Inequity Averse Optimization of Railway Traffic Management Considering Passenger Route Choice and Gini Coefficient

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***Abstract*—Train rescheduling is important in railway traffic management, as it helps to minimize the negative impact on passengers caused by train delays. A train rescheduling problem can have different goals. A common goal is to minimize the average train delay time. Focusing only on this performance measure overlooks how the delay is distributed among subjects. [1] consider both the average train delay time and the equity level between train operating companies as objectives by means of a mixed integer linear programming (MILP) model, to understand the trade-offs between the two. This thesis takes this approach a step further by considering the problem from the passenger's perspective. Moreover, it compares results for two different equity measures, namely the Gini Coefficient and the maximal deviation method. Real-life data from the Netherlands are used for the experiment. The results show that the average passenger delay time can be further decreased in the new model, and using Gini Coefficient as the equity measure results in a lower average delay time than the maximal deviation method.**

***Index Terms*—Passenger Delay, Equity, Train Rescheduling, Gini Coefficient, Optimization**

I. INTRODUCTION

Public rail transport is one of the most important transport modes in today's society. For passengers, it provides a more affordable solution compared to private cars. Passengers also have disposable time to conduct other activities such as reading or working while taking the trains. From the society's perspective, trains are more environmentally friendly than private cars, as they consume less energy per passenger-kilometer ($0.07 - 0.13 kWh \cdot pkm^{-1}$ for trains and $0.33 - 0.49 kWh \cdot pkm^{-1}$ for cars) [7]. Train is also considered to be the safest transport mode in terms of fatality rate ($1/12770 Mio \cdot pkm$ for trains and $1/556 Mio \cdot pkm$ for cars) [8].

To further improve the attractiveness of public rail transport, a high quality of service should be kept. It requires a safe, punctual and reliable operation, which relies not only on the offline timetabling stage before the operation, but also on the rescheduling stage when the planned timetable cannot be fulfilled due to unexpected disruptions. The rescheduling stage requires the operators to bring the operation back to normality in a limited time. Rescheduling plans should also be feasible given the capacity constraints of the railway tracks, which further increases the complexity of rescheduling tasks.

Most train rescheduling problems aim at bringing the trains back to their scheduled timetables. One of the most common goals is to minimize the total train delay times. However, it has the following two drawbacks.

Focusing on the performance of trains fails to examine the rescheduling problems from passengers' perspective. For passengers, being able to reach their destination on time makes sure that the activities following up can be done. For train operating companies, passenger punctuality is also one of the most important factors to ensure. In Switzerland, SBB uses "customer punctuality" as one of the indicators to measure the quality of its service, and considers "customer punctuality" of "the utmost importance" [9]. It is worth noting that minimizing the delay of single trains is not equal to minimizing the delay of passengers, especially in the case when transfers are required. Thus, it is of great interest to investigate the rescheduling problems from the passengers' perspective.

Focusing on the performance of the whole system overlooks equity among individuals. It is possible that some trains would have to afford a much longer delay time than other trains so that the total train delay time of the whole system is minimal. It is an undesirable situation for passengers on the trains with longer delays. Not only for passengers, but some train operating companies might also afford more delays than their competitors, which does not comply with the Detective 2001/14/EC that the access to train infrastructures to all train operating companies should be distributed in a non-discriminatory manner. Thus, train rescheduling problems should also be examined from the perspective of equity.

This thesis aims answering this research question: What are the possible benefits for passengers and operators, when passenger route choice and Gini Coefficient (as a second equity indicator) are included in the process of train rescheduling?

II. LITERATURE REVIEW

A. *Rail Traffic Management*

Train rescheduling problem is also known as ”train scheduling problem”, ”train dispatching problem”, ”railway traffic control”, or ”real-time railway traffic management problem”, as pointed out by [2]. It is at the microscopic level, where the tracks are examined as signaling blocks. Train rescheduling problem aims at bringing back the trains to their scheduled plan, in order to minimize the any possible negative impacts on the traffic (such as delay propagation) and passengers (late arrivals).

Train rescheduling problem can be formalized as an optimization problem: given the safety constraints between signaling blocks and trains, how to reschedule the trains so that the total loss is the minimal. One of the common goals of such problem is ”to minimize the total deviation time of involved trains”, as used by [3]. Some studies also consider one or multiple goals, such as train delay cost, keeping as many train connections for passengers as possible, passenger satisfaction rate, etc. [4] determined two objectives for delay management on railways, to minimize both the train delay time and the missed connections. [1] were the first ones to consider both the average train delay time and the equity level between transport operation companies.

There are also several different methods to examine rescheduling problems. Using mixed-integer linear programming (MILP) model is one of the most commonly used approach. [3] developes a MILP model with the big-M method to formalize constraints on safety headway between trains and track capacities. This method gives a better result on system performance than the approached previously used. [5] examines rescheduling problem with a MILP formulation with certain ”granularity” on the rail infrastructure, and studies how such ”granularity” have an impacts on the delay propagation is studies.

There are studies that considered the rescheduling problem and delay management problem together, aiming at providing a dispatching solution that is feasible given the track constraints from passengers perspective. [6] combined the microscopic train rescheduling problem with the macroscopic delay management problem with a MILP model. In this study, heuristic algorithms are proposed and some lower bounds with promising solutions are found. [2] examined the problem with a consideration of multiple stakeholders such as ”passengers” and ”infrastructure managers” from a game theory perspective. This study provided some numerical results on the trade-offs between passenger delay and train delay.

B. *Inequity Averse Optimization*

Equity is one of the most important topics in operation research. It describes a state when certain resource is allocated in a system to different entities in a fair and just way given some constraints. In an ideal case, every user should have the exact same accessibility to the resource. This ideal case is however hardly seen in reality. Despite that, the concept of equity is still widely used in many practices in real life. It can be used either to measure the performance of a system, or to help make decisions so that the utmost equity could be reached in the system. The concept of equity is also widely used in transport field. However, only very few studies considered equity with train rescheduling problem or delay management problem. Thus, this thesis aims at examining the train rescheduling problem from the perspective of passenger and with equity concerns at the same time.

One example where the equity is used to measure the performance is the Gini Coefficient in economics: it measures the equity regarding income distribution in a region. There exist many cases where equity is also used in the problem solving stage, in order to get a more fair solutions, such as allocation and location problems [10]. In this paper, we used the Gini Coefficient as an indicator of equity. Gini Coefficient is can be used to measure the performance of a system regarding equity and be included in an optimisation process (as constraints or objectives).

III. MATHEMATICAL MODEL

The mathematical model introduced in this thesis is based on the MILP model proposed by [1]. Compared with the original model, the new model considers passenger route choices and Gini Coefficient as a second equity indicator.

A. *Gini Transformation*

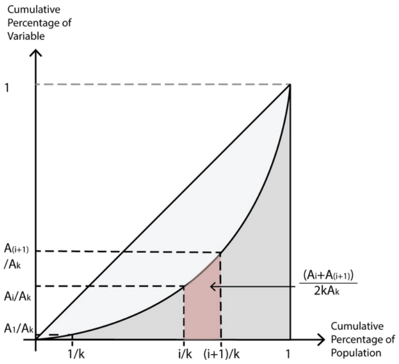

Fig. 1. Gini Coefficient and Lorenz Curve

Gini Coefficient is first brought up by the Italian statistician Corrado Gini in 1912 [11].The Gini Coefficient is calculated based on the Lorenz Curve as shown in 1. It is often time used to measure the equity level of income distribution in a population, and can also be used to measure the equity of other variables. The value of Gini Coefficient ranges from 0 to 1, and is not related to the absolute value of the variables.

As Gini Coefficient is a non-linear fractional expression and cannot be directly applied into the MILP model, the Charnes-Cooper Transformation [12] is used to transform the Gini Coefficient into a linear form.

B. Passenger Route Choice

The mathematical model concerning the passenger route choice consist of following groups of constraints.

Group 1: Possible Train Connections for Passengers

This group of constraints aims at finding all transferring possibilities within stations. Eq 1 and Eq 2 calculate the departure and arrival time of each train at the stations where they are scheduled to stop by, based on the spatial relation between nodes and stations. Eq 3 and Eq 4 determine whether it is possible to transfer from train f to train g at station s. It is only possible temporally when the departure time of train g at station s is later than the arrival time of train f at station s plus the required transferring time inside of station s.

$$t^{arr}_{f,s} = t^{dep}_{f,i,j}, \quad \forall f \in F, \forall s \in S_f, \forall ij \in L_s \tag{1}$$

$$t^{dep}_{f,s} = t^{dep}_{f,i,j}, \quad \forall f \in F, \forall s \in S_f, \forall ij \in L_s \tag{2}$$

$$w_{f,g,s} \geq (t^{dep}_{g,s} - t^{arr}_{f,s} - t^{trans})/M, \forall s \in S_f, s \in S_g \tag{3}$$

$$w_{f,g,s} \leq (t^{dep}_{g,s} - t^{arr}_{f,s} - t^{trans})/M + 1, \forall s \in S_f, s \in S_g \tag{4}$$

Group2: Spatial Train Choice

This group of constraints determines a series of trains that are able to transport passengers from their origin stations to destination stations spatially. Eq 5 and Eq 6 ensure that passenger group p will only take one train to leave their origin station s^{orig}_p and they do not take any train to arrive at their origin station. Analogously, Eq 7 and Eq 8 make sure that passenger group p are only allowed to take one train to arrive at their destination. Eq 9 and Eq 10 ensure that for one station s that is neither the origin nor the destination for passenger group p, this group of passenger will at most arrive at station s once. Moreover, once passenger group p takes one train to arrive at station s, they should also take one train (stay in the the same train or transfer to another train) to leave station s.

$$\sum_{f\in F}\sum_{t\in S\setminus s^{orig}_p} v_{p,f,s^{orig}_p,t} = 1, \forall p \in P \tag{5}$$

$$\sum_{f\in F}\sum_{t\in S\setminus s^{orig}_p} v_{p,f,t,s^{orig}_p} = 0, \forall p \in P \tag{6}$$

$$\sum_{f\in F}\sum_{t\in S\setminus s^{dest}_p} v_{p,f,t,s^{dest}_p} = 1, \forall p \in P \tag{7}$$

$$\sum_{f\in F}\sum_{t\in S\setminus s^{dest}_p} v_{p,f,s^{dest}_p,t} = 0, \forall p \in P \tag{8}$$

$$\sum_{f\in F}\sum_{t\in S\setminus s} v_{p,f,t,s} = \sum_{f\in F}\sum_{r\in S\setminus s} v_{p,f,s,r}, \forall p \in P, \forall s \in S, t \neq r \tag{9}$$

$$\sum_{f\in F}\sum_{t\in S\setminus s} v_{p,f,t,s} \leq 0, \forall p \in P, \forall s \in S \tag{10}$$

Group3: Temporal Train Choice

This group of constraints ensure that passenger train choice is also temporally possible. In Eq 11 passenger group p will only take train f if it departs later than the arrival time of passenger group p at their origin station. Eq 12 makes sure that passenger group p are only allowed transfer from train f to train g at station s when time allows.

$$v_{p,f,s^{orig}_p,t} \leq (t^{dep}_{f,s^{orig}_p} - t^{orig}_p)/M + 1, \forall p \in P \tag{11}$$

$$v_{p,f,t,s} + v_{p,g,s,r} \leq w_{f,g,s} + 1, \forall p \in P, s \neq s^{orig}_p \tag{12}$$

IV. Experiment Setup

Real-life data are used to conduct the experiment. The railway section between Utrecht and Den Bosch in the Netherlands is chosen. The railway section between station Utrecht and Den Bosch in the Netherlands is about $50km$ long, consisting of 9 stations, 80 nodes, and 84 signaling blocks. In the experiment, traffic of both directions are considered and are independent of each other, i.e., each signaling block only allows traffic of one direction, and trains of one direction will not share any block with train of the other direction.

Train service used in the experiments can be categorized into two: Intercity (IC) train service and Sprinter (SP) train service. IC trains stop at major stations and run at a higher average speed. SP trains stop at every station that they pass by. In total, 24 trains are used for the experiment. 12 trains run on each direction. For each direction, there are 6 IC trains and 6 SP trains. These two types of train service are provided by two operators, IC train operating company (IC TOC) and SP train operating company (SP TOC).

In the experiment, we consider 10 sets of primary delay scenarios where are randomly generated. For each delay scenario, each train is given a randomly generated primary delay time at its origin. The primary delay times follow a 3-parameter Weibull distribution. The parameters of the distribution are fitted based on real life data according to [13]. The used parameters are as follows:

- IC trains: scale = 394, shape = 2.27, shift = 315;
- SP trains: scale = 235, shape = 3.00, shift = 186.

Real-life passenger numbers is used for the experiment. In total, eight groups of passengers with different origin and destination (O-D) pairs are chosen. Those eight O-D groups are then separated into two or four smaller groups with different departure time, according to the passenger number size of each O-D group. The experiment time horizon is one hour. The departure time is randomly generated and distributed within the first 45 minutes of an hour time, to make sure that all passengers have at least one train to take to reach their destination. Table I shows the experiment scheme.

TABLE I
EXPERIMENT SCHEME

	Objective	Constraints	w
1	Avg. Train Delay Time		
2	Avg. Passenger Delay Time (Absolute)		
3	Avg. Passenger Delay Time (Relative)		
4	Avg. Train Delay Time	$Gini_f \leq w \cdot Gini_f^*$	0.75
5			0.50
6			0.25
7	Avg. Train Delay Time	$Max_f \leq w \cdot Max_f^*$	0.75
8			0.50
9			0.25
10	Avg. Train Delay Time	$Gini_{toc} \leq w \cdot Gini_{toc}^*$	0.75
11			0.50
12			0.25
13	Avg. Train Delay Time	$Max_{toc} \leq w \cdot Max_{toc}^*$	0.75
14			0.50
15			0.25

$Gini_f$: Gini Coefficient of train delay times.
Max_f: Maximal deviation of train delay times.
$Gini_{toc}$: Gini Coefficient of TOCs, loss measured by train delays.
Max_{toc}: Maximal deviation of TOCs, loss measured by train delays.
Superscript ($*$): the corresponding value calculated in case 1.

Cases 1-3 The first group of experiments includes cases 1- 3. These three cases have no constraints on the equity level of the system, but have different objective functions. The goal of this group is to determine the impact of different objectives on passenger and train travel time.

Cases 4-9 The second group of experiments includes case 4-9. This group of cases has the same objective function as case 1, i.e. to minimize the average train delay time. Equity consideration on all trains in terms of delay time is included in form of constraints. Case 1 serves as a benchmark, in order to examine the impact when equity is required. Cases 4-6 use Gini Coefficient as the equity indicators, and cases 7-9 the maximal deviation method. By comparing cases 4-6 to cases 7-9, the impact of two equity measures can be analyzed.

Cases 10-15 The third group includes case 10- 15. These cases also aim at minimizing the average train delay, the same as cases 1 and 4-9. Equity level between the train operating companies are considered in constraints. Case 1 also serves as a benchmark. This group of cases is designed to examine the impact of equity requirements on the delay time (of both trains and passengers) and the loss of train operating companies themselves.

V. RESULTS AND DISCUSSION

A. *Computation Time*

In cases 2 and 3, where passenger delay time is used as objectives to be minimized, no proven optimal solutions are declared. For all other cases, it is possible to get proven integer optimal solutions within at most six minutes on average.

B. *Neglecting Equity: Cases 1-3*

Compared with case 1, passengers' delay time of case 2 has decreased by 13 seconds per passenger. However, the average passenger delay time in case 3 doubles its size to more than one minute. Case 3 also has a higher maximal deviation and a higher Gini Coefficient in passenger delay time than the other two cases, denoting a less equal situation. Meanwhile, case 3 shows the lowest value in average passenger delay ratio.

Fig 2 helps us to investigate the reasons behind those numbers. Note that each point in the figure represents one group of passengers who intend to take the same train from the same origin station to the same destination. The size of the entry in the figure denotes the relative number of passengers in that group.

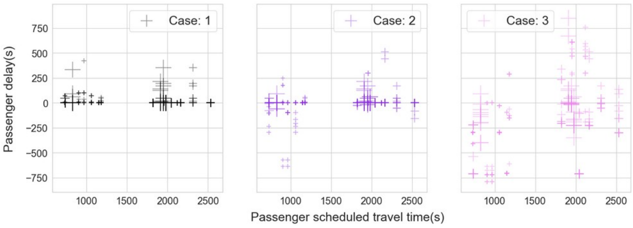

Fig. 2. Passenger Delay: Cases 1-3

One can observe that while in case 1 almost all passengers arrive at their destination stations on time or slightly later (within 9 minutes), in case 2 there are more instances when passengers arrive early. In case 3, the delay time distribution is more disperse. More passengers are facing either delay or early arrivals. Moreover, in case 3, passengers with longer travel times are allocated with longer delays. It explains why case 3 has a higher average passenger delay time but a low delay ratio. However, such result does not indicate that considering the ratio is of no use. One possible way is to consider the relative delay of passengers in the constraints while minimizing the average delay time.

C. *Neglecting Equity: Cases 4-9*

The results of cases 4-9 are analyzed. Firstly, more equal the situation is forced to be, the higher the train delay time is. Secondly, using Gini Coefficient and maximal deviation methods as measures for equity yield different results. The group using Gini Coefficient shows a much less average train delay time than the group using the maximal deviation method, i.e., a better system performance, but a higher maximal train delay value. Both groups give similar results on Gini Coefficient of train delays. Thirdly, it is also worth noting that the maximal train delay time sees no big changes in all cases.

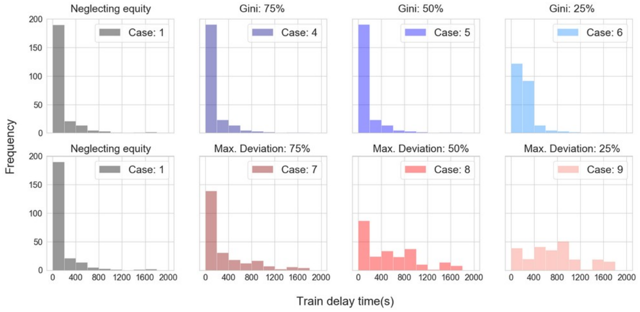

Fig. 3. Train Delay Distribution: Cases 4-9

Fig 3 shows the train delay time distribution in cases 1 and 4-9. The first row shows the impact when using Gini Coefficient as the equity measure, and the second row shows results from maximal deviation measure. One should keep in mind that 3 includes all trains under all ten sets of primary delay scenarios and provides an intuitive comparison of the two equity methods on the train delay distribution. The low maximal train delay deviation values in cases 7-9 are attributed to the increase of average delay time, instead of decreasing the extreme delay times.

Why do cases 4-6 and cases 7-9 give similar Gini Coefficient values while the distribution of train delays shows different patterns? Fig 4 shows the Lorenz Curves of train delay times in cases 4-6 and 7-9 and provides an insight into the difference between Gini Coefficient and maximal deviation method. Note that each line in the figure represents the result under one set of primary delay scenario. Recall that case 4 and case 7 yield a similar value of Gini Coefficient of train delays, and case 4 has a much lower average train delay time and a higher maximal train delay time than case 7, as shown in 4. Same applies to case 5 and 8, and case 6 and case 7.

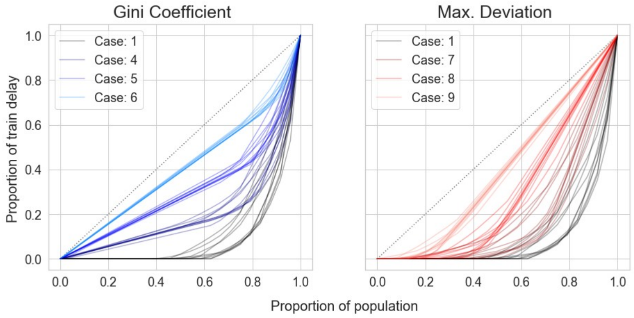

Fig. 4. Train Delay Lorenz Curve: Cases 4-9

Here, we take the Lorenz Curves of case 6 and case 9 as examples to analyze. As can be observed in Fig 4, the Lorenz Curve of case 6 shows a small but rather constant slope from 0% to around 90% of the population, and a greater slope from around 90% to 100% of the population. For first 90% of the population, the slope of the Lorenz Curve is smaller than one, indicating that they have a smaller delay time than the average delay of all trains. The Lorenz Curve of case 9 shows a very different pattern: about 20% of the trains are facing almost no delay, while the other 80% of the trains face a rather high delay.

Having no constraints on the maximal deviation in delay time gives cases 4, 5 and 6 such a freedom to keep the average delay time of all trains at a low level. By making a small amount of population worse off to the rest of the population, the majority of the population can get a better result. One should keep in mind that the maximal train delay time values of cases 4-6 and cases 7-9 are almost the same, meaning that cases 4-6 do not increase any unnecessary extra delays. However, in cases 7, 8 and 9, as the maximal deviation in delay time is strictly restricted, when the extreme values cannot be decreased any further, the average value will then increase, leading to a less favorable situation overall.

It is clear that cases 7-9 yield better results in terms of maximal deviation of train delays, and similar results in terms of Gini Coefficient. Thus, cases 7-9 dominate cases 4-6 in terms of equity level. However, the average and maximal delay times in cases 7-9 are no better than case 4-6. From the perspective of individual train delays, cases 4-6 dominate cases 7-9. Such an equity in cases 7-9 is reached by adding loss to those who have benefited more, but fails at making those who has suffered more less better off. Whether such an equity improvement is worthwhile, is questionable.

D. *Neglecting Equity: Cases 10-15*

The train delay distribution in these six cases are very close to that in case 1 (as can be seen in Fig 5). The average train delay time in cases 10-15 increases no more than half a minute. Compared to cases 4-9, including the equity constraints on train operating companies generates a smaller impact on train delays. What's more, when the equity level between train operating companies is not required to be high (in cases 10, 11, 13 and 14), the extreme train delay time decreases. Only when the equity level is required to be high (in cases 12 and 15), an increase in the extreme train delay time can be seen. It can also be observed that the Gini Coefficient for all six cases see a very slight decrease (0.01-0.02 less) compared to case 1, meaning that an equity improvement between the train operating companies could also increase the equity level between trains, though not on a very high level.

As for passengers, the average passenger delay time in case 10-15 are very close to that in case 1, as there is no significant change in the train delay time can be observed. What's more, the Gini Coefficient of passenger delay time sees no change. This means an increase in equity level between train operating companies has almost no impact on

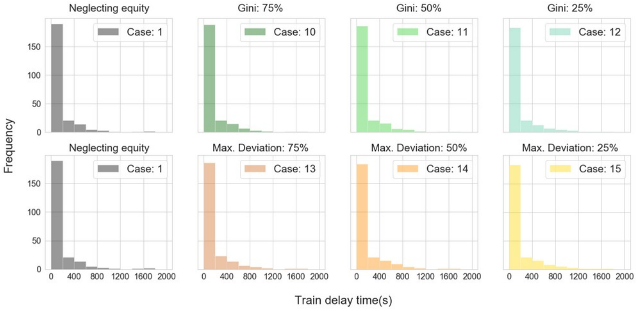

Fig. 5. Train Delay Distribution: Cases 10-15

passengers' equity in terms of travel time.

The results from these six cases reveal a possibility to reach a higher equity level between train operating companies with almost no impact on passengers' travel time and their equity level.

VI. CONCLUSION

To sum up, this thesis demonstrates the possibility to examine the train rescheduling problems from the perspective of passengers. It proves that passenger's delay time can be further improved from this new perspective. Furthermore, this thesis shows how to transform and integrate the Gini Coefficient in the mathematical model. It reveals that the average delay time increases when the a higher equity level is required in the system, regardless of which equity indicator is used. However, different equity indicators have result in different delay distribution. The present findings confirm that using Gini Coefficient as the equity indicator results in less total delay time than the maximal deviation method, while keeping the extreme delay values at a similar level. Finally, the experiment results find out that forcing higher equity level between train operations companies leads to less delay extension for both trains and passengers, compared to a higher equity level between individual trains.

The answer to the research question is as follows. It is observed that less delay tension for passengers could be achieved by having the passenger delay in the objective function to minimize. However, from the perspective of operation, the delay time of trains increases by this change. By setting a lower bound for the equity level of train delays, the average delay time increases, regardless of which equity indicator is used. However, compared to the maximal deviation method, Gini Coefficient results in a lower average delay time and a similar value in extreme delay cases.

A. Limitations

Long calculation time required to minimize passenger delay time. Firstly, when passengers travel time is in the objective value (case 2 and 3), the calculation time is too long making it hard to get the optimal results. Out of the consideration of calculation time and the possibility to reach an optimal or feasible solution, the following cases only include the equity constraints on trains' delay time.

Equity level of train operating companies was already at a high level when equity was ignored. The number of train operating companies is rather small and the equity level between these two train operating companies was already relatively high, when no equity constraint is considered. Thus, the impact on the equity constraints can hardly be observed.

Passengers are separated in groups. Passengers are assumed to arrive at the origin stations in groups at randomly-generated discrete time points. This leads to such a result that when the average travel time of passengers is the objective function to be minimized, the trains are dispatched just to adapt the travel demand of these passenger groups. Also, the randomness in the arrival time creates noise to observe the real passenger delay time.

B. Future Work

Consider other equity indicators As shown previously, Gini Coefficient has the advantage of keeping the average train delay time at a lower level when used as an equity indicator in the constraints. It provides a good starting point to compare the effect of different equity measure in train rescheduling problems. It will be of interest to examine whether other equity constraints would result in similar or even better solutions.

Test equity indicators with other data In this thesis, we show that there exists such a risk that unnecessary increase in average delay might occur when the maximal deviation method is used as the equity constraints. However, this result is concluded based on one railway section in the Netherlands. It is not clear yet whether this risk exists in other railway sections. Future study could continue to investigate whether the advantage of Gini Coefficient revealed in this thesis would still hold on other railway sections.

Impact on passengers with/without transfers As one of the major drawback of examining the rescheduling problem by minimizing the train delay time is that passengers' transferring needs are ignore, it is thus of interest to investigate whether transferring passengers would benefit more with the new objectives.

Better algorithms for minimizing passenger delay The long calculating time for cases 2 and 3 (with the objective to minimize the passenger delay) has shown the complexity of such problems which try to combine the train rescheduling feasibility together with passengers' benefit. Some improvements in the mathematical model or experiment design are needed. Future study could continue to explore better algorithms or methods on this topic.

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