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# Ab initio calculation of femtosecond-time-resolved photoelectron spectra of $\mathrm{NO}_{2}$ after excitation to the A-band 

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#### Abstract

We present calculations of time-dependent photoelectron spectra of $\mathrm{NO}_{2}$ after excitation to the A-band for comparison with extreme-ultraviolet (XUV) time-resolved photoelectron spectroscopy. We employ newly calculated potential energy surfaces of the two lowest-lying coupled ${ }^{2} \mathrm{~A}^{\prime}$ states obtained from multi-reference configuration-interaction calculations to propagate the photo-excited wave packet using a split-step-operator method. The propagation includes the nonadiabatic coupling of the potential surfaces as well as the explicit interaction with the pump pulse centered at $3.1 \mathrm{eV}(400 \mathrm{~nm})$. A semiclassical approach to calculate the time-dependent photoelectron spectrum arising from the ionization to the eight energetically lowest-lying states of the cation allows us to reproduce the static experimental spectrum up to a binding energy of 16 eV and enables direct comparisons with XUV time-resolved photoelectron spectroscopy. Published by AIP Publishing. https://doi.org/10.1063/1.5029365


## I. INTRODUCTION

Conical intersections play a central role in the photochemistry of most molecules, determining the pathway of reactions in molecules of all sizes ranging from DNA to small molecules, such as 1,3-cyclohexadiene, iodomethane, or nitrogen dioxide. The latter is a prototypical system to study the effect of a conical intersection, which, in $\mathrm{NO}_{2}$, connects the energetically lowest lying excited state to the ground state. Its presence leads to complex femtosecond dynamics in the electronically excited state after excitation around 400 nm and enables the dissociation of the molecule after internal conversion to the groundstate surface. These properties as well as the limited number of electrons and nuclei make $\mathrm{NO}_{2}$ an ideal candidate to benchmark high-level $a b$ initio calculations against suitable experimental observables. The spectroscopic properties of $\mathrm{NO}_{2}$ have been determined with high accuracy. ${ }^{1-9}$ However, femtosecond time-resolved experiments have only recently become possible with the introduction of multiphoton time-resolved photoelectron spectroscopy (MP-TRPES) ${ }^{10-15}$ (for a review, see Ref. 16) and time-resolved high-harmonic spectroscopy (TRHHS). ${ }^{17-22}$ Although these techniques could be realized with state-of-the-art femtosecond lasers, their interpretation is sometimes challenging because of the high peak intensities associated with femtosecond laser pulses. The previous experiments on $\mathrm{NO}_{2}$ nicely illustrate this statement because substantially different observations were reported with respect to the dynamics created by single-photon excitation at 400 nm . For example, oscillations in the yield of $\mathrm{NO}^{+}$and slow (nearzero eV ) photoelectrons were reported with periods ranging

[^0]from 500 to 750 fs in pump-probe experiments involving $400-\mathrm{nm}$ and $267-\mathrm{nm}$ laser pulses. ${ }^{10,13,14}$ In contrast to this, oscillations with a period of $\sim 220$ fs were observed in timeresolved mass spectrometry. ${ }^{23}$ Time-resolved high-harmonic spectroscopy revealed 1-2 oscillations with a period close to $100 \mathrm{fs} .{ }^{18-20}$ Notably, all of these observations were attributed to dynamics induced by single-photon excitation in the vicinity of 400 nm . A distinctive feature of the TRHHS results ${ }^{18}$ was the unequivocal demonstration of dominant single-photon excitation through the wavelength dependence of the signals, i.e., the presence of the characteristic few-picosecond dissociation time scale of $\mathrm{NO}_{2}$ for excitation above the dissociation threshold at 3.11 eV ( 398 nm ) and its absence for excitation below the threshold. This hallmark of singlephoton excitation in $\mathrm{NO}_{2}$ has not been reported in any of the MP-TRPES experiments, although a small number of studies based on coincidence detection have established the contribution of single-photon excitation at long pump-probe delays ( $>400-500 \mathrm{fs}$ ). ${ }^{12,14}$ Recently, some of the present authors have applied extreme-ultraviolet (XUV)-TRPES to the same excited-state dynamics in $\mathrm{NO}_{2} .{ }^{24}$ The application of singlephoton ionization removes the complications induced by the high peak intensities and reveals the genuine dynamics of the photo-excited molecular system over its complete reaction path. The main motivation for the present (theoretical) work was the interpretation of these novel experimental results, as well as a possible future modeling of TRHHS or MP-TRPES results.

On the theoretical side, $\mathrm{NO}_{2}$ has also been extensively studied. Potential-energy surfaces of the energetically lowest lying electronic states of the neutral molecule ${ }^{25-33}$ and the cation ${ }^{34,35}$ have been calculated for a long time and have reached a high level of accuracy. The surfaces
with the highest density of points so far were Davidsoncorrected multi-reference configuration-interaction (MRCI) calculations with the Dunning's correlation-consistent polarized valence quadruple zeta (cc-pVQZ) basis set ${ }^{36}$ for the two lowest electronic states of the neutral molecule ${ }^{32}$ and MRCI cc-pVDZ ${ }^{36}$ calculations for the cationic molecule. ${ }^{35}$ Some of the surfaces have been used to calculate time-dependent wave packets after excitation, leading to insights into the shorttime dynamics ${ }^{37}$ and enabling the calculation of experimental observables, such as photoelectron spectra and photoelectron angular distributions. ${ }^{38}$ However, due to the strong dependence of the potential energies of the different states of $\mathrm{NO}_{2}$ on the nuclear coordinates (both in the cation as well as in the neutral molecule), there are many accessible cationic states for a given binding energy after excitation at 400 nm . This makes high-level calculations of photoelectron spectra very expensive, which were therefore limited to the lowest two electronic states of the cation. ${ }^{37,38}$ In this work, we extend previous wave-packet calculations of two of the present authors ${ }^{37,38}$ to a larger grid, enabling us to accurately describe the photodissociation. Simultaneously, we improve the level of theory from complete active space self-consistent field (CASSCF) to MRCI, which allows us to reach quantitative accuracy with respect to the dissociation threshold. We present the results of full-dimensional quantum-mechanical wave-packet calculations on these novel surfaces. Subsequently, we report the calculation of time-dependent photoelectron spectra for comparison with recent experimental data from XUV-TRPES ${ }^{24}$ in an extended range of binding energies for the channels leading to each of the two lowest lying states of ${ }^{1} \mathrm{~A}^{\prime},{ }^{1} \mathrm{~A}^{\prime \prime},{ }^{3} \mathrm{~A}^{\prime}$, and ${ }^{3} \mathrm{~A}^{\prime \prime}$ characters of the cationic molecule within a semiclassical model.

## II. METHOD

## A. Calculation of potential-energy surfaces

To describe the nuclear arrangement of $\mathrm{NO}_{2}$, we use three different coordinate systems: internal coordinates, Jacobi coordinates, ${ }^{39}$ and Cartesian coordinates (see Fig. 1).

The potentials, which are used in the wave-packet calculations and the photoelectron calculations, are determined on the following grid:

$$
\begin{equation*}
r=0.9 \AA+2.625 \cdot 10^{-2} \AA i_{r}, \quad \text { with } \quad i_{r}=0, \ldots, 90, \tag{1}
\end{equation*}
$$



FIG. 1. Coordinate systems for $\mathrm{NO}_{2}$ : Black: internal coordinates with the NO bond lengths $r_{1}$ and $r_{2}$, as well as the bond angle $\beta$. Green: Jacobi coordinates $r, R$, and $\theta$. The coordinate $R$ is measured from the center of mass along the NO-bond, i.e., $r_{2} m=r_{2} m_{\mathrm{O}} /\left(m_{\mathrm{O}}+m_{\mathrm{N}}\right)$ from the nitrogen nucleus, where $m_{\mathrm{O}}$ and $m_{\mathrm{N}}$ are the atomic masses of oxygen and nitrogen, respectively. Orange: molecular-frame Cartesian coordinates. The molecule lies in the $y z$-plane. The origin is located in the center of charge and $z$ is pointing along the $\beta$-angle bisector.

$$
\begin{align*}
R= & 0.484375 \AA+2.21875 \cdot 10^{-2} \AA i_{R} \\
& \text { with } i_{R}=0, \ldots, 155  \tag{2}\\
\theta= & 180^{\circ}-(45 / 64)^{\circ} i_{\theta}, \quad \text { with } i_{R}=0, \ldots, 250 . \tag{3}
\end{align*}
$$

This grid consists of $3.6 \times 10^{6}$ points. For the electronic states of the neutral molecule, 28250 points were calculated and the remaining ones were interpolated. The calculated points were not equidistant. Their density was chosen to be high in the potential minimum, around the conical intersection, and where problems with the interpolation on the grid occurred. The number of points was increased until satisfactory potential surfaces up to an energy of 3.5 eV of the $(1)^{2} \mathrm{~A}^{\prime}$ state above the lowest energy in the grid were achieved. The quantum-chemistry calculations were performed using the Molpro suite. ${ }^{40}$ We first determined the two-state-averaged wave functions with the multi-configurational self-consistent field (MC-SCF) approach with a $\operatorname{CAS}(13,10)^{41,42}$ in a simplified configuration, namely, $r_{1}^{\text {init }}=r_{2}^{\text {init }}=1 / 2\left(r_{1}+r_{2}\right)$. This was taken as an initial guess for a two-state-averaged $\operatorname{CASSCF}(13,10)$ calculation at the final configuration with a subsequent $\operatorname{MRCI}(13,10)^{43-45}$ calculation. Here, $(13,10)$ refers to the chosen active space: we used 10 open orbitals with 13 active electrons. In the case of CASSCF calculations, the five orbitals with the lowest energies were closed. For the MRCI calculation, two orbitals were closed and three were frozen (core) orbitals. All calculations used the optimized augmented Dunning's correlation-consistent polarized valence quadruple zeta (aug-cc-pVQZ) basis set. ${ }^{36}$ The output was checked for convergence problems and, in the case of errors, recalculated. Configurations which could not be converged or which resulted in energies which were significantly different from the surrounding points were excluded. For the potential surfaces of the neutral states, we tried to calculate the potential at 29174 grid points (i.e., 924 calculations did not converge). For the potential surfaces of the cationic states, the number of points was decreased (between 5864 and 6167 points), as they were not needed for the propagation. Further details are given in Appendix A.

The diabatization of the neutral adiabatic potentials was performed on the basis of a phenomenological approach ${ }^{46,47}$ by maximizing the transition dipole moment $\mu_{\mathrm{D}, 12}^{y}$ between the diabatic states. The reliability of this diabatization scheme has been numerically verified in Ref. 46. This leads to a diabatization angle

$$
\begin{equation*}
\alpha_{\mu}=\frac{1}{2} \operatorname{atan} 2\left(\mu_{11}^{y}-\mu_{22}^{y}, 2 \mu_{12}^{y}\right) \tag{4}
\end{equation*}
$$

where $\mu_{11}^{y}$ and $\mu_{22}^{y}$ are the permanent dipole moments along the $y$-axis of the $(1)^{2} \mathrm{~A}^{\prime}$ and $(2)^{2} \mathrm{~A}^{\prime}$ states, respectively, and $\mu_{12}^{y}$ is the transition dipole moment between them (see Fig. 1 for the definition of the $y$-axis). This method relies on the properties of the $(1)^{2} \mathrm{~A}_{1}$ and $(1)^{2} \mathrm{~B}_{2}$ states (labeled in the $\mathrm{C}_{2 \mathrm{v}}$ symmetry of the ground state) and fails when a third state interacts significantly with these states. In these parts of the coordinate space, the mixing angle was approximated. Generally, the angles were visually inspected for cuts along $\theta$ and smoothly continued to $0, \pi / 2$, or $-\pi / 2$, respectively. Figure 2 illustrates the diabatization process. The treatment of areas with non-negligible coupling of a third state is further discussed in Appendix B.


FIG. 2. Illustration of the diabatization process. All plots are cuts through the hypersurfaces of the lowest-lying electronic states of $\mathrm{NO}_{2}$ with $r=1.1625 \AA$ and $R=1.7047 \AA$. (a) Calculated permanent dipole moments of $(1)^{2} \mathrm{~A}^{\prime}$ and $(2)^{2} \mathrm{~A}^{\prime}$ along the $y$-axis (blue and red) as well as their transition dipole moment (black). (b) Mixing angles: The black curve illustrates the mixing angle calculated according to Eq. (4) and the green curve is the modified diabatization angle which was used in the diabatization. (c) Final result of the diabatization: The adiabatic potentials are colored in red and blue and the diabatic potentials in yellow and violet. The additional avoided crossings around $\theta=80^{\circ}$ (between the first and second adiabatic state), $105^{\circ}$, and $165^{\circ}$ (both between the second and third adiabatic state) were not diabatized.

The raw data available in the supplementary material includes the adiabatic energies, the diabatic energies and coupling constants, the permanent dipole moments and the transition dipole moments in the $y$-direction between the two lowest ${ }^{2} \mathrm{~A}^{\prime}$ states of the neutral molecule, and the adiabatic energies of each of the two lowest ${ }^{1} \mathrm{~A}^{\prime},{ }^{1} \mathrm{~A}^{\prime \prime},{ }^{3} \mathrm{~A}^{\prime}$, and ${ }^{3} \mathrm{~A}^{\prime \prime}$ states of the cationic molecule. The interpolation method is described in Appendix C.

## B. Calculation of the vibronic wave packet

The vibronic wave packet was calculated with a split-stepoperator method presented before Ref. 37. As the wave packet is calculated on a grid, the dissociating parts of the wave function could not be fully described. Absorbing barriers were constructed ${ }^{48}$ and tested for minimal reflection of the wave function at the edges of the grid. The optical potential has the shape

$$
\begin{equation*}
V_{\mathrm{opt}}(\theta, R, r)=\max \left(A(\theta, R, r), A\left(\theta^{\prime}, R^{\prime}, r^{\prime}\right)\right) \tag{5}
\end{equation*}
$$

with
$A(\theta, R, r)=\max \left(0, m_{r}\left(r-r_{\mathrm{abs}}\right), m_{R}\left(R-R_{\mathrm{abs}}\right), m_{\theta}\left(\theta-\theta_{\mathrm{abs}}\right)\right)$,
where $r_{\mathrm{abs}}=3.0263 \AA, m_{r}=1.2425 \mathrm{eV} \AA^{-1}, R_{\mathrm{abs}}=3.7016 \AA$, $m_{R}=1.1828 \mathrm{eV} \AA^{-1}, \theta_{\text {abs }}=21.7969^{\circ}$, and $m_{\theta}=-0.0139 \mathrm{eV}$ $\mathrm{deg}^{-1}$. The primed parameters are defined as the set of Jacobicoordinates using the center of mass along the other bond of $\mathrm{NO}_{2}$ [see Eqs. (B1)-(B3)]. This definition ensures that the symmetry of the generated wave packet is not broken due to the absorption. The initial ground state wave function was found by calculating an energy spectrum of a Gaussian centered at the equilibrium position ${ }^{49}$ and subsequent filtering of the wave function with the lowest energy. ${ }^{50}$ The energy of the ground state wave function was determined to be 0.233 eV above the minimal energy on the grid, which is in good agreement with the zero-point-energy derived from experimental vibrational frequencies using the harmonic-oscillator approximation $(0.228 \mathrm{eV})$. The wave function was propagated with a split-step operator method, ${ }^{49}$ explicitly including the dipole-coupling of the two lowest-lying states due to the pump pulse. ${ }^{37}$ The step size of a typical calculation was 0.02 fs . The wave functions were saved every 1 fs to serve as inputs for further calculations. The following approximations had to be made; first, the rotational motion of the molecules was not included. They were assumed to be aligned with their y-axis (cf. Fig. 1) along the polarization of the pump pulse. Second, the transition from the ${ }^{2} \mathrm{~A}_{1}$ ground state to the ${ }^{2} \mathrm{~B}_{1}$ second excited state was neglected. Previous work ${ }^{51}$ showed this transition to be weak compared to the transition to the ${ }^{2} \mathrm{~B}_{2}$ state at excitation energies above 2.9 eV . Third, the Renner-Teller coupling of the ${ }^{2} \mathrm{~A}_{1}$ with the ${ }^{2} \mathrm{~B}_{1}$ state around $\theta=0^{\circ}$ and $\theta=180^{\circ}$ was not treated.

To depict the results of a propagation, we project the ground state vibronic wave function $\chi_{0}$ out of the total wave function according to

$$
\begin{equation*}
\chi^{\mathrm{proj}}(t)=\chi(t)-\int_{\mathbf{R}} \chi_{0} \chi(t) \mathrm{d} \mathbf{R} \chi(t) \tag{7}
\end{equation*}
$$

and calculate partial densities along specific coordinates with

$$
\begin{align*}
& \rho_{r}=\rho_{r_{2}}=\int\left|\chi_{i}^{\mathrm{proj}}(r, R, \theta)\right|^{2} \sin \theta \mathrm{~d} R \mathrm{~d} \theta  \tag{8}\\
& \rho_{\beta}=\int\left|\chi_{i}^{\mathrm{proj}}\left(r_{1}, r_{2}, \beta\right)\right|^{2} \frac{\sin \theta r_{1}}{R} \mathrm{~d} r_{1} \mathrm{~d} r_{2} \tag{9}
\end{align*}
$$

where $\chi_{i}^{\text {proj }}$ denotes the vibrational wave function of state $i$ without contributions of the initial state.

## C. Calculation of photoelectron spectra

Due to the number of the involved cationic states as well as their complexity, it was not possible to calculate the photoelectron spectra by direct propagation of the wave functions on the ionic surfaces. ${ }^{38}$ Instead, we turned to a semiclassical model. The cross section $\sigma$ for a transition at the energy $E_{\text {tr }}$ can generally be written as
$\left.\sigma\left(E_{\mathrm{tr}}\right)=\frac{\pi e^{2} E_{\mathrm{tr}}}{3 \epsilon_{0} c} \sum_{a, b} P_{a}\left|\left\langle\Psi_{b}\right| \hat{\mathbf{r}}\right| \Psi_{a}\right\rangle\left.\right|^{2} \delta\left(E_{\mathrm{tr}}-\left(E_{b}-E_{a}\right)\right)$,
where $P_{a}$ is the population of the initial state, $\hat{\mathbf{r}}$ is the dipole operator, $\Psi_{a}$ and $\Psi_{b}$ are the initial and final molecular wave
functions, respectively, and $E_{a}$ and $E_{b}$ are their energies. ${ }^{52}$ Following the literature, ${ }^{53}$ one can express Eq. (10) in terms of the nuclear coordinates $\mathbf{R}$ and the electronic coordinates $\mathbf{r}$ by using the Born-Oppenheimer approximation as

$$
\begin{align*}
\sigma\left(E_{\mathrm{tr}}\right)= & C \sum_{i, k, f, l} P_{i k} \iint\left\langle\psi_{i}\left(\mathbf{R}^{\prime}\right)\right| \hat{\mathbf{r}}\left|\psi_{f}\left(\mathbf{R}^{\prime}\right)\right\rangle \chi_{i k}^{*}\left(\mathbf{R}^{\prime}\right) \\
& \times \chi_{f l}\left(\mathbf{R}^{\prime}\right) \chi_{f l}^{*}(\mathbf{R}) \chi_{i k}(\mathbf{R})\left\langle\psi_{f}(\mathbf{R})\right| \hat{\mathbf{r}}\left|\psi_{i}(\mathbf{R})\right\rangle \\
& \times \delta\left(E_{\mathrm{tr}}-\left(E_{f l}-E_{i k}\right)\right) \mathrm{d} \mathbf{R}^{\prime} \mathrm{d} \mathbf{R} \tag{11}
\end{align*}
$$

where $\psi_{i}$ and $\psi_{f}$ are the initial and final electronic wave functions, respectively, $\chi_{i k}$ and $\chi_{f}$ are the initial and final nuclear wave functions with their vibrational quantum numbers collectively described by the indices $k$ and $l$, and we collected the prefactors in $C$. Inserting the semiclassical approximation ${ }^{54}$

$$
\begin{equation*}
E_{f l}-E_{i k} \approx V_{f}(\mathbf{R})-V_{i}(\mathbf{R}) \tag{12}
\end{equation*}
$$

where $V_{i}$ and $V_{f}$ are the potential energies at coordinate $\mathbf{R}$ of the involved electronic states, and the identity

$$
\begin{equation*}
\sum_{l} \chi_{f}\left(\mathbf{R}^{\prime}\right) \chi_{f}^{*}(\mathbf{R})=\delta\left(\mathbf{R}^{\prime}-\mathbf{R}\right) \tag{13}
\end{equation*}
$$

one obtains

$$
\begin{align*}
\sigma\left(E_{\mathrm{tr}}\right)= & \left.C \sum_{i, k_{s} f} P_{i k} \int\left|\left\langle\psi_{f}(\mathbf{R})\right| \hat{\mathbf{r}}\right| \psi_{i}(\mathbf{R})\right\rangle\left.\right|^{2} \chi_{i k}^{*}(\mathbf{R}) \\
& \times \chi_{i k}(\mathbf{R}) \delta\left(E_{\mathrm{tr}}-\left(V_{f}(\mathbf{R})-V_{i}(\mathbf{R})\right)\right) \mathrm{d} \mathbf{R} \tag{14}
\end{align*}
$$

This is known as the multidimensional reflection principle ${ }^{55,56}$ without the kinetic energy correction ${ }^{57,58}$ because the approximation in Eq. (12) neglects effects of the vibronic energy ${ }^{53}$ and thus assumes that the kinetic energies of the nuclei do not change during the transition. As the electronic transition dipole moment $\mu_{i f}(\mathbf{R})=\left\langle\psi_{f}(\mathbf{R})\right| \hat{\mathbf{r}}\left|\psi_{i}(\mathbf{R})\right\rangle$ as well as the $\delta$-function in Eq. (14) are independent of $k$, it can also be written in a time-dependent form

$$
\begin{align*}
\sigma\left(E_{\mathrm{tr}}, t\right)= & C \sum_{i, f} \int\left|\mu_{i f}(\mathbf{R})\right|^{2} \delta\left(E_{\mathrm{tr}}-\left(V_{f}(\mathbf{R})-V_{i}(\mathbf{R})\right)\right) \\
& \times \sum_{k} P_{i k}(t) \chi_{i k}^{*}(\mathbf{R}) \chi_{i k}(\mathbf{R}) \mathrm{d} \mathbf{R} \\
= & C \sum_{i, f} \int\left|\mu_{i f}(\mathbf{R})\right|^{2} \delta\left(E_{\mathrm{tr}}-\left(V_{f}(\mathbf{R})-V_{i}(\mathbf{R})\right)\right) \\
& \times\left|\chi_{i}(\mathbf{R}, t)\right|^{2} \mathrm{~d} \mathbf{R} . \tag{15}
\end{align*}
$$

For simplicity, we contracted $P_{i}$ into $\chi_{i}$ such that

$$
\begin{equation*}
P_{i}(t)=\int\left|\chi_{i}(\mathbf{R}, t)\right|^{2} \mathrm{~d} \mathbf{R} \tag{16}
\end{equation*}
$$

For photoionization, we define $V_{f}=E_{\text {kin }}^{\mathrm{e}}+V_{\text {cat }, f}$, where $E_{\text {kin }}^{\mathrm{e}}$ is the kinetic energy of the measured electron and $V_{\mathrm{cat}, f}$ the electronic energy of the cationic state, to get

$$
\begin{align*}
\sigma\left(E_{\mathrm{tr}}\right)= & C \sum_{i, f} \int\left|\mu_{i f}(\mathbf{R})\right|^{2}\left|\chi_{i}(\mathbf{R}, t)\right|^{2} \\
& \times \delta\left(E_{\mathrm{tr}}-E_{\text {kin }}^{\mathrm{e}}-V_{\text {cat } f}(\mathbf{R})+V_{i}(\mathbf{R})\right) \mathrm{d} \mathbf{R} \tag{17}
\end{align*}
$$

As $E_{\mathrm{tr}}$ is typically constant in an experiment, we substitute it with the binding energy $E_{\mathrm{bind}}=E_{\mathrm{tr}}-E_{\text {kin }}^{\mathrm{e}}$,

$$
\begin{align*}
\sigma\left(E_{\text {bind }}\right)= & C \sum_{i, f} \int\left|\mu_{i f}(\mathbf{R})\right|^{2}\left|\chi_{i}(\mathbf{R}, t)\right|^{2} \\
& \times \delta\left(E_{\text {bind }}-V_{\text {cat }, f}(\mathbf{R})+V_{i}(\mathbf{R})\right) \mathrm{d} \mathbf{R} \tag{18}
\end{align*}
$$

Because we did not diabatize the cationic potential energy surfaces, the adiabatic representation of $\chi_{i}, V_{\text {cat }, f}$, and $V_{i}$ is used to evaluate Eq. (18). As the electronic structure of the involved states is changing in this representation, $\left|\mu_{i f}\right|^{2}$ is strongly configuration dependent. We approximate the dipole matrix element as the norm of the Dyson orbital ${ }^{59-61}$ by using the sudden approximation, i.e., we disregard any continuum effects. The norm was determined by calculating a three-stateaveraged $\operatorname{CASSCF}(13,10)$ CI-expansion of the lowest lying ${ }^{2} \mathrm{~A}^{\prime}$ states of the neutral molecule with the aug-cc-pVQZ basis set, with a subsequent optimization of the CI-coefficients of all relevant states of the cation with invariant molecular orbitals. See Appendix D for more details. In Fig. 3, the behavior of $\left|\mu_{i f}\right|^{2}$ for a cut along $\theta$ is shown.

Using this approximation and defining the local ionization potential $I_{p}^{i f}(\mathbf{R})=V_{\text {cat }, f}(\mathbf{R})-V_{i}(\mathbf{R})$, the photoelectron spectrum is given by
$\sigma\left(E_{\mathrm{bind}}, t\right)=C \sum_{i, f} \int\left|\chi_{i}(\mathbf{R}, t)\right|^{2}\left|\mu_{i f}(\mathbf{R})\right|^{2} \delta\left(E_{\mathrm{bind}}-I_{p}^{i f}(\mathbf{R})\right) \mathrm{d} \mathbf{R}$.

The cationic potential energies were shifted collectively by 0.32 eV in order to maximally overlap the photoelectron bands with the experimental spectrum of $\mathrm{NO}_{2}$. Using the calculated time-dependent wave functions, photoelectron spectra of individual channels can be calculated according to Eq. (19).

Depending on the excitation energy, a significant part of the wave function runs into the absorption barrier at large $R$ and $r$. To include the absorbed population into our model, its spectrum $\sigma_{\text {abs }}$ has to be approximated. To take the configuration dependence of the spectra of the absorbed parts of the wave functions into account, we calculated the time-integrated density

$$
\begin{equation*}
P_{\mathrm{int}}(r, R, \theta)=\int_{t}\left|\chi^{\mathrm{proj}}(r, R, \theta, t)\right|^{2} \mathrm{~d} t \tag{20}
\end{equation*}
$$

and from this the conditional probability density in $r$,

$$
\begin{equation*}
P_{\text {cond }}(R, \theta \mid r)=\frac{P_{\mathrm{int}}(r, R, \theta)}{\int_{R, \theta} P_{\mathrm{int}}(r, R, \theta) \sin \theta \mathrm{d} R \mathrm{~d} \theta} . \tag{21}
\end{equation*}
$$

The photoelectron signal of the absorbed wave packet can then be approximated by

$$
\begin{equation*}
\sigma_{\mathrm{abs}}^{i f}=C \int_{\mathbf{R}, r \geq r_{\mathrm{abs}}} P_{\mathrm{cond}}(\mathbf{R})\left|\mu_{i f}(\mathbf{R})\right|^{2} \delta\left(E_{\mathrm{bind}}-I_{p}^{i f}(\mathbf{R})\right) \mathrm{d} \mathbf{R} . \tag{22}
\end{equation*}
$$



FIG. 3. Illustration of the calculated transition-matrix element $\left|\mu_{i f}(\mathbf{R})\right|^{2}$ for the example of the transitions from the two energetically lowest adiabatic ${ }^{2} \mathrm{~A}^{\prime}$ states [black curves in (b)] to the cationic (1) ${ }^{3} \mathrm{~A}^{\prime}$ (blue), (1) ${ }^{3} \mathrm{~A}^{\prime \prime}$ (red), and (2) ${ }^{3} \mathrm{~A}^{\prime \prime}$ (yellow) states in a cut through $r=1.1625 \AA$ and $R=1.7047 \AA$. (a) Dyson norm. The three colors (blue, red, and yellow) indicate the final state and the line style the initial state. [The legend is given in (b).] (b) Cut through the potential surfaces of the states. The magnitude of the overlap is strongly dependent on the electronic structure of the initial and the final state. Avoided crossings in the involved states can lead to sharp changes in the Dyson norm, as can be observed for the case of the transition from (2) $)^{2} \mathrm{~A}^{\prime}$ to $(2)^{3} \mathrm{~A}^{\prime \prime}$ (dashed yellow line) due to the crossing at $\theta=140^{\circ}$ in the final state. However, not all crossings lead to a change in the Dyson norm, as can be observed for the crossing of the neutral states at $\theta=130^{\circ}$ in the transition to the $(1)^{3} \mathrm{~A}^{\prime}$ state (solid blue line).

The total photoelectron signal was then calculated using

$$
\begin{align*}
\sigma^{\mathrm{tot}} & =\sum_{i, f} \sigma^{i f}\left(E_{\mathrm{bind}}, t\right) \\
& =\sum_{i, f} \sigma_{0}^{i f}\left(E_{\mathrm{bind}}, t\right)+(1-P(t)) \sigma_{\mathrm{abs}}^{i f}\left(E_{\mathrm{bind}}, t\right) \tag{23}
\end{align*}
$$

where $\sigma_{0}^{i f}\left(E_{\mathrm{bind}}, t\right)$ is the photoelectron spectrum without absorbed contributions from state $i$ to state $f$, and $P$ is the total population in the grid. We simulated the limited energy resolution of the experiment by convolving the resulting spectrum with a Gaussian with a FWHM of 0.35 eV . To simulate the probe pulse, the spectra were convolved with a Gaussian with a FWHM of 8 fs (electric field envelope).

## III. RESULTS

## A. Potential-energy surfaces

The potential-energy surfaces used in the propagation agree very well with earlier high-level calculations and experimental values (see Table I). The dissociation threshold, which is shifted by $-0.2 \mathrm{eV}(-6.25 \%)$ from the experimental value, shows the largest deviation. A cut through the calculated potential energy surfaces with $r_{\mathrm{NO}}=1.625 \AA$ is shown in Fig. 4.

## B. Vibronic wave packets

The result of a wave packet propagation with a Fourierlimited excitation pulse of 8 fs duration (FWHM of the electric field envelope), centered at 400 nm , and a peak intensity of $1 \cdot 10^{12} \mathrm{~W} / \mathrm{cm}^{2}$ is illustrated in Fig. 5 (Multimedia view) and Fig. 6. Figure 5 (Multimedia view) shows the density of the excited wave packets with isosurfaces. It is a static representation of the linked animation of the densities in steps of 1 fs . Figure 6 shows the partial densities in $r$ and $\beta$ as defined in Eqs. (8) and (9) and the behavior of the populations in the electronic states. Note that the partial density in $r$ has to remain in the $C_{2 \mathrm{v}}$ symmetry of the vibronic ground state equilibrium of the molecule, i.e., $\rho_{r}=\rho_{r_{2}}=\rho_{r_{1}}$. Thus, a molecule which is strongly asymmetrically stretched is associated with density at small and large $r$ at the same time.

The excited wave packet initially moves symmetrically toward longer $r_{1}$ and $r_{2}$ and smaller $\beta$ and reaches the conical intersection within 12 fs after its excitation, where most of the population transfers to the lower adiabatic state. The remaining part of the wave packet scatters at the conical intersection and reaches a turning point at $\theta \approx 100^{\circ}$ and 14 fs , moving back toward the conical intersection [cf. the upper panels of Figs. 5(a)-5(d) (Multimedia view) and Fig. 6(b)]. The excited wave packet on the lower adiabatic state reaches the turning point at $\beta \approx 88^{\circ}$ later ( 20 fs ) and afterwards moves toward $180^{\circ}$ (lower panels). In contrast to this, the turning point in $r$

TABLE I. Comparison of the calculated potential surface with the work of Kurkal et al. ${ }^{32}$ and experimental values.

|  | This work |  |  | Kurkal et al. ${ }^{32}$ |  |  | Experiment |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $V(\mathrm{eV})$ | $r_{1}=r_{2} / a_{0}$ | $\beta$ (deg) | $V(\mathrm{eV})$ | $r_{1}=r_{2} / a_{0}$ | $\beta$ (deg) | $V(\mathrm{eV})$ | $r_{1}=r_{2} / a_{0}$ | $\beta$ (deg) |
| $(1)^{2} \mathrm{~A}_{1}$ minimum | 0 | 2.258 | 134.0 | 0 | 2.2609 | 134.3 | 0 | $2.255^{1}$ | $133.9^{2}$ |
| (1) ${ }^{2} \mathrm{~B}_{2}$ minimum | 1.248 | 2.380 | 101.9 | 1.31 | 2.3659 | 101.9 | $1.21{ }^{3}$ | $2.351^{3}$ | $102.6{ }^{3}$ |
| (1) ${ }^{2} \Pi_{u}$ minimum | 1.766 | 2.266 | 180 | 1.726 | 2.263 | 180 | $1.83{ }^{4.5}$ | $2.324^{4,5}$ | 180 |
| (1) $)^{2} \mathrm{~A}_{1} /(1)^{2} \mathrm{~B}_{2}$ cusp | 1.313 | 2.374 | 107.4 | 1.28 | 2.3590 | 106.6 | $1.21 \pm 0.09^{1}$ | $2.355^{1}$ | $103.1{ }^{1}$ |
| $(1)^{2} \mathrm{~A}^{\prime}: \mathrm{NO}+\mathrm{O}$ | 3.028 |  |  | 3.11 |  |  | $3.23{ }^{6,7}$ |  |  |



FIG. 4. Cut at $r=1.625 \AA$ through the ${ }^{2} \mathrm{~A}^{\prime}$ potential energy surfaces of neutral $\mathrm{NO}_{2}$ in Jacobi coordinates. (a) shows the adiabatic potentials and the result of the diabatization is shown in (b). In the back planes, cuts through $R=1.35 \AA$ are shown, and on the right, cuts through $\theta=128^{\circ}$ are illustrated.
occurs earlier ( 16 fs ), leading to a maximal contraction of the molecule (around 24 fs ) and a subsequent expansion [Fig. 6(a), bottom]. Only during this extension is the asymmetric stretch mode significantly excited, where one NO bond length oscillates around the equilibrium distance, while the other oxygen
moves toward the absorption barrier (at 3.0263 Å). Simultaneously, the wave packet spreads in the angle dimension. The leading edge of the wave packet reaches the absorbing barrier after 87 fs [Figs. 5(g)-5(j) (Multimedia view) and Fig. 6]. The second crossing of the upper adiabatic wave packet over the conical intersection around 20 fs after the excitation [Figs. 5(b)-5(d) (Multimedia view)] leads to a second distinct wave packet on the lower surface, which does not lead to fast dissociation. While the coherent vibration of one of the bond lengths is visible until the end of the simulation [Fig. 6(a)], the wave packet clearly disperses very rapidly such that no distinct variations in the wave-packet density can be observed for times longer than $\sim 150 \mathrm{fs}$. The loss of population due to absorption is mainly caused by the elongation of the bond length into the absorbing barrier at large $r$, rather than at small $\beta$.

The chosen pump pulse excites $2.58 \%$ of the sample. The adiabatic population dynamics show that most of the excited population relaxes to the adiabatic ground state with the first crossing of the conical intersection. The significant net transfer to the upper adiabatic state around 38 fs correlates with the second crossing of the initially formed wave packet across the conical intersection, visible in Figs. 5(e) and 5(f) (Multimedia view) and Fig. 6(b).

The results of our new calculations closely resemble the original work over the first $50-100$ fs, i.e., before the wave packet has started to explore the dissociative regions of the potential energy surface. This can be seen by comparing the population dynamics shown in Fig. 6(c) of the present article with Fig. 3 of Ref. 37. The population dynamics for longer delays are also similar, i.e., the population of the upper diabatic state [red curve in Fig. 6(c)] shows local maxima at 100 and 170 fs in both calculations. However, the local maxima are somewhat less pronounced in the new calculations compared to those in Ref. 37.


FIG. 5. Isosurfaces of the timedependent difference in adiabatic nuclear densities in internal coordinates calculated as $\rho_{i}(t)=\left|\Psi_{i}(t, \mathbf{R})\right|^{2}$ - $\left|\Psi_{i}(t=0, \mathbf{R})\right|^{2}$. Blue are negative contributions, i.e., where population was removed due to the excitation, and red are positive contributions. The black line indicates the seam of the conical intersection. The animated version of this figure is linked in steps of 1 fs . An animation of the isosurfaces shown from another perspective is part of the supplementary material. Multimedia view: https://doi.org/10.1063/ 1.5029365 .1


Experimentally, pump pulses of 8 fs are difficult to achieve and very challenging to apply to single-photon excitation. The total fraction of excited molecules indeed scales linearly with the total pump fluence, whereas multi-photon excitation and ionization both scale faster than linearly with the peak intensity. Therefore, it is challenging to achieve a sufficient excitation fraction (e.g., $>1 \%$ ), while avoiding multi-photon processes with a short pump pulse. Recent experimental work ${ }^{24}$ therefore used 56 fs long (FWHM of the electric field envelope, i.e., 40 fs FWHM of the intensity envelope) excitation pulses. The longer excitation duration leads to a significant broadening of the wave packets in configuration space, as can be seen in Fig. 7. In particular, the longer pump pulse blurs the fine details of the fast wave-packet motion in the bond-length coordinate, especially the predicted coherent vibrations in the lower adiabatic state [Fig. 6(a)]. The slower dynamics in the bondangle coordinate are less affected by the long pump pulse. In this case, the dominant part of the wave packet moving to larger bond angles on the lower adiabatic surface is preserved [Fig. 7(b)]. The main difference between the dynamics induced by the short and long pulses is the blurring of the second wave-packet component created on the lower adiabatic surface around $t=20 \mathrm{fs}$ when the conical intersection is approached for the second time. However, the relative amplitude of this second wave-packet component is small in Fig. 6(b), explaining why the dominant features observed in the case of the 8 -fs pulse are retained in the case of the $56-\mathrm{fs}$ pulse. A similar conclusion is reached when comparing the population dynamics in Figs. 6(c) and 7 (c). The main features in the population dynamics
induced by the long pump pulse can be rationalized as a temporal convolution of the short-pulse dynamics with a Gaussian envelope.

In the picosecond time range, the dynamics are known to be complex. Individual resonances show strongly varying dissociation constants close to the dissociation threshold, leading to excitation-energy-dependent dissociation times. ${ }^{62,63}$ In picosecond and femtosecond time-resolved experiments, due to the bandwidth of the excitation pulse and the associated spectral averaging, a monotonic increase of the dissociation rates with increasing excitation energies ${ }^{64,65}$ has been reported [see crosses in Fig. 8(b)]. This trend is reproduced by our calculations. Figure 8(a) shows the adiabatic and total populations as a function of time for different excitation energies. For all excitation energies, we observe an initial fast decay due to the coherent movement of a wave packet to the absorption barrier around 150 fs after excitation. Subsequently, the dissociation can be approximated by an exponential decay. The extracted dissociation times $\tau$, which are determined by exponential fits to the total population at $250 \mathrm{fs}<t<10 \mathrm{ps}$, are shown as black circles in Fig. 8(a). These results cannot directly be compared with the experimental dissociation rates because the latter were obtained with picosecond pump pulses with correspondingly narrower spectra. We have therefore converted the decay rates from the literature (crosses) into expected decay rates induced by 56 -fs pulses, as described in Appendix E. The result is displayed as a red line in Fig. 8(b). Our calculations reproduce the general trend of an increasing dissociation rate with increasing excess energy and agree with the experimental results for


FIG. 7. Result of a wave packet calculation with a 56 fs long excitation pulse. (a) Adiabatic projected densities $\rho_{r}$ for the $(2)^{2} \mathrm{~A}_{\mathrm{A}}^{\prime}$ and $(1)^{2} \mathrm{~A}_{\mathrm{A}}^{\prime}$ states in the upper and lower panel, respectively. (b) Adiabatic projected densities $\rho_{\beta}$. (c) Populations: dotted lines represent the diabatic representation and solid lines the adiabatic representation. The total population in the grid is shown with the black dashed curve (left scale).


FIG. 8. Simulation of the picosecond dissociation dynamics of $\mathrm{NO}_{2}$ : Results of wave packet calculations with a 56 fs long excitation pulse centered at $2.95,3.05$, and 3.15 eV with a peak intensity of $1 \cdot 10^{12} \mathrm{~W} / \mathrm{cm}^{2}$ in the top, middle, and bottom panel, respectively. The total population (black) and the population of the lower adiabatic state (blue) are indicated with the left scale and the population of the upper adiabatic state (red) is indicated with the right scale. The parameter $\tau$ is the lifetime of the excited molecule determined by a fit to $P_{\text {tot }}=a \exp (-t / \tau)+b$ with $t \in[0.25,10] \mathrm{ps}$. (b) Decay constants vs. the energy offset of the central pump energy $E_{\text {pump }}$ with respect to the calculated dissociation energy $E_{0}=3.028 \mathrm{eV}$. The expected experimental femtosecond response of the system was simulated with an exponential fit to the population when exciting with a Fourier-limited laser pulse characterized by a Gaussian spectrum centered at the pump energy. ${ }^{63}$ Experimental values were taken from picosecond time-resolved measurements. ${ }^{64}$
excess energies above 0.1 eV . Our calculations overestimate the dissociation rate at lower excess energies and in particular for the cases where the spectrum of the pump pulse is centered below the dissociation threshold.

## C. Photoelectron spectra

The time-dependent wave functions can be used to calculate extreme-ultraviolet time-resolved photoelectron
spectra ${ }^{24,66}$ according to Eq. (23). The time-dependent spectrum of $\mathrm{NO}_{2}$, assuming an 8 -fs long pump pulse centered at 400 nm and an 8 -fs long probe pulse centered at 45.75 nm (27.1 eV ) with a bandwidth of 0.33 eV , is depicted in Fig. 9(a). In this article, we concentrate on the analysis of the time-dependent photoelectron spectra generated by these short pulses. The comparison of calculations with longer pulses and experimental data will be shown in a future publication. ${ }^{24}$ We consider all transitions from the neutral $(1)^{2} \mathrm{~A}^{\prime}$ and $(2)^{2} \mathrm{~A}^{\prime}$ states to each of the energetically lowest-lying two states of ${ }^{1} \mathrm{~A}^{\prime},{ }^{1} \mathrm{~A}^{\prime \prime},{ }^{3} \mathrm{~A}^{\prime}$, and ${ }^{3} \mathrm{~A}^{\prime \prime}$ characters of the cationic molecule. The spatially confined


FIG. 9. Calculated time-dependent photoelectron spectrum of $\mathrm{NO}_{2}$. (a) Total time-dependent photoelectron spectrum of the excited part of the wave function. (b) Comparison of the static spectrum calculated from the ground-state wave function with experimental spectra ( $\operatorname{red}^{24}$ and blue ${ }^{8}$ ). The calculation using a constant photoionization matrix element $\mu_{\text {if }}$ (yellow) shows a significantly worse agreement with the experiment than the calculation including variable matrix elements (violet). (c) Individual contributions from different channels to the photoelectron spectrum at a pump-probe delay $\Delta t$ of 50 fs . The contributions from different channels are differentiated by their initial states (solid and dashed) and final states (colors). All spectra are normalized with respect to the maximum of the cross section of the first photoelectron peak [assigned to the transition $(1)^{2} \mathrm{~A}_{1} \rightarrow(1)^{1} \mathrm{~A}_{1}$ around $E_{\text {bind }}=11.2 \mathrm{eV}$ ] of the static spectrum.
character of the wave packets up to 70 fs after excitation leads to strong modulations in the spectrum. For delays larger than 70 fs , i.e., when the wave packet covers a dominant fraction of the available configuration space and one NO-bond is significantly elongated, the spectrum can be structured into three parts: ionization of the NO-fragment leading to a band around 9.3 eV , ionization of the O-fragment leading to a sharp band around 13.3 eV , and the hot ground state of $\mathrm{NO}_{2}$ leading to a broad band ranging from 10 to 14 eV , peaking at 12.3 eV . Note that significant loss of population in the grid only occurs for delays longer than 100 fs [cf. Fig. 6]. To illustrate the accuracy of the method, Fig. 9(b) compares the calculated spectrum of unexcited $\mathrm{NO}_{2}$ to the corresponding experimental spectra. The blue line shows the experimental spectrum reported by Baltzer et al. ${ }^{8}$ using He-I radiation ( 21.2 eV ), whereas the red line shows the experimental spectrum obtained in our laboratory using a 27.1 eV high-harmonic source. ${ }^{24}$ The yellow line shows the calculated photoelectron spectrum with all photoionization matrix elements $\mu_{i f}$ set to unity, whereas the black line shows the full calculation that uses the Dyson norm as an approximation to these matrix elements. A good agreement is obtained between the full calculation and our experimental spectrum. In particular, the relative intensities are all reasonably well reproduced, the largest deviation (of less than a factor of two) occurring for the band at 13.0 eV binding energy, i.e., the $(1)^{3} \mathrm{~B}_{2}\left((1)^{3} \mathrm{~A}^{\prime}\right)$ state of $\mathrm{NO}_{2}^{+}$. This agreement motivates our use of the Dyson-norm approximation in the present work. The comparison of the experimental results further shows the significant variation of the photoionization cross section into the $(1)^{3} \mathrm{~B}_{2}\left((1)^{3} \mathrm{~A}^{\prime}\right)$ continuum as a function of the photon energy. The comparison of the two calculations further shows the importance of including the photoionization matrix elements as the latter entirely suppress the broad photoelectron band centered around 16 eV because the corresponding photoionizing transitions are forbidden by Koopman's correlations. The absence of the photoelectron band with an onset at $\sim 17 \mathrm{eV}$ from both calculations is due to the fact that the corresponding states of the cation [(3) $)^{3} \mathrm{~A}^{\prime}$ and $(3)^{3} \mathrm{~A}^{\prime \prime}$ according to Ref. 8] were not included in our calculation.

These calculations reveal the complex nature of the timedependent photoelectron spectra of the excited-state dynamics in $\mathrm{NO}_{2}$. The present calculations are therefore essential for a detailed interpretation of experimental XUV-TRPES spectra because they enable an identification of the contributions of individual channels. The contributions of the different channels are exemplarily shown for a pump-probe delay of 50 fs in Fig. 9(c).

Figure 10 shows the signal arising from the ionization channel from the coupled states of the neutral molecule to the $(1)^{1} \mathrm{~A}^{\prime}$ state of the cation in more detail. While the binding energy of the photoelectron is determined by the local ionization potential $I_{\mathrm{p}}$ at the position of the wave packet, the probability to ionize is determined by the photoionization matrix element. Cuts through the potential surfaces along $r$ and $\beta$ are illustrated in Fig. 10(a), which give rise to the $I_{\mathrm{p}}$ shown in panel b . The electronic structures of the states lead to a strongly varying matrix element along these cuts (panel b, right scale). A calculation with the transition matrix elements set to 1 , as illustrated in panel c , shows the strong


FIG. 10. Illustration of the photoionization channels $(1)^{2} \mathrm{~A}^{\prime} \rightarrow(1)^{1} \mathrm{~A}^{\prime}$ and $(2)^{2} \mathrm{~A}^{\prime} \rightarrow(1)^{1} \mathrm{~A}^{\prime}$. (a) shows the potential surfaces of the involved states in cuts along $r_{1}$ with $r_{2}=1.190 \AA$ and $\beta=134^{\circ}$ (left) and $\beta$ with $r_{1}=r_{2}$ $=1.190 \AA$ (right). The potentials of the neutral molecule are only shown in regions which are energetically accessible to the wave packet. Both the ionization potentials depicted in (b) (left scale, blue) as well as the norm of the Dyson orbital (red, right scale) vary strongly along the cuts. Panel (c) depicts the calculated photoelectron spectrum of the channels with the matrix elements set to 1 , while panel d illustrates the complete implementation of Eq. (19).
modulations due to the movement of the wave packet. For example, the excitation in the Franck-Condon region leads to the appearance of a peak at 8.1 eV , which is offset from the binding energy of the corresponding band in the static photoelectron spectrum by the excitation energy. The peak shifts fast toward higher binding energies as the wave packet moves toward the conical intersection (mainly along $\beta$ ). The crossing of the conical intersection leads to the appearance of the transition from the lower adiabatic state, which initially shows strong modulations due to the oscillation in $\beta$ and $r$ (see Fig. 6). The intensity maximizes due to accumulation of density at turning points of the dynamics or, on longer time scales, when the wave packet reaches areas where the local $I_{\mathrm{p}}$ is similar for many configurations (e.g., when the molecule reaches configurations with an elongated $r$ ). The spread of the wave packet in the potential leads to a spectrum with a 4 eV width. If one includes the ionization probability [Fig. 10(d)], the direct interpretation of the spectra becomes more complicated.

Exemplarily, the ionization of the initially formed wave packet is strongly suppressed, leading to the near-vanishing of the expected peak at 8.1 eV around $\Delta t=0$. In the case of the transition to the $(1)^{1} \mathrm{~A}^{\prime}$ state, the ionization of the NO fragment is Koopman's forbidden such that the band is losing intensity when the molecule reaches strongly stretched configurations. The large range of local $I_{\mathrm{p}} \mathrm{s}$ covered by the wave packet, which leads to the overlap of contributions from many ionization channels, as well as the strong influence of the matrix element make it challenging to interpret experimental time-resolved photoelectron spectra without extended theoretical modeling of the system. ${ }^{24,66}$

## IV. CONCLUSION

The potential energy surfaces presented here are, to our knowledge, the most spatially extended (for $\mathrm{NO}_{2}^{+}$) and are obtained at the highest level of theory (for $\mathrm{NO}_{2}$ ) published so far. We performed full-dimensional wave-packet calculations on the electronic surfaces of the neutral molecule, revealing the signatures of the conical intersection and extending the time scale of previous studies to the picosecond range. This enabled us to report time-dependent photoelectron spectra for a broad range of binding energies, involving many ionization channels which are needed for a direct comparison to recent experimental results in XUV time-dependent photoelectron spectroscopy (XUV-TRPES). ${ }^{24}$ The comparison allows for a stringent test of the $a b$ initio method presented here. The calculations allow interpretations of time-dependent spectra of XUV-TRPES, laying the foundation for a quantitative evaluation of such experiments. They also open the perspective for the quantitative interpretation of time-resolved highharmonic spectroscopy and several other experimental techniques, such as core-level transient absorption spectroscopy, ${ }^{67}$ by using similar techniques. Finally, $\mathrm{NO}_{2}$ is an ideal candidate to demonstrate electronic control over non-adiabatic wave-packet dynamics, ${ }^{68-70}$ one of the major perspectives of attosecond science.

## SUPPLEMENTARY MATERIAL

See supplementary material for the following files: NO2_2Ap.txt-Adiabatic and diabatic energies of the doublet $\mathrm{A}^{\prime}$ states of $\mathrm{NO}_{2}$. Table of potential energies and dipole moments along the $y$-axis of the two energetically lowest lying doublet A' states of the neutral $\mathrm{NO}_{2}$ molecule in dependence of the Jacobi coordinates. The table is organized in 14 columns and has one header row. The first three columns are the indices $i_{r}, i_{R}$, and $i_{\theta}$, according to Eqs. (1)-(3) in the main text of the article. The columns 4-6 are the associated Jacobi coordinates $r, R$, and $\theta$. The distances are given in Angstrom and the angle is given in degrees. The subsequent two columns, 7 and 8 , are the calculated adiabatic energies in eV of the $(1)^{2} \mathrm{~A}^{\prime}$ and $(2)^{2} \mathrm{~A}^{\prime}$ states, respectively. Columns 9 and 10 describe the diabatic potential energies in electron Volts. Column 11 is the coupling matrix element between the two surfaces in eV . The columns 12 and 13 are the dipole moments along the y -axis of the two states in debye. Column 14 is the transition dipole along the y -axis between the states in debye. The dipole
moments are direct outputs of the Molpro program. The sign of the transition dipole moment is adjusted to correct for the arbitrary phase-flipping in the calculation. The axis convention is given in Fig. 1 of the article. NO2_1Ap.txt: Adiabatic energies of the singlet A' states of $\mathrm{NO}_{2}^{+}$. Table of the adiabatic potential energies of the two energetically lowest lying singlet $\mathrm{A}^{\prime}$ states of $\mathrm{NO}_{2}^{+}$in dependence of Jacobi coordinates. The table is organized in 8 columns and has one header row. The first five columns contain the same information as in the paragraph above. The subsequent two columns, 7 and 8 , are the calculated adiabatic energies in eV of the (1) ${ }^{1} \mathrm{~A}^{\prime}$ and (2) ${ }^{1} \mathrm{~A}^{\prime}$ states, respectively. NO2_1App.txt: Adiabatic energies of the singlet $\mathrm{A}^{\prime \prime}$ states of $\mathrm{NO}_{2}^{+}$. The same as the previous item for the two energetically lowest lying singlet $\mathrm{A}^{\prime \prime}$ states of $\mathrm{NO}_{2}^{+}$. NO2_3Ap.txt: Adiabatic energies of the triplet $\mathrm{A}^{\prime}$ states of $\mathrm{NO}_{2}^{+}$. The same as the previous item for the two energetically lowest lying triplet $\mathrm{A}^{\prime}$ states of $\mathrm{NO}_{2}^{+}$. NO2_3App.txt: Adiabatic energies of the triplet $\mathrm{A}^{\prime \prime}$ states of $\mathrm{NO}_{2}^{+}$. The same as the previous item for the two energetically lowest lying triplet $\mathrm{A}^{\prime \prime}$ states of $\mathrm{NO}_{2}^{+}$. WP8fs $\_$_ 200 fs .avi: An animation of the time-dependent wave-packet dynamics viewed from a different perspective compared to Fig. 5 (Multimedia view).

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## APPENDIX A: POTENTIAL SURFACE CALCULATIONS

For the calculation of the potential energy surfaces of the neutral molecule, two main difficulties had to be treated. First, as can be seen in Fig. 11, a sharp avoided crossing of the


FIG. 11. Illustration of the artifact at large $\theta$ and small $r$ in the adiabatic potentials: Blue and red: state-averaged potential surfaces. Yellow: single state calculation of the lower surface, shifted for minimal deviation between $152^{\circ}$ and $161^{\circ}$.
second with the third ${ }^{2} \mathrm{~A}^{\prime}$ state at large $\theta$ and $r$ below $1.5 \AA$ led to an artificial jump in the calculated energy of the lower state (indicated by an arrow). This was corrected by calculating the relevant volume without state-averaging and shifting the energy to maximize the overlap at smaller $\theta$. As the energy of the $(2)^{2} \mathrm{~A}^{\prime}$ state is larger than 4 eV and the diabatic coupling of the lowest two states is negligible, no accurate energy of the higher state is needed at these structures.

Second, due to the approximation of the $(3)^{2} \mathrm{~A}^{\prime}$ state to the lower states at large $R$, three-state averaged calculations had to be performed in these regions. The energy was shifted for minimal deviation along $R=3.1469 \AA$.

## APPENDIX B: DIABATIZATION DETAILS

For $r>r_{\text {ref }}=1.8188 \AA$, the third ${ }^{2} \mathrm{~A}^{\prime}$ state couples too strongly to the other states to use the phenomenological diabatization method. Instead, the angle was taken to be constant [i.e., $\alpha_{\mathrm{D}}\left(\theta, R, r>r_{\text {ref }}\right)=\alpha_{\mathrm{D}}\left(\theta, R, r=r_{\text {ref }}\right)$ ], which was checked by comparing it to cuts at larger $r$. An example is shown in Fig. 12. For large $R$, the same problem occurred.


FIG. 12. Illustration of the diabatization process at long $r$. Shown are the cuts through the hypersurfaces with $R=1.9266 \AA$ and with $r=r_{\text {ref }}=1.8188 \AA$ and $r=r_{1}=1.9500 \AA$, respectively. (a) Mixing angles: The black curve shows the mixing angle calculated from the dipole moments at $r=r_{\text {ref }}$ [according to Eq. (4)] and the green curve its modification for the diabatization procedure. The black dashed curve illustrates the calculated diabatization angle for $r=r_{1}$. (b) Potential surfaces for $r=r_{\text {ref }}$ : the adiabatic potentials are in red and blue and the diabatic potentials are in yellow and violet. (c) Potential surfaces for $r=r_{1}$ : the same color code as in (b). Additionally, the dashed curves indicate the diabatic surfaces calculated with the dipole-dependent mixing angle $\alpha_{\mu}$ (shown as the black dashed curve in panel a). The proximity of the third adiabatic state leads to problems with the phenomenological model of the diabatization. The mixing angle was approximated by the values at $r=r_{\text {ref }}$.

Here, the construction of the diabatic potentials was more involved. We used an inherent property of the Jacobi coordinates; for every set of Jacobi-coordinates $(\theta, R, r)$, there is a second set $\left(\theta^{\prime}, R^{\prime}, r^{\prime}\right)$ with identical properties, where

$$
\begin{align*}
r^{\prime} & =\sqrt{(m r+R \cos \theta)^{2}+R^{2} \sin \theta^{2}}  \tag{B1}\\
R^{\prime} & =\sqrt{m^{2} R^{2}+2 m\left(m^{2}-1\right) r R \cos \theta+\left(m^{2}-1\right)^{2} r^{2}}  \tag{B2}\\
\theta^{\prime} & =\operatorname{atan} 2\left(\frac{r R \sin \theta}{m r^{2}-m^{3} r^{2}-m R^{2}+\left(1-2 m^{2}\right) r R \cos \theta}\right)  \tag{B3}\\
m & =\frac{m_{\mathrm{O}}}{m_{\mathrm{O}}+m_{\mathrm{N}}} \tag{B4}
\end{align*}
$$

Here, $m_{\mathrm{O}}$ and $m_{\mathrm{N}}$ are the atomic masses of O and N . The diabatic potentials around $\left(\theta^{\prime}, R^{\prime}, r^{\prime}\right)$ were used to interpolate the potential and extract a mixing angle for $(\theta, R, r)$. For a set of coordinates with $2.0375 \AA \leq R \leq 2.2594 \AA$, this enabled us to make an educated guess how to consistently extrapolate the mixing angle into areas (at small $\theta$ ) where three states interact. Furthermore, with $R>2.2594 \AA$, the interpolated mixing angles were fitted with a heuristic function $\alpha=a \arctan (b \theta+c)+\pi / 4$ for each cut along $\theta$.

## APPENDIX C: INTERPOLATION

The adiabatic and diabatic energies as well as the dipole moments at the calculated points were interpolated on the complete grid by iterative one-dimensional Akimainterpolations. ${ }^{71}$ Sections of the surface with at least six subsequent points with a maximal distance of $d$ between them were searched and the missing points in between them interpolated (only interpolating up to the third point from each end of the array). The new set of points was included in the similar next interpolation along the second dimension $(R)$. This was repeated along the first two dimensions with increasing $d$. Then, the interpolation along the third dimension $(r)$ was performed and the process repeated until no additional points were generated anymore. Finally, the interpolation was extended to the data points at the edge. For the interpolation along $\theta$, the mirror plane at $\theta=180^{\circ}$ was taken into account. As the interpolations of the diabatic and adiabatic surfaces were performed independently, small errors had to be corrected afterwards: e.g., if the lowest diabatic energy at a certain geometry was smaller than the lowest adiabatic energy, the diabatic energy was adjusted to coincide with the adiabatic energy. The amplitudes of the diabatic coupling constant were calculated from the interpolated surfaces. Its sign was taken from a direct interpolation of the defined coupling constants at the calculated points of the grid.

## APPENDIX D: CALCULATION OF IONIZATION PROBABILITY

The calculation of the structure dependent transition strength is motivated by the work of Åberg. ${ }^{59,72,73}$ Within the MC-SCF approach, ${ }^{41}$ electronic states $\Psi$ can be expanded on a basis of Slater determinants $\phi_{i}$ as

$$
\begin{equation*}
\Psi=\sum_{k} c_{k} \phi_{k} \tag{D1}
\end{equation*}
$$

where $c_{k}$ are CI coefficients. The Slater determinants can be expressed as a sequence of creation operators associated with the molecular orbitals (acting on the empty space), ${ }^{74}$ resulting in expressions for the initial $i$ and final states $f$,

$$
\begin{align*}
\Psi^{i} & =\sum_{k} c_{k} \phi_{k}=\sum_{k} c_{k} \prod_{r=1}^{N}\left(\hat{a}_{r}^{+}\right)^{S_{r, k}}|\mathrm{vac}\rangle  \tag{D2}\\
\Psi^{f} & =\sum_{l} c_{l} \phi_{l}=\sum_{l} c_{l} \prod_{r=1}^{N}\left(\hat{a}_{r}^{+}\right)^{S_{r, l}}|\mathrm{vac}\rangle \tag{D3}
\end{align*}
$$

where $r$ runs over the number of (spin-)orbitals in the CI calculation and $S_{r, n}$ are the occupation-number vectors of the individual orbitals in the $n$th determinant. If we choose a biorthonormal basis, the operators are the same for the neutral and cationic molecule. We approximate the ionization probability within the sudden approximation as the norm of the Dyson orbital of the transition from state $i$ to state $f,{ }^{60,61}$

$$
\begin{equation*}
\left.\left|\Phi_{i f}\right|^{2}=\left|\sum_{q}\left\langle\Psi^{f}\right| \hat{a}_{q}\right| \Psi^{i}\right\rangle\left.\varphi_{q}\right|^{2} \tag{D4}
\end{equation*}
$$

where $\varphi_{q}$ are the individual orbitals associated with the annihilation operators $\hat{a}_{q}$. Using the Eqs. (D2) and (D3), this results in
$\left.\left|\Phi_{i f}\right|^{2}=\sum_{q}\left|\sum_{l} \sum_{k} c_{k} c_{l}\langle\operatorname{vac}| \prod_{r=N}^{1}\left(\hat{a}_{r}\right)^{S_{r, l}} \hat{a}_{q} \prod_{r=1}^{N}\left(\hat{a}_{r}^{+}\right)^{S_{r, k}}\right| \operatorname{vac}\right\rangle\left.\right|^{2}$,
where we used the orthonormality of the individual orbitals $\varphi_{q}$. The bra-ket evaluates to $\pm 1$ or 0 , depending on the occupation of the initial state and the number of permutations needed to annihilate the $q$ th electron from the initial state. ${ }^{74}$ In practice, we evaluated Eq. (D5) by calculating the state averaged electronic structure of the three lowest ${ }^{2} \mathrm{~A}^{\prime}$ states with $\operatorname{CASSCF}(13,10)$ within the Molpro suite ${ }^{40}$ and, subsequently, while not optimizing the orbitals anymore, the three lowest cationic ${ }^{1} \mathrm{~A}^{\prime},{ }^{1} \mathrm{~A}^{\prime \prime},{ }^{3} \mathrm{~A}^{\prime}$, and ${ }^{3} \mathrm{~A}^{\prime \prime}$ states. All determinants with $c_{k}, c_{l} \geq 0.0001$ were included.

## APPENDIX E: CALCULATION OF EXPECTED EXPERIMENTAL fs RESPONSE

The femtosecond response illustrated in Fig. 8(b) was calculated similar to a previously presented method. ${ }^{63}$ The picosecond decay rates from jet-cooled $\mathrm{NO}_{2}{ }^{64}$ were interpolated to a function $k_{\mathrm{ps}}(E)$ with the excitation energy $E$. The expected bound population $P_{\mathrm{B}}$ after a broadband excitation was then calculated as

$$
\begin{equation*}
P_{\mathrm{B}}\left(E_{\mathrm{pump}}, t\right)=\int_{E} I_{\mathrm{pump}}\left(E, E_{\mathrm{pump}}\right)\left(1-\exp \left(-k_{\mathrm{ps}}(E) t\right)\right) \mathrm{d} E, \tag{E1}
\end{equation*}
$$

where $I_{\text {pump }}$ describes the intensity spectrum of the excitation pulse. We assumed a Fourier-limited Gaussian pulse centered at $E_{\text {pump }}$ with a FWHM of 56 fs. Subsequent monoexponential fits to the population $P_{\mathrm{B}}\left(E_{\text {pump }}, t\right)$ result in the energy-dependent femtosecond decay constants.
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