1 LTE of the implicit Euler method

Given the 1st order initial value problem

$$\begin{aligned} \dot{y}(t) &= f(t, y(t)) \\ y(t_0) &= y_0 \end{aligned} \tag{1}$$

calculate the local truncation error (LTE) of the implicit Euler method $y_{j+1} = y_j + hf(t, y_{j+1})$.

2 2nd Order ODEs

The equation of motion for a damped harmonic oscillator (mass on a spring including friction) reads:

$$m\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = -ky - a\frac{\mathrm{d}y}{\mathrm{d}t} \tag{2}$$

1. Rewrite (2) as an explicit, 1^{st} order ODE
2. Use `ode45` to solve the ODE
   - Set up a function to calculate the derivatives. Its header should read something like `function dy = harm_osc(t, y, m, k, a)`
   - Use m = 1; k = 10; a = 0.5; $y(t_0) = 1$; $\dot{y}(t_0) = 0$; tSpan = [0, 20];

Plot the position of the mass $y(t)$ and its velocity $\dot{y}(t)$.

3 2nd Order ODEs

The equation of motion for an ideal harmonic oscillator reads:

$$m\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = -ky \tag{3}$$

Set m = 10; k = 1; t_0 = 0; tEnd = 100; $y(t_0) = 1$; $\dot{y}(t_0) = 0$; and use `ode45` and `ode15s` to solve the ODE.

- Calculate the total energy $E_{\text{tot}} = E_{\text{kin}} + E_{\text{pot}} = \frac{1}{2}ky^2 + \frac{1}{2}m\dot{(y)}^2$
- Plot the total energy against time for the two methods. What do you observe (remember, this time there is no friction and the system is conservative)?