Responsible TA:
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1 Programming for fun: Conway's Game of Life

We consider a 8×8 grid of quadratic cells in a periodic domain that evolves according to Conway's Game of Life[1]. Every cell has two possible states (*dead* or *alive*) and interacts only with its eight neighbour cells. Beginning from an initial configuration, the system evolves at each step in time according to the following rules:

1. Any alive cell with fewer than two alive neighbours dies, as if by underpopulation.
2. Any alive cell with two or three alive neighbours lives on to the next generation.
3. Any alive cell with more than three alive neighbours dies, as if by overpopulation.
4. Any dead cell with exactly three alive neighbours becomes a living cell, as if by reproduction.

Implement this simple evolution algorithm in the MATLAB template `conwayGameOfLife.m` that contains the initial configuration. In addition, the function mfile `plotBoard.m` is provided for plotting.

2 Linear Interpolation

We are given the following measurements of the velocity of a particle at certain times

Time [s]	0	1.2	1.7	2	3
Velocity [m/s]	0	10.8	15.3	18	27

1. Estimate by linear interpolation the velocity of the particle at time $t = 2.5$.
2. Estimate by linear interpolation the distance travelled by the particle between times $t = 2$ and $t = 3$.
3. Estimate by linear interpolation the instant acceleration of the particle a time $t = 2.5$.

[1] https://en.wikipedia.org/wiki/Conway%27s_Game_of_Life

3 Numerical Differentiation

Consider the function

$$f(x) = \log(x) \tag{1}$$

$$\left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x=x_0} = \frac{1}{x_0} \tag{2}$$

$$\text{Forward finite difference: } \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x=x_0} \approx \frac{f(x_0+h) - f(x_0)}{h} \tag{3}$$

$$\text{Centered finite difference: } \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x=x_0} \approx \frac{f(x_0+h) - f(x_0-h)}{2h} \tag{4}$$

1. Derive analytically the order of accuracy for both methods. Use the following Taylor series in your second calculation:
$$f(x_0 - h) = f(x_0) + \sum_{k=1}^{\infty} f^{(k)}(x_0)\,\frac{(-h)^k}{k!}$$
2. Use the method of forward finite difference to approximate the derivative of (1) at `x = 1`. Vary h between 10^{-15} and 10^{-1} using `logspace(-15, -1, 200)`, and calculate the relative error of the finite difference approximation compared to (2) for each `h`.
3. Plot the error vs. `h` using `loglog`. What do you observe? What could be the cause for this behavior? Use the *degree of accuracy* in your explanation.
4. Repeat the calculations of 1. and 2. using the method of centered finite difference. Compare the two `loglog` plots.