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Statistical and Numerical Methods
Exercise 1, HS24

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# 1 Linear Interpolation

We are given the following measurements of the velocity of a particle at certain times

| Time [s] | 0 | 1.2 | 1.7 | 2 | 3 |
|---|---|---|---|---|---|
| Velocity [m/s] | 0 | 10.8 | 15.3 | 18 | 27 |

1. Estimate by linear interpolation the velocity of the particle at time t = 2.5.
2. Estimate by linear interpolation the distance travelled by the particle between times t = 2 and t = 3.
3. Estimate by linear interpolation the instant acceleration of the particle a time t = 2.5.

# 2 Polynomial Interpolation (Core)

Some modeling considerations have mandated a search for a function[1]

$$u(x) = k_0 e^{k_1 x + k_2 x^2}, \quad (1)$$

where the unknown coefficients $k_1$ and $k_2$ are expected to be nonpositive. Given are data pairs to be interpolated, $(x_0, z_0)$, $(x_1, z_1)$, and $(x_2, z_2)$, where $z_i > 0$, $i = 0, 1, 2$. Thus, we require $u(x_i) = z_i$.
The function $u(x)$ is not linear in its coefficients, but $v(x) = ln(u(x))$ is linear in its coefficients.

1. Find a quadratic polynomial $v(x)$ that interpolates appropriately defined three data pairs, and then give a formula for $u(x)$ in terms of the original data.
2. Write a script to find $u$ for the data (0, 1), (1, 0.9), (3, 0.5). Give the coefficients $k_i$ and plot the resulting interpolant over the interval [0, 6]. In what way does the curve behave qualitatively differently from a quadratic?

[1] Ascher et al. *A first course on numerical methods.* Society for Industrial and Applied Mathematics, 2011.

# 3 Numerical Differentiation (Core)

Consider the function

$$f(x) = \log(x) \tag{2}$$

$$\left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x=x_0} = \frac{1}{x_0} \tag{3}$$

$$\text{Forward finite difference: } \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x=x_0} \approx \frac{f(x_0+h) - f(x_0)}{h} \tag{4}$$

$$\text{Centered finite difference: } \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x=x_0} \approx \frac{f(x_0+h) - f(x_0-h)}{2h} \tag{5}$$

1. Derive analytically the order of accuracy for both methods. Use the following Taylor series in your second calculation:
$$f(x_0 - h) = f(x_0) + \sum_{k=1}^{\infty} f^{(k)}(x_0)\,\frac{(-h)^k}{k!}$$
2. Use the method of forward finite difference to approximate the derivative of (2) at x = 1. Vary h between $10^{-15}$ and $10^{-1}$ using `logspace(-15, -1, 200)`, and calculate the relative error of the finite difference approximation compared to (3) for each `h`.
3. Plot the error vs. `h` using `loglog`. What do you observe? What could be the cause for this behavior? Use the *degree of accuracy* in your explanation.
4. Repeat the calculations of 1. and 2. using the method of centered finite difference. Compare the two `loglog` plots.