1 Degree of Exactness

In the following paragraphs the degree of exactness of midpoint, trapezoidal and Simpson rules :

The easiest way to compute the degree of exactness (DOE) is to explicitly calculate the integral of polynomials on a given interval (e.g. [0,1]) with the corresponding quadrature rule and compare the value with the analytical result. Given that :

$$I[x^k] = \int_0^1 x^k dx = \frac{1}{k+1} \tag{1}$$

The equivalence for mid point rule :

$$Q_0[f] = (b-a)f(\frac{b+a}{2}) \tag{2}$$

Therefore, the degree of exactness is calculated as follows

$$\begin{aligned} I[1] &= \frac{1}{0+1} = 1 \qquad & Q_0[1] &= 1 \\ I[x] &= \frac{1}{1+1} = \frac{1}{2} \qquad & Q_0[x] &= \frac{1}{2} \\ I[x^2] &= \frac{1}{2+1} = \frac{1}{3} \qquad & Q_0[x^2] &= \frac{1}{4} \end{aligned}$$

Thus, $I[x^2] \neq Q_0[x^2]$. The degree of exactness of the midpoint rule is **1**.

The equivalence for trapezoidal rule :

$$Q_1[f] = \frac{1}{2}(b-a)(f(b)+f(a)) \tag{3}$$

Therefore, the degree of exactness is calculated as follows

$$\begin{aligned} I[1] &= \frac{1}{0+1} = 1 \qquad & Q_1[1] &= 1 \\ I[x] &= \frac{1}{1+1} = \frac{1}{2} \qquad & Q_1[x] &= \frac{1}{2} \\ I[x^2] &= \frac{1}{2+1} = \frac{1}{3} \qquad & Q_1[x^2] &= \frac{1}{2} \end{aligned}$$

Thus, $I[x^2] \neq Q_0[x^2]$. The degree of exactness of the trapezoidal rule is **1**.

Finally, he equivalence for simpson rule :

$$Q_2[f] = \frac{b-a}{6}\big(f(a) + 4f(\frac{b+a}{2}) + f(b)\big) \tag{4}$$

$$\begin{aligned} I[1] &= \frac{1}{0+1} = 1 \qquad & Q_2[1] &= 1 \\ I[x] &= \frac{1}{1+1} = \frac{1}{2} \qquad & Q_2[x] &= \frac{1}{2} \\ I[x^2] &= \frac{1}{2+1} = \frac{1}{3} \qquad & Q_2[x^2] &= \frac{1}{3} \\ I[x^3] &= \frac{1}{3+1} = \frac{1}{4} \qquad & Q_2[x^4] &= \frac{1}{4} \\ I[x^4] &= \frac{1}{4+1} = \frac{1}{5} \qquad & Q_2[x^4] &= \frac{5}{24} \end{aligned}$$

Thus, $I[x^4] \neq Q_0[x^4]$. The degree of exactness of the simpson rule is **3**.