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Statistical and Numerical Methods
Exercise 5, HS 2024

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1 LTE of the implicit Euler method

Given the 1st order initial value problem

$$\begin{aligned} \dot{y}(t) &= f(t, y(t)) \\ y(t_0) &= y_0 \end{aligned} \tag{1}$$

calculate the local truncation error (LTE) of the implicit Euler method $y_{j+1} = y_j + hf(t_{j+1}, y_{j+1})$.

2 Damped harmonic oscillator (Core)

The equation of motion for a damped harmonic oscillator (mass on a spring including friction) reads:

$$m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -ky - a\frac{\mathrm{d}y}{\mathrm{d}t} \tag{2}$$

1. Rewrite (2) as an explicit 1st order ODE in your note.

$$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} y \\ \dot{y} \end{pmatrix} \tag{3}$$

2. Use `ode45` to solve the ODE numerically:
   - Set up a function to calculate the derivatives. Use the result from 1. Its header should read something like `function dz = damp_harmonic(t, z, m, k, a)`
   - Use m = 1; k = 10; a = 0.5; $y(t_0) = 1$; $\dot{y}(t_0) = 0$; tSpan = [0, 20];
   - Solve the system using the following script to obtain the position and velocity of a mass:
     `[t, z] = ode45(@(t,z)damp_harmonic(t, z, m, k, a), tSpan, z0);`
   - Plot the position of the mass $y(t)$ and its velocity $\dot{y}(t)$ against time, respectively.

3 Ideal harmonic oscillator (Core)

The equation of motion for an ideal harmonic oscillator reads:

$$m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -ky \tag{4}$$

1. Rewrite (4) as an explicit 1st order ODE in your note.
2. Use various solvers: `ode23`, `ode45`, `ode15s` to solve the ODE and compare them.
   - Set up a function the calculate the derivatives. Use the result from 1. The function may have a form `function dz = ideal_harmonic(t, z, m, k, a)`
   - Use m = 10; k = 1; $y(t_0) = 1$; $\dot{y}(t_0) = 0$; tSpan = [0, 100];
   - Set up a function to calculate the total energy $E_{\text{tot}} = E_{\text{kin}} + E_{\text{pot}} = \frac{1}{2}ky^2 + \frac{1}{2}m\dot{y}^2$.
     `function E = E_total(z,m,k)`
   - Plot the total energy against time resulting from all three solvers in one figure and compare them. What do you observe and what is the cause of discrepancies if any (remember, this time there is no friction and the total energy should be conserved)?