

Cryptocurrencies, Currency Competition, and the Impossible Trinity*

Pierpaolo Benigno[†], Linda M. Schilling[‡] and Harald Uhlig[§]

August 7, 2020

Abstract

We analyze a two-country economy with complete markets, featuring two national currencies as well as a global (crypto)currency. If the global currency is used in both countries, the national nominal interest rates must be equal and the exchange rate between the national currencies is a risk-adjusted martingale. Deviation from interest rate equality implies the risk of approaching the zero lower bound or the abandonment of the national currency. We call this result *Crypto-Enforced Monetary Policy Synchronization (CEMPS)*. If the global currency is backed by interest-bearing assets, additional and tight restrictions on monetary policy arise. Thus, the classic Impossible Trinity becomes even less reconcilable.

Key words: currency competition, cryptocurrency, impossible trinity, exchange rates, uncovered interest parity, independent monetary policy

JEL Classification: E4, F31, D53, G12

*Financial support from the Fondation Banque de France is gratefully acknowledged.

[†]University of Bern, CEPR, pierpaolo.benigno@vwi.unibe.ch.

[‡]Ecole Polytechnique CREST, CEPR, linda.schilling@polytechnique.edu. Linda's work was conducted in the framework of the Chair 'Blockchain and B2B Platforms' at Ecole Polytechnique. The project partially evolved during a research stay of Linda Schilling at the Simons Institute at UC Berkeley. Support and hospitality is gratefully acknowledged.

[§]University of Chicago, NBER, CEPR, huhlig@uchicago.edu. We thank Peter DeMarzo, Jonathan Berk, Maurice Obstfeld, and participants of the 2019 ESSFM Gerzensee conference for very insightful comments.

1 Motivation

Globally usable cryptocurrencies are on the rise. 10 years after the introduction of Bitcoin, Facebook is seeking to launch Libra, designed to appeal to its more than 2 billion world-wide members. Other companies are not far behind. While other means of payment have reached worldwide usage before, the ease of use and the scope of these new cryptocurrencies are about to create global currencies of an altogether different quality. How will they alter the financial landscape? How will this affect exchange rates and monetary policies of traditional currencies?

Global currencies are not a new phenomenon. The Spanish Dollar in the 17th and 18th centuries, gold during the gold standard period, the Pound Sterling prior to 1944, and the U.S. Dollar since then served as an internationally accepted unit of account. The new cryptocurrencies, however, seek to become an internationally accepted means of payment, thus directly competing with national currencies for transaction purposes. We argue that this feature, together with the consequences for national monetary policies, is an entirely new phenomenon, see Section 6.5.

We thus analyze a two-country economy featuring a home, a foreign and a global (crypto)currency. We allow for interest-bearing bonds competing against money as a store of value. Our focus is on the function of money as medium of exchange. Therefore, we adopt a general, minimalistic framework and assume that these currencies provide liquidity services. In Section 6, we show that our framework nests a number of standard approaches in the monetary economics literature. For the benchmark case that markets are complete, that liquidity services on currency are rendered immediately and that the global currency is used in both countries, we show that nominal interest rates must be equal (Proposition 4.1). We call this phenomenon a *crypto-enforced monetary policy synchronization (CEMPS)*. One might pause here for a moment and argue that interest rates are policy instruments of independent central banks. The home central bank may seek to move interest rates down or up relative to the foreign interest rate. We can however show, that while deterring entrance of the global currency, lowering interest rates risks being trapped in too low-interest-rate policies, approaching the zero lower bound. When increasing interest rates relative to the foreign interest rate, we show that the central bank risks the abandonment of its own national currency as a medium of exchange. If the global currency is backed by interest-bearing assets, additional and tight restrictions on monetary policy arise,

see Section 5. In particular, the central bank may be forced to the zero lower bound when the global currency consortium seeks to keep its currency in use per selecting appropriately low and competitive fees.

For intuition on why a global currency can impose restrictions on exchange rates and interest rates, compare the returns in the money and bond markets. A global currency can circulate as a medium of exchange in local markets, competing with local currencies, only if its expected return is equal to that on the local currency. The equality of returns implies that the expected exchange-rate change between the global currency and any local currency should be zero. The immediate and surprising consequence of this result is that the expected change of the exchange rate between any pair of local currencies, which would otherwise be disconnected, should also be zero. Note that, in a world of integrated and frictionless capital markets, uncovered interest parity links cross-country interest rate differentials to the expected, albeit risk adjusted, change of the nominal exchange rate between the same two currencies. Via the previous result, this variation is zero and therefore nominal interest rates across countries should also be equalized. In this way, it can be intuited why lowering nominal interest rates can be the escape hatch for a central bank to crowd out a global currency: the opportunity costs of holding the domestic currency is reduced, making it more attractive than the global currency as a means of payment.

Our results can be understood as a strengthened version of the Mundell-Fleming Trilemma (Fleming, 1962; Mundell, 1960, 1963), or the *Impossible Trinity*. According to this cornerstone in international economics, it is impossible to ensure a fixed exchange rate, free capital flows and an independent monetary policy all at the same time. In our framework, we allow for a flexible exchange rate and assume free capital flows: nevertheless, to defend the usage of their own national currency, presumably independent central banks have to coordinate their monetary policies. More broadly, our results are reminiscent of Rey (2015), where the Trilemma is transformed into a “Dilemma” or an “irreconcilable” duo. While the global financial cycle is the culprit in her analysis, on ours, it is the worldwide diffusion of a global currency. Furthermore, we contribute to the debate on how currency competition through globalization influences the central bank’s capacity to impact the economy, see Romer (2007).

1.1 Literature

Our paper contributes to three literature strands in particular. The first is the literature on the international role of currencies and the interdependence of monetary policies. The second is the literature on currency competition and the emerging literature on cryptocurrencies. Finally, we contribute to the monetary economics literature examining the role of money as a medium of exchange.

As for the first, the literature on international currencies and the interdependence of monetary policy, our paper is related to the classic contributions by [Mundell \(1960\)](#), [Fleming \(1962\)](#) and [Mundell \(1963\)](#), and the vast literature following it (see [Boughton \(2003\)](#), [Obstfeld et al. \(2003\)](#) for reviews and [Obstfeld and Rogoff \(1996\)](#) for a textbook treatment). Our result can be read as a sharp contrast to [Obstfeld and Rogoff \(2002\)](#), who argue that in an economy with integrated international financial markets, monetary policymakers have the ability to control their monetary instruments to achieve their target. Instead, our work shows that, under the same assumptions, having a global currency can constrain central banks in pursuing their objectives by limiting the impact of their monetary policy instruments. [Krugman \(1979\)](#), [Goldberg and Tille \(2008\)](#), [Rey \(2001\)](#), [Eichengreen et al. \(2017\)](#), [Amiti et al. \(2018\)](#), [Gopinath and Stein \(2018\)](#), [Maggiore et al. \(2019\)](#), [Gourinchas et al. \(2019\)](#), [Ilzetzki et al. \(2020\)](#), [Gopinath et al. \(2020\)](#) and [Bahaj and Reis \(2020\)](#) study the role of vehicle currencies, international currencies, global currencies and dominant currencies, emphasizing the unit of account function as well as the liquidity role played by securities denominated in these currencies. [Gopinath and Stein \(2018\)](#) justify the dominance of a currency on the basis of a higher share of trade settled in that currency. By contrast, we emphasize the medium of exchange function of money and the direct competition between the national and global currencies in that regard. Financial considerations are, instead, the reason that justifies, in our context, the dominance of a currency through the comparison of return differentials and other asset-pricing relationships. Another contribution of our work with respect to the above-mentioned literature is the analysis of the restrictions imposed by a global currency on international financial markets, relating them to the monetary policy followed by the single countries. Along these lines, [Ilzetzki et al. \(2020\)](#) argue that the limited role of the euro in international financial markets can be explained by the policy of the European Central Bank. [Benigno \(2019\)](#) focuses on a one-country model and shows that under competition to cryptocurrencies, the central bank can face some restrictions

on interest rates and inflation if the government currency has to retain a role as a medium of exchange. We differ from his analysis by analyzing the consequences of cryptocurrency competition for the international monetary system by building on a general stochastic framework.

Our paper is closely related to the literature on currency competition. Classic contributions are by [Girton and Roper \(1981\)](#), who consider the impact of currency substitution on exchange rates, [Matsuyama et al. \(1993\)](#), who consider currency substitution in a two-country, two-currency model of random matching, the exchange rate indeterminacy result due to [Kareken and Wallace \(1981\)](#), and its stochastic version by [Manuelli and Peck \(1990\)](#). These analyses have found a modern echo in the emerging literature on cryptocurrencies. [Berentsen \(1998\)](#) is an early example. [Garratt and Wallace \(2018\)](#) provide an extension of [Kareken and Wallace \(1981\)](#) to cryptocurrencies. [Schilling and Uhlig \(2018\)](#) focus on implications of competition between a cryptocurrency and traditional fiat money, while [Schilling and Uhlig \(2019\)](#) analyze the implications of goods-specific transaction costs. [Fernández-Villaverde and Sanches \(2016\)](#) and [Chiu and Koepl \(2019\)](#) analyze currency competition and monetary policy in a Lagos-Wright model. Our framework is considerably more general than all these contributions, allowing for interest-bearing bonds and nesting a number of classic monetary models. Our paper is close in spirit to [Chahrour and Valchev \(2019\)](#), who likewise emphasize the importance of an international medium of exchange. [Brunnermeier and Niepelt \(2019\)](#) pursue the implications of the equivalence between private and public money, in units of the same currency, while our emphasis is on the international context, on different currencies and, thus, has a different focus.

Finally, we contribute to the monetary economics literature examining the role of money as medium of exchange. There are a variety of benchmark approaches that discuss a role of money, see, e.g., [Walsh \(2010\)](#) for an excellent textbook treatment. We take up several of these approaches in [Section 6](#). Concerning the role of money as medium of exchange, the New Monetarist framework developed by [Lagos and Wright \(2005\)](#) has become the benchmark and has spawned a considerable literature. [Lagos et al. \(2017\)](#) provide an excellent survey and assessment. Among recent contributions, [Fernández-Villaverde and Sanches \(2016\)](#) and [Chiu and Koepl \(2019\)](#) employ the Lagos-Wright framework to analyze currency competition and cryptocurrencies. For our general and minimalistic framework, we abstract from the details of how money is used as a medium of exchange. All that we require is money offering

“liquidity services”. In section 6, we show that many benchmark approaches in the monetary economics literature, amongst others the Lagos-Wright approach, feature such liquidity services. Thus, the results derived in our abstract and minimalistic structure here apply to a large variety of classic settings.

There, moreover, exists a growing literature that analyze the functionality, feasibility and microincentives of individual cryptocurrencies and blockchain. [Biais et al. \(2019a,b\)](#) analyze equilibria in proof-of-work protocols such as Bitcoin, [Ebrahimi et al. \(2019\)](#) consider robust consensus protocols for blockchain-based distributed ledgers, [Huberman et al. \(2017\)](#) analyze revenue generation in the Bitcoin system, [Leshno and Strack \(2020\)](#) characterize Bitcoin as the unique reward scheme that satisfies anonymity, while neither giving incentive for consolidation nor for assuming fake identities, [Prat and Walter \(2018\)](#) use the Bitcoin-Dollar exchange rate for predicting the computing power of the Bitcoin network. [Cong and He \(2019\)](#) analyze implications of decentralized consensus via distributed ledger technology on competition. [Garratt and van Oordt \(2019\)](#) analyze the role of cryptocurrency specific mining equipment for avoiding double spending attacks. This paper abstracts from microincentives and the possibility of attacks, and instead assumes full functionality and reliability of all currencies in this paper.

2 A simple framework

This section uses a simple framework with a minimalist and non-stochastic structure to provide some intuition, and to preview the main results.

There are two countries, home and foreign, and three currencies: currency h and f in their respective countries and a global (crypto) currency g . While currency h can be used for transactions only in country h and the currency f only in country f , the global currency can be used in both countries. Money, either in a physical or digital form, provides non-pecuniary benefits, which we call liquidity services and yield liquidity premia. Let us assume that the two currencies are perfect substitutes in providing liquidity services and that these services are delivered at the same time in which money is held.¹

¹In a non-stochastic economy, it does not matter whether the liquidity services for holding money in t are provided at time t or at time $t + 1$. As it will be shown in the next sections and in Appendix A.1, results in the stochastic economy are different and specific to the timing of liquidity services.

Let S_t be the exchange rate between currency h and f in date t , i.e. the amount of currency h needed to buy one unit of currency f . Let Q_t denote the amount of currency h needed to buy one unit of the global (crypto) currency. Likewise, let Q_t^* denote the amount of currency f needed to buy one unit of the global (crypto) currency. Therefore,

$$Q_t = S_t Q_t^* \quad (1)$$

At a generic time t , a resident in country h can acquire $M_{h,t}$ units in currency h and $M_{g,t}$ units in the global currency at the exchange rate Q_t implying an overall expenditure or total money holding

$$M_{h,t}^{\text{tot}} = M_{h,t} + Q_t M_{g,t}, \quad (2)$$

expressed in units of the domestic currency. Note that we are assuming perfect substitutability here and in the main text. However, currencies are perfect substitutes only when they are both used and the key aspect of our analysis is to study corner solutions where one currency crowds out the other as a medium of exchange. The extension to imperfect substitutability is taken up in Appendix B and in [Irizawa \(2020\)](#). Since liquidity services are delivered immediately in t , the investor in country h receives non-pecuniary benefits from the overall money expenditure $M_{h,t}^{\text{tot}}$ deflated by the price of some generic consumption good (either tradeable or non-tradeable) for which money is exchanged. At time $t+1$, the two monies deliver an overall payoff $M_{h,t} + Q_{t+1} M_{g,t}$, in units of the domestic currency. Since liquidity services provided by each currency are substitutes, the amount of services received is independent of the portfolio choice. Only if the returns on money are equal then agents are willing to hold both currencies in their portfolio. This is equivalent to saying that the exchange rate Q should be constant, $Q_{t+1} = Q_t$. Otherwise, one currency would dominate the other as a means of payment. This result is nothing more than a restatement of [Kareken and Wallace \(1981\)](#), additionally allowing the monies to provide liquidity services. The analysis can equivalently be applied to country f to obtain that the exchange rate Q^* should also be constant.

Our first result in the paper follows directly from the above analysis: when a global currency is used in both local markets, the exchange rate, S , between currency h and f has to be constant too, although h and f do not compete directly since h and f are not simultaneously accepted in the same local market. The monies h and

f , however, compete indirectly through the global currency g which has worldwide acceptance, thereby creating a link between the two local currencies. This indirect competition then enforces equality of returns on h and f . To see this result, apply the constancy of Q and Q^* into (1).

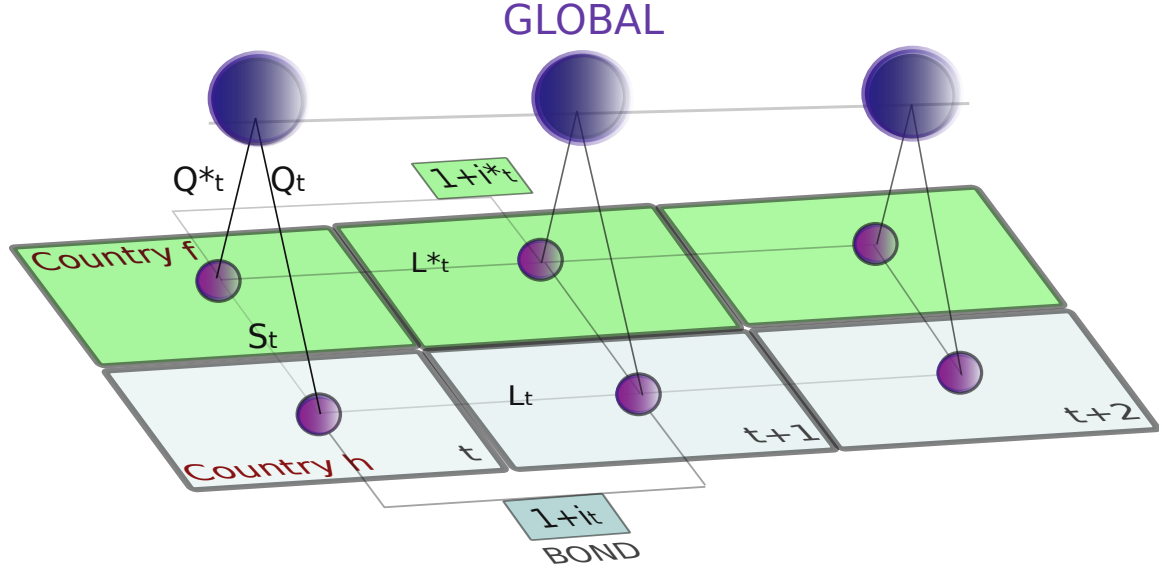


Figure 1: International trade and money flow in time

Our second result states that simultaneous trade in global and local currencies requires the synchronization of monetary policies across countries, i.e., the nominal interest rates are equalized across countries. To see this result, we allow investors in each country to trade also in two nominal bonds denominated in currency h and f , respectively. In a non-stochastic economy, with frictionless capital markets, uncovered interest rate parity holds

$$\frac{1 + i_t}{1 + i_t^*} = \frac{S_{t+1}}{S_t} \quad (3)$$

in which i_t and i_t^* denote, respectively, the nominal interest rate in country h and f from period t to $t + 1$ on one-period bonds denominated in the respective currencies. Since the exchange rate S is constant, interest rates should be equal. Figure 1 summarizes the key relationship between interest rates and exchange rates.

As the next section will show, the result of equal nominal interest rates extends unchanged to a stochastic economy in the case where liquidity services of money are delivered at the same time money is held in the portfolio. The result of constant exchange rates generalizes to a stochastic economy with the qualification that the

exchange rate between currency h and f follows instead a martingale in the risk-adjusted measure. In the stochastic setting, we will further show the equalization of the liquidity premia of money across countries.²

3 A general framework

We present our main results through a general framework relying only on asset-pricing considerations, in a stochastic multi-period economy. Our structure is broad enough to encompass a large variety of models.³ Both agents can trade either bond and can hold the global currency. The agent in country h can, in addition, hold currency h but not currency f . Vice versa, the agent in country f can hold currency f but cannot trade currency h .

The key assumption for obtaining our result is that markets are complete, arbitrage-free and frictionless. As a consequence, a stochastic discount factor exists and is unique. Let \mathcal{M}_{t+1} denote the nominal stochastic discount factor in units of currency h for the agent in country h , and likewise let \mathcal{M}_{t+1}^* denote the nominal stochastic discount factor in units of currency f for the agent in country f . An implication of complete markets is that the nominal discount factors in units of the two local currencies are connected through their exchange rate since they are equalized once expressed in the same unit of account.⁴

Assumption 3.1 (Complete Markets:).

$$\mathcal{M}_{t+1} = \mathcal{M}_{t+1}^* \frac{S_t}{S_{t+1}}. \quad (4)$$

Consider a (non-monetary) asset offering a (possibly random) nominal return R_{t+1} in currency h . Since returns have a price of one (Cochrane, 2009), the indifference condition of an intertemporal utility maximizing agent implies the following standard

²Liquidity premia are in general monotone in the opportunity cost of holding money, i.e. the interest rate. Equal interest rates directly imply equal liquidity premia.

³The framework applies to one or multi-good exchange or production economies. Thus, we do not pin down these features specifically. Agents may live for two periods in an OLG model or be infinitively lived. In Section 6 we map our general framework into specific examples drawn from classical monetary models in which we specify the preferences, constraints and maximization problem. These classical monetary models have been examined in a large body of literature, including existence of equilibria and their properties: for these reasons, we can sidestep these issues here.

⁴For the generality of this result see Obstfeld (2007).

asset pricing equation to value a random return R_{t+1} ,

$$1 = E_t[\mathcal{M}_{t+1}R_{t+1}]. \quad (5)$$

where \mathcal{M}_{t+1} denotes the nominal stochastic discount factor. Thus, since a nominal one-period bond in country h pays a return $R_{t+1} = 1 + i_t$,

$$\frac{1}{1 + i_t} = E_t[\mathcal{M}_{t+1}] \quad (6)$$

and likewise for the bond in country f

$$\frac{1}{1 + i_t^*} = E_t[\mathcal{M}_{t+1}^*]. \quad (7)$$

Bonds and currencies are used for the intertemporal transfer of resources. Money needs to offer some liquidity services beyond the intertemporal transfer to be able to compete with interest-bearing bonds for investors. We shall therefore assume that currency h , as well as the global currency, pays a non-monetary liquidity service L_t to agents in country h per unit of currency, in addition to the intertemporal payoff. Likewise, we assume that currency f pays a liquidity premium L_t^* to agents in country f per unit of currency. For clarity and simplicity, we assume here that currency h and g in country h , as well as currency f and g in country g , are perfect substitutes, postponing the generalization and discussion of imperfect substitutability to Appendix B, see also Irizawa (2020).

In a full model specification such as given in Section 6, these liquidity services are endogenously determined through optimal consumption choices of households under, for instance, cash-in advance constraints or money-in-the-utility function. In all of these models, money is held across periods from t to $t + 1$, and the particular model structure determines whether the services are rendered in period t (“immediately”) or in $t + 1$ (“with delay”). For the benchmark case here we assume the former, but return to the latter in Appendix A.1. In contrast, we shall think of nominal interest rates on bonds as exogenously set policy instruments.

Assumption 3.2 (Liquidity immediacy). *The purchase of currency h and g in country h at t yields an immediate liquidity premium L_t receivable in t . Analogously, the time t purchase of currency f and g in country f at t yields an immediate liquidity*

premium L_t^* receivable in t .

The date- t (post-liquidity) price of a unit of currency h , expressed in units of the same currency equals unity, by definition. Standard asset pricing considerations then deliver

$$1 \geq L_t + E_t[\mathcal{M}_{t+1}]. \quad (8)$$

Whenever (8) holds with equality, agents in country h are willing to accept currency h at its unitary price, since they are exactly compensated by the liquidity premium plus the discounted future value of the payoff, see terms on the right hand side of equation (8). In case of a strict inequality, the current price of currency h is too high compared to the expectations on future price developments such that agents are not willing to hold or purchase the currency. Note that we do not allow for short sale.

If a national currency is valued, then liquidity services provided by this currency stand in a one-to-one relationship with the nominal interest rate paid on bonds, compare (8) to (6), because the nominal interest is the opportunity cost of holding money for transaction purposes.

Likewise, for a unit of the global (or crypto) currency, trading at a price of Q_t in terms of units of currency h , we obtain

$$Q_t \geq L_t Q_t + E_t[\mathcal{M}_{t+1} Q_{t+1}], \quad (9)$$

where this equation holds with equality, if the global currency is used in country h , i.e. if agents are compensated for the price Q_t of a global currency exactly by the liquidity premium plus the discounted future value of the payoff, on the right hand side of equation (9). The price cannot be lower than the right hand side, since otherwise agents in country h would seek to acquire the currency and bid up its value. The price can be higher, however, if the global currency is not used in country h . We implicitly rule out short sales or, more precisely, rule out that short-sold currencies render negative liquidity premia.

Combining (6) and (8), we obtain

$$\frac{i_t}{1 + i_t} \geq L_t, \quad (10)$$

which holds with equality when currency h is used and therefore describes a monotone relationship between the nominal interest rate and the liquidity services.

For the foreign country, we likewise obtain

$$1 \geq L_t^* + E_t[\mathcal{M}_{t+1}^*] \quad (11)$$

$$Q_t^* \geq L_t^* Q_t^* + E_t[\mathcal{M}_{t+1}^* Q_{t+1}^*], \quad (12)$$

$$\frac{i_t^*}{1 + i_t^*} \geq L_t^*. \quad (13)$$

In the analysis that follows we stick to the following assumption.

Assumption 3.3 (Non-negative liquidity premia). *The liquidity premia are non-negative, i.e. $L_t \geq 0$ and $L_t^* \geq 0$.*

This assumption, together with equations (10) and (13), implies that $i_t \geq 0$ and $i_t^* \geq 0$, i.e., imply a zero lower bound for nominal interest rates. Moreover, we assume that at least one currency is used in each country while bonds cannot serve as medium of exchange.

Assumption 3.4 (Currency usage). *In each country, at least one currency is used. That is, in country h , at least one out of inequalities (8) and (9) holds with equality. In country f , at least one out of inequalities (11) and (12) holds with equality.*

Additionally, it is reasonable to impose that at least one out of (9) or (12) holds with equality, but we are not making use of that restriction. We make the assumption that the global currency has a positive value in the time period t under consideration.

Assumption 3.5 (Global currency is valued).

$$Q_t > 0 \text{ and } Q_t^* > 0 \quad (14)$$

Given the triangular relationship among exchange rates, $Q = SQ^*$, it follows that $Q > 0$ if and only if $Q^* > 0$. Thus, the currency being valued in one country necessarily spills over to the other country.

Some additional terminology shall prove useful. For a generic random variable X_{t+1} , define the risk adjusted expectation $\tilde{E}_t[X_{t+1}]$ in country h as

$$\tilde{E}_t[X_{t+1}] \equiv \frac{E_t[\mathcal{M}_{t+1} X_{t+1}]}{E_t[\mathcal{M}_{t+1}]}, \quad (15)$$

and the risk adjusted expectation $\tilde{E}_t^*[X_{t+1}]$ in country f as

$$\tilde{E}_t^*[X_{t+1}] \equiv \frac{E_t[\mathcal{M}_{t+1}^* X_{t+1}]}{E_t[\mathcal{M}_{t+1}^*]}. \quad (16)$$

As a consequence of market completeness and bond pricing, (4), (6) and (7), we obtain uncovered interest parity (UIP) under the risk-adjusted measure,

$$\frac{\tilde{E}_t[S_{t+1}]}{S_t} = \frac{1 + i_t}{1 + i_t^*} \quad (17)$$

$$\frac{\tilde{E}_t^*[S_{t+1}^*]}{S_t^*} = \frac{1 + i_t^*}{1 + i_t} \quad (18)$$

where $S_t^* = 1/S_t$. Note that UIP holds irrespective of whether there is a global currency or not.

4 Main Results

We now obtain our main result.

Proposition 4.1 (Stochastic Economy under Liquidity Immediacy)

Assume liquidity immediacy, complete markets, and that the global currency is valued. If all currencies are used in both countries, i.e. if equations (9), (12) and (8), (11) hold with equality, then

1. *the nominal interest rates are equalized $i_t = i_t^*$;*
2. *the liquidity premia are equal $L_t = L_t^*$;*
3. *the nominal exchange rate S_t between currency h and f follows a martingale, using risk adjusted expectations of country h ;*
4. *the nominal exchange rate $S_t^* = 1/S_t$ between currency f and h follows a martingale, using risk adjusted expectations of country f ;*

Proof. [Proposition 4.1] The competition between currency h and the global currency, i.e. (8) and (9) with equality, the complete-market assumption (4) and finally the

competition between currency f and the global currency, i.e. (11) and (12) with equality, deliver

$$E_t[\mathcal{M}_{t+1}] = E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = E_t \left[\mathcal{M}_{t+1}^* \frac{Q_{t+1}^*}{Q_t^*} \right] = E_t[\mathcal{M}_{t+1}^*] \quad (19)$$

Equations (8) and (11) now imply $L_t = L_t^*$ and thus $i_t = i_t^*$, per equations (10) and (13). Exploiting equality (19) and using the complete market assumption (4) to obtain

$$E_t[\mathcal{M}_{t+1}] = E_t \left[\mathcal{M}_{t+1} \frac{S_{t+1}}{S_t} \right] \quad (20)$$

implies the country- h risk adjusted martingale property for the exchange rate,

$$S_t = \tilde{E}[S_{t+1}].$$

When proceeding instead per replacing \mathcal{M}_{t+1} on the left hand side of (19) with $\mathcal{M}_{t+1}^* S_t / S_{t+1}$ implies the country- f risk adjusted martingale property for $S_t^* = 1/S_t$,

$$S_t^* = \tilde{E}^*[S_{t+1}^*]. \quad \square$$

Proposition 4.1 says that, with complete markets, global usage of the global currency, and simultaneous usage of the local currency, monetary policies must be perfectly synchronized.

But does Proposition 4.1 mean that the central banks in the two countries have no choice but to accept this fate of coordinated monetary policy? When central banks in Home and Foreign are independent, they can set their interest rates distinctly from one another. Proposition 4.1 can also be read the other way around. If $i_t \neq i_t^*$, then at least one of the presumptions has to be violated, either the global currency is not used in at least one country or one of the national currencies is not in use or both. The central bank in country h may then contemplate pursuing a policy that makes sure that the global currency is not used in country h , i.e. that (9) remains an inequality.

Proposition 4.2 (Escaping global currency adoption)

Assume liquidity immediacy, complete markets, and that the global currency is valued. Assume that both local currencies are used in their corresponding countries, i.e. equations (8) and (11) hold with equality. Independently of whether the global currency is

used or not in country f , if $i_t < i_t^*$, then

1. the global currency is not adopted in country h ;
2. the liquidity premia satisfy $L_t < L_t^*$;
3. the nominal exchange rate S_t between currency h and f follows a supermartingale, using risk adjusted expectations in country h ;
4. the nominal exchange rate $S_t^* = 1/S_t$ between currency f and h follows a submartingale, using risk adjusted expectations in country f ;

Proof. [Proposition 4.2] Proof in Appendix 8. □

To understand the economics behind this result, it is important to acknowledge not only the competition between the currency h respectively f and the global currency but also the countrywise competition between the bond and currency and the role of the frictionless foreign exchange-rate market. The proof has three parts. First, since the nominal interest rate in country f is higher than the nominal interest rate in country h , liquidity services in country f are higher than in country h . Second, the competition between the national currencies and the global currency yields upper bounds on the risk-adjusted return of the global currency. The bound is sharper, if the nominal interest rate is higher, i.e. in country f , and it binds, if the global currency is adopted. Third, by frictionless foreign exchange-rate markets and the no arbitrage condition, the risk-adjusted return on the global currency has to be equal in countries h and f . As a consequence, the country with the weaker constraint on that return does not adopt the global currency.

The proposition shows that there is an escape hatch indeed, but only to one side. Starting from an equilibrium in which the global currency is used in both countries, by lowering the risk-free interest rate in currency h below that in currency f , the central bank in country h lowers the opportunity costs of holding the domestic currency and thus makes it more attractive than the global currency as a means of payment, crowding out the global currency in country h .

This escape hatch is not particularly attractive, however. Nominal interest rates can only be lowered to zero. Furthermore, a rat race between the two central banks may eventually force both to stick to the zero lower bound forever or at low interest

rates.⁵ Some may applaud this as the ultimate and global implementation of the Friedman rule, while others may fear deflationary spirals and macroeconomic damage. Either way, these are surely dramatic consequences of the circulation of a global currency.

What forces central banks to lower rather than raise interest rates is the risk of entering unknown territories in which their currency is abandoned as mean of exchange in favor of the global currency. These worries can significantly limit the central bank's room of manoeuvring for stabilizing the economy. The next Proposition depicts a case in which the global currency is used in country f and spreads to country h when its central bank raises rates above the foreign ones.

Proposition 4.3 (Losing medium-of-exchange property)

Assume liquidity immediacy, complete markets, and that the global currency is used in country f , i.e. equation (12) holding with equality. If the central bank in country h sets $i_t > i_t^$, then currency h is abandoned in country h and the global currency takes over (currency substitution). Currency h would also be abandoned in country h if the central bank sets $i_t = i_t^*$ and only currency g is used in country f .*

Proof. [Proposition 4.3] Proof in Appendix 8. □

We call the collection of the three results in Propositions 4.1, 4.2 and 4.3 *Crypto-Enforced Monetary Policy Synchronization (CEMPS)*. They demonstrate that introducing a global currency in a free international capital market constitutes a constraint on the Impossible Trinity. Under free capital flows and without a global currency, uncovered interest parity and the classic Impossible Trinity result provides the home central bank with a choice: it can give up on either a pegged exchange rate or the monetary policy independence. Our result shows that introducing a global currency implies a further restriction, when it becomes a perfect substitute for the local currencies. Either the monetary policy of the central banks can no longer be independent or central banks risk the crowding out of their own currency. Additionally, the exchange rate is now a risk-adjusted martingale and not necessarily a peg, see also [Manuelli and Peck \(1990\)](#) and [Schilling and Uhlig \(2018\)](#). The classical Impossible Trinity thus becomes even less reconcilable. With currency substitution, the countries' nominal

⁵In a one-country model [Benigno \(2019\)](#) shows that if the central bank keeps the inflation target below the growth rate of private currency, then it can maintain the monopoly power on the medium of exchange. However, cryptocurrencies' issuance is in general engineered with quite low, or zero, growth rates so that inflation targets set by central banks should be close to zero or below.

interest rates are equalized independently of whether the economy is stochastic or deterministic.

5 Special case: Asset-backed global currencies

This section is motivated by the recent proposal that Facebook is going to launch a new global currency, Libra. We therefore analyze the consequences of introducing a global currency backed by a basket of risk-free securities denominated in government currencies. In our framework, suppose that the issuing consortium backs the global currency by safe bonds denominated in currency h . Moreover, assume that the consortium is ready to buy and sell any amount of the global currency at a fixed price Q_t . When issuing the amount Δ_t of the global currency at some date t , the consortium invests the proceeds $\Delta_t Q_t$ in the safe bonds of country h . In period $t + 1$, the consortium receives the interest payments on the bonds. The consortium keeps a portion of the date $t + 1$ portfolio value as a per-period asset management fee, assumed to be $\phi_t \Delta_t Q_t$ for some $\phi_t \geq 0$ set in t . One may wish to think of these fees as profits paid to the shareholders of the consortium. The consortium then sets the new price Q_{t+1} , again trading any amount of the global currency at that price and investing their client's funds in home safe bonds. The return that accrues to the global currency between t and $t + 1$, i.e., the bond return after applying the management fee, can be redeemed at the global currency's price Q_{t+1} or is reinvested. In order to credibly promise the repurchase of the global currency for a price Q_{t+1} at $t + 1$ and assuming no profits beyond the asset management fee, assets and liabilities have to grow at the same rate,

$$Q_{t+1} = (1 + i_t - \phi_t) Q_t \quad (21)$$

Note that for $i_t \geq \phi_t$ the price of the global currency then increases over time $Q_{t+1} \geq Q_t$.

Proposition 5.1 (Asset backed global currency)

Assume that the global currency is valued.

- (i) *If $\phi_t < i_t$, then currency h is crowded out and only the global currency is used in country h . Moreover, $L_t = \frac{\phi_t}{1+i_t}$.*
- (ii) *If $\phi_t = i_t$, both currency h and the global currency coexist in country h .*
- (iii) *If $\phi_t > i_t$, then only currency h is used in country h .*

Proof. Proof in Appendix 8. □

From the results in Proposition 5.1, we can generate more striking implications if we assume the fee to apply in the form of a fixed portion of the interest payments, $\phi_t = \kappa i_t$ for some parameter $0 \leq \kappa \leq 1$. Then

1. If $\kappa < 1$, then $i_t \leq \phi_t$ only holds for $i_t = 0$. Moreover, $i_t = 0$ implies $\phi_t = 0$ and the global currency is used together with the local currency in country h .
2. If $\kappa = 1$ (or $\phi_t = i_t$), then the price Q_t for the global currency is fixed (Stable-coin) and both currencies are used.

A useful reading of the above results from a central-banking perspective is the following. For local currency h to remain in usage, the nominal interest rate has to undercut or match the management fee ϕ . The proposition therefore suggests that an interesting Bertrand-type game could unfold. The home central bank may seek to undercut the fee charged by the consortium, in order to drive the global currency out of usage at home. But without usage, the global currency consortium cannot earn any revenue from the fees: it would be better off by lowering its fees in response.⁶ In the limit, this dynamic could result in both parties ratcheting down the “price” for their currencies to their marginal costs of issuance. If these marginal costs are zero or near zero, an assumption often made in the literature, then one obtains a zero interest rate policy and a zero fee. Put differently, currency competition between currency h and the global currency leads to the establishment of the celebrated Friedman rule to keep interest rates at zero, thereby setting the private costs of holding the currencies equal to the social cost of its production. There is a large literature establishing conditions under which the Friedman rule is optimal, see [Woodford \(1990\)](#). More generally, if one currency has higher marginal production costs than the other, then the resulting zero profit condition for this higher-cost currency will dictate the resulting limit. From the consortium’s perspective, the nominal interest rate on the backing asset provides an upper bound on the fee that can be charged while maintaining usage of the global currency.

These results are also reminiscent of the view in [Hayek \(1978\)](#), that unfettered competition can align private incentives with social objectives. To extract rents from

⁶The consortium may not care if country h is small. It presumably would care, though, if the country was large and economically important or a large and important currency union.

liquidity services, currency issuers have to supply better money than others by keeping its value high and, therefore, inflation low. But then competition kicks, driving rents to zero and eliminating liquidity premia so that the better money also serves the social benefits. Benigno (2019) presents a model of currency competition obtaining the same result under free entry. Our insights are related to the analysis in Marimon et al. (2003), who likewise emphasize that cheap inside monies place tight upper bounds on inflation rates there or nominal interest rates here.

In a nutshell, Libra may push central banks back to the zero lower bound. In essence, an asset backed global currency employs bonds to finance liquidity services, thus combining both the advantages of the liquidity services of money with the interest payments of bonds. Now, using the home currency cannot be more costly than the asset management fee charged by the consortium.

6 Examples

In the previous sections, we presented our results using a general framework with a generic notation for the stochastic discount factors and the liquidity services. We now provide several examples of models which put more structure on preferences and constraints. We consider four different models: 1) a Lagos-Wright monetary model; 2) a money-in-utility function model; 3) a cash-in-advance-constraint model in which the “credit” market opens before the “cash” market; 4) a cash-in-advance-constraint model in which the “cash” market opens before the “credit” market. The first three models can be cast in the framework of Section 3 in which liquidity services are received at the same time money is held in the agents’ portfolio. Model 4) deals with the case of delayed liquidity services, which is discussed in its more general form in Appendix A.1.

6.1 Lagos-Wright model

We describe the home country: the description of the foreign country is exactly parallel. There are infinitely many periods. Each period has two subperiods. In the morning, there is a centralized market (CM), while there is a decentralized market (DM) in the afternoon. There is a continuum of agents. In the CM market, all agents meet and trade monies, assets, as well as a morning consumption good enjoyed ac-

according to a linear utility function. Denote the morning consumption by c_t . Negative consumption denotes production. In the afternoon, agents randomly meet pairwise. Each agent chooses the quantity $q \geq 0$ of a good they wish to produce, experiencing disutility $-w(q)$ in doing so. We normalize $w(0) = 0$. Production of a strictly positive quantity is only useful, if the agent they meet happens to like that good. From the perspective of each agent, this happens with probability σ . In that case, we call the producing agent the “seller”, and the other agent the “buyer” in this decentralized market (DM). We assume that buyers can only trade money against goods in the DM; they cannot use other assets such as bonds. Agents therefore have to decide on the quantities $M_{h,t}$ and $M_{g,t}$ of the home and global monies to acquire earlier in the CM to allow trading in the DM. If they do not meet a seller, agents will hold their monies until the CM in the next period. If they meet a seller, they will make the seller a take-it-or-leave-it offer (TIOLI) of $(q_t, D_{h,t}, D_{g,t})$. That is, the buyer offers to purchase a quantity q_t in return for currency amounts $D_{h,t}$ and $D_{g,t}$. The seller can either accept or reject that offer. Periods are discounted at rate β . We assume that aggregate shocks will be drawn at “dawn”, before the CM opens. There are no further shocks within a period. Given the stochastic sequence $(c_t, q_t)_{t=t_0}^{\infty}$ for some agent, with $c_t \in \mathbf{R}$ and $q_t \geq 0$, a buyer agent enjoys utility

$$E_0 \sum_{t=0}^{\infty} \beta^t (c_t + u(q_t))$$

while the seller enjoys utility

$$E_0 \sum_{t=0}^{\infty} \beta^t (c_t - w(q_t)).$$

Note here that q_t is consumption from the perspective of the buyer and production from the perspective of the seller. Let $1/P_t$ be the CM price in terms of the morning good of a unit of home currency. Put differently, one unit of the morning good costs P_t units of the home currency, which is the conventional notation. One unit of the global currency costs Q_t units of the home currency in the morning market. We assume that $u'(0) = \infty$ and $w'(0) < \infty$: this assures that buyers and sellers will strike a deal at a strictly positive quantity $q_t > 0$. Given the linearity of preferences in c_t , the nominal stochastic discount factor relevant for pricing assets from one morning to the next is

therefore

$$\mathcal{M}_{t+1} = \beta \frac{P_t}{P_{t+1}}. \quad (22)$$

As usual, a riskless nominal bond will offer a nominal interest rate i_t , satisfying

$$1 = E_t[\mathcal{M}_{t+1}(1 + i_t)]$$

Consider the decentralized market or DM in t and a seller. For a unit of the home currency, he can buy $1/P_{t+1}$ morning goods in $t + 1$, yielding expected utility $E_t[\beta/P_{t+1}]$ when discounted to the DM. Similarly, a unit of the global currency yields discounted utility $E_t[\beta Q_{t+1}/P_{t+1}]$. The seller is therefore indifferent between rejecting the offer versus accepting to produce q_t units of the good for receiving $D_{h,t}$ units of home currency as well as $D_{g,t}$ units of the global currency, iff

$$w(q_t) = E_t \left[\beta \frac{1}{P_{t+1}} \right] D_{h,t} + E_t \left[\beta \frac{Q_{t+1}}{P_{t+1}} \right] D_{g,t}. \quad (23)$$

The buyer enters the DM, holding $M_{h,t}$ units of the home currency and $M_{g,t}$ units of the global currency. He values the remaining currencies after transacting with the seller just as much as the seller values received currencies. The buyer makes a TIOLI offer $(q_t, D_{g,t}, D_{g,t})$, evaluating the tradeoff between purchasing goods now against the opportunity costs of spending tomorrow. The buyer thus solves

$$\max_{q_t, D_{h,t}, D_{g,t}} u(q_t) - E_t \left[\beta \frac{1}{P_{t+1}} \right] D_{h,t} - E_t \left[\beta \frac{Q_{t+1}}{P_{t+1}} \right] D_{g,t}, \quad (24)$$

subject to keeping the seller at his indifference point (23) and subject to not spending more cash than is available,

$$0 \leq D_{h,t} \leq M_{h,t}, \quad (25)$$

$$0 \leq D_{g,t} \leq M_{g,t}. \quad (26)$$

It is immediately clear that we only need to keep track of the total utility equivalent of available currency and spent currency,

$$m_t = E_t \left[\beta \frac{1}{P_{t+1}} \right] M_{h,t} + E_t \left[\beta \frac{Q_{t+1}}{P_{t+1}} \right] M_{g,t}, \quad (27)$$

$$d_t = E_t \left[\beta \frac{1}{P_{t+1}} \right] D_{h,t} + E_t \left[\beta \frac{Q_{t+1}}{P_{t+1}} \right] D_{g,t}. \quad (28)$$

Therefore, the buyer's problem can be written as

$$v(m_t) = \max_{q_t, d_t} u(q_t) - d_t \quad (29)$$

$$\text{s.t. } w(q_t) = d_t \quad (30)$$

$$d_t \leq m_t \quad (31)$$

The solution is

$$u'(q_t) = w'(q_t) \quad (32)$$

if the currency constraint $d_t \leq m_t$ is slack and

$$w(q_t) = m_t$$

if not. In that case, the latter equation provides an implicit function $q_t = q(m_t)$, and implies

$$w'(q_t)q'(m_t) = 1. \quad (33)$$

Armed with these insights, we see that

$$v'(m_t) = 0 \quad (34)$$

if the currency constraint is slack and

$$v'(m_t) = u'(q(m_t))q'(m_t) - 1 = \frac{u'(q_t)}{w'(q_t)} - 1 \quad (35)$$

if not. With (32), we see that (35) holds, regardless of whether the currency constraint is slack or not. In the CM, the buyer can obtain home currency of quantity $M_{h,t}$ at a utility cost $M_{h,t}/P_t$ and global currency of quantity $M_{g,t}$ at a utility cost $Q_t M_{g,t}/P_t$. We can thus proceed to use the indirect utility function $v(m)$ to formulate the CM

problem for the buyer as maximizing

$$\begin{aligned}
& -\frac{M_{h,t}}{P_t} - \frac{Q_t M_{g,t}}{P_t} + \sigma v \left(E_t \left[\beta \frac{1}{P_{t+1}} \right] M_{h,t} + E_t \left[\beta \frac{Q_{t+1}}{P_{t+1}} \right] M_{g,t} \right) \\
& + \left(E_t \left[\beta \frac{1}{P_{t+1}} \right] M_{h,t} + E_t \left[\beta \frac{Q_{t+1}}{P_{t+1}} \right] M_{g,t} \right)
\end{aligned} \tag{36}$$

Define

$$L_t = \sigma E_t[\mathcal{M}_{t+1}] \left(\frac{u'(q_t)}{w'(q_t)} - 1 \right) \tag{37}$$

Differentiating (36) with respect to $M_{h,t}$ as well as $M_{g,t}$ and exploiting (22) yields the two first order conditions as claimed.

$$\begin{aligned}
1 &= L_t + E_t[\mathcal{M}_{t+1}] \\
Q_t &= L_t Q_t + E_t[\mathcal{M}_{t+1} Q_{t+1}]
\end{aligned}$$

6.2 Money-in-the-utility-function model

The model follows the Sidrauski-Brock framework extended to allow for multiple currencies. Consumers preferences in Home country have the form

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ U(C_t) + V \left(\frac{M_{h,t}^{\text{tot}}}{P_t} \right) \right\} \tag{38}$$

where $M_{h,t}^{\text{tot}} = M_{h,t} + Q_t M_{g,t}$ as in equation (2), where β is the rate of time preferences with $0 < \beta < 1$, C is a consumption good and P its price in units of currency h . We can also assume more generally that C represents a bundle of goods. Consumers enjoy utility from consumption through a concave function $U(\cdot)$ strictly increasing in C and from real money balances by holding currency h , M_h , and the global currency M_g . The utility $V(\cdot)$ increases weakly in real money balances but may exhibit a satiation point at a finite level of real money balances; Q_t is the price of the global currency in units of currency h . Consistently with the general framework of Section 3, consumers can invest in four securities: *i*) a risk-free bond denominated in currency h , B_h , paying an interest rate i ; *ii*) a risk-free bond denominated in currency f , B_f , paying an interest rate i^* ; *iii*) money in units of currency h , M_f , and *iv*) the global money, M_g . Consumers can also trade in a complete set of state-contingent securities,

spanning all states of nature. We omit these securities from the presentation of the consumer's budget constraint. The nominal exchange rate between currency h and f is denoted by S , as in the main text; let T denote lump-sum transfers received from the government in units of currency h while T_g are the transfers from the issuer of global money in units of the global currency. Finally, Y is the home endowment of good C . Preferences in country f are specular, with appropriate starred variables. Consumers are subject to the following budget constraint

$$B_{h,t} + S_t B_{f,t} + M_{h,t} + Q_t M_{g,t} = W_t + P_t(Y_t - C_t) + T_t + Q_t T_{g,t},$$

in which

$$W_t \equiv M_{h,t-1} + Q_t M_{g,t-1} + (1 + i_{t-1})B_{h,t-1} + (1 + i_{t-1}^*)S_t B_{f,t-1}.$$

In the preferences (38) domestic and global monies are perfect substitutes. While we allow the short sales of bonds, as in the main text, we impose a short-sale constraint on the global currency and currency h , i.e. $M_g \geq 0$ and $M_h \geq 0$. The first-order conditions with respect to B_h , B_f , M_h , M_g are

$$\begin{aligned} \frac{U_C(C_t)}{P_t} \frac{1}{1 + i_t} &= E_t \left\{ \beta \frac{U_C(C_{t+1})}{P_{t+1}} \right\} \\ \frac{U_C(C_t)}{P_t} \frac{1}{1 + i_t^*} &= E_t \left\{ \beta \frac{U_C(C_{t+1}) S_{t+1}}{P_{t+1} S_t} \right\} \\ \frac{U_C(C_t)}{P_t} &\geq \frac{1}{P_t} V_m \left(\frac{M_{h,t}^{\text{tot}}}{P_t} \right) + E_t \left\{ \beta \frac{U_C(C_{t+1})}{P_{t+1}} \right\} \\ \frac{Q_t U_C(C_t)}{P_t} &\geq \frac{Q_t}{P_t} V_m \left(\frac{M_{h,t}^{\text{tot}}}{P_t} \right) + E_t \left\{ \beta \frac{Q_{t+1} U_C(C_{t+1})}{P_{t+1}} \right\}, \end{aligned}$$

with the last two equations holding with equality for an interior solution in which $M_h \geq 0$ and $M_G \geq 0$, respectively. As in the main text, at least one should hold with equality. In the above conditions, $U_C(\cdot)$ and $V_m(\cdot)$ are the partial derivatives of the respective functions. These equations can be cast in the notation of Section 3 by

noting that the stochastic discount factors are

$$\mathcal{M}_{t+1} = \beta \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{P_t}{P_{t+1}} \quad \mathcal{M}_{t+1}^* = \beta \frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{P_t^*}{P_{t+1}^*}$$

and the liquidity premia are

$$L_t = \frac{V_m \left(\frac{M_{h,t}^{\text{tot}}}{P_t} \right)}{U_C(C_t)} \quad L_t^* = \frac{V_m \left(\frac{M_{f,t}^{\text{tot},*}}{P_t^*} \right)}{U_C(C_t^*)},$$

where $M_{f,t}^{\text{tot},*} = M_{f,t}^* + Q_t^* M_{g,t}^*$, analogously to (2). Note that liquidity services endogenously satisfy additivity and immediacy. Note that complete markets imply that

$$\frac{U_C(C_t)}{P_t} = k \frac{U_C(C_t^*)}{S_t P_t^*}$$

for some positive parameter k which can be set equal to one. In the case where purchasing power parity holds, $P_t = S_t P_t^*$, marginal utilities of consumption are proportional across countries. When all currencies are used, Proposition 4.1 applies and therefore $L_t = L_t^*$. Another implication is that the marginal utilities of real money balances $V_m(\cdot)$ are equalized across countries.

6.3 Cash-in-advance model, type I

Consider a cash-in-advance model with the timing of Lucas and Stokey (1987), in which the “credit” market opens before the “cash” market. Consumers living in country h have the following preferences

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(C_{T,t}, C_{N,t}) \quad (39)$$

in which C_T and C_N are, respectively, a traded and non-traded good and β , with $0 < \beta < 1$, is the intertemporal discount factor; $U(\cdot, \cdot)$ is a concave function, strictly increasing in both arguments. Preferences in country f are similar with variables denoted by a star. Each period is divided into two sub-periods. In the first sub-

period financial markets are open and the consumer's budget constraint is given by

$$B_{h,t} + S_t B_{f,t} + M_{h,t} + Q_t M_{g,t} = W_t + T_t + Q_t T_{g,t} \quad (40)$$

in which W_t is the nominal wealth, which remains after taking into account the purchases of goods in the previous period

$$\begin{aligned} W_t = & (1 + i_{t-1})B_{h,t-1} + (1 + i_{t-1}^*)S_t B_{f,t-1} + M_{h,t-1} + Q_t M_{g,t-1} + \\ & + P_{T,t-1}(Y_{T,t-1} - C_{T,t-1}) + P_{N,t-1}(Y_{N,t-1} - C_{N,t-1}). \end{aligned} \quad (41)$$

Y_T and Y_N are the endowments of the traded and non-traded goods, and P_T and P_N the respective prices. In the second subperiod of period t , the ‘‘cash’’ market opens and non-traded goods can be purchased following this constraint

$$M_{h,t}^{\text{tot}} \geq P_{N,t} C_{N,t}. \quad (42)$$

where $M_{h,t}^{\text{tot}} = M_{h,t} + Q_t M_{g,t}$ as in equation (2). Budget constraints can be written specularly for the consumers living in country f .

The first-order conditions with respect to B_h , B_f , M_h , M_g are

$$\frac{\lambda_t}{1 + i_t} = E_t \{ \beta \lambda_{t+1} \} \quad (43)$$

$$\frac{\lambda_t}{1 + i_t^*} = E_t \left\{ \beta \lambda_{t+1} \frac{S_{t+1}}{S_t} \right\} \quad (44)$$

$$\lambda_t \geq \mu_t + \beta E_t \{ \lambda_{t+1} \} \quad (45)$$

$$\lambda_t Q_t \geq \mu_t Q_t + \beta E_t \{ \lambda_{t+1} Q_{t+1} \} \quad (46)$$

with the last two equations holding with equality for an interior solution $M_{h,t} > 0$ and $M_{g,t} > 0$, respectively; λ_t and μ_t are the multipliers associated with constraints (40) and (42), respectively. Moreover, the first-order conditions with respect to C_N and C_T imply that

$$\frac{U_{C_N}(C_{T,t}, C_{N,t})}{P_{N,t}} = \mu_t + \beta E_t \{ \lambda_{t+1} \}, \quad (47)$$

$$\frac{U_{C_T}(C_{T,t}, C_{N,t})}{P_{T,t}} = \beta E_t \{ \lambda_{t+1} \}, \quad (48)$$

where $U_{C_T}(\cdot, \cdot)$ and $U_{C_N}(\cdot, \cdot)$ are the derivatives of function $U(\cdot, \cdot)$ with respect to the first and second arguments, respectively. We can now map this model in the notation of the general framework of Section 3 by noting that the stochastic discount factors are given by

$$\mathcal{M}_{t+1} = \frac{\beta\lambda_{t+1}}{\lambda_t} \quad \mathcal{M}_{t+1}^* = \frac{\beta\lambda_{t+1}^*}{\lambda_t^*}$$

while the liquidity premia can be written instead as

$$L_t = \frac{\mu_t}{\lambda_t} \quad L_t^* = \frac{\mu_t^*}{\lambda_t^*}.$$

Using first-order conditions (45), (47) and (48), we can also write the nominal stochastic discount factors

$$\begin{aligned} \mathcal{M}_{t+1} &= \beta \frac{U_{C_N}(C_{T,t+1}, C_{N,t+1})}{U_{C_N}(C_{T,t}, C_{N,t})} \frac{P_{N,t}}{P_{N,t+1}} \\ \mathcal{M}_{t+1}^* &= \beta \frac{U_{C_N}(C_{T,t+1}^*, C_{N,t+1}^*)}{U_{C_N}(C_{T,t}^*, C_{N,t}^*)} \frac{P_{N,t}^*}{P_{N,t+1}^*} \end{aligned}$$

and the liquidity premia as

$$\begin{aligned} L_t &= \frac{U_{C_N}(C_{T,t}, C_{N,t}) - \frac{P_{N,t}}{P_{T,t}} U_{C_T}(C_{T,t}, C_{N,t})}{U_{C_N}(C_{T,t}, C_{N,t})} \\ L_t^* &= \frac{U_{C_N}(C_{T,t}^*, C_{N,t}^*) - \frac{P_{N,t}^*}{P_{T,t}^*} U_{C_T}(C_{T,t}^*, C_{N,t}^*)}{U_{C_N}(C_{T,t}^*, C_{N,t}^*)}. \end{aligned}$$

As in the case of money-in-the-utility function, liquidity services endogenously satisfy additivity and immediacy. The results of Proposition 4.1 apply in the case that all currencies are used. Additional results can be derived in this particular example. Note first that market completeness implies that $\lambda_t = \kappa\lambda_t^*$ for some positive constant κ and at all t , which in the context of the above model can also be written as

$$\frac{U_{C_N}(C_{T,t}, Y_{N,t})}{P_{N,t}} = k \frac{U_{C_N}(C_{T,t}^*, Y_{N,t}^*)}{S_t P_{N,t}^*}. \quad (49)$$

Under appropriate assumptions on the initial distribution of wealth, the constant k can be set equal to 1.⁷ In (49), we have substituted equilibrium in the non-traded

⁷The result that $\lambda_t = \kappa\lambda_t^*$ implies (49) is driven by the fact that money allows the insurance of

goods market, $C_{N,t} = Y_{N,t}$ and $C_{N,t}^* = Y_{N,t}^*$. Moreover, combining first-order conditions (43), (45), (47) and (48) it is possible to obtain that

$$\frac{U_{C_N}(C_{T,t}, Y_{N,t})}{U_{C_T}(C_{T,t}, Y_{N,t})} = (1 + i_t) \frac{P_{N,t}}{P_{T,t}} \quad \frac{U_{C_N}(C_{T,t}^*, Y_{N,t}^*)}{U_{C_T}(C_{T,t}^*, Y_{N,t}^*)} = (1 + i_t^*) \frac{P_{N,t}^*}{P_{T,t}^*},$$

Using $i_t = i_t^*$ and (49) with $k = 1$ in the above conditions, we obtain that

$$\frac{U_{C_T}(C_{T,t}, Y_{N,t})}{U_{C_T}(C_{T,t}^*, Y_{N,t}^*)} = \frac{P_{T,t}}{S_t P_{T,t}^*}. \quad (50)$$

Assume that the law-of-one price holds for traded goods, $P_{T,t} = S_t P_{T,t}^*$, and consider the special case in which $Y_{N,t} = Y_{N,t}^*$. Then (50) implies perfect cross-country risk-sharing of the consumption of traded goods, $C_{T,t} = C_{T,t}^*$. Using this result in (49), we also obtain that the law-of-one price holds for non-traded goods $P_{N,t} = S_t P_{N,t}^*$, for which the equalization of the nominal interest rates is key.

6.4 Cash-in-advance model, type II

Consider a cash-in-advance model with a different timing, in which the “cash” market now opens before the “credit” market. Preferences of consumers living in country h are similar to (39). Each period is divided into two sub-periods. In the first sub-period the non-traded good can be purchased subject to the following constraint,

$$M_{h,t-1} + Q_t M_{g,t-1} \geq P_{N,t} C_{N,t} \quad (51)$$

in which variables follow previous definitions. After the “cash” market closes, in the second sub-period of period t the “credit” market opens and consumers are subject to the following constraint,

$$\begin{aligned} B_{h,t} + S_t B_{f,t} + M_{h,t} + Q_t M_{g,t} + P_{T,t} C_{T,t} + P_{N,t} C_{N,t} = \\ + P_{T,t} Y_{T,t} + P_{N,t} Y_{N,t} + T_t + Q_t T_{g,t} + W_t \end{aligned} \quad (52)$$

where

$$W_t \equiv (1 + i_{t-1}) B_{h,t-1} + (1 + i_{t-1}^*) S_t B_{f,t-1} + M_{h,t-1} + Q_t M_{g,t-1}.$$

any movement in the price of non-traded goods in the cash constraint (42).

Budget constraints can be specularly written for the consumers in country f . The first-order conditions with respect to B_h , B_f , M_h , M_g are

$$\begin{aligned}\frac{\lambda_t}{1+i_t} &= E_t \{ \beta \lambda_{t+1} \} \\ \frac{\lambda_t}{1+i_t^*} &= E_t \left\{ \beta \lambda_{t+1} \frac{S_{t+1}}{S_t} \right\} \\ \lambda_t &\geq \beta E_t \{ \mu_{t+1} + \lambda_{t+1} \} \\ \lambda_t Q_t &\geq \beta E_t \{ (\mu_{t+1} + \lambda_{t+1}) Q_{t+1} \}\end{aligned}$$

with the last two equations holding with equality for an interior solution $M_{h,t} > 0$ and $M_{g,t} > 0$, respectively. In the above conditions, λ_t and μ_t are the multipliers associated with constraints (52) and (51), respectively. Moreover, the first-order conditions with respect to C_N and C_T imply that

$$\frac{U_{C_N}(C_{T,t}, C_{N,t})}{P_{N,t}} = \mu_t + \lambda_t, \quad (53)$$

$$\frac{U_{C_T}(C_{T,t}, C_{N,t})}{P_{T,t}} = \lambda_t. \quad (54)$$

Note that in this model liquidity premia are received with a one-period delay. Therefore, this example can be mapped in the notation of the general framework presented in Appendix A.1 by noting that the stochastic discount factors are given by

$$\mathcal{M}_{t+1} = \frac{\beta \lambda_{t+1}}{\lambda_t} \quad \mathcal{M}_{t+1}^* = \frac{\beta \lambda_{t+1}^*}{\lambda_t^*}$$

while liquidity premia are

$$L_{t+1} = \frac{\mu_{t+1}}{\lambda_{t+1}} \quad L_{t+1}^* = \frac{\mu_{t+1}^*}{\lambda_{t+1}^*}.$$

Using first-order conditions (53) and (54), we can further write the stochastic discount factors and the liquidity premia as

$$\begin{aligned}\mathcal{M}_{t+1} &= \beta \frac{U_{C_T}(C_{T,t+1}, C_{N,t+1})}{U_{C_T}(C_{T,t}, C_{N,t})} \frac{P_{T,t}}{P_{T,t+1}} \\ \mathcal{M}_{t+1}^* &= \beta \frac{U_{C_T}(C_{T,t+1}^*, C_{N,t+1}^*)}{U_{C_T}(C_{T,t}^*, C_{N,t}^*)} \frac{P_{T,t}^*}{P_{T,t+1}^*}\end{aligned}$$

and

$$\begin{aligned}1 + L_{t+1} &= \frac{U_{C_N}(C_{T,t+1}, C_{N,t+1})}{U_{C_T}(C_{T,t+1}, C_{N,t+1})} \frac{P_{T,t+1}}{P_{N,t+1}} \\ 1 + L_{t+1}^* &= \frac{U_{C_N}(C_{T,t+1}^*, C_{N,t+1}^*)}{U_{C_T}(C_{T,t+1}^*, C_{N,t+1}^*)} \frac{P_{T,t+1}^*}{P_{N,t+1}^*}.\end{aligned}$$

The results of Proposition A.1 and Corollary A.1 apply to this model.

6.5 Are Gold, Dollar and Credit Cards global currencies?

One may wonder whether the emphasis on cryptocurrencies as emerging global currencies is misplaced. Is gold not already a globally acceptable means of payment? Isn't the Dollar already a global currency? What about credit cards: if payments abroad can be made with a credit card charged to a domestic bank account, shouldn't all our results already go through? The purpose of this section is to shed some light on these questions.

First, consider the case of gold. Nowadays, it is hard or even impossible to make payments directly with gold: it is rare to find a shop which would accept it as a means of payment. The reasons can be verification issues (measurement of purity), the risk of fraud, the lack of normalization (size of a gold bar), or the inconvenience due to its weight. Moreover, gold offers benefits beyond those of a fiat currency: gold can be turned into jewelry or used for a variety of medical or industrial purposes. In addition, random findings of gold act as exogenous shocks to the gold supply and thus its price. This may also impede the gold's store of value functionality (see the abandonment of the Gold Standard). In sum, gold is rarely used as a medium of exchange, which is the focus and the basis for our analysis. For that reason, gold does not constitute a global currency, as analyzed in this paper.

Next, consider the Dollar. Indeed, international invoicing is often done in terms

of Dollar, see [Gopinath et al. \(2020\)](#), and the Dollar serves as a vehicle currency, see e.g. [Rey \(2001\)](#). These papers, however, emphasize the “unit of account” function of money. Moreover, “Dollarization” often means only the invoicing in terms of Dollar, thus concerning the numeraire function of money, and not its usage as a medium of exchange. However, in some countries, physical Dollars are regularly used as a means of payment. In this case, our analysis on restraining monetary policy applies. For the local currency to circulate with the Dollar, its interest rate should be capped by the Dollar interest rate. Monetary policy in the dollarized economy then becomes constrained, but not the U.S. monetary policy. For all practical purposes, however, Dollars do not circulate as a medium of exchange in Europe, and the Euro does not circulate as a medium of exchange in the U.S. By contrast, a crypto-currency, which may become an accepted medium of exchange in advanced countries, may dramatically alter the financial landscape.

Finally, consider credit cards, which we discuss extensively in [Appendix C](#). We distinguish two cases. If the credit-card market functions efficiently, i.e. if the interest paid on credit-card debt equals the interest received on extending credit-card credit, and if credit cards can be used to make purchases, then there can be full satiation of liquidity in the economy. The reason is that competition in the credit-card market makes the effective cost of paying through credit cards zero. In this way, consumers are able to fully overcome the constraint imposed by the need of holding cash. The Friedman rule is attained and $i_t = 0$. Applying this result to the foreign economy too, we conclude that $i_t = i_t^* = 0$. However, the equalization of interest rates to zero is different in nature from that implied by our framework when introducing a global currency. A global currency solely equates interest rates across countries, and not necessarily to zero, because of competition among the returns on different currencies. In a frictionless credit market instead, the credit card is an efficient means of payment, which completely relaxes the liquidity constraint. The competition coming from this payment instrument drives to zero all rents in the liquidity market. Therefore, if this happens in all countries, interest rates are equalized to zero globally. A global currency can achieve the same outcome, as we have shown in [Section 4](#), if it competes with government currencies for the control of the medium-of-exchange function.

At the other extreme, credit card payments are a perfect substitute for money when the seller in a credit card transaction is not remunerated, leaving then some profits to the credit card company. In this case, the effective cost of paying via credit

card is the nominal interest rate and therefore credit cards are a perfect substitute for money. Under this assumption, credit card transactions do not add anything to our model, they are not acting like a global currency since they do not put any pressure on interest rates. What makes a credit card similar to a global currency, in this case, is instead the possibility for the domestic household to have credit cards linked to accounts denominated in different currencies which can be used to purchase the same goods. This possibility would create competition between currencies and produce the same effects as the global currency of our model. In practice, this solution requires households to have accounts denominated in different currencies, which is not always feasible in all countries. Moreover, competition should be created among all possible pairs of national currencies. A global currency like Libra, which, in contrast, could be easily adopted in the world economy, can achieve the same outcome in a more straightforward and natural way.

7 Conclusion

Starting from a general framework, we analyze a two-country economy featuring a home, a foreign and a global (crypto)currency. For the benchmark case that markets are complete, that the global currency is used in both countries and that currency liquidity services are immediate, we show that nominal interest rates must be equal and that the exchange rate between the home and the foreign currency is a risk-adjusted martingale. We call this phenomenon *Crypto-Enforced Monetary Policy Synchronization (CEMPS)*. It adds a further restriction to the classic Impossible Trinity. We discuss the dangers for monetary policies that seek to circumvent this restriction. We characterize the implications for the exchange rate dynamics and the pricing dynamics of the global currency. If the global currency is backed by interest-bearing assets, additional and tight restrictions on monetary policy arise. We demonstrate that our general framework encompasses a number of classic monetary models, such as the Lagos-Wright model, models featuring money-in-the-utility function, and cash-in-advance models. In the appendix, we extend our results to the case of delayed liquidity services, and where currencies are not perfect substitutes. We conclude that the introduction of a globally used currency may substantially change the landscape of international monetary policy.

8 Appendix

In this Appendix, we collect the proofs of the Propositions.

8.1 Proof of Proposition 4.2

Proof. [Proposition 4.2] It holds

$$E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = E_t \left[\mathcal{M}_{t+1}^* \frac{Q_{t+1}^*}{Q_t^*} \right] \quad (55)$$

$$\leq E_t[\mathcal{M}_{t+1}^*] = \frac{1}{1+i_t^*} \quad (56)$$

$$< \frac{1}{1+i_t} = E_t[\mathcal{M}_{t+1}] = 1 - L_t \quad (57)$$

The first step follows by market completeness, the second step holds since the global currency may or may not be in use in country f , thereby yielding a weakly lower return than currency f in country f . The third step uses equation (7). The fourth step, the inequality sign, is implied by the assumption $i_t < i_t^*$, the fifth step uses equation (6) and the final step follows from the assumption that currency h is used in country h , i.e. equation (8) with the equality sign. Thus,

$$E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] < 1 - L_t \quad (58)$$

and the global currency is not used in country h . We directly see that $L_t < L_t^*$ from our derivation. Thus, by market completeness

$$E_t \left[\mathcal{M}_{t+1} \frac{S_{t+1}}{S_t} \right] = E_t[\mathcal{M}_{t+1}^*] < E_t[\mathcal{M}_{t+1}] \quad (59)$$

where the last step follows from the derivation above. Therefore, S_t follows a supermartingale in the country- h risk-adjusted measure. Vice versa,

$$E_t[\mathcal{M}_{t+1}^*] < E_t[\mathcal{M}_{t+1}] = E_t \left[\mathcal{M}_{t+1}^* \frac{S_t}{S_{t+1}} \right] \quad (60)$$

Thus, with $S^* = 1/S$, $E_t[\mathcal{M}_{t+1}^*] < E_t \left[\mathcal{M}_{t+1}^* \frac{S_{t+1}^*}{S_t^*} \right]$ and also the exchange rate from the perspective of country f follows a submartingale. \square

8.2 Proof of Proposition 4.3

Proof. [Proposition 4.3] We have

$$E_t[\mathcal{M}_{t+1}] = \frac{1}{1+i_t} < \frac{1}{1+i_t^*} = E_t[\mathcal{M}_{t+1}^*] \quad (61)$$

$$\leq 1 - L_t^* = E_t \left[\mathcal{M}_{t+1}^* \frac{Q_{t+1}^*}{Q_t^*} \right] \quad (62)$$

$$= E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] \leq 1 - L_t \quad (63)$$

Here the first step uses equation (6), the second step uses the policy set in the two countries, $i_t \geq i_t^*$, the third step equation (7). The fourth step and inequality follows because currency f may or may not be used in country f . The fifth step uses that the global currency is used in country f , the sixth step uses completeness of markets and the last step uses that the global currency may or may not be adopted in country h . Altogether, $E_t[\mathcal{M}_{t+1}] < 1 - L_t$ for $i > i^*$. Alternatively, $E_t[\mathcal{M}_{t+1}] < 1 - L_t$ for $i = i^*$ if currency f is not used in country f , $E_t[\mathcal{M}_{t+1}^*] < 1 - L_t^*$. \square

8.3 Proof of Proposition 5.1

Proof. [Proposition 5.1] (i) Assume $\phi_t < i_t$. Then

$$1 - L_t \geq E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = (1 + i_t - \phi_t) E_t[\mathcal{M}_{t+1}] > E_t[\mathcal{M}_{t+1}]. \quad (64)$$

The first inequality holds by (9), the second step holds by (21), the third step follows from $i_t > \phi_t$. Since $1 - L_t > E_t[\mathcal{M}_{t+1}]$, local currency h is not used. Given the assumption that at least one currency is used in country h , (9) has to hold with equality, $1 - L_t = (1 + i_t - \phi_t) E_t[\mathcal{M}_{t+1}]$, and the global currency is used in h . By no arbitrage, a comparison between the return on the global currency and the bond through (6) yields

$$\frac{1 - L_t}{1 + i_t - \phi_t} = \frac{1}{1 + i_t} \quad (65)$$

and thus $L_t = \frac{\phi_t}{1+i_t}$.

(ii) Assume $\phi_t = i_t$, then

$$1 - L_t \geq E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = (1 + i_t - \phi_t) E_t[\mathcal{M}_{t+1}] = E_t[\mathcal{M}_{t+1}] \quad (66)$$

and since at least one currency has to be in use, we have

$$1 - L_t = E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = E_t[\mathcal{M}_{t+1}] \quad (67)$$

implying that both currencies are used.

(iii) Assume $\phi_t > i_t$, then

$$1 - L_t \geq E_t[\mathcal{M}_{t+1}] > (1 + i_t - \phi_t) E_t[\mathcal{M}_{t+1}] = E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] \quad (68)$$

Thus, the global currency is not used. But since one currency has to be used, it has to be currency h , $1 - L_t = E_t[\mathcal{M}_{t+1}]$. \square

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FOR ONLINE PUBLICATION APPENDIX

A Robustness analysis

In this section, we present several robustness analysis of our main results. First, we investigate the case in which liquidity services are delayed by one period with respect to when money is held in the agents' portfolio. Second, we sketch out the implications of imperfect substitutability between currencies. Finally, we provide a detailed model involving credit cards.

A.1 Delayed liquidity services

An important assumption of our framework is liquidity immediacy, i.e. that the liquidity services provided by a currency occur at the same date t that money is added to the agent's portfolio. However, some models, such as the third example in Section 6, postulate instead that liquidity premia are to be received a period after portfolio choices are made, i.e. with delay in $t + 1$:

Assumption A.1 (Liquidity delay). *The purchase of the global currency and currency h in country h at t yields delayed liquidity premia L_{t+1} receivable in $t + 1$. Analogously, the time t purchase of global currency and currency f in country f at t yields delayed liquidity premia L_{t+1}^* receivable in $t + 1$.*

In this case, equations (8), (9) and (10) need to be replaced with

$$1 \geq E_t[\mathcal{M}_{t+1}(1 + L_{t+1})], \quad (\text{A.1})$$

$$Q_t \geq E_t[\mathcal{M}_{t+1}(1 + L_{t+1})Q_{t+1}], \quad (\text{A.2})$$

$$\frac{i_t}{1 + i_t} \geq E_t[\mathcal{M}_{t+1}L_{t+1}]. \quad (\text{A.3})$$

The liquidity premia are appropriately discounted by the stochastic discount factor. Since we focus on equilibria in which all currencies are used, we set (A.1), (A.2), (A.3) with an equality sign.

In country f , one must likewise replace (11), (12) and (13) with

$$1 \geq E_t[\mathcal{M}_{t+1}^*(1 + L_{t+1}^*)], \quad (\text{A.4})$$

$$Q_t^* \geq E_t[\mathcal{M}_{t+1}^*(1 + L_{t+1}^*)Q_{t+1}^*], \quad (\text{A.5})$$

$$\frac{i_t^*}{1 + i_t^*} \geq E_t[\mathcal{M}_{t+1}^*L_{t+1}^*]. \quad (\text{A.6})$$

Again, in what follows, we will assume that the above equations hold with an equality sign. Define the conditional covariance under the home country risk-adjusted measure as

$$\widetilde{cov}_t(X, Y) \equiv \widetilde{E}_t[XY] - \widetilde{E}_t[X]\widetilde{E}_t[Y] \quad (\text{A.7})$$

For a random variable X , define the risk-adjusted expectation in country f as the equivalent to $\widetilde{E}_t[\cdot]$ via

$$\widetilde{E}_t^*[X] \equiv \frac{E_t[\mathcal{M}_{t+1}^*X]}{E_t[\mathcal{M}_{t+1}^*]} \quad (\text{A.8})$$

Let

$$\Delta_t \equiv i_t - i_t^*$$

be the differences between the nominal interest rates. Maintaining all other assumptions, we next turn to deriving implications for the exchange rate. The next results apply independently of whether liquidity premia are delayed, and they need as input solely the interest rate differential, like in (A.11).

Proposition A.1 (Delayed Liquidity Services and Exchange Rates)

In a stochastic economy, assuming liquidity delay, complete markets, and all currencies being used: the expected liquidity services differences and exchange rates then satisfy

$$\Delta_t = \widetilde{E}_t[L_{t+1}] - \widetilde{E}_t^*[L_{t+1}^*] \quad (\text{A.9})$$

and

$$\frac{\widetilde{E}_t[S_{t+1}]}{S_t} = 1 + \frac{\Delta_t}{1 + i_t^*} \quad (\text{A.10})$$

This corollary is a strict consequence of the given interest differential: the presence of the global currency is not necessary to establish these consequences. Note how the results here are adjusted relative to the expressions in our benchmark result. The (expected) liquidity services now differ by the interest rate differential. If the rate is zero, as in the main result, so is the (expected) liquidity service difference. The

exchange rate is no longer a risk-adjusted martingale: instead, there is an adjustment term that depends on the interest rate differential. If that interest rate differential is zero, as in the main result, we are back to the risk-adjusted martingale.

Proof. [Proposition A.1] Note that (A.1) and (A.3) can be written as

$$i_t = \tilde{E}_t[L_{t+1}].$$

Likewise, (A.4) and (A.6) can be written as

$$i_t^* = \tilde{E}_t^*[L_{t+1}^*].$$

The combination yields (A.9). Finally, consider the uncovered-interest-parity relationship (17) to obtain (A.10). \square

Corollary A.1 (Stochastic Economy under Delayed Liquidity Premia)

In a stochastic economy, assuming liquidity delay, complete markets, and all currencies being used, the nominal interest rate differential satisfies

$$i_t^* - i_t = \frac{\widetilde{cov}_t(L_{t+1} - L_{t+1}^*, Q_{t+1})}{\tilde{E}_t[Q_{t+1}]} + \frac{\widetilde{cov}_t(L_{t+1}^*, S_{t+1})}{\tilde{E}_t[S_{t+1}]} \quad (\text{A.11})$$

Note that the benchmark result of interest rate equality in case of liquidity immediacy is a direct consequence of (A.11), since the conditional covariance terms must be zero, if L_{t+1} and L_{t+1}^* are known in t . In the general case, nonzero covariance terms arise and equation (A.11) informs us, in which direction one needs to adjust the interest differential.

Proof. [Corollary A.1.] Since all currencies are used, (A.3) and (A.6) hold with equality. With (6) and (7), rewrite (A.3) and (A.6) using the risk-adjusted measures as

$$i_t = \tilde{E}_t[L_{t+1}] \quad (\text{A.12})$$

and

$$i_t^* = \tilde{E}_t^*[L_{t+1}^*] = \frac{\tilde{E}_t[L_{t+1}^* S_{t+1}]}{\tilde{E}_t[S_{t+1}]} \quad (\text{A.13})$$

where in the latter we have also used the assumption of complete markets. Combining

the two equations above, we can write the interest-rate differential as

$$i_t^* - i_t = \tilde{E}_t [L_{t+1}^*] - \tilde{E}_t [L_{t+1}] + \frac{\widetilde{\text{cov}}_t(L_{t+1}^*, S_{t+1})}{\tilde{E}_t[S_{t+1}]}, \quad (\text{A.14})$$

Note that this equation holds, regardless of whether there is a global currency or not. The presence of the global currency, however, delivers a restriction on the difference between the expected liquidity services. Use (A.5) together with the assumption of complete markets and the equivalence $Q_t = S_t Q_t^*$ to obtain

$$Q_t = E_t[\mathcal{M}_{t+1}(1 + L_{t+1}^*)Q_{t+1}] \quad (\text{A.15})$$

This can be written under the risk-adjusted measure as

$$(1 + i_t)Q_t = \tilde{E}_t [(1 + L_{t+1}^*)Q_{t+1}]. \quad (\text{A.16})$$

Writing (A.2) using the risk-adjusted measure

$$(1 + i_t)Q_t = \tilde{E}_t [(1 + L_{t+1})Q_{t+1}], \quad (\text{A.17})$$

and compare it with the equation above to obtain that

$$0 = \tilde{E}_t [(L_{t+1}^* - L_{t+1})Q_{t+1}] \quad (\text{A.18})$$

and thus

$$\tilde{E}_t [L_{t+1}^*] - \tilde{E}_t [L_{t+1}] = \frac{\widetilde{\text{cov}}_t(L_{t+1} - L_{t+1}^*, Q_{t+1})}{\tilde{E}_t[Q_{t+1}]}, \quad (\text{A.19})$$

Plugging (A.19) into (A.14) delivers (A.11). \square

Note that equation (A.19) determines the expected difference in the liquidity premia, by which we can retrieve the result of the benchmark case of equal liquidity premia when L_{t+1} and L_{t+1}^* are known at time t .

B Imperfect substitutability of currencies

Our analysis easily generalizes, with suitable modification, to the situation where the currencies are not perfect substitutes. We outline the general approach here. A more

detailed analysis can be found in [Irizawa \(2020\)](#). As in [Section 2](#), let $M_{h,t}^{\text{tot}}$ denote the total money holdings in country h at time t , expressed in units of the domestic currency. In [Section 2](#) and implicitly in the general framework of [Section 3](#), we have assumed that $M_{h,t}^{\text{tot}}$ is the sum of the nominal value of the home currency, as well as, the global currency used at home,

$$M_{h,t}^{\text{tot}} = M_{h,t} + Q_t M_{g,t}, \quad (\text{B.20})$$

see [equation 2](#). More generally, assume that

$$M_{h,t}^{\text{tot}} = f(M_{h,t}, Q_t M_{g,t}) \quad (\text{B.21})$$

for some constant returns to scale function $f(\cdot, \cdot)$. This captures the idea that the national currency may be relatively more useful for certain transactions, while the global currency is more useful for others. A full-fledged version of this idea is contained in [Schilling and Uhlig \(2019\)](#). Due to constant returns to scale, [\(B.21\)](#) can alternatively be written in terms of real units as

$$\frac{M_{h,t}^{\text{tot}}}{P_t} = f\left(\frac{M_{h,t}}{P_t}, \frac{Q_t M_{g,t}}{P_t}\right) \quad (\text{B.22})$$

[Equation \(B.20\)](#) arises for the linear specification

$$f(M_{h,t}, Q_t M_{g,t}) = M_{h,t} + Q_t M_{g,t}.$$

Total home money holdings provide the total liquidity services $L_t M_{ht}^{\text{tot}}$. Via [\(B.21\)](#), a marginal unit of home currency therefore provides liquidity services $L_t f_{1,t}$, while a marginal unit of global currency provides liquidity services $L_t f_{2,t} Q_t$, where $f_{1,t}$ and $f_{2,t}$ are the partial derivatives of the function f with respect to their first and second argument, evaluated at $(M_{h,t}, Q_t M_{g,t})$. [Equations \(8\) and \(9\)](#) now become

$$1 \geq L_t f_{1,t} + E_t[\mathcal{M}_{t+1}] \quad (\text{B.23})$$

and

$$1 \geq L_t f_{2,t} + E_t\left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t}\right] \quad (\text{B.24})$$

These equations and the usual properties of constant-returns-to-scale functions clarify that marginal liquidity services $L_t f_{1,t}$ and $L_t f_{2,t}$ provided by either currency now depend on the ratio⁸ of their nominal values

$$\rho_t = \frac{Q_t M_{g,t}}{M_{h,t}} \quad (\text{B.25})$$

Propositions 4.1, 4.2 and 4.3 require appropriate modification. If f is not linear, then $i_t < i_t^*$ generally results in a tilt towards the home currency and a decrease in ρ_t rather than complete elimination of the global currency though, the latter is still a possibility, if $f_2(\cdot \cdot \cdot, 0) < \infty$, i.e. if the two currencies are substitutes, and the interest rate difference $i_t^* - i_t$ is sufficiently large. Likewise, $i_t > i_t^*$ generally results in a tilt towards the global currency and an increase in ρ_t , rather than complete elimination of the home currency. Once again, the latter can happen in the economically plausible case of substitutes and $f_1 < \infty$ as well as a sufficiently large interest rate differential $i_t - i_t^*$. These considerations add nuance to the main analysis, without changing its core message.

C Credit Cards

When considering credit cards instead of a global currency in a cash-in-advance model, the precise “story telling” and timing matters. We assume that there are nominal (home) bonds paying interest rate i_t while credit card debt commands an interest rate x_t . A representative household consumes, as well as, sells a cash good and a credit good in period t . It may be best to think of a household as a group of two agents, a buyer and a seller. There is a buyer who can use money M_t to buy cash goods c_t^M and credit card debt D_t to buy credit goods c_t^D . There is a seller with an endowment y_t who is indifferent between selling in exchange for money or credit-card credit. The household gets together at the end of the period, enjoying consumption according to preferences $u(c_t^M, c_t^D)$. A special case is perfect substitutability between the cash and the credit good. In that case (and with slight abuse of notation of using the same $u(\cdot)$), the household cares about $u(c_t)$, where $c_t = c_t^M + c_t^D$, choosing the cheaper course of action, if possible. It is easy to generalize this setting to allowing

⁸For completeness and as usual, define the function $g(\rho) = f(1, \rho)$. Note that $M_{h,t}^{\text{tot}} = g(\rho_t)M_{h,t}$. Calculate that $f_{1,t} = g(\rho_t) - g'(\rho_t)\rho_t$ and $f_{2,t} = g'(\rho_t)$.

for less than perfect substitution, but we wish to keep things simple here.

We shall allow the price P_t on the goods-for-cash market to differ from the price P_t^D on the goods for credit-card-debt market. In terms of the credit card market, we will have the sellers providing the credit to the buyers (as well as buyers keeping some balances as credit provision to other buyers): net credit card debt in the economy is zero. Except for allowing for a difference between the credit card interest x_t paid by buyers and \hat{x}_t received by sellers, we assume that this market for credit card debt functions without friction.

The period has a “morning”, where the asset markets operate, and an “afternoon”, where the goods market operates.

As for the *morning*, the household starts the period with total nominal wealth W_t . It then decides how to split W_t into bond and cash holdings

$$B_t + M_t = W_t \tag{C.26}$$

with $M_t \geq 0$, $B_t \in \mathbb{R}$. Note that the household does not buy or sell credit card debt at this point: this all takes place on the spot later in the goods markets. Later we will clarify, that it is the buyer who decides on the splitting of wealth and therefore how much money to bring to the goods market. We will circle back to this equation at the end of this “accounting exercise”, see equation (C.36).

As for the *afternoon*, the following timing applies.

1. The buyer brings M_t units of cash to buy goods with cash. Sellers sell one unit of the consumption good for P_t units of cash. Thus, the buyer spends

$$0 \leq \tilde{M}_t = P_t c_t^M \tag{C.27}$$

units of cash to acquire the consumption goods c_t^M . The cash-in-advance constraint reads

$$\tilde{M}_t \leq M_t \tag{C.28}$$

or, combined,

$$P_t c_t^M \leq M_t \tag{C.29}$$

Exiting the goods market, the buyer holds $M_t - \tilde{M}_t$ cash, to be kept “under the mattress” until the beginning of period $t + 1$, and c_t^M units of the “cash good”.

2. The buyer charges debt D_t units of cash to his credit card when paying for consumption goods c_t^D ,

$$P_t^D c_t^D = D_t \quad (\text{C.30})$$

Note, in contrast to money and equation (C.28), there is no ‘credit-in-advance constraint’ that would require positive credit balances upfront. Exiting the goods market, the buyer owes the credit card debt D_t which carries the interest rate x_t and has consumption goods c_t^D .

3. The seller obtains

$$\hat{M}_t = P_t c_t^M \quad (\text{C.31})$$

units of cash for the goods c_t^M sold against cash and keep the cash “under the mattress” until the beginning of period $t + 1$.

4. Likewise the seller obtains

$$\hat{D}_t = P_t^D c_t^D \quad (\text{C.32})$$

as positive credit card balances where credit carries an interest rate \hat{x}_t or claims on borrowing buyers.

5. Prices adjust to clear goods markets

$$y_t = c_t^M + c_t^D \quad (\text{C.33})$$

6. At the end of the period, the household consumes the purchased consumption goods c_t^M and c_t^D , enjoying utility $u(c_t^M, c_t^D)$ or, in case of perfect substitutes, $u(c_t)$, where $c_t = c_t^M + c_t^D$. The four members of the household carry the cash balances and credit card balances forward to the next period.

Then, period t ends. At the beginning of period $t + 1$, the household tallies total wealth W_{t+1} . It consists of the following parts:

1. The buyer brings unspent cash balance $M_t - \tilde{M}_t$. This balance is zero, if (C.29) holds with equality.

2. The buyer carries a credit card debt of D_t , on which interest x_t needs to be repaid: the total repayment is $(1 + x_t)D_t$.
3. The seller brings \hat{M}_t units of cash.
4. The seller holds a claim of $(1 + \hat{x}_t)\hat{D}_t$.
5. The household will have earned principal and interest $(1 + i_t)B_t$ on bonds B_t purchased in the previous period t .

The household therefore carries total cash holdings

$$(M_t - \tilde{M}_t) + \hat{M}_t = M_t \quad (\text{C.34})$$

into period $t+1$, since $\hat{M}_t = \tilde{M}_t$ in equilibrium. Likewise, the household carries a total credit card balance (debt when positive, credit when negative), including interest,

$$(1 + x_t)D_t - (1 + \hat{x}_t)\hat{D}_t = (x_t - \hat{x}_t)D_t \quad (\text{C.35})$$

into $t + 1$ (before rebalancing the portfolio), since $D_t = \hat{D}_t$. The household also receives principal and interest on bond investments totalling $(1 + i_t)B_t$ in nominal terms. Finally, the household receives (and takes as given) transfers T_t , which include the per-household difference between credit card interests paid by buyers and received by sellers and the government's transfers.

The total nominal wealth W_{t+1} at the beginning of period $t + 1$, therefore, equals

$$W_{t+1} = M_t - (x_t - \hat{x}_t)D_t + (1 + i_t)B_t + T_t.$$

With this wealth, the morning of $t + 1$ starts. The household buys bonds and keeps the rest in cash,

$$B_{t+1} + M_{t+1} = W_{t+1}. \quad (\text{C.36})$$

C.1 Analysis

We seek to understand the solution to the maximization problem of the household, taking prices and interest rates as given. To keep matters simple, let us assume that we have perfect substitutability between the cash and the credit good, i.e. that the

preferences of the household are given by

$$U = \sum_{t=0}^{\infty} u(c_t) \quad (\text{C.37})$$

where

$$c_t = c_t^M + c_t^D \quad (\text{C.38})$$

To understand the optimization problem of the household we have to distinguish between the problem of the buyer and that of the seller. Let us start with the buyer.

The buyer neither internalize that ultimately, his/her money holdings go to the household, through the seller, nor that his/her credit card debt is also the household's credit, through the seller. Therefore, the household's wealth from his point of view is

$$W_{t+1}^b = M_t - P_t c_t^M + \hat{M}_t - (1 + x_t)D_t + (1 + \hat{x}_t)\hat{D}_t + (1 + i_t)B_t + T_t$$

in which we can substitute the constraint $D_t = P_t^D c_t^D$ for D_t .

The buyer maximizes his indirect utility $V(W_t^b)$, given by

$$V(W_t^b) = u(c_t) + \beta V(W_{t+1}^b),$$

by choosing B_t , M_t , c_t^M and c_t^D subject to the following constraints.

$$M_t + B_t = W_t^b,$$

$$M_t \geq P_t c_t^M, \quad (\text{C.39})$$

$$W_{t+1}^b = M_t - P_t c_t^M + \hat{M}_t - (1 + x_t)P_t^D c_t^D + (1 + \hat{x}_t)\hat{D}_t + (1 + i_t)B_t + T_t.$$

Defining the marginal utility of nominal wealth as $\lambda_t \equiv V_w(W_t^b)$ and attaching the

Lagrange multipliers μ_t to (C.39), we obtain the first-order conditions

$$\left(\frac{\partial}{\partial c_t^M} : \right) \quad u'(c_t) = (\mu_t + \beta\lambda_{t+1})P_t, \quad (\text{C.40})$$

$$\left(\frac{\partial}{\partial c_t^D} : \right) \quad u'(c_t) = \lambda_{t+1}\beta(1 + x_t)P_t^D, \quad (\text{C.41})$$

$$\left(\frac{\partial}{\partial M_t} : \right) \quad \lambda_t = \mu_t + \beta\lambda_{t+1}, \quad (\text{C.42})$$

$$\left(\frac{\partial}{\partial B_t} : \right) \quad \lambda_t = \beta\lambda_{t+1}(1 + i_t). \quad (\text{C.43})$$

assuming existence of an interior solution. The second equation is novel and says that the marginal utility benefit of consuming an extra unit obtained by paying with the credit card equals the discounted costs of paying for that debt in the next period valued through the marginal utility of next-period wealth. Note that $\mu_t \geq 0$ implies that $i_t \geq 0$ by comparing (C.42) and (C.43). We can further combine (C.40) and (C.42) to obtain $\lambda_t = u'(c_t)/P_t$. Using it together, plugging into (C.41) and comparing with (C.43), we obtain

$$(1 + x_t)\frac{P_t^D}{P_t} = 1 + i_t. \quad (\text{C.44})$$

Further, note that, given the zero-lower bound on the nominal interest rate, it should be that

$$(1 + x_t)\frac{P_t^D}{P_t} \geq 1.$$

Let us look at the problem of the seller. From her point of view, wealth at time $t + 1$ is given by

$$W_{t+1}^s = M_t - \tilde{M}_t + P_t c_t^M - (1 + x_t)D_t + (1 + \hat{x}_t)P_t^D c_t^D + (1 + i_t)B_t + T_t.$$

A unit of the good sold for cash contributes P_t units of cash to nominal wealth W_{t+1}^s . A unit of the good sold on credit card debt contributes $(1 + \hat{x}_t)P_t^D$. Assuming that both markets operate, i.e. that equilibrium prices and interest rates are such so as to keep the seller indifferent, it follows that

$$P_t = (1 + \hat{x}_t)P_t^D. \quad (\text{C.45})$$

Implicit is here the assumption that there is no default on credit card debt. If $\hat{x}_t = 0$, then the prices of the two goods should be the same from the point of view of the seller.

Combining (C.45) with (C.44), we obtain

$$1 + x_t = (1 + i_t)(1 + \hat{x}_t) \quad (\text{C.46})$$

which is essentially an indifference condition stemming from the competition between investment in credit card debt versus the bond.

Note that even in a version without bonds, we obtain interesting implications. Assume the morning and afternoon are integrated and that money is in positive net supply, i.e. (C.42) holds with equality. Only cash and credit cards are used for goods purchases and savings. In such a model, we still have the first order conditions (C.40)-(C.42) but not (C.43). Plugging (C.42) into the condition (C.40), then rolling that constraint forward by one period and plugging it into the second constraint (C.41), we obtain

$$1 = \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{P_t^D}{P_{t+1}} (1 + x_t). \quad (\text{C.47})$$

The equation says if an agent has an extra endowment tomorrow, then the marginal utility of consuming already today by buying goods at price P_t^D via credit at $1 + x_t$ must equal the marginal utility of consumption tomorrow, when using cash instead at price P_{t+1} . With (C.45), we then have

$$\frac{1 + \hat{x}_t}{1 + x_t} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{P_t}{P_{t+1}} = \mathcal{M}_{t+1}. \quad (\text{C.48})$$

These two equations hold regardless of whether there are bonds or not. Note that for $x_t = \hat{x}_t$ we obtain $\mathcal{M}_{t+1} = 1$. That is, the liquidity services of holding money are valued at zero, $L_t = 0$. The latter is visible in constraint (C.42), which implies $L_t = \mu_t/\lambda_t$ for liquidity services on money.

Equation (C.46) together with (C.45) provides some interesting conclusions concerning credit card payments. First, every credit card interest rates x_t and \hat{x}_t are compatible with equilibrium as long as they satisfy relationship (C.46), and $x_t \geq \hat{x}_t$.

From a seller's perspective, higher interest rates \hat{x}_t earned on extended credit are offset by lower prices P_t^D charged, compared to P_t , see (C.45). From a buyer's perspective and with (C.46), purchases with a credit card together with the interest-implied discount of (C.45) leave the buyer indifferent between using credit cards or cash. In general, there are two cases of special interest.

First Case (credit cards are the efficient means of payment): For the first case, suppose that the credit card debt market functions under free competition and therefore that no profits are left in the market, i.e. that $x_t = \hat{x}_t$. In this case, (C.46) implies that $i_t = 0$ and the liquidity constraint is completely relaxed. Although borrowing through the credit card carries an interest rate x_t , the effective cost of borrowing is zero since competition in the credit-card market lowers the price of the credit-card good to the point that P_t^D/P_t completely offsets the interest rate paid on the credit card. Buyers are effectively borrowing at zero cost and therefore they will use the credit card up to the point of satiating their liquidity needs. The credit card interest rate $x_t = \hat{x}_t$ can still be arbitrary, as it is offset by the appropriate pricing (C.45). If one additionally imposes $P_t = P_t^D$, then $x_t = \hat{x}_t = i_t = 0$.

Second Case (credit cards are like cash): For the second case, suppose that $\hat{x}_t = 0$, i.e. the seller does not earn interest on credit card debt extended to buyers. In this case, there are intermediation profits in the credit-card market. It follows that $x_t = i_t$ and $P_t = P_t^D$, via equation (C.46). This is akin to the buyer borrowing via his credit card to obtain cash and then giving that cash to the seller to hold until the next period.

C.2 Credit Cards: a global currency?

Armed with the structure above, we can now make progress on the question as to whether credit cards might constitute a global currency. Consider a two-country version of the model sketched above, using the notation of our main analysis. Consider the first case: If the credit card debt market functions efficiently and there is no credit card default risk, then $x_t = \hat{x}_t$ and the nominal interest rates in both countries must be identical $i_t = i_t^* = 0$. Recall that the use of a global currency, in our framework, implied the equalization of interest rates but not necessarily a zero nominal interest

rate. A global currency created competition among local monies by equalizing their returns. Things are different with credit cards. In a frictionless credit-card market, a credit card, even when denominated in the same units of domestic money, allows the relaxation of the liquidity constraint in each country. It then happens that interest rates are zero in each country and therefore equal across countries. This result does, however, not occur because of currency competition but because of the efficiency brought about by credit cards in the national payment system. In this case, the cost of borrowing through the credit card, x_t , is completely offset by the lower price P_t^D that the buyer faces when paying with the credit card. Therefore, the effective cost of borrowing is zero and the buyer can fully relax his cash constraint.

Consider, thus, the second special case, where sellers do not earn interest on credit card debt extended to buyers, and where, therefore, credit card and cash prices are the same, $P_t = P_t^D$. If domestic buyers only purchase goods at home, using their credit cards and being charged i_t , and foreign buyers only purchase goods in the foreign country, using their credit cards and being charged i_t^* , there is no pressure for these interest rates to become equal. The credit card, here, is a perfect substitute for domestic money. Our analysis would then go through as in the main text, since a global currency would be needed to imply the equalization of interest rates across countries. What can make a credit card act like a global currency, in this framework, is the possibility of the domestic household to attach a credit card to an account denominated in units of foreign currency and to be able to purchase goods with credit cards in units of two different currencies. This would create competition between currencies and produce the same effects as the global currency in our framework. In practice, this solution requires households to have accounts denominated in different currencies, which is not always feasible in all countries. Moreover, competition should be created among all possible pairs of currencies. To the contrary, a global currency which could be easily adopted in the world economy may achieve the same outcome in a more direct and straightforward way.