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1 Concentration evolution via ODE

A decaying radioactive element changes its concentration according to the ODE:

$$\frac{dy}{dt} = -ky \tag{1}$$

The analytical solution reads:

$$y(t) = y_0 \exp(-kt) \tag{2}$$

1. Plot the behavior of the radioactive decay problem as a vector field in the y vs t plane
   - Find online the function vector_field.m. It plots the solutions and derivatives of first order initial value problems (IVPs) for different initial conditions
   - Plot the vector field for different initial values between 0 and 1, time between 0 and 10 and $k = 1$. Can you see what a solver has to do?
   - Is it possible to switch from one trajectory in the vector field to another? What follows for the uniqueness of the solutions?
2. The forward Euler method reads for this problem

   $$y_{n+1} = y_n + h * f(t_n, y_n) = y_n - h * k * y_n \tag{3}$$

   - First define a function defining the successor of yn with a header like function ynp = Forward_stepper(k, yn, h)
   - Use the conditions y0 = 1, k = 1 and h = 0.1 to solve the radioactive decay problem from t0 = 0 to tEnd = 10 by defining an integrating function of the form function [T,Y] = stepper_integrate(@stepper,k,t0,tEnd,y0,h)
3. The backward Euler method uses the following step formula

   $$y_{n+1} = y_n + h * f(t_{n+1}, y_{n+1}) \tag{4}$$

   - Rearrange this equation so that you can define a function defining the successor of yn with a header like function ynp = Backward_stepper(k, yn, h)
   - Use the prior defined integration function to solve the problem with the backward method.
4. Plot the obtained solutions comparing them to the analytical solution. Use the subplot method to produce two subplots in a single figure. What happens if you increase the step size h? What happens if you increase the step size above 2?

2 High order ODEs

Convert the following fourth order initial value problem

$$y^{(4)}(t) = \cos(\ddot{y}(t)) + \dot{y}(t)e^{-5t} \tag{5}$$

with initial conditions

$$y(t_0) = 0,\ \dot{y}(t_0) = 3,\ \ddot{y}(t_0) = -1,\ \ddot{y}(t_0) = -1,\ \dddot{y}(t_0) = 0 \tag{6}$$

into a first order initial value problem.