A Dual Approach to Ambiguity Aversion

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Policyholder Behavior in Insurance Decision Making
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Outline of the presentation

- Ellsberg's expermiment and ambiguity aversion
- Usual ambiguity models
- A thought experiment
- Dual ambiguity models
  - Foundation
  - Main features
  - Intuition
- Applications
  - Saving behavior
  - Portfolio choice

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- Results show that most people strictly prefer to bet on "red" than to bet on "black" or on "white".
- Interpretation: agents do not know like bets for which they do not precisely know the probability of winning.

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  - A strong heritage from Anscombe Aumann (1963) who provided an axiomatization of Subjective Expected Utility.
  - All models keep Anscombe Aumann monotonicity assumption as a basic principle of rationality.

$$U(l_1, ..., l_N) = I(u(l_1), ..., u(l_N))$$

RISK REDUCTION $\longrightarrow$

AMBIGUITY REDUCTION $\downarrow$

Ambiguous act

$L = (l_1, ..., l_N)$ $\longrightarrow$ Ambiguous utility distribution $(u(l_1), ..., u(l_N))$

$\nearrow$ AA-monotone models $\nearrow$

utility: $U_{\text{AA-mon}}(L) = I(u(l_1), ..., u(l_N))$

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- A very natural and logical assumption if accepting the principle of "separability of preferences across mutually exclusive events".
- From **Machina (2009)**

> *If there is a general lesson to be learned from Ellsberg´s examples and the examples here, it is that the phenomenon of ambiguity aversion is intrinsically one of non-separable preferences across mutually exclusive events, and that* **models that exhibit full, or even partial, event-separability cannot capture all aspects of this phenomenon.**

A thought experiment

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- **An urn** C obtained by putting all the balls of A and B into a single urn.
- The decision maker has to choose an urn. A single ball is drawn from that urn, and the DM is payed the number written on the drawn ball.

A thought experiment-(cont'd)

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- Urn A pays 100 with $p = \frac{1}{2}$ and 300 with $p = \frac{1}{2}$.
- Urn B pays 150 with $p = \frac{1}{2}$ and 240 with $p = \frac{1}{2}$.

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  - 300 with $p = \frac{n_A}{2(n_A+n_B)}$
- **Choosing urn A (or choosing urn B) generates a constant act, while choosing C generates an uncertain act.**

A thought experiment-(cont'd)

- The ambiguity models of Schmeidler (1989), Gilboa and Schmeidler (1989), Klibanoff, Marinacci and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), Siniscalchi (2009), Segal (1987), and all MBA preferences of Cerreia Vioglio & al (2011) predict that

$$A \sim B \Rightarrow A \sim B \sim C$$

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  whatever the agents's degree of ambiguity aversion.
- This is because these models uses the horse-roulette setting of Anscombe Aumann (1963) and keep **Anscombe-Aumann monotonicity assumption as a basic principle of rationality.**

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- **Assumptions**
  1. Completeness, transitivity, continuity
  2. **Monotonicity with respect to first-order stochastic dominance:**

$$l_i \text{ FSD } m_i \text{ for all } i \Rightarrow (l_1, ..., l_N) \succeq (m_1, ..., m_N)$$

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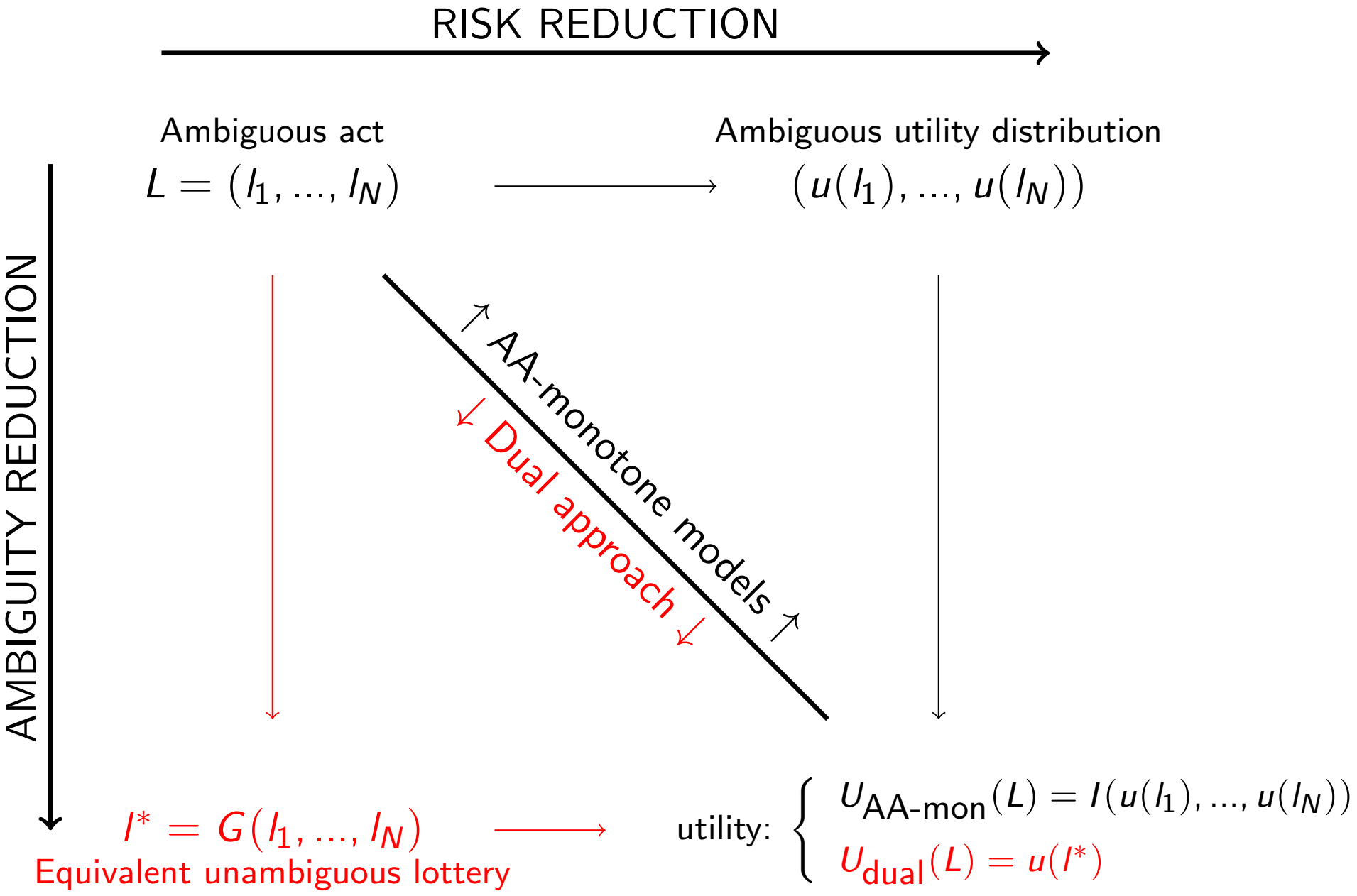
  3. Comonotonic sure-thing principle
- **Representation result** A model where:
  1. **in a first stage** ambiguous beliefs are aggregated to form an "equivalent unambiguous belief"
  2. in the **second stage**, this "equivalent unambiguous belief" is plugged into (rank dependent) risk preferences to obtain a final evaluation.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AA-monotone models
Dual approach
$l^* = G(l_1, ..., l_N)$
Equivalent unambiguous lottery
utility:
$U_{\text{AA-mon}}(L) = I(u(l_1), ..., u(l_N))$
$U_{\text{dual}}(L) = u(l^*)$

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- **As effective as AA-monotone models to explain Ellsberg paradox:** the dual approach is in fact indistinguishable from the traditional approach when the set of consequences contains only two elements
- **Tractability** Extremely simple to use in applications: An increase in ambiguity aversion is equivalent to using more pessimistic beliefs in the sense of first-order stochastic dominance

Intuition: representing acts as **matrices**

$$
\begin{array}{cc}
\begin{array}{ccccc} l_1 & \cdots & l_i & \cdots & l_N \end{array} & \leftarrow \textit{lotteries} \\
 & \textit{outcomes} \downarrow \\
L = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P(l_1 \geq x_j) & \cdots & P(l_i \geq x_j) & \cdots & P(l_N \geq x_j) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P(l_1 \geq x_n) & \cdots & P(l_i \geq x_n) & \cdots & P(l_N \geq x_n) \end{pmatrix} & \begin{array}{c} x_1 \\ \cdots \\ x_j \\ \cdots \\ x_n \end{array}
\end{array}
$$

It is here assumed that $x_n \geq ... \geq x_1$ and $supp(L) \subset \{x_1, ..., x_n\}$
The column i of the matrix provides the decumulative distribution function of lottery l_i , obtained in state s_i.

Intuition: monotonicity with respect to *FSD*

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Monotonicity with respect to *FSD* $\Leftrightarrow$ componentwise monotonicity

Intuition: AA-monotone vs. Dual Approach

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- **AA Monotonicity $\Leftrightarrow$ separability across columns**
- **Dual approach $\Leftrightarrow$ separability across (ordered) rows** (technically obtained from the comonotonic sure-thing principle).

Intuition: "aggregating probabilities, rather than utilities"

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  In the EU framework:

$$
\begin{aligned}
V(l) &= \textstyle\sum_{j=1}^{n} Prob(l = x_j)u(x_j) && \rightarrow \text{aggregation of utilities} \\
&= \textstyle\sum_{j=1}^{n-1} \sigma_j Prob(l \geq x_j) && \rightarrow \text{aggregation of probabilities} \\
&= Prob(l \geq \textstyle\sum_{j=1}^{n} \sigma_j \delta_{x_j}) && \rightarrow \text{comparison with a benchmark}
\end{aligned}
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(assuming $u(x_1) = 0$, $u(x_n) = 1$, and $\sigma_j = u(x_j) - u(x_{j-1})$).

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  (assuming $u(x_1) = 0$, $u(x_n) = 1$, and $\sigma_j = u(x_j) - u(x_{j-1})$).
- In the EU framework, the utility of a lottery is given by the probability to be above the benchmark.
- Agent's risk preferences are embedded in the distribution of the benchmark.

Interpretation: two-stage aggregation of probabilities

$$
\begin{array}{cccccc}
l_1 & \cdots & l_i & \cdots & l_N & \leftarrow \textit{lotteries} \\
 & & & & & \textit{benchmark's} \\
 & & & & & \textit{realizations} \downarrow \\
\end{array}
$$

$$
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1 & 1 & 1 & 1 & 1 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
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- **AA Monotonicity $\Leftrightarrow$ aggregating probabilities across benchmark's realizations and then across states**
- **Dual approach $\Leftrightarrow$ aggregating probabilities across states and then across benchmark' s realizations**

Interpretation: A version for economists....

$$
\begin{array}{cc}
\begin{array}{ccccc} l_1 & \cdots & l_i & \cdots & l_N \end{array} & \begin{array}{r} \leftarrow \textit{referees} \\ \textit{editor's} \\ \textit{acceptance} \\ \textit{bar} \downarrow \end{array} \\
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1 & 1 & 1 & 1 & 1 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
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- **AA Monotonicity $\Leftrightarrow$ aggregating acceptance probabilities across editor's bars and then across referees**
- **Dual approach $\Leftrightarrow$ aggregating acceptance probabilities across referees and then across editor's bars**

MBA preferences and their dual

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The dual version is

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$$U^{MBA}_{dual}(L) = \sum_{i=1}^{n} \sigma_j I\left(Prob(L \geq x_j)\right)$$

Remark that if I is concave, then $U^{MBA}_{dual}(L) \leq U^{MBA}(L)$ for all L, with equality when L is a constant act.

Example 1 : the SEU model and its dual

SEU model:

$$U^{\text{SEU}}(L) = \sum_{i=1}^{N} \pi_i \left(\sum_{j=1}^{n} \sigma_j Prob(l_i \geq x_j) \right)$$

Dual version:

$$U^{\text{SEU}}_{\text{dual}}(L) = \sum_{j=1}^{n} \sigma_j \left(\sum_{i=1}^{N} \pi_i Prob(l_i \geq x_j) \right)$$

$\rightarrow$AA-monotone and dual versions of the SEU model are identical.

Example 2: the max-min model and its dual

max-min model (Gilboa and Schmeidler, 1989):

$$U^{\text{MEU}}(L) = \min_{(\pi_i)\in\Pi} \left(\sum_{i=1}^{N} \pi_i \left(\sum_{j=1}^{n} \sigma_j Prob(l_i \geq x_j) \right) \right)$$

Dual version:

$$U^{\text{MEU}}_{\text{dual}}(L) = \sum_{j=1}^{n} \sigma_j \left(\min_{(\pi_i)\in\Pi} \sum_{i=1}^{N} \pi_i Prob(l_i \geq x_j) \right)$$

Example 3 : The smooth ambiguity model and its dual

Smooth model (Klibanoff, Marinacci and Mukerji,2005):

$$U^{\mathrm{KMM}}(L) = \phi^{-1}\left(E_{(\pi_i)\in\Pi}\left[\phi\left(\sum_{i=1}^{N}\pi_i\left(\sum_{j=i}^{n}\sigma_j Prob(l_i \geq x_j)\right)\right)\right]\right)$$

Dual version of the smooth model:

$$U^{\mathrm{KMM}}_{\mathrm{dual}}(L) = \sum_{j=1}^{n}\sigma_j\phi^{-1}\left(E_{(\pi_i)\in\Pi}\left[\phi\left(\sum_{i=1}^{N}\pi_i Prob(l_i \geq x_j)\right)\right]\right)$$

This includes the EUUP specification used in Izhakian (2015).

Application to saving behavior

An agent has to decide about how much to save.

- First period income (y_1) and interest rate (r) are known.
- Second period income is uncertain and ambiguous. More precisely, if the state of the world $s_i \in \{s_1, ..., s_N\}$ realizes, the second period income is distributed as $\tilde{y}_2^i$
- We assume EU additively separable risk preferences, and level independent ambiguity aversion.

Agent's problem is to find:

$$s^* = \arg\max_{s \in \mathbb{R}} U((y_1 - s, \tilde{y}_2^1 + rs), ..., (y_1 - s, \tilde{y}_2^N + rs))$$

Application to saving behavior: results

- An increase of ambiguity aversion (holding risk preferences fixed) generate an increase in savings
  ($\rightarrow$ **Ambiguity aversion increases precautionary savings**)

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- An increase of ambiguity aversion (holding risk preferences fixed) generate an increase in savings
  ($\rightarrow$ **Ambiguity aversion increases precautionary savings**)
- **Comparison with AA-monotone models**. In order to derive a similar result, Osaki and Schlesinger (2014) assume:
  - KMM preferences with constant absolute ambiguity aversion
  - Beliefs about second period income (the $\tilde{y}_2^i$) that can be ranked in terms of second order stochastic dominance.

Application to Portfolio Choice

An agent has to allocate her wealth W_0/R between:

- A risk-free asset, whose return is equal to $R > 0$
- A risky asset, whose return is uncertain and ambiguous. More precisely, if the state of the world $s_i \in \{s_1, ..., s_N\}$ realizes, the return of that second asset is $R + \tilde{R}_i$
- We assume:

$$U(L) = \sum_{x \in S(L)} u(x) \left(I(\text{Prob}(L \geq x)) - I(\text{Prob}(L > x)) \right)$$

(thus EU risk preferences, and level independent ambiguity aversion).

Agent's problem is to find:

$$\alpha = \arg\max_{\alpha \in \mathbb{R}} U((W_0 + \alpha \tilde{R}_1), ..., (W_0 + \alpha \tilde{R}_N))$$

Application to Portfolio Choice: Result

1. **Stock market participation**
   Define

$$\begin{aligned} Prob(\widetilde{R}_- &\geq x) = I(Prob(\widetilde{R}_1 \geq x), ..., Prob(\widetilde{R}_N \geq x)) \\ Prob(\widetilde{R}_+ &\geq x) = 1 - I(Prob(\widetilde{R}_1 < x), ..., Prob(\widetilde{R}_N < x)) \end{aligned}$$

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   $\rightarrow$ Result similar to that of Gollier (2011) **BUT** obtained:
   - without assuming that the $\widetilde{R}_i$ are comparable in terms of first- or second-order stochastic dominance
   - without assuming a particular form the certainty equivalent function I

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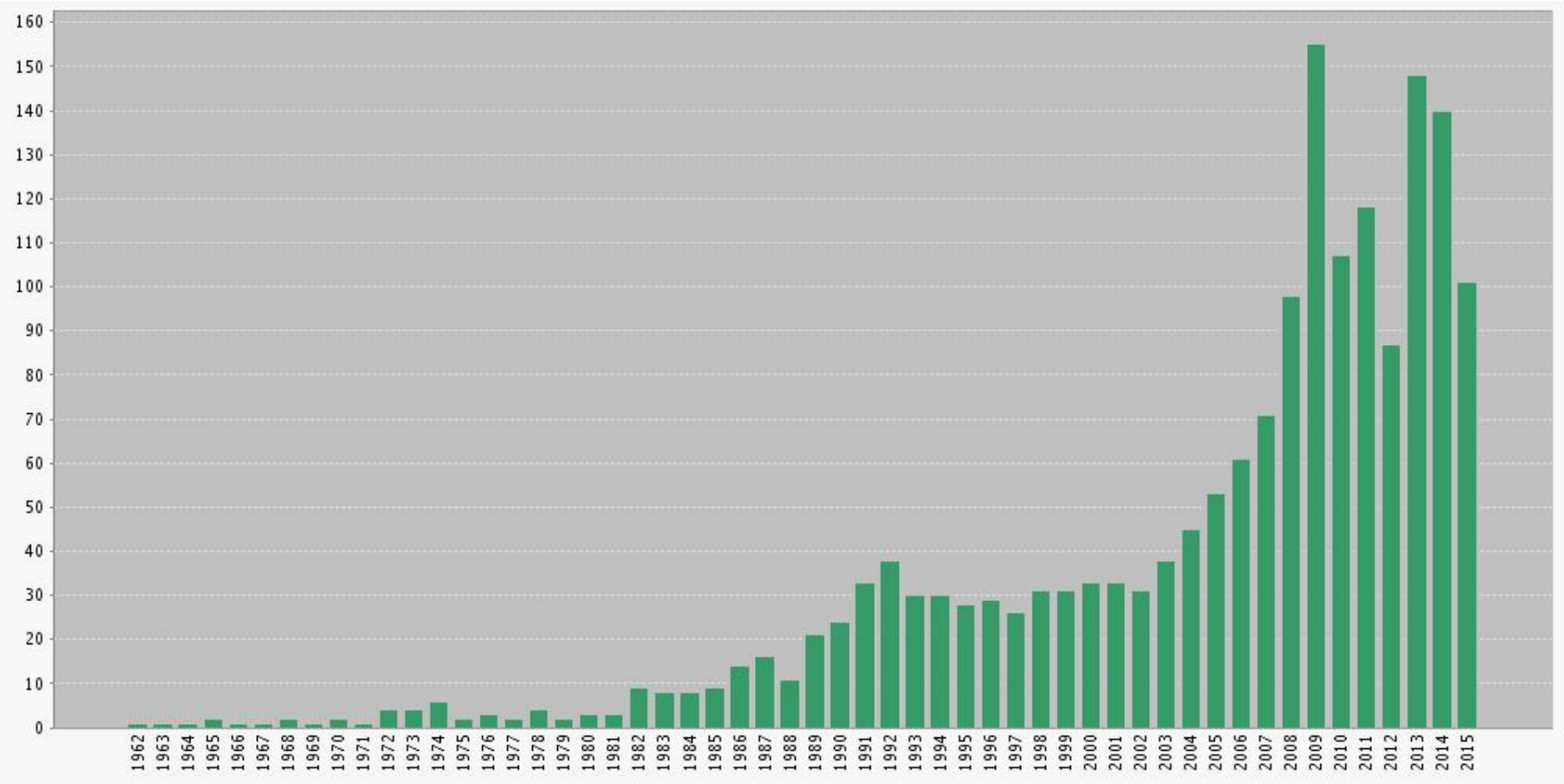
Number of articles citing Ellsberg (1961)



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- A very particular approach however. The current paper provides another ambiguity model, just as good as standard models to explain Ellsberg's Paradox, with quite distinct predictions.
- Some common features however: ambiguity aversion resembles a form of pessimism. But which specific form?

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- Answers will be collected during coffee break. Late electronic submissions are welcome (abommier@ethz.ch).

THANK YOU !