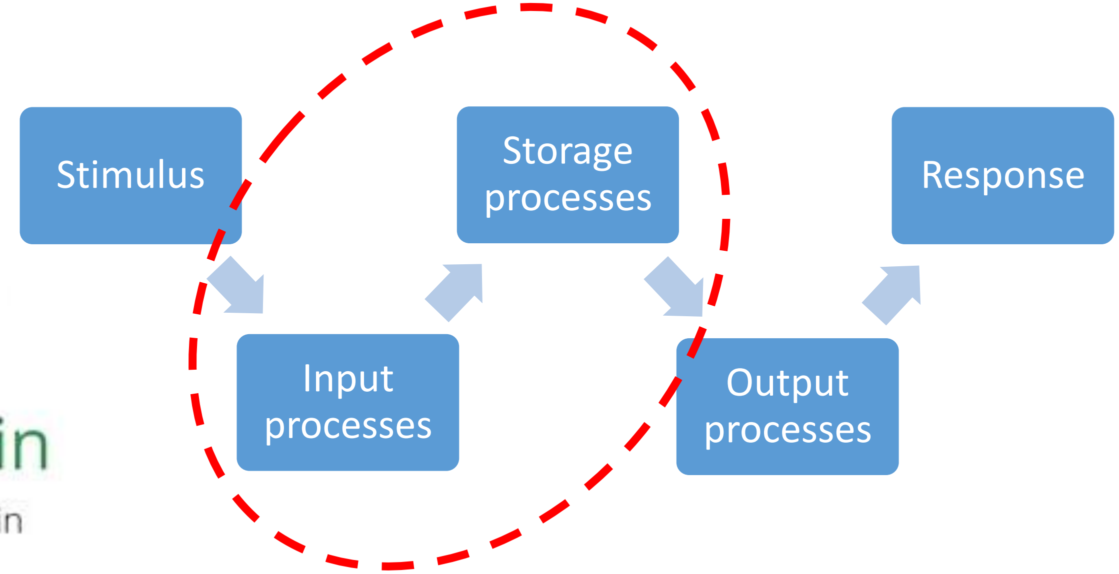






How does the brain process information?

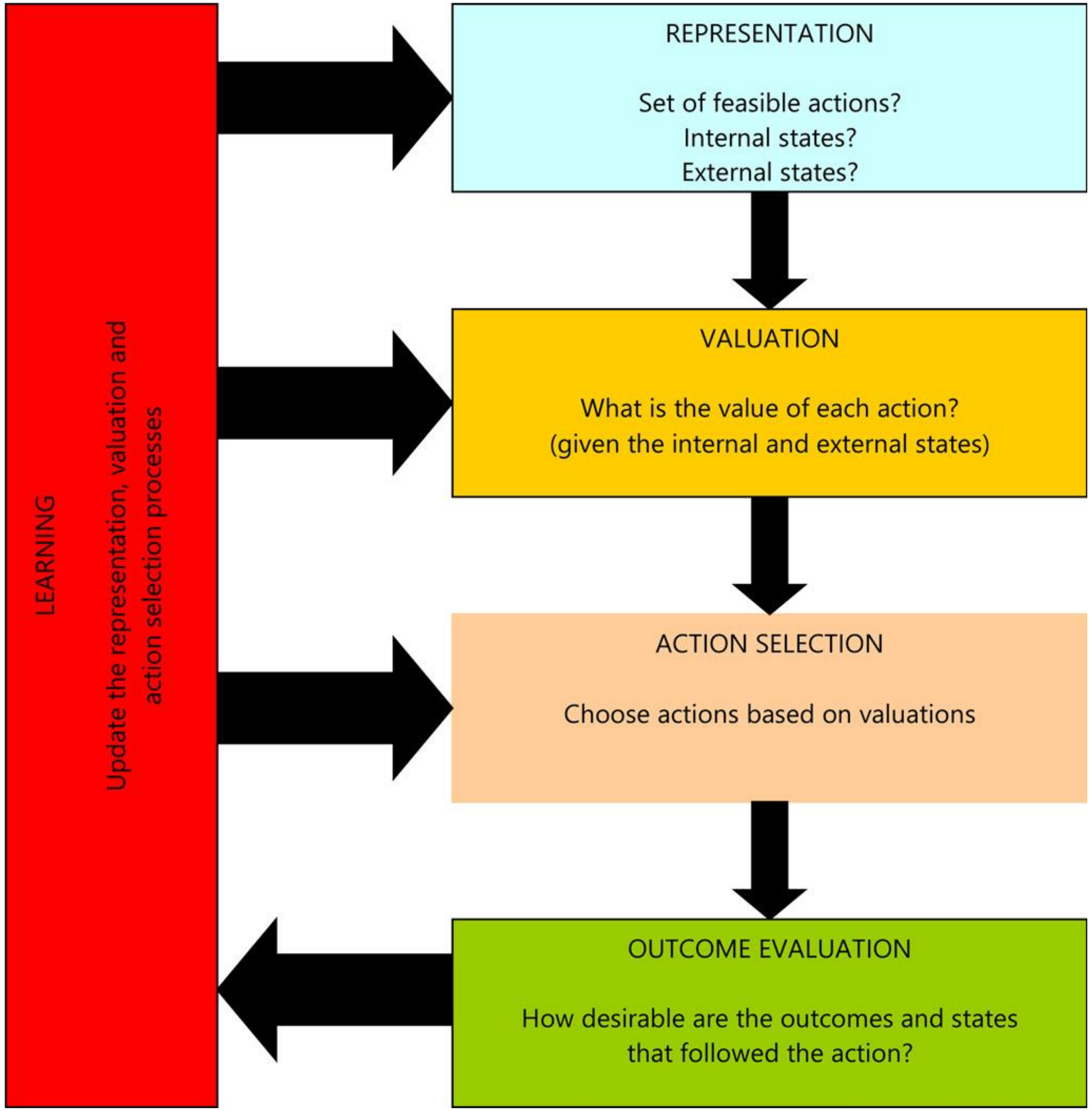




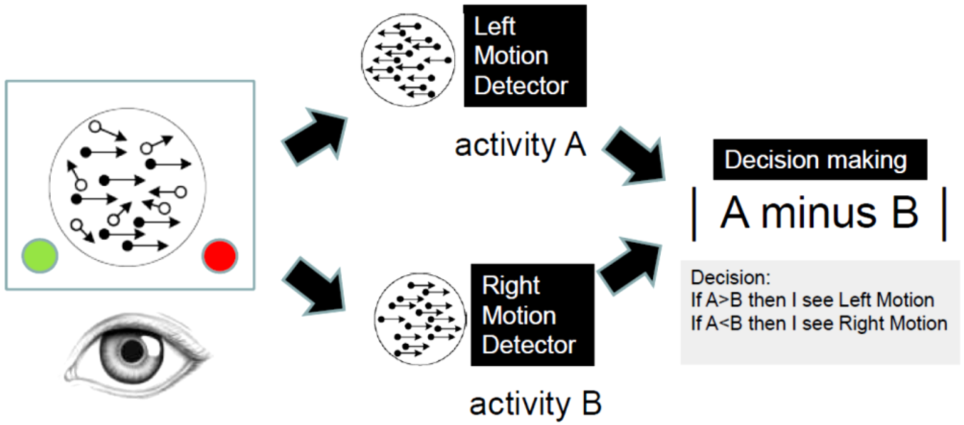
- Simplest circuit is a reflex
  - sensory stimulus directly triggers immediate motor response (milliseconds-seconds)
- Complex responses
  - brain integrates information from many circuits to generate response (can take months)

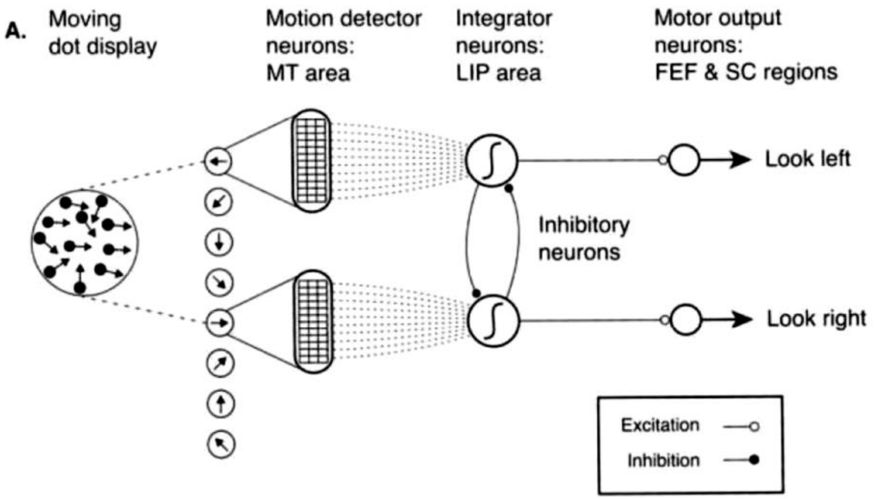
How does the brain make decisions?

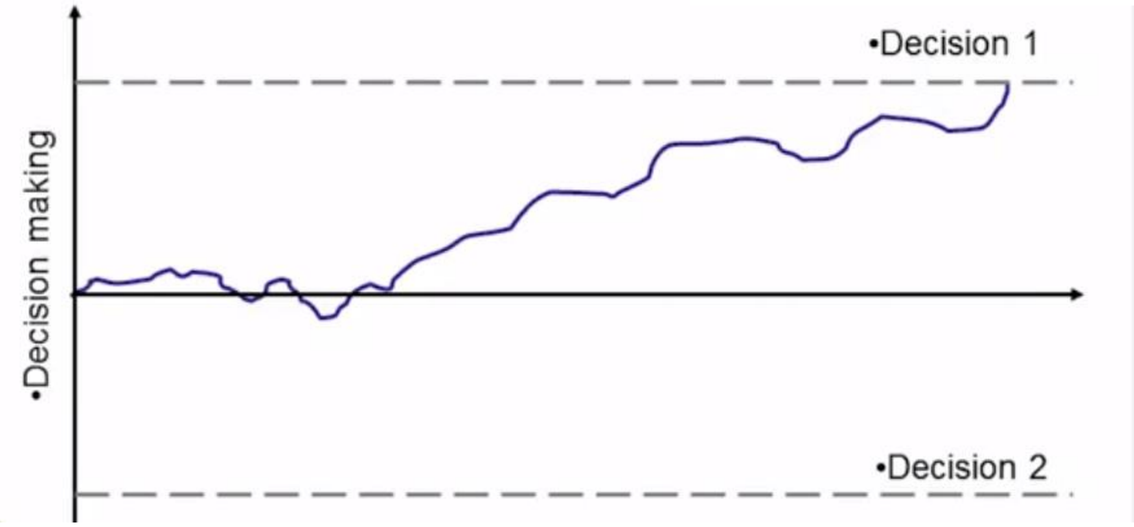
1) Representation of decision problem
   - internal or external state (e.g. hunger)
   - possible courses of action
2) Valuation of different actions based on analysis of anticipated cost and benefits
3) Based on valuation, one action is chosen
4) After implementation, action is assessed in terms of outcome desirability
   - feeds into learning to ensure quality of future decisions



Drift-diffusion model: gather evidence and make decision when threshold is reached







Issues with drift-diffusion model

- Only used for estimating reaction times for multiple 'correct/incorrect' decisions (e.g. dot motion perception)
- Multi-alternative context:
  - many simulations required
  - each simulating evolution of preference with given β
- Psychologists often run 1,000s of simulations to calculate probs for each alternative and each set of parameters
- Our motivation for looking at mathematical psychology!

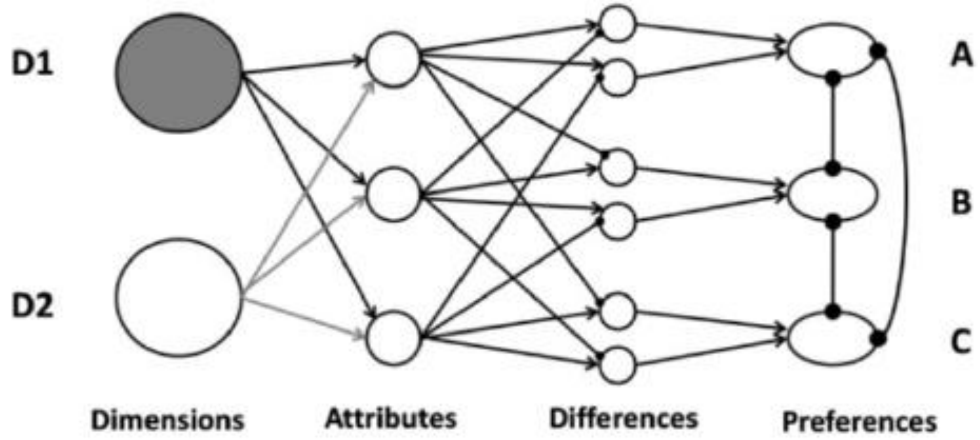
Part 2: adventures in mathematical psychology

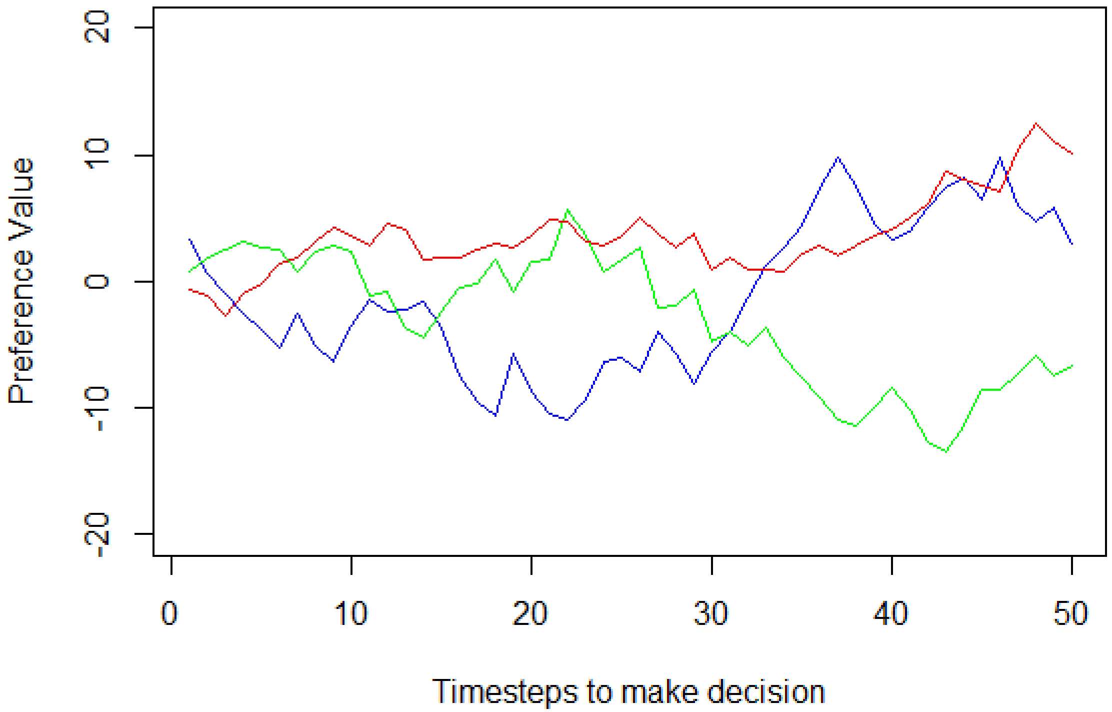
Background

- Mathematical psychology is a very active field of research
- Many similarities (especially in terms of interest) with choice modelling
- But they speak a different language!
- Also very little emphasis on translating models into practice
- Two key aims in our work:
  - Operationalising and improving models from mathematical psychology
  - Contrasts with more "typical" approaches
- Focus today only on Decision Field Theory (DFT)
  - Also worked with e.g. multi-attribute linear ballistic accumulator model (MLBA)

Models from mathematical psychology

- Dynamic models of preference creation
- Consider different attributes of the alternatives at different points in time





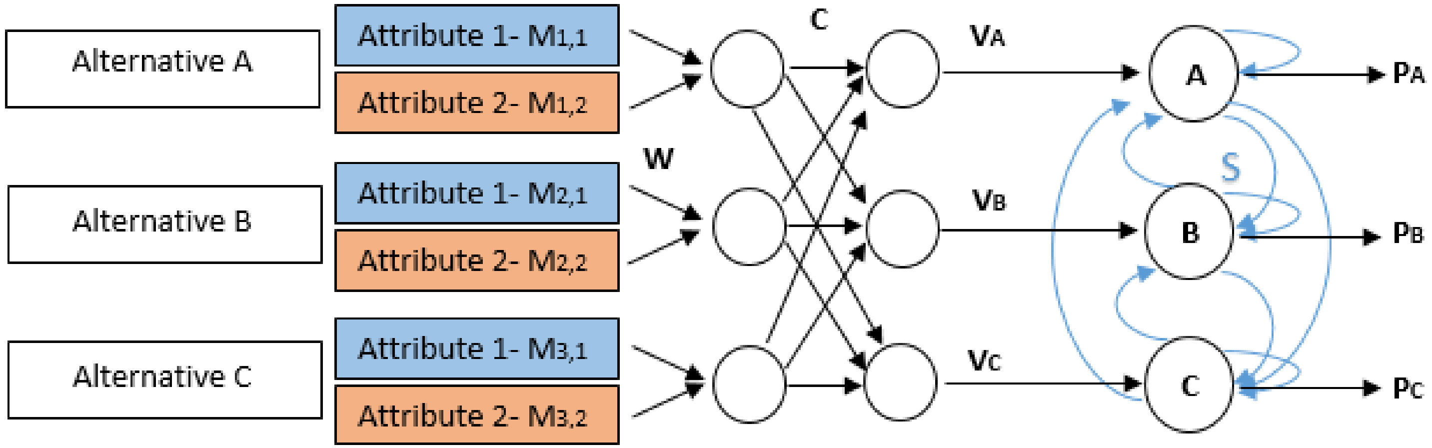
	train	bus	car
cost	high	low	average
time	low	high	average
environment	average	good	poor

Basic DFT equations

$$P_t = S \cdot P_{t-1} + V_t$$

$$V_t = C \cdot M \cdot W_t + \varepsilon_t$$

- Preference vector P_t at a given timestep t, updates over time
  - Preference vector ≠ probability
- S = feedback matrix
- P_0 = initial preference vector
- V_t = valence vector (how much preferences update at t)
- M = attribute matrix
- C = contrast matrix (to centre the values around zero)
- W_t = weights vector
- ε_t = error (drawn from a normal distribution with mean zero and a variance which is estimated)



DFT feedback matrix

$$S = I - \phi_2 \times exp(-\phi_1 \times D^2)$$

- $\emptyset_1$ = sensitivity parameter – how much do similar alternatives compete?
  - 0.05 in this example
- $\emptyset_2$ = memory parameter – is initial or later information more important?
  - 0.1 in this example, so later information is slightly more important
- D = distance in attribute space between alternatives, sum of squared differences across attributes
  - Hess and Santa Rita wines are most similar therefore compete most

S	Hess	Campo	Santa	Tesco
Hess	0.900	-0.050	-0.074	-0.030
Campo	-0.050	0.900	-0.061	-0.045
Santa	-0.074	-0.061	0.900	-0.067
Tesco	-0.030	-0.045	-0.067	0.900

Valence calculation and updating of P

Attribute matrix M	name	type	country	year	cost	P0	P1
Hess Select	3	3	3	1	-3	0.0	-1.0
Campo Viejo	2	1	1	3	-2	2.0	2.4
Santa Rita	1	3	2	1	-2	-1.0	-1.1
Tesco Australian	0	2	0	0	-1	-2.0	-0.4

$$P_t = S \cdot P_{t-1} + V_t$$

$$V_t = C \cdot M \cdot W_t + \varepsilon_t$$

$$E[V_t] = \boldsymbol{\mu} = C \cdot M \cdot W$$

W	0.125	0.125	0.125	0.25	0.375
ε	1				

In time period t=1, cost is the attribute attended to

$$W_1 = [0,0,0,0,1]'$$

Draws with mean 0, s.d =1.

$$\varepsilon_1 = [0.3,0.5,-0.2,0.1]'$$

C	1	-1/3	-1/3	-1/3
	-1/3	1	-1/3	-1/3
	-1/3	-1/3	1	-1/3
	-1/3	-1/3	-1/3	1

$$V_1 = [-1.3,0,0,1.3]' + \varepsilon_1$$

$$V_1 = [-1,0.5,-0.2,1.4]'$$

Estimated parameters

$$P_1 = S \cdot P_0 + V_1$$

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DFT probabilities

- P_t converges to a multivariate normal distribution, e.g. with 3 alternatives:

$$Pr[P_t[A] - P_t[B] > 0 \cap P_t[A] - P_t[C] > 0]$$

$$= \int_{X>0} exp\left[-(X-\Gamma)'\Lambda^{-1}(X-\Gamma)/2\right]/(2\pi\,|\Lambda|^{0.5})dX$$

with $X = [P_t[A] - P_t[B], P_t[A] - P_t[C]]'$, $\Gamma = L\xi_t$, $\Lambda = L\Omega_t L'$ and

$$L = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

- Need mean and covariance

$$E[P_t] = \xi_t = \sum_{k=0}^{t-1} S^k \cdot \mu + S^t \cdot P_0$$

$$= (I-S)^{-1}(I-S^t) \cdot \mu + S^t \cdot P_0$$

$$Cov[P_t] = \Omega_t = Cov\left[\sum_{k=0}^{t-1} S^k \cdot V_{t-k} + S^t \cdot P_0\right]$$

Key limitation in existing DFT work

- Mathematical psychologists:
  - 'computationally dissatisfying' process of summing over timesteps (and hence powers of *S*) to get the covariance matrix

$$Cov[P_t] = \Omega_t = Cov\left[\sum_{k=0}^{t-1} S^k \cdot V_{t-k} + S^t \cdot P_0\right]$$

- They avoid this by by assuming that $t \rightarrow \infty$
- This loses the timestep element of the model!
- Possible to solve this problem and calculate probability at given timestep

$$\overline{S\Phi S'} = Z\overline{\Phi} \tag{13}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix} C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{bmatrix} \tag{14}$$

$$[\overline{D}]_{(j-1)n+i} = \sum_{k=1}^{n}\sum_{l=1}^{n} [a_{il} b_{lk} c_{kj}] \tag{15}$$

$$Z = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1n^2} \\ z_{21} & z_{22} & \dots & z_{2n^2} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n^21} & z_{n^22} & \dots & z_{n^2n^2} \end{bmatrix} \overline{B} = \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \\ b_{12} \\ \vdots \\ b_{nn} \end{bmatrix} \tag{16}$$

$$[Z\overline{B}]_{(j-1)n+i} = \sum_{k=1}^{n}\sum_{l=1}^{n} [z_{(j-1)n+i,(k-1)n+l} b_{lk}] \tag{17}$$

$$z_{(j-1)n+i,(k-1)n+l} = a_{il}\, c_{kj}.$$

$A = S, B = \Phi$ and $C = S'$

$$\overline{S^n \Phi S^{n\prime}} = Z^n \overline{\Phi} \tag{18}$$

$$\overline{S^{n+1} \Phi S^{n+1\prime}} = Z^{n+1} \overline{\Phi} \tag{19}$$

$A^n = X$, $C^n = Y$ and $Z^n = W$

$$[A^{n+1} B C^{n+1}]_{ij} = [AXBCY]_{ij} = \sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{r=1}^{n}\sum_{s=1}^{n} [a_{ir}\, x_{rl}\, b_{lk}\, y_{ks}\, c_{sj}] \tag{20}$$

$$\Rightarrow \overline{[AXBCY]}_{(j-1)n+i} = \sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{r=1}^{n}\sum_{s=1}^{n} [a_{ir}\, x_{rl}\, b_{lk}\, y_{ks}\, c_{sj}] \tag{21}$$

$$z_{(j-1)n+i,(k-1)n+l} = a_{il}\, c_{kj} \tag{22a}$$

$$w_{(j-1)n+i,(k-1)n+l} = x_{il}\, y_{kj} \tag{22b}$$

$$[ZW]_{uv} = \sum_{r=1}^{n}\sum_{s=1}^{n} [z_{u,(s-1)n+r}\, w_{(s-1)n+r,v}] \tag{23}$$

$$[Z\overline{B}]_i = \sum_{k=1}^{n}\sum_{l=1}^{n} [z_{i,(k-1)n+l} b_{lk}] \tag{24}$$

$$[ZW\overline{B}]_i = \sum_{k=1}^{n}\sum_{l=1}^{n} \left[[ZW]_{i,(k-1)n+l}\, b_{lk}\right] \tag{25}$$

$$[ZW\overline{B}]_i = \sum_{k=1}^{n}\sum_{l=1}^{n} \left[\sum_{r=1}^{n}\sum_{s=1}^{n} [z_{i,(s-1)n+r}\, w_{(s-1)n+r,(k-1)n+l}]\, b_{lk}\right] \tag{26}$$

$$[ZW\overline{B}]_{(j-1)n+i} = \sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{r=1}^{n}\sum_{s=1}^{n} [z_{(j-1)n+i,(s-1)n+r}\, w_{(s-1)n+r,(k-1)n+l}\, b_{lk}] \tag{27a}$$

$$= \sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{r=1}^{n}\sum_{s=1}^{n} [a_{ir}\, c_{sj}\, x_{rl}\, y_{ks}\, b_{lk}] \tag{27b}$$

$$= \overline{[AXBCY]}_{(j-1)n+i} \tag{27c}$$

$$Cov[P_t] = \Omega_t = \sum_{k=0}^{t-1} \left[S^k \cdot \Phi \cdot S^{k'}\right] \tag{28a}$$

$$= \sum_{k=0}^{t-1} \left[Z^k \cdot \overline{\Phi}\right] \tag{28b}$$

$$= (I-Z)^{-1}\,(I-Z^t)\,\overline{\Phi} \tag{28c}$$

Calculating probabilities

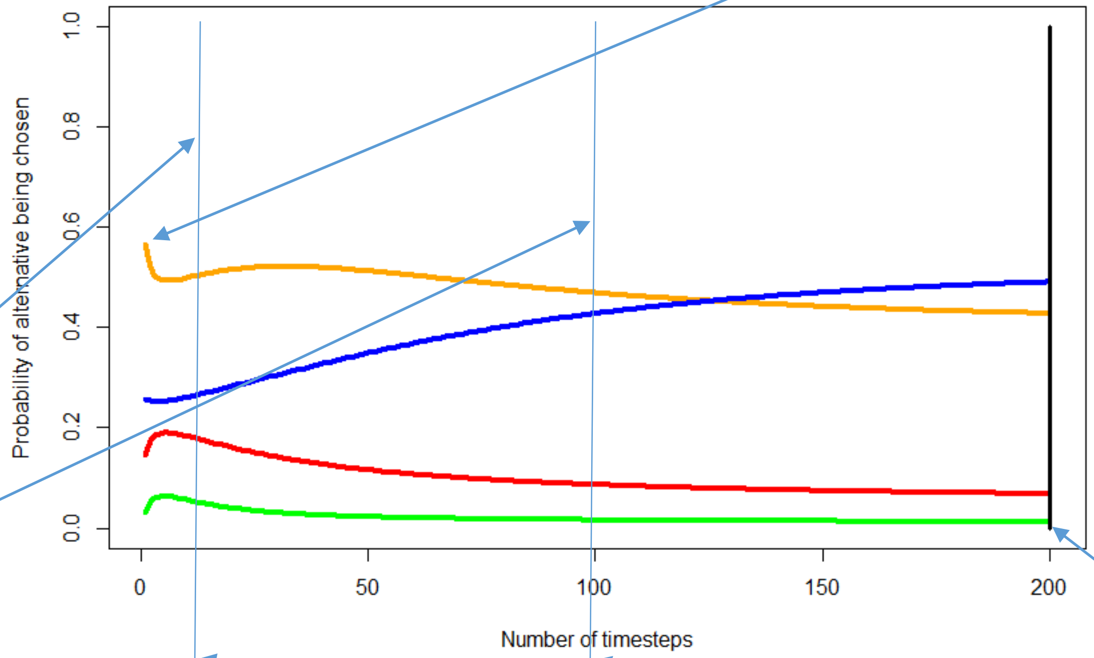
This results in the simplification:

$$Cov[P_t] = \Omega_t = \sum_{k=0}^{t-1} \left[S^k \cdot \Phi \cdot S^{k'}\right]$$

$$= \sum_{k=0}^{t-1} \left[Z^k \cdot \overline{\Phi}\right]$$

$$= (I - Z)^{-1}\,(I - Z^t)\,\overline{\Phi}$$

New method allows for influence of initial preferences



Previously was only ever calculated at (t=∞), when values stabilised

	P(t=10)	P(t=100)
Hess Select	0.184	0.086
Campo Viejo	0.499	0.469
Santa Rita	0.261	0.428
Tesco Australian	0.056	0.016

Summary of DFT changes:

	DFT-2014	DFT-2018
Model Fit		Always at least as good as DFT-2014
$E[P_t]$	$E[P_\infty] = (I - S)^{-1} \cdot \mu$	$E[P_t] = (I - S)^{-1}(I - S^t) \cdot \mu + S^t \cdot P_0$
$Cov[P_t]$	$\overline{Cov[P_\infty]} = (1 - Z)^{-1}\overline{\Phi}$	$Cov[P_t] = (I - Z)^{-1}(I - Z^t)\overline{\Phi}$
Timesteps	Assumed to be infinite	Can be related to, for example, response time
Initial Pref	Cannot be included	Explicitly captured
Memory	Must deteriorate over time	Can inflate or deteriorate
Parameters	x	x+1

Danish value of time dataset

- 2 alternatives described by cost and time:

- MNL : LL = -2,301.53
- Non-linear MNL : LL = -2,212.10
- DFT : LL = **-2,015.35**

UK commuter dataset

- 3 alternatives, described by cost, time, rate of delays, average length of delays, crowding and provision of a delay information service:

- MNL : LL = -3,391.79
- RRM : LL = -3,379.96
- DFT : LL = **-3,346.23**

Swiss value of time survey

- MNL: LL = -1,667.97
- DFT: LL = -1,595.85

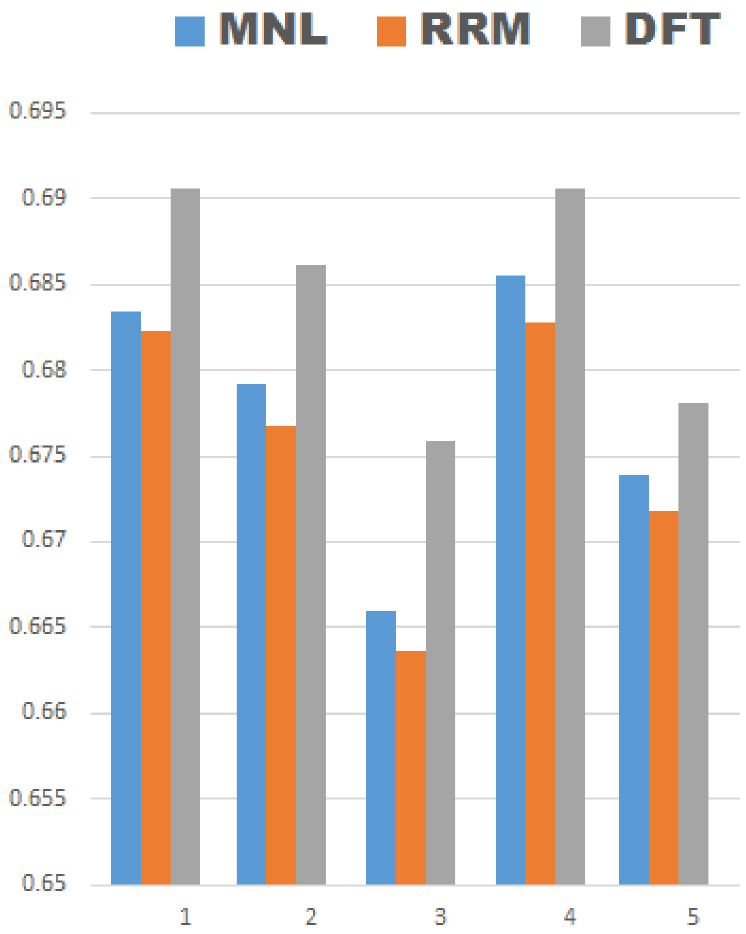
- Can also do a DFT with random parameters: LL = -1430.41

RP data

Results from UK value of travel time study

- MNL: -370.05
- RRM: -373.31
- DFT: -363.31

average probability of chosen alternatives for each forecasting subset



Including response time in DFT

- So far, we simply estimated the number of timestep parameters
- Can be linked to response time instead

$$\tau = 1 + e^{(t0 + t1 * srt + t2 * \log(mrt))}$$

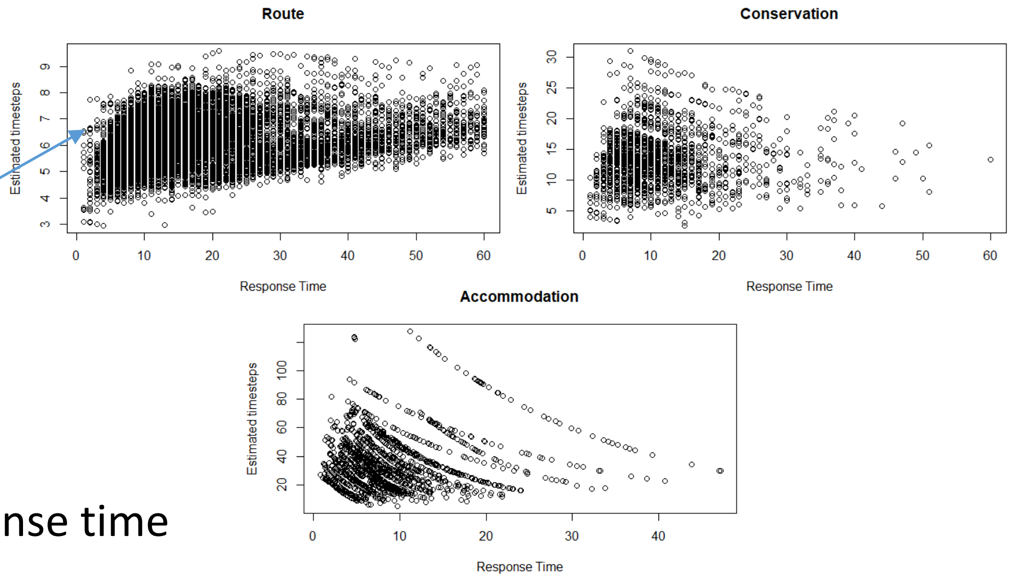
where

- τ = number of timesteps
- *mrt* is the mean response time for the individual
- *srt* is the number of standard deviations the response time for a given choice is away from the individual's mean

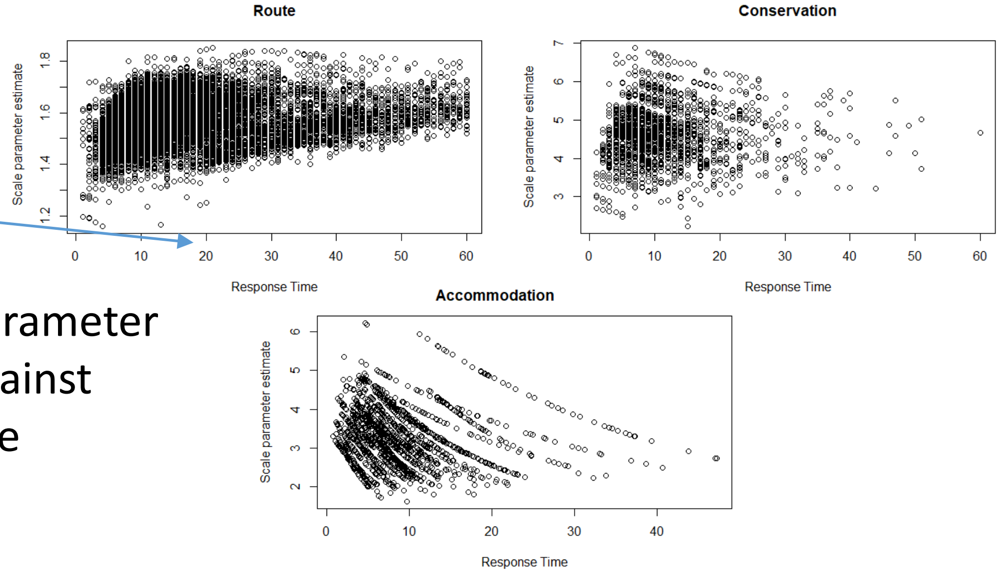
Timestep vs scale parameter estimates

- Timestep parameter appears to be equivalent to a MNL's scale parameter…

DFT timestep parameter estimates, against response time



MNL scale parameter estimates, against response time



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Response time results

Choice		DFT without response time	DFT with response time
Route	Log-likelihood	**-6,883.18**	**-6,874.37**
	t0 estimate	1.58 (12.04)	0.16 (0.36)
	t1 estimate		-0.02 (-0.58)
	t2 estimate		0.54 (3.22)
Accommodation	Log-likelihood	**-1,324.55**	**-1,313.08**
	t0 estimate	3.68 (14.43)	1.77 (3.57)
	t1 estimate		-0.40 (-2.60)
	t2 estimate		0.76 (3.85)
Conservation	Log-likelihood	**-1,960.24**	**-1,935.06**
	t0 estimate	2.62 (5.74)	0.12 (0.14)
	t1 estimate		-0.15 (-2.06)
	t2 estimate		1.01 (3.22)

- T1 -> always negative:
  Not in line with DFT
  A longer response time from an individual compared to their own mean response time results in a less deterministic choice

- T2 -> always positive:
  In line with DFT
  An individual who has a longer mean response time is on average more deterministic

Meaning of psychological parameters in DFT

‘Timesteps to make a decision’

- appears to be equivalent to MNL scale parameter

‘Attention weights’

- could use eye-tracking data as indicators

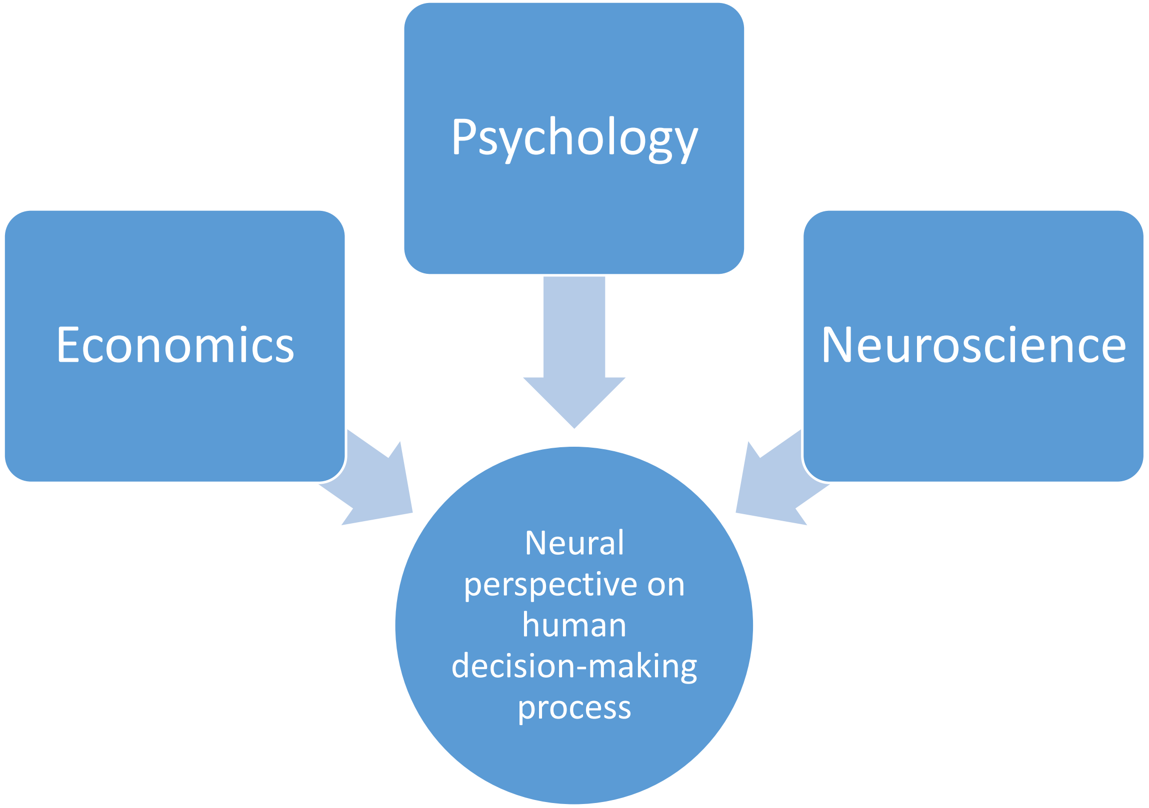
‘Memory parameter’

- meaningless if we only observe the final choice

Part 3: looking into the brain *(and other parts of the body)*

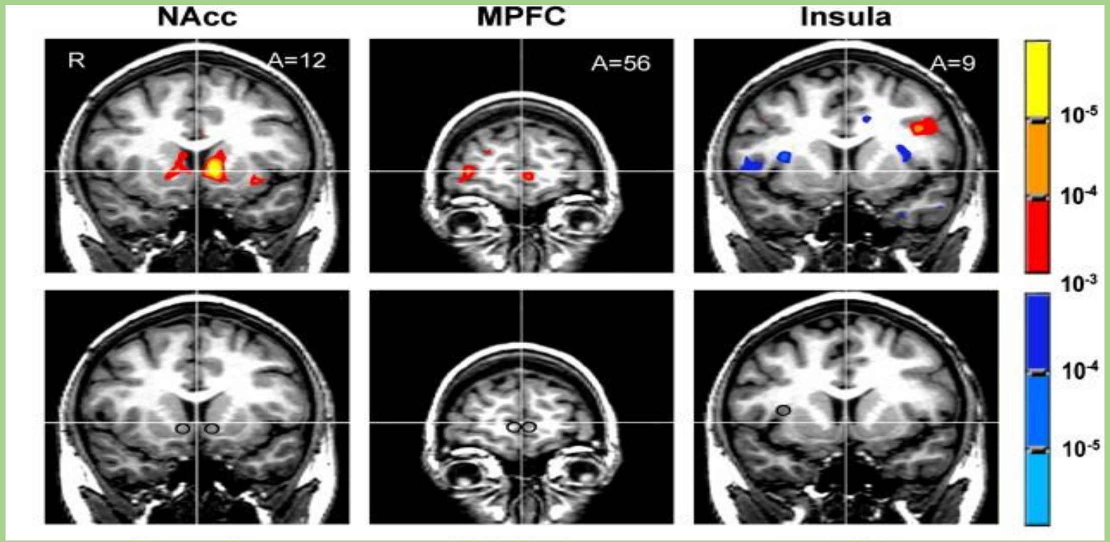
What is neuroeconomics and why do we need it?

- Biological foundations of decision making vs. classical economic theory
- Focus on process rather than outcome



Strengths and limitations of neuroeconomics

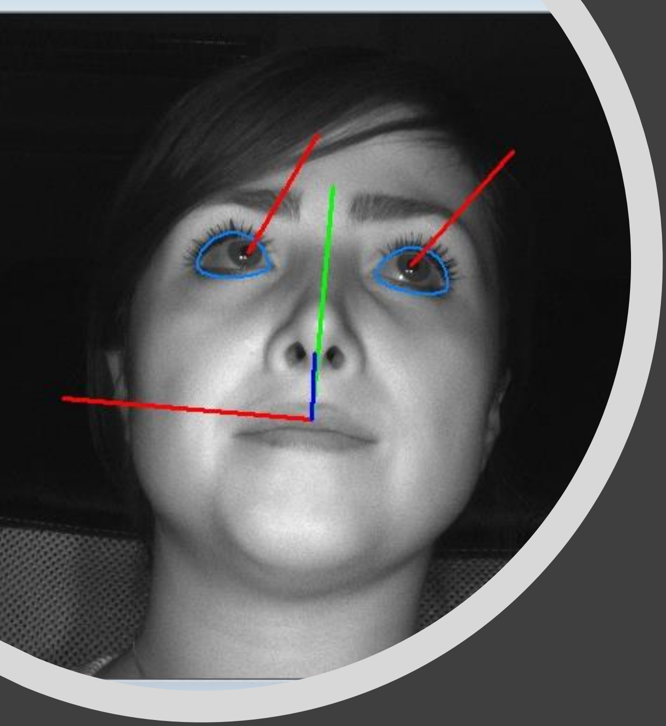
- Extensive work on understanding brain processes
- And on capturing these using scanners etc



- Very limited modelling effort
- Very simplistic choice settings, partly constrained by use of scanners etc
- Little cross-disciplinary influence
- Weak connection between neuroscience and the real world

What can we do about it?

- Look for middle ground solution to increase the applicability of neuroscience in a real world context
  - use virtual reality setting
- Use neuroscience data:
  - Improve models through using additional information about process
    - Especially useful for dynamic models like DFT
  - Help with model selection when mathematical bases have been exhausted



Aim is to capture decision process information without ability by the respondent to bias this

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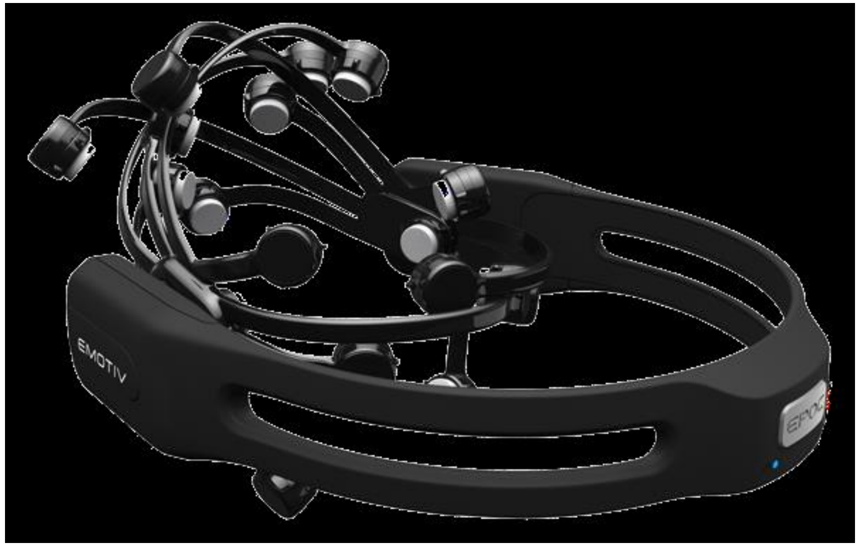
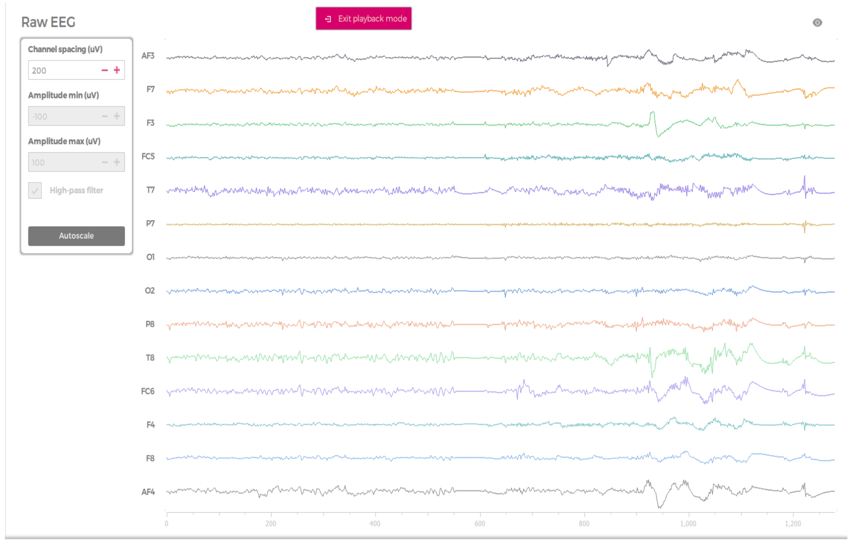
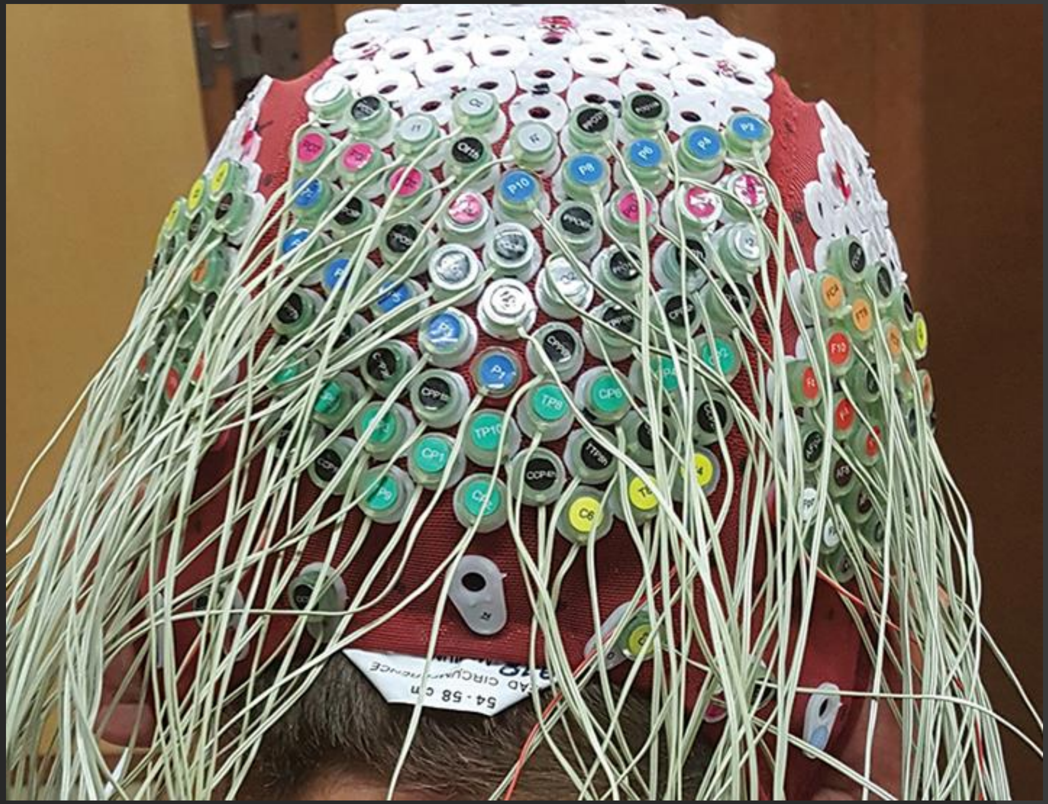
Our current work relies on VR and EEG

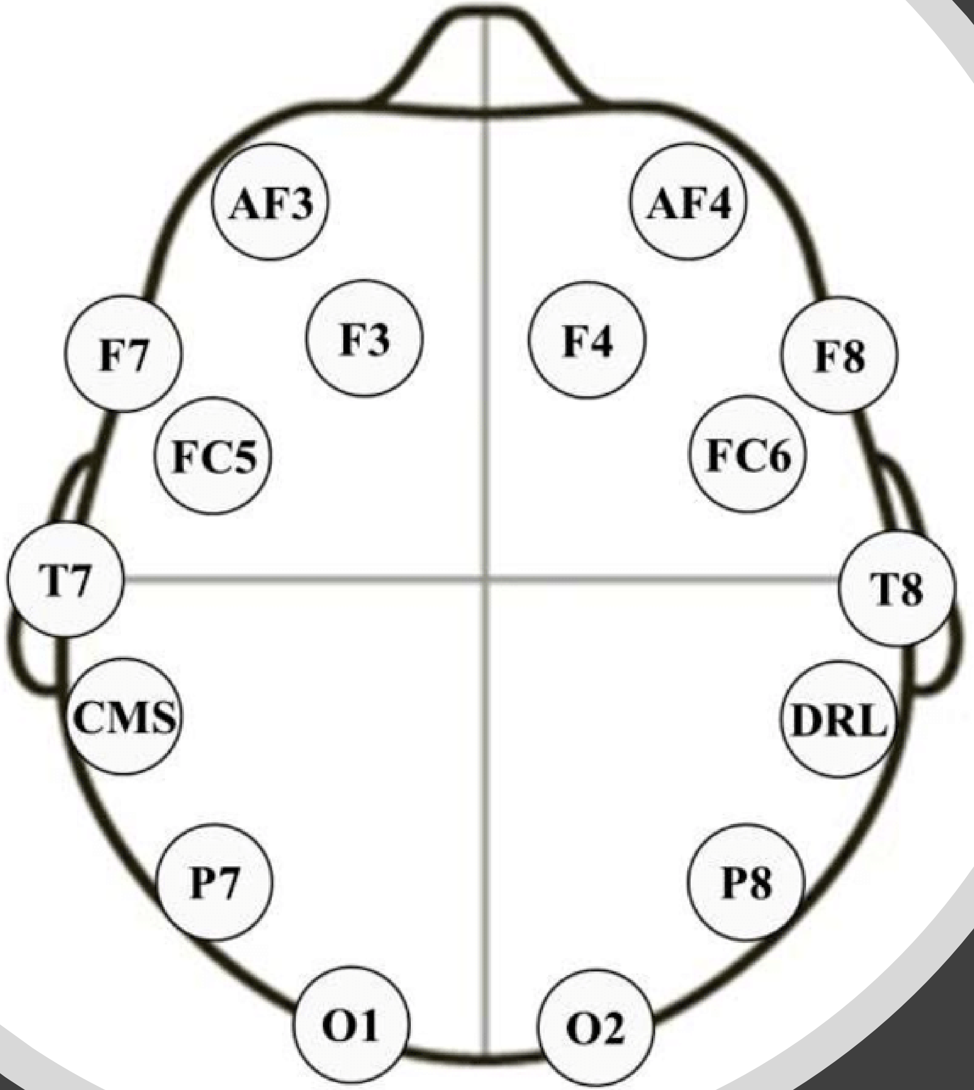




EEG

- Oldest neuroscientific technique
- Measures voltage fluctuations resulting from electrical current within neurons
- Records brain's spontaneous electrical activity over time
- Multiple electrodes placed on scalp
- Less spatially accurate than fMRI (which relies on blood flow) but much finer temporal resolution
    - Also easier to use in practice!

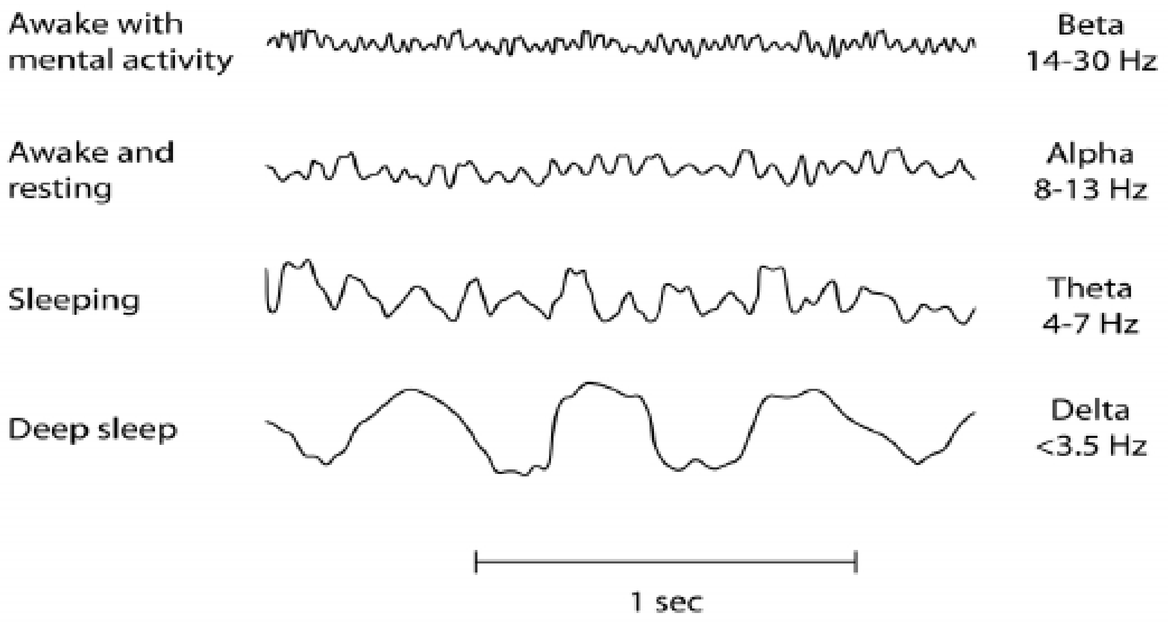




EEG Channels

- Number of electrodes in the EEG headsets can range from 5 to 264 electrodes.
- **We use 14 electrode headset (Emotiv EPOC)**
- **Moving streams of data, with very fine temporal resolution**
- Location of electrodes is important as brain performs different functions in different parts

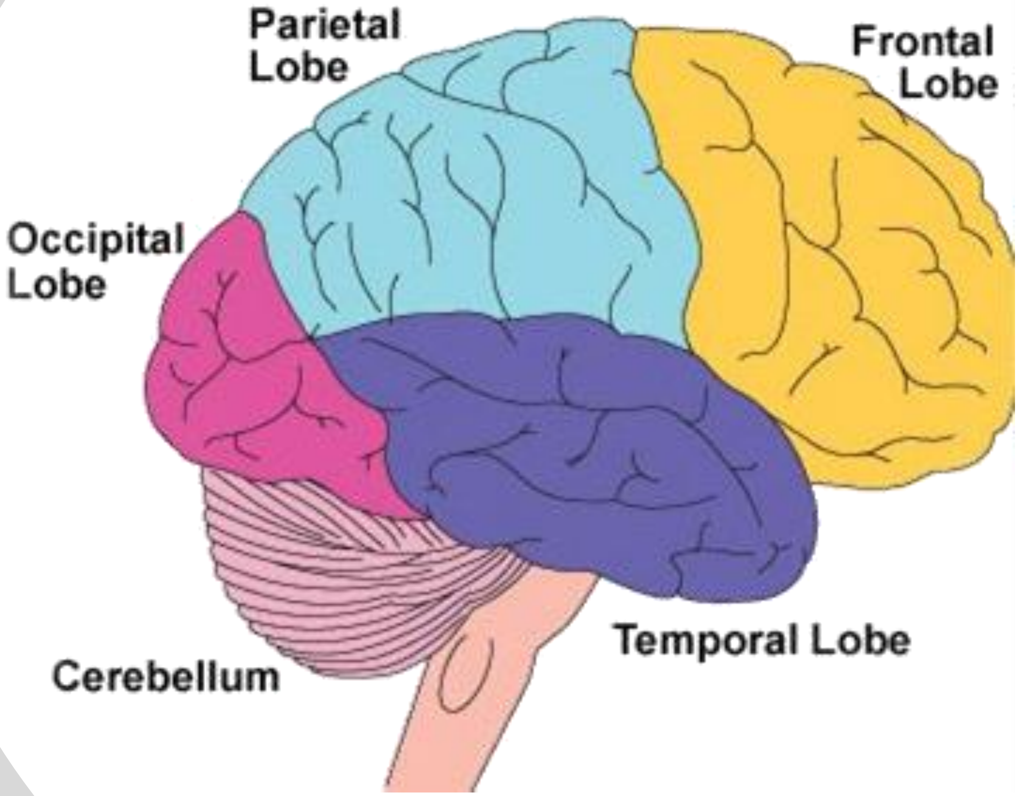
EEG waves


Awake with mental activity
Beta
14-30 Hz
Awake and resting
Alpha
8-13 Hz
Sleeping
Theta
4-7 Hz
Deep sleep
Delta
<3.5 Hz
1 sec


Raw EEG
Exit playback mode
Channel spacing (uV)
200
Amplitude min (uV)
-100
Amplitude max (uV)
100
High-pass filter
Autoscale
AF3
F7
F3
FC5
T7
P7
O1
O2
P8
T8
FC6
F4
F8
AF4
0
200
400
600
800
1,000
1,200

Brain parts

- **Focus on frontal lobe and occipital lobe**
- **Frontal lobe**:
  - emotions, reasoning, movement
  - also purposeful acts such as creativity, judgment, problem solving, planning
- **Occipital lobe**:
  - brain's ability to recognise objects
  - responsible for our vision
- **Extract theta waves from frontal electrodes to investigate cognitive functions**
- **Alpha waves from electrodes placed on the occipital lobe to explore impact of visual stimuli**



VR experimental procedure

- 24 simulations of risky road scenarios for cyclists
- 3 behavioural responses (acceleration, braking, freewheeling)
- Also stated assessment of riskiness of scenarios and willingness to cycle (1-7 scale)

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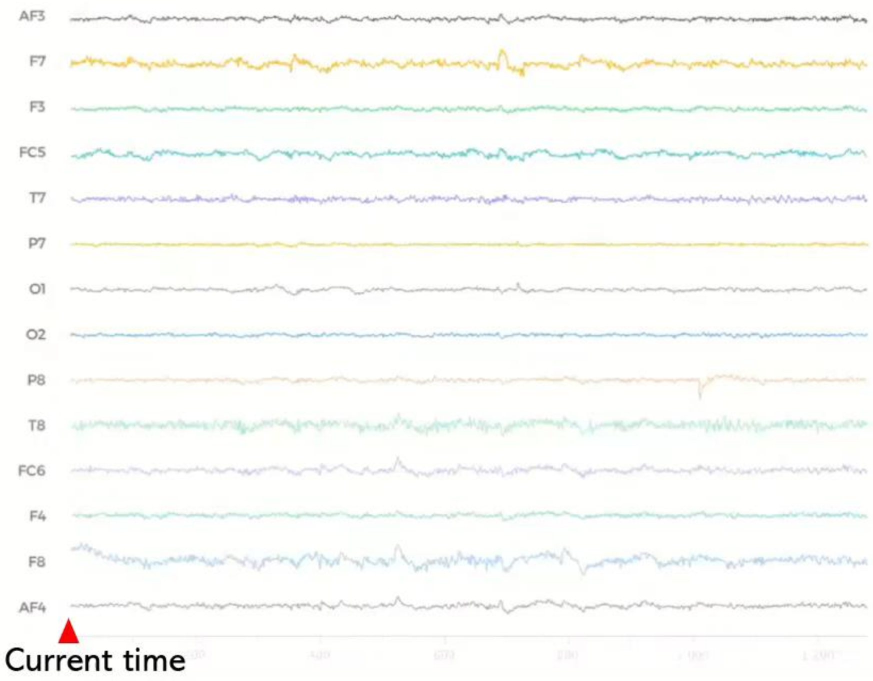
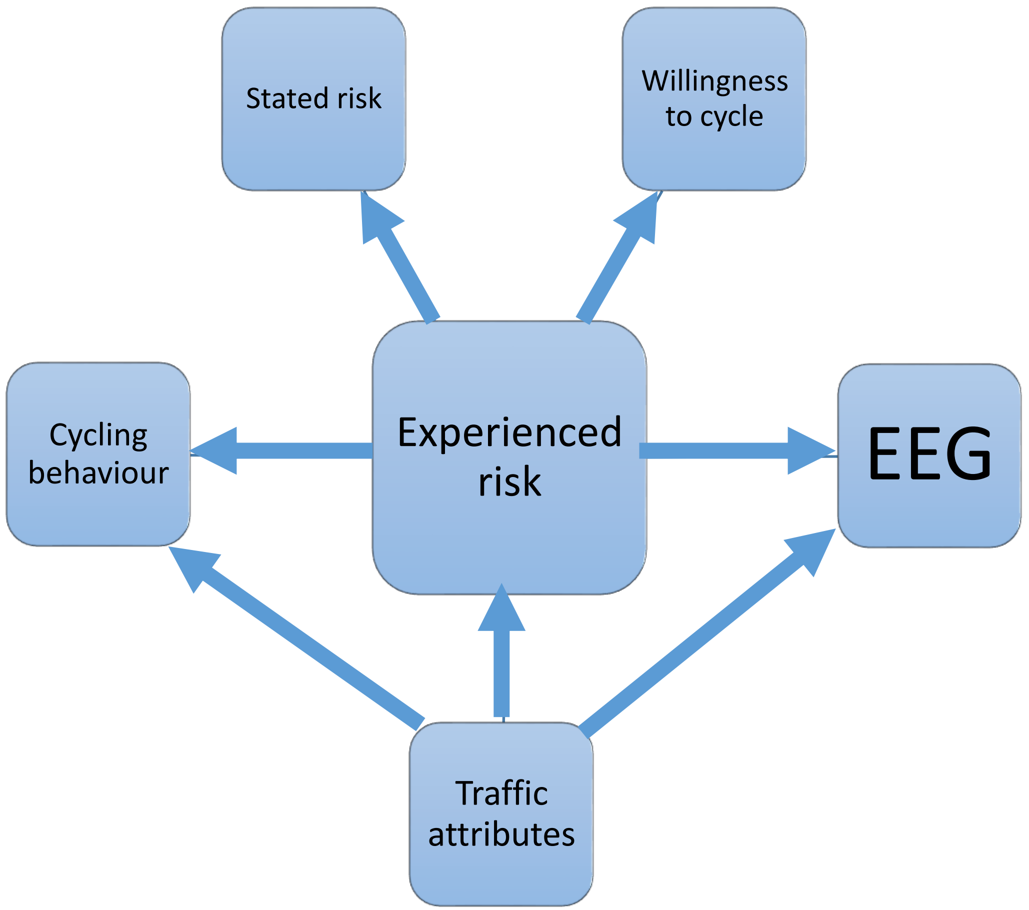
Example of pavement scenario



Proposed model framework



Correlations between stated variables

- Inverse relationship between risk and willingness to cycle (1)
- Positive correlation between scenario riskiness and stated risk (2)
- Negative relation between scenario riskiness and willingness to cycle (3)

	Stated risk	Willingness to cycle
Willingness to cycle	-0.55	
Scenario riskiness	0.17	-0.15

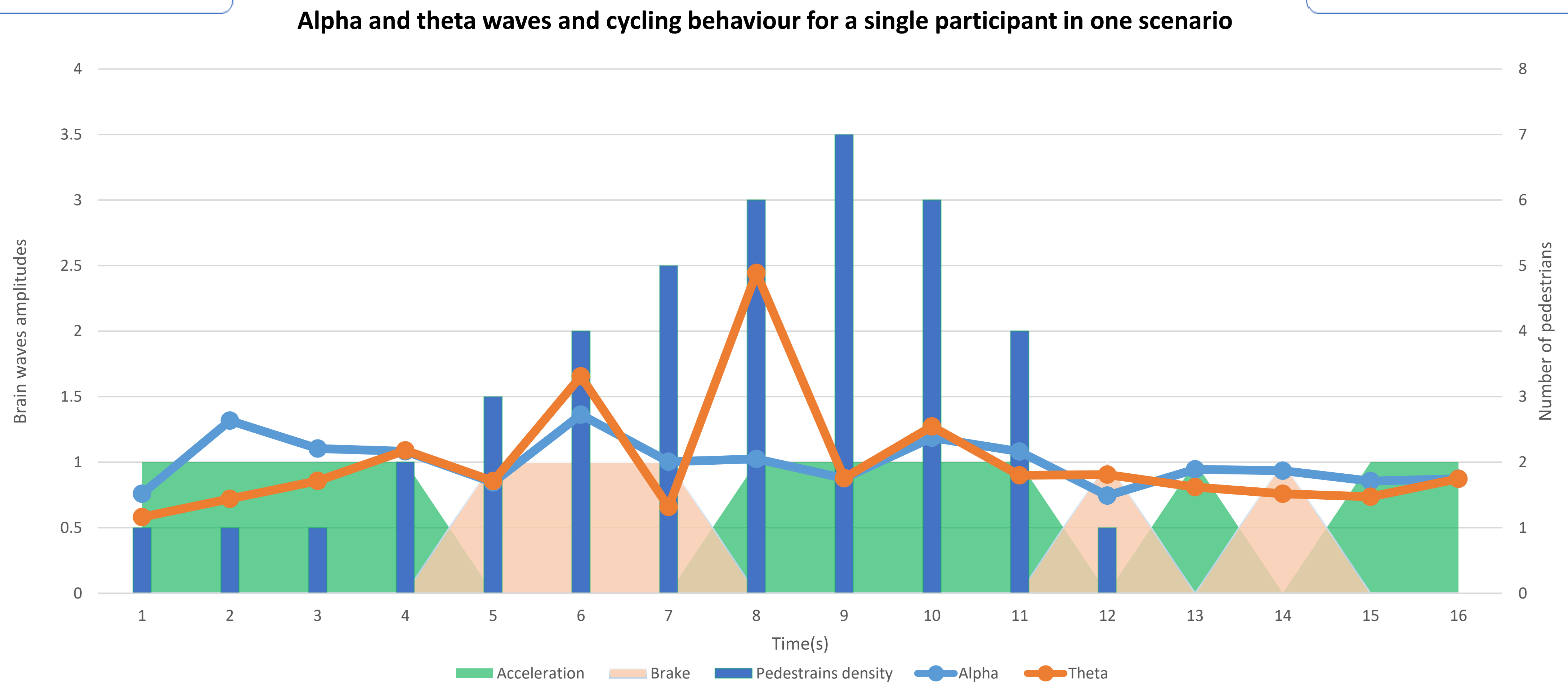
DCM example: MNL model pavement

		Estimate	Rob.std.err.	Rob.t.ratio(0)
currently accelerating	ASC for accelerating	0.2593	0.095	2.73
	ASC for braking	-2.4971	0.2606	-9.58
	ASC for freewheeling	0	-	-
currently braking	ASC for accelerating	2.826	0.151	18.72
	ASC for braking	3.7918	0.132	28.73
	ASC for freewheeling	0	-	-
currently freewheeling	ASC for accelerating	0.1443	0.0767	1.88
	ASC for braking	-3.1188	0.1757	-17.75
	ASC for freewheeling	0	-	-
shifts for 3D	ASC for accelerating	0.0124	0.0292	0.42
	ASC for braking	0.1577	0.0593	2.66
	ASC for freewheeling	0	-	-
pedestrians within 3 metres in front	gain in utitlity for accelerating	-0.0058	0.0009	-6.48
	gain in utitlity for braking	-0.0101	0.0022	-4.62
	gain in utitlity for freewheeling	0	-	-
pedestrians within 3 metres behind	gain in utitlity for accelerating	-0.001	0.0012	-0.89
	gain in utitlity for braking	0.0035	0.0025	1.39
	gain in utility for freewheeling	0	-	-

Dynamic EEG and behaviour

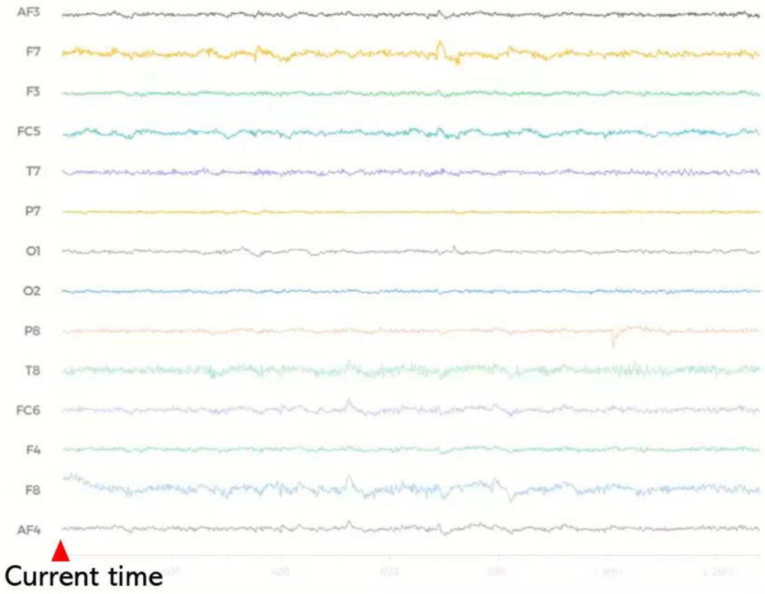
ALPHA WAVE AMPLITUDE ↓ COGNITIVE WORKLOAD ↑

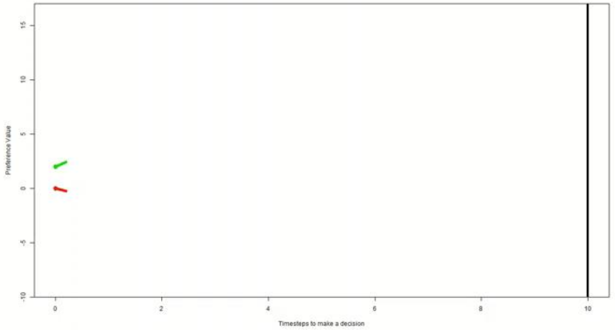
THETA WAVE AMPLITUDE ↑ COGNITIVE WORKLOAD ↑



Part 4: bringing it all together

First step: combining math psych model with neuro-science





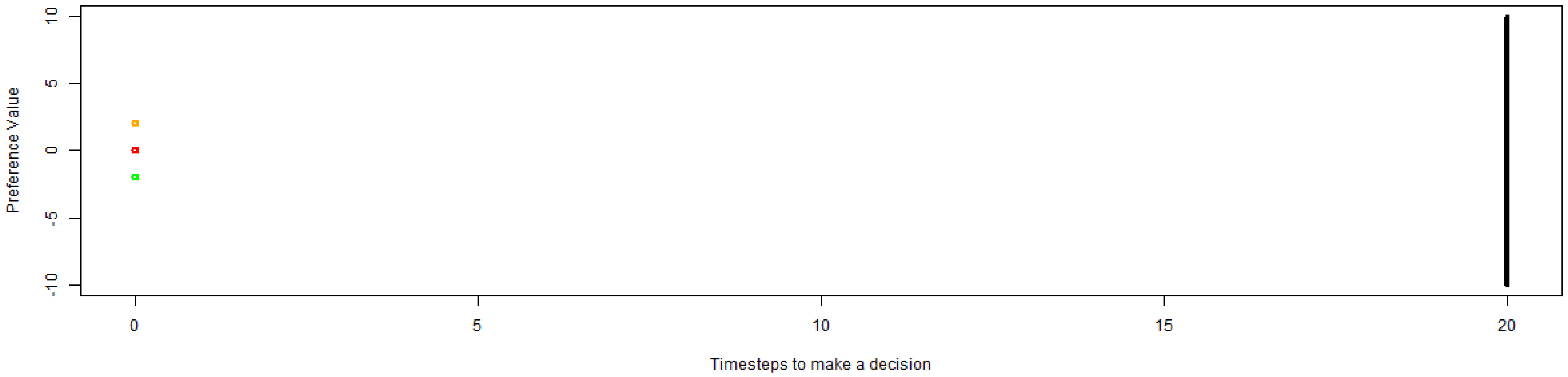
Issues we're still facing

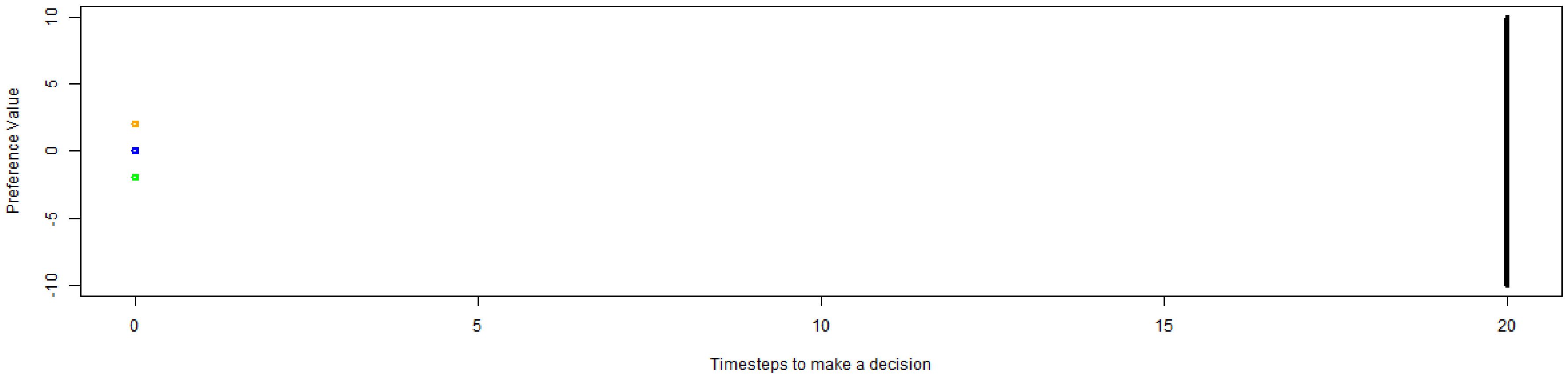
- EEG data uses very fine temporal resolution
- Need to work on making the link between EEG and choices
- What brain activity matters?
  - Just before the choice?
  - Also some remaining impact of earlier processes, with temporal discounting?
  - Full accumulation over time, without discounting?
- Last option seems to be ruled out by our results, which is reasonable

Making DFT truly dynamic

- Evaluation of alternatives is a dynamic process already
- But existing version of DFT assumes that attributes are constant within a given choice context
- This is not what happens in reality
  - Short term choices: environment changes, e.g. traffic
  - Long term choices: new information, new experiences, etc

Example 1: Santa Rita becomes available halfway





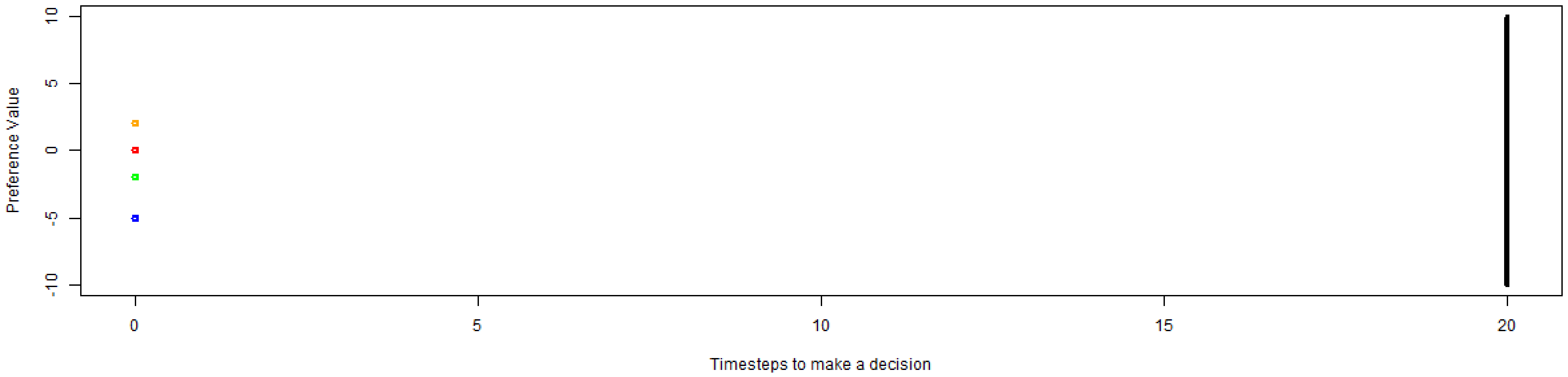
Hess Select

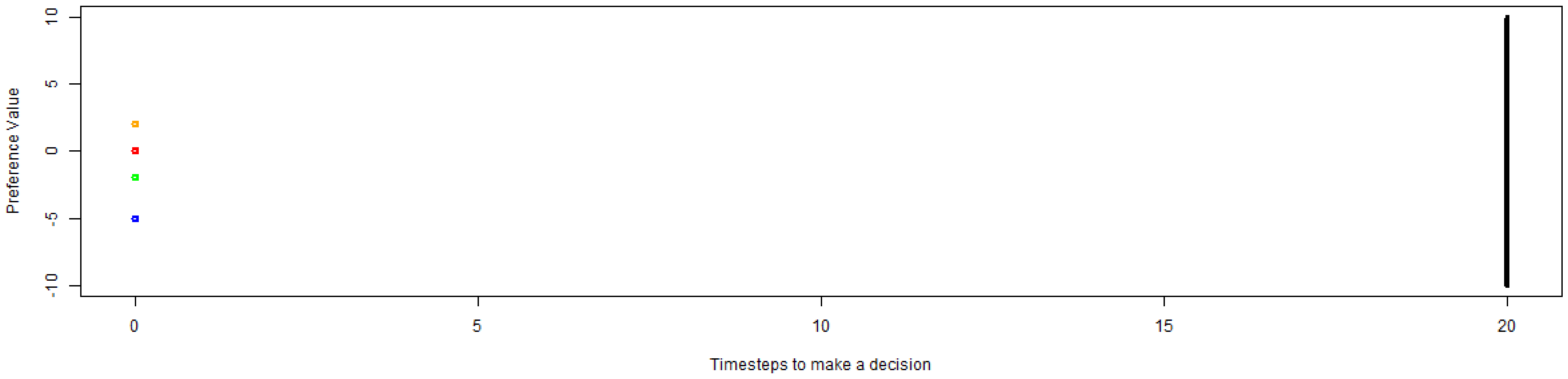
Campo Viejo

Santa Rita

Tesco Australian Red

Example 2: Santa Rita sells out (and competes more with Tesco)





Hess Select
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