A Theory of Bounded Utility for Insurance: Experimental Evidence and Implications for China's Personal Property Market

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Confluence of Two Research Interests

Heavy-Tailed Insurance Loss Distributions

- Identify appropriate risk measures (Powers and Zanjani, 2013, *ARFE*; Powers and Powers, 2014, *SAJ*)
- Study impact on transfer/diversification alternatives (Powers, Powers, and Gao, 2012, *RA*)
- Apply expected-utility theory (Powers, 2015, *Risks*)

Cultural Attitudes and Insurance Demand (Supported by AXA Research Fund)

- Explain limited Chinese property insurance market
- Explore preferences for transfer/diversification in China

Current Research

Assume:

- Insurance buyers accurately modeled by expected-utility theory
- If insurance losses can have infinite means, then buyers' utility functions must be bounded, sigmoid ("S-shaped"; follows from St. Petersburg Two-Envelope Paradox)
- Infinite-mean models appropriate for some insurance losses

Note:

- Bounded, sigmoid utility functions not well studied in insurance
- Conventional results concerning full insurance (Mossin, 1968) and deductibles (Arrow, 1971) do not apply

Results:

- Experiments using bounded, sigmoid utility functions suggest IARA for Chinese decision makers
- Reasonable to anticipate growth of personal property insurance market as wealth increases

Part I: Experiments with Bounded, Sigmoid Utility

Model A: 2-Parameter Double-Exponential Utility

Let

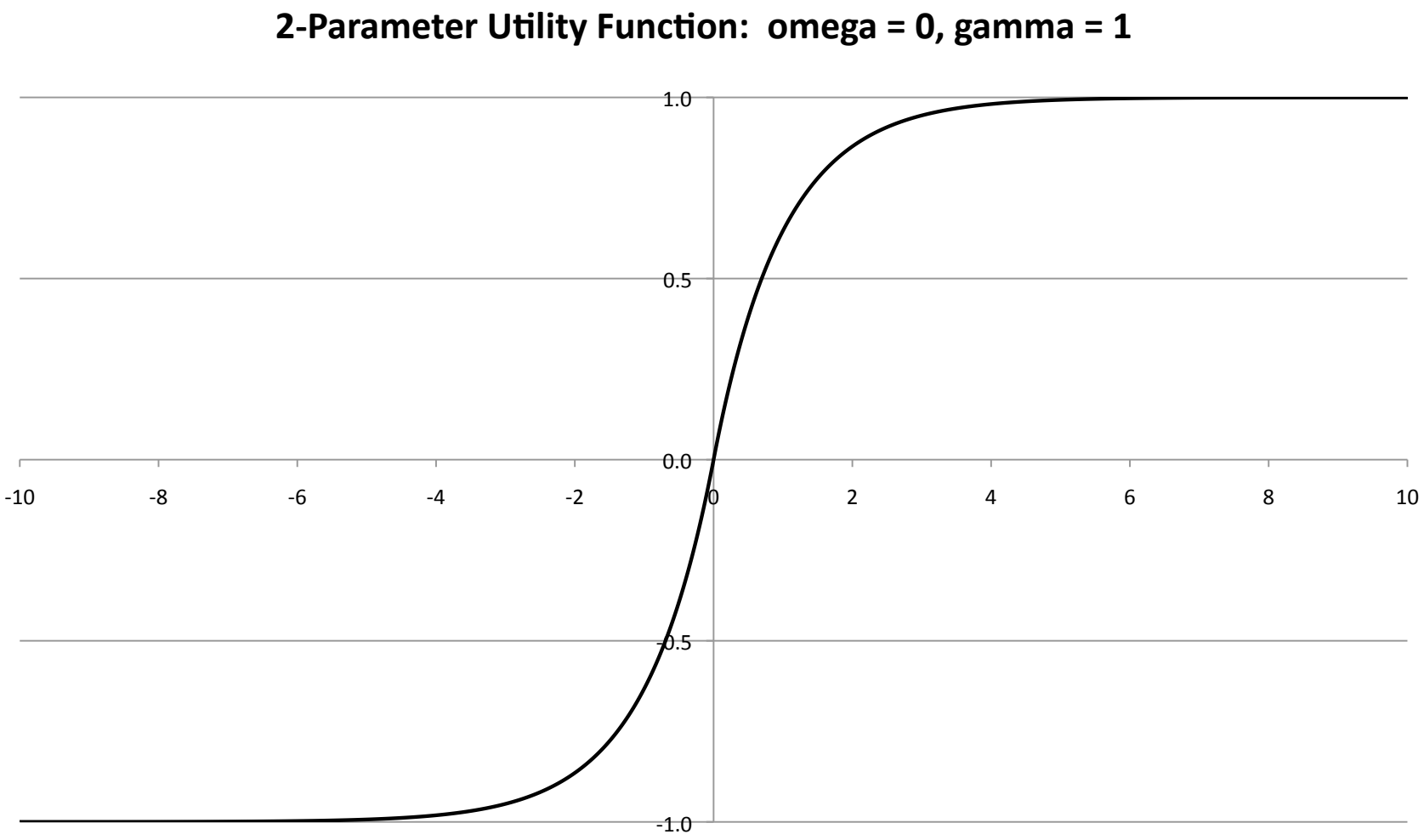
$$u(w) = \begin{cases} 1 - e^{-\gamma(w-\omega)}, & w \geq \omega \\ -1 + e^{\gamma(w-\omega)}, & w < \omega \end{cases}$$

where:

- $\omega \in (-\infty, \infty)$ denotes "reference" (inflection) point;
- $\gamma > 0$ denotes absolute risk-aversion coefficient for $w \geq \omega$; and
- $\gamma > 0$ denotes absolute risk-proneness coefficient for $w < \omega$

Both $u(w)$ and $u'(w)$ continuous at $w = \omega$

Parameter Effects: More Vulnerable, More Risk Sensitive



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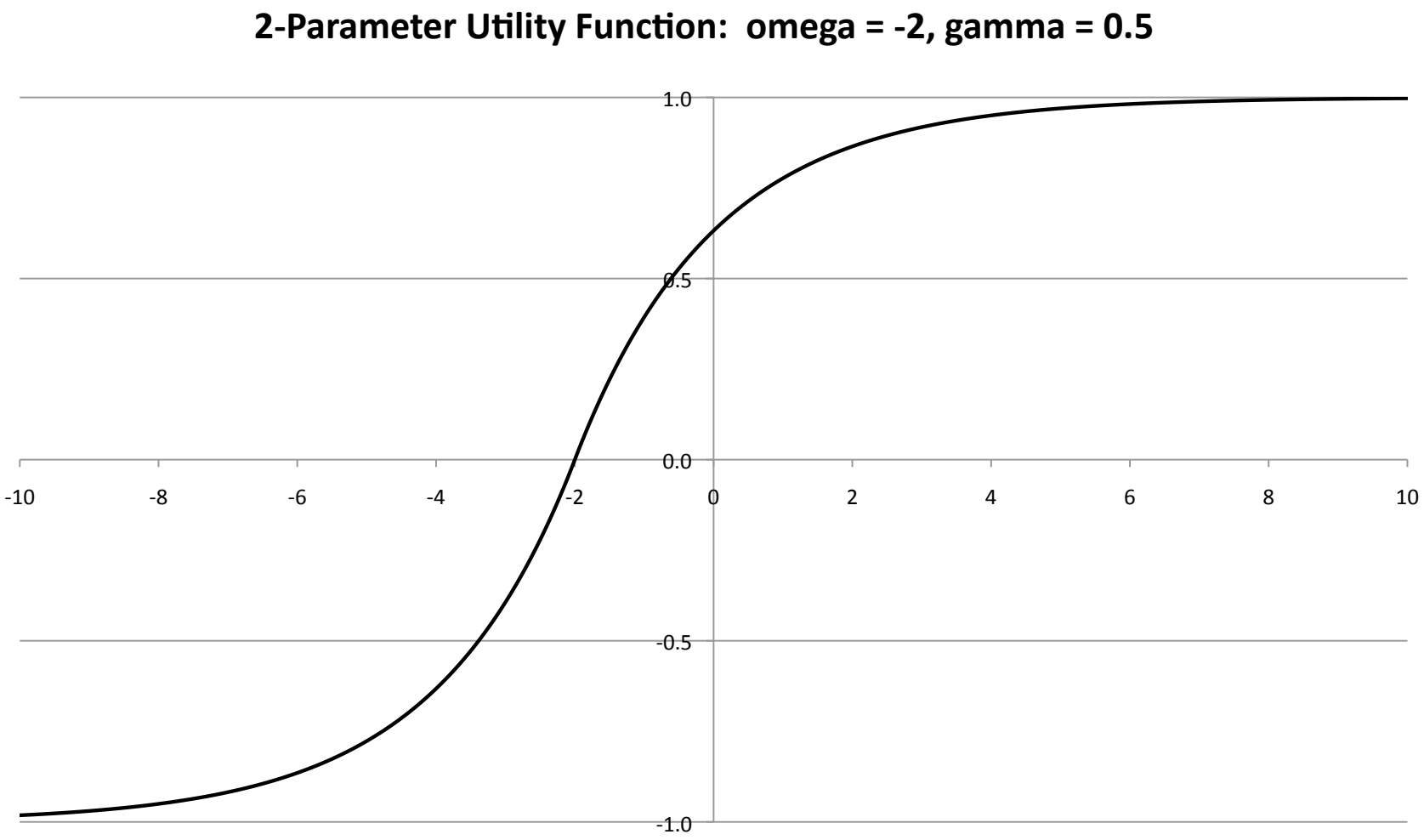


Model A: Experimental Results

Experiment involved 109 Tsinghua University student subjects

Students completed questionnaire to elicit 21 points along utility function (using certainty-equivalence chain method; Torgerson, 1958)

Individual 2-parameter double-exponential model fit to each student i to obtain $\hat{\omega}_i$ and $\hat{\gamma}_i$

101 of 109 regressions yielded significant F statistic at $\alpha = 0.10$

Model A: Experimental Results (Continued)

Covariates included gender, age, business major, master's program, doctoral program, financial-aid status, part-time job status, and home province

Parameter estimates $\hat{\omega}_i$ and $\hat{\gamma}_i$ separately regressed against covariates using multiple linear regression

Significant effects –

- $\hat{\omega}_i$: age (+), master's program (-), doctoral program (-), southwestern region (+)
- $\hat{\gamma}_i$: age (+), master's program (-), doctoral program (-), northern region (+)

Model B: 3-Parameter Double-Exponential Utility

Let

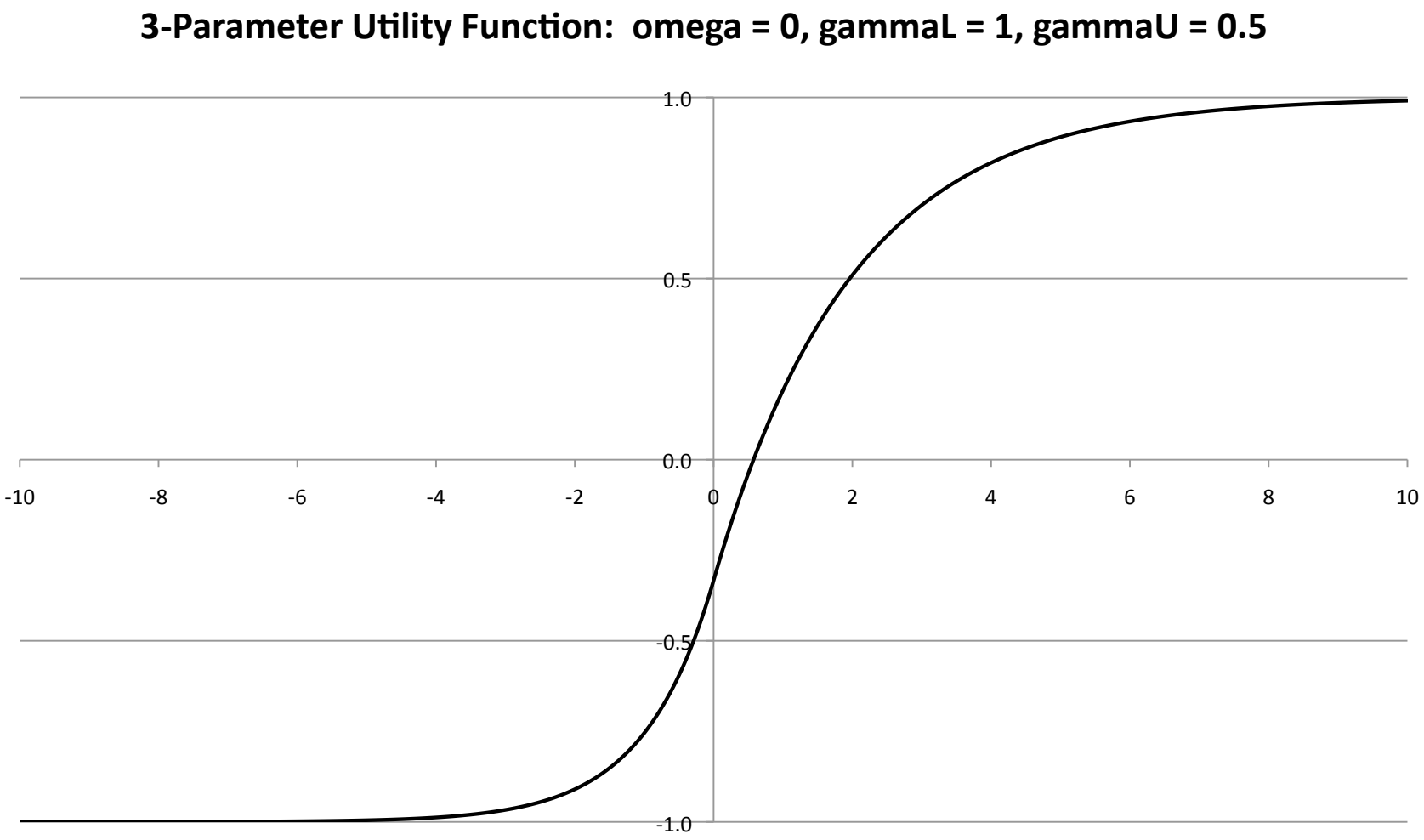
$$u(w) = \begin{cases} 1 - \left(\dfrac{2}{1+\gamma_{\mathrm{U}}/\gamma_{\mathrm{L}}}\right) \exp\left(-\gamma_{\mathrm{U}}\left(w-\omega\right)\right), & w \geq \omega \\ -1 + \left(\dfrac{2}{1+\gamma_{\mathrm{L}}/\gamma_{\mathrm{U}}}\right) \exp\left(\gamma_{\mathrm{L}}\left(w-\omega\right)\right), & w < \omega \end{cases}$$

where:

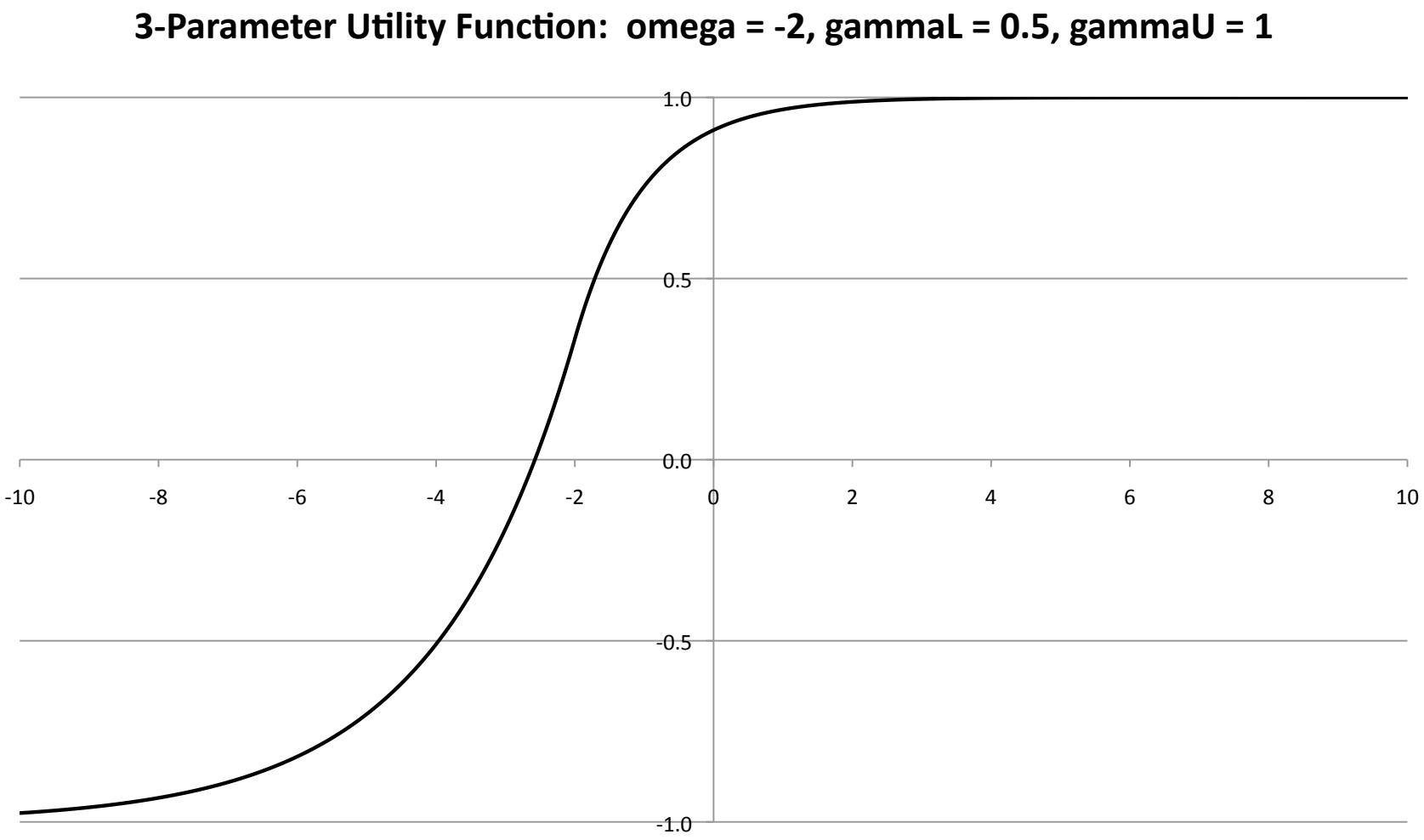
- $\omega \in (-\infty, \infty)$ denotes "reference" (inflection) point;
- $\gamma_{\mathrm{U}} > 0$ denotes absolute risk-aversion coefficient for $w \geq \omega$; and
- $\gamma_{\mathrm{L}} > 0$ denotes absolute risk-proneness coefficient for $w < \omega$

Both $u(w)$ and $u'(w)$ continuous at $w = \omega$

Parameter Effects: More Vulnerable, More Loss Averse



Parameter Effects: Less Vulnerable, Less Loss Averse



Model B: Experimental Results

Experiment involved 84 Tsinghua University student subjects

Students completed questionnaire to elicit 21 points along utility function (using certainty-equivalence chain method)

Individual 3-parameter double-exponential model fit to each student i to obtain $\hat{\omega}_i$, $\hat{\gamma}_{\mathrm{L},i}$, and $\hat{\gamma}_{\mathrm{U},i}$

84 of 84 regressions yielded significant F statistic at $\alpha = 0.10$

Model B: Experimental Results (Continued)

Covariates included gender, age, business major, years driving, family income, financial-aid status, part-time job status, and urban status

Parameter estimates $\hat{\omega}_i$, $\hat{\gamma}_{\mathrm{L},i}$, and $\hat{\gamma}_{\mathrm{U},i}$ separately regressed against covariates using multiple linear regression

Significant effects –

- $\hat{\omega}_i$: none [not significantly different from 0 for most students]
- $\hat{\gamma}_{\mathrm{U},i}$: family income (+), urban status (+), family income $\times$ urban status (-) [family income not significant for urban students]
- $\hat{\gamma}_{\mathrm{L},i}$: family income (+)
- $\hat{\gamma}_{\mathrm{L},i}/\hat{\gamma}_{\mathrm{U},i}$: family income (+) [test of loss aversion]

Part I Conclusions

Greater financial security ("wealth") appears to cause greater sensitivity to both risk and loss

Possibly explained by "cushion hypothesis" of communal culture (Hsee and Weber, 1999; Zhang, 2012)

Also possibly explained by disproportionately large holdings of risk-exposed property (e.g., cars and apartments), rather than cash and other liquid assets

Both explanations suggest robust growth of personal property insurance market as wealth increases

What is current state of China's personal property insurance market? and how has it developed?

Part II: China's Personal Property Insurance Market

Current Market Characteristics

China's personal property insurance market highly concentrated: 5 largest companies compose 79% of market

Personal property insurance only 5% of total property market

Personal property insurance only 0.6% of total property-liability market

Explanations for Limited Market

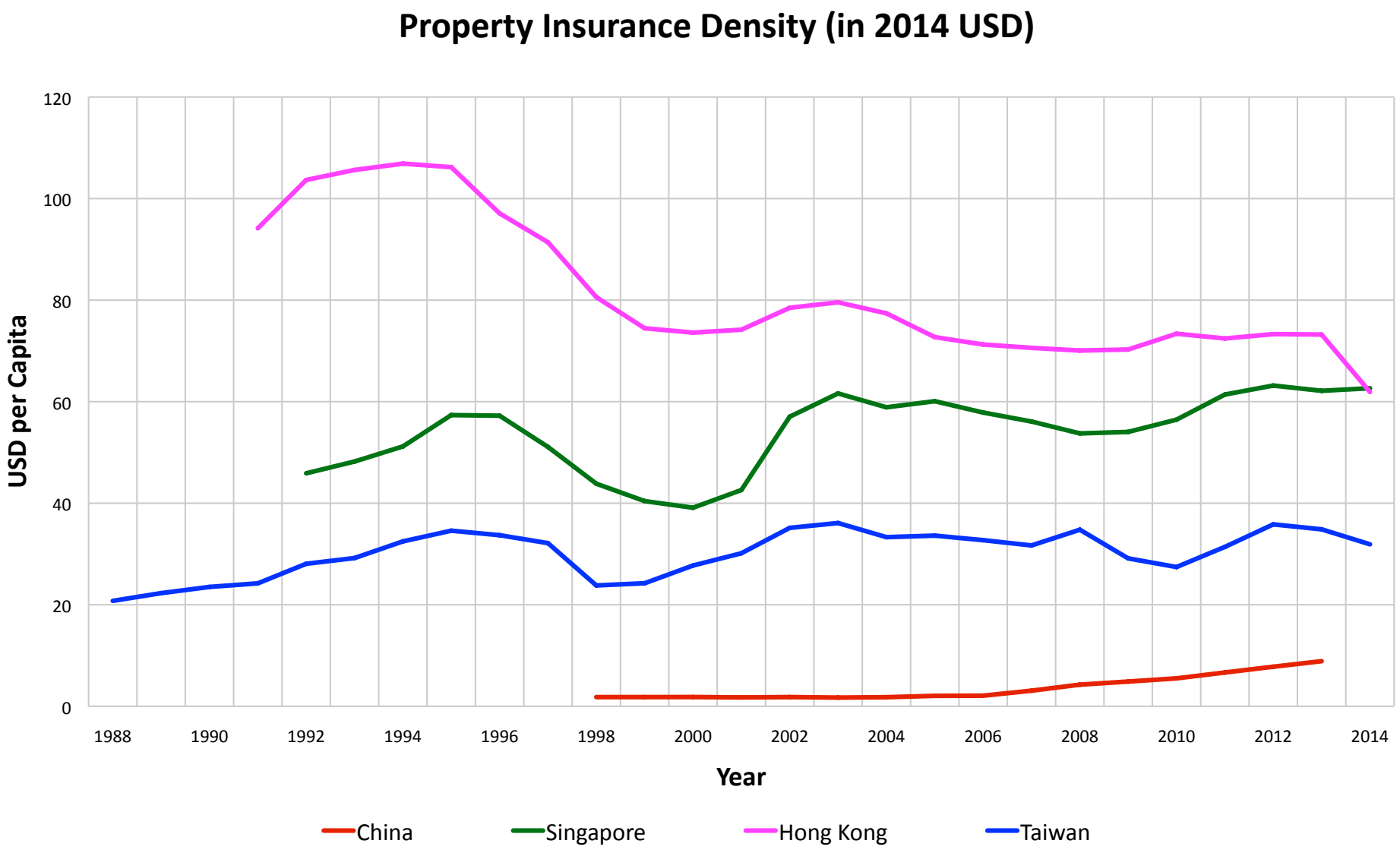
Customer-related explanations:

- lack of awareness of [trust in] insurance;
- unwillingness to pay for product without investment return;
- fatalism (i.e., belief in inability to escape one's fate); and
- reliance on family/community for financial support

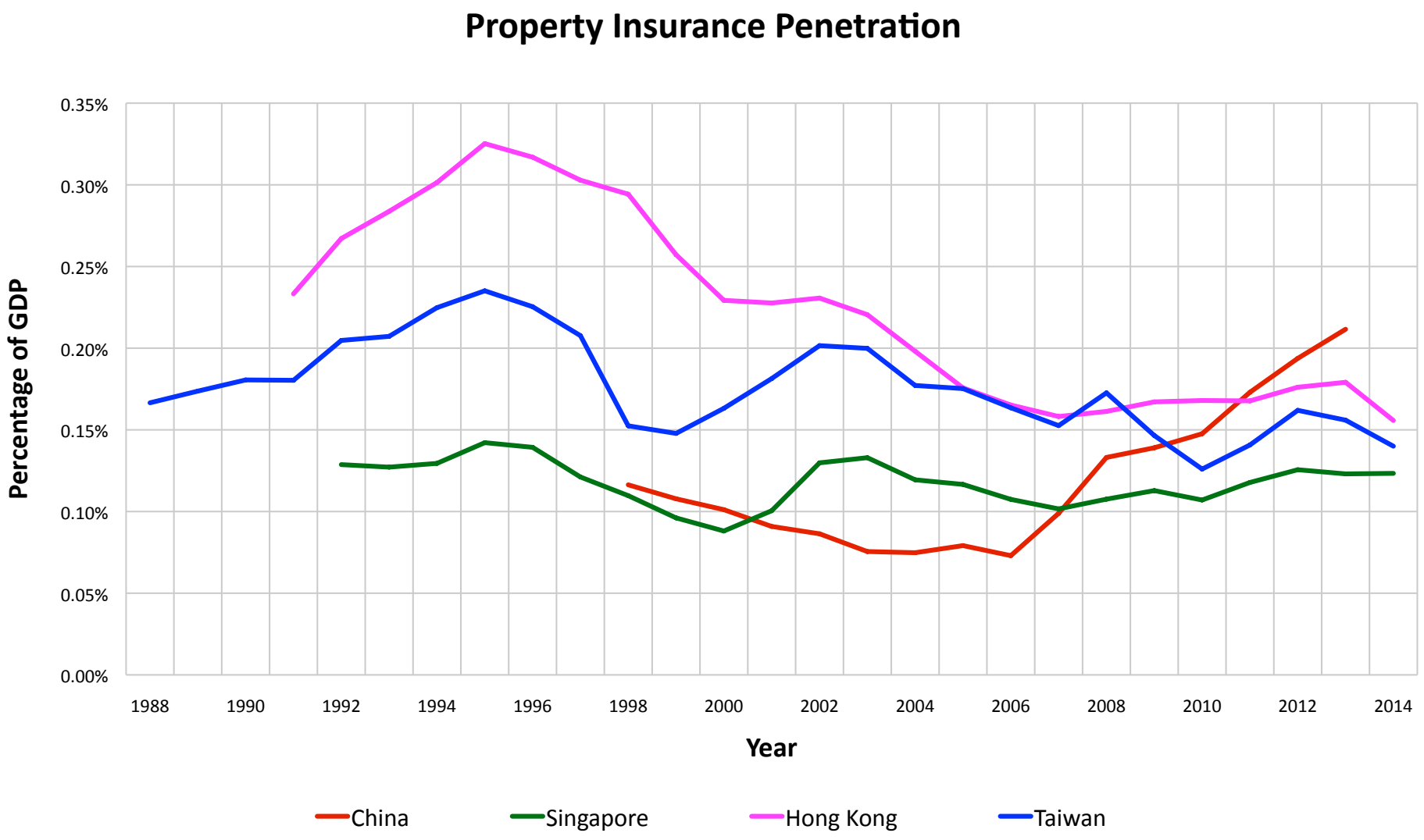
Industry-related explanations:

- inability of banks to require property coverage for mortgages;
- general unavailability of earthquake coverage; and
- lack of insurance company interest/innovation because of low premiums

Historical Market Growth



Historical Market Growth (Continued)



Part II Conclusions

Personal property insurance premiums grew by 90% from 2009 to 2012

All customer-related explanations for limited market diminishing over time

Ultimately, government subsidies may be necessary to kickstart personal property market

Do Infinite-Mean Losses Exist?

Reasonable to question existence of infinite-mean insurance losses in real world:

1. If losses always bounded above (i.e., $X < B < \infty$), then must possess finite mean
2. Insurance companies invariably apply policy limits (i.e., indemnity caps) to loss payments
3. Even without policy limits, actual loss payments still bounded above (e.g., by national GDP)

Do Infinite-Mean Losses Exist? (Continued)

Point (1) true: Upper bounds impose finite means

Point (2) true, but not necessarily effective: Policy limits can be nullified by courts (e.g., for "bad faith" settlements)

Point (3) true, but not necessarily of practical importance: Current focus

Common Model – Pareto II Distribution

Let $L \sim \text{Pareto II}\,(\alpha, \theta)$, characterized by PDF,

$$f_L(\ell) = \frac{\alpha}{\theta}\left(\frac{\theta}{\ell+\theta}\right)^{\alpha+1}, \ \ell > 0,$$

and CDF,

$$F_L(\ell) = 1 - \left(\frac{\theta}{\ell+\theta}\right)^{\alpha}, \ \ell > 0,$$

where $\alpha > 0$ and $\theta > 0$. Then

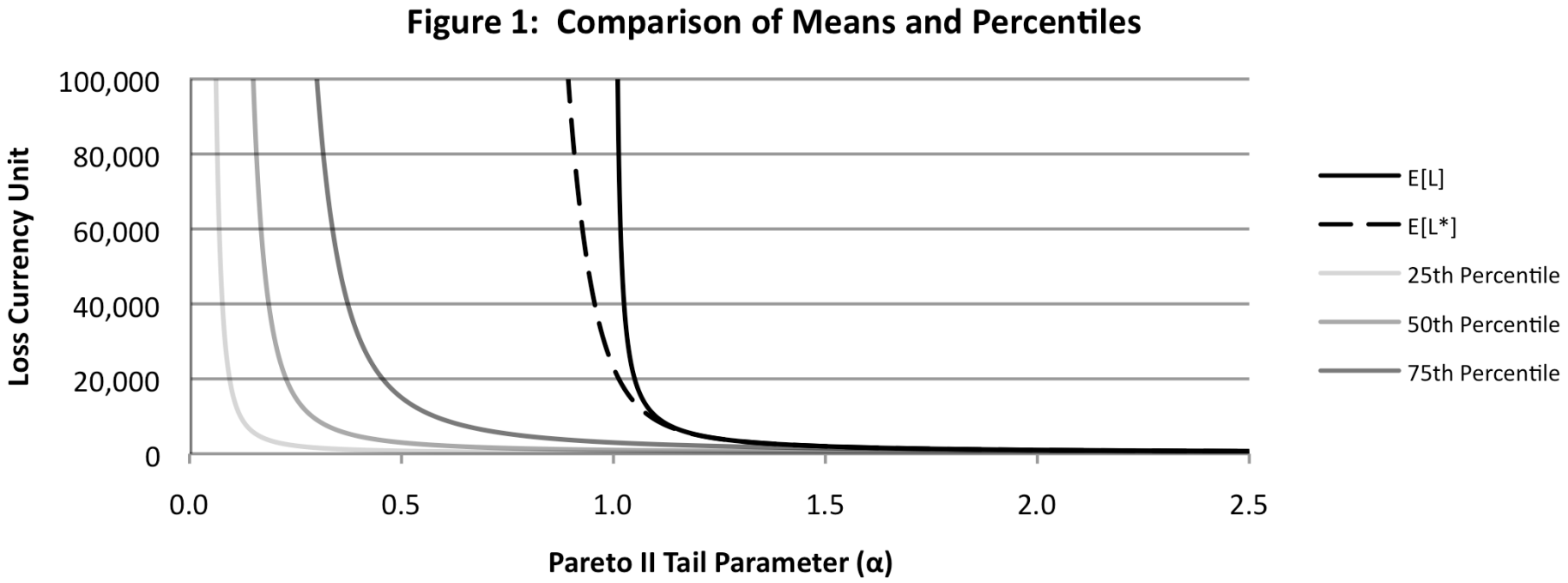
$$E[L] = \begin{cases} \dfrac{\theta}{\alpha - 1} & \text{for } \alpha > 1 \\ \infty & \text{for } \alpha \leq 1 \end{cases}$$

Assume Large Upper Bound, B

Let $L^* = \min\{L, B\}$, so

$$E[L^*] = \begin{cases} \dfrac{\theta}{\alpha - 1}\left[1 - \left(\dfrac{\theta}{B+\theta}\right)^{\alpha-1}\right] & \text{for } \alpha \neq 1 \\ -\theta \ln\left(\dfrac{\theta}{B+\theta}\right) & \text{for } \alpha = 1 \end{cases}$$

Impact on Mean of B = 10^13 (for θ = 10^3)



Impact on Probability of Ruin (Insolvency)

Let

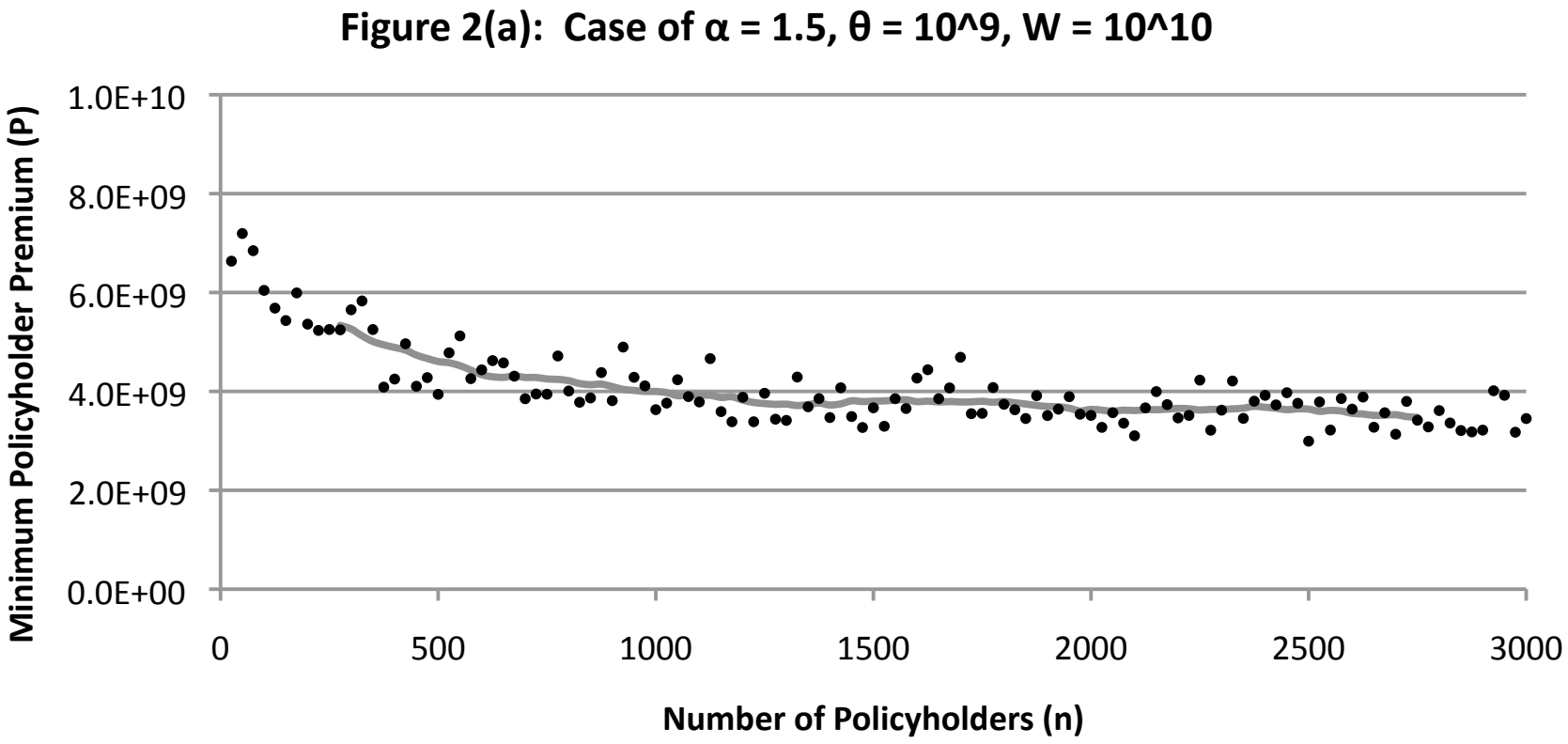
$$\psi = \Pr\left\{ W + nP - \sum_{i=1}^{n} L_i < 0 \right\}$$

denote insurance company's one-period ruin probability for n policyholders, where:

- $W = 10^{10}$ is company's initial net wealth;
- L_i are i.i.d. losses distributed as L or L^*; and
- P is per-policyholder premium

For selected α and θ, use statistical simulation to compute minimum P for $\psi \leq 0.01$

Case of $\alpha = 1.5$, $B = \infty$



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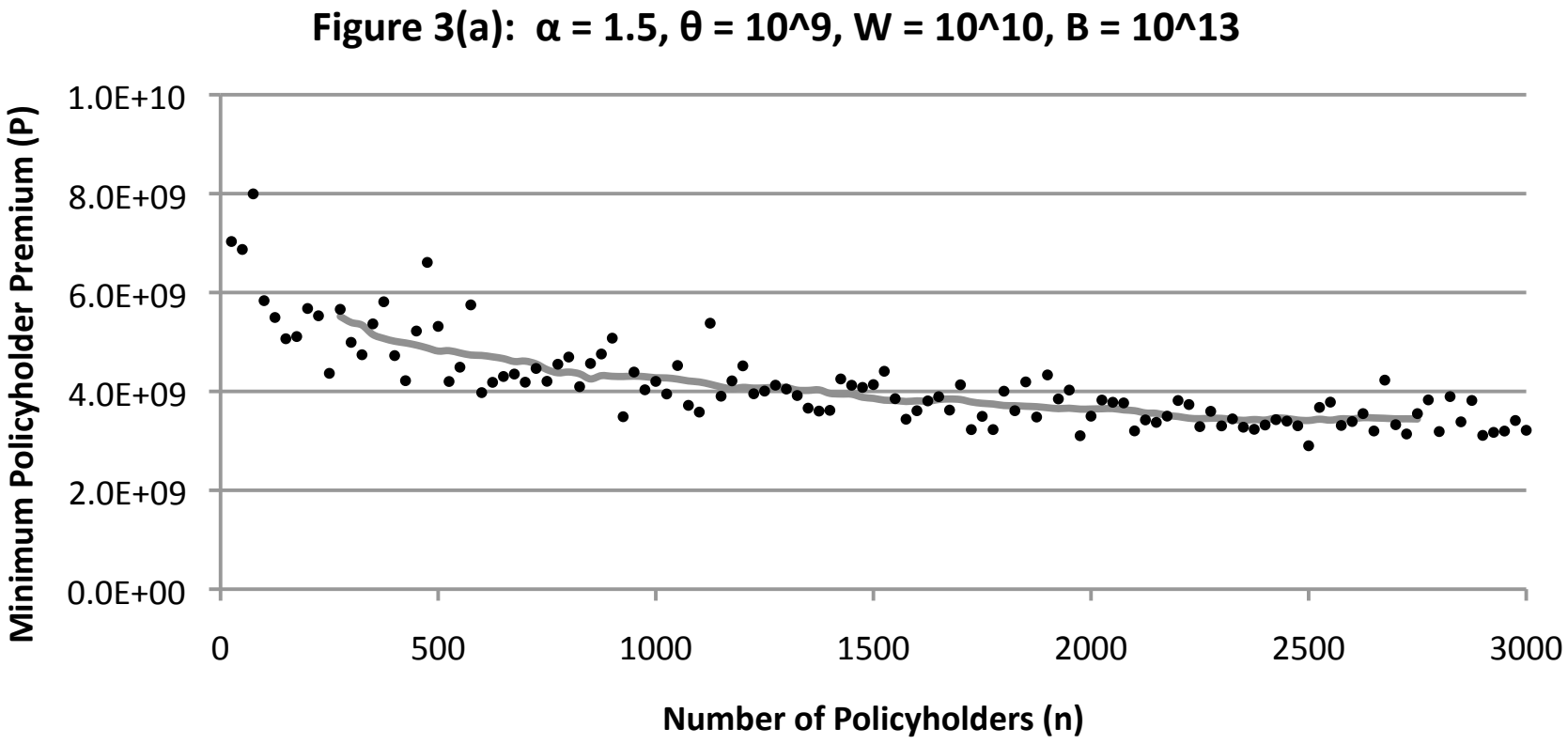
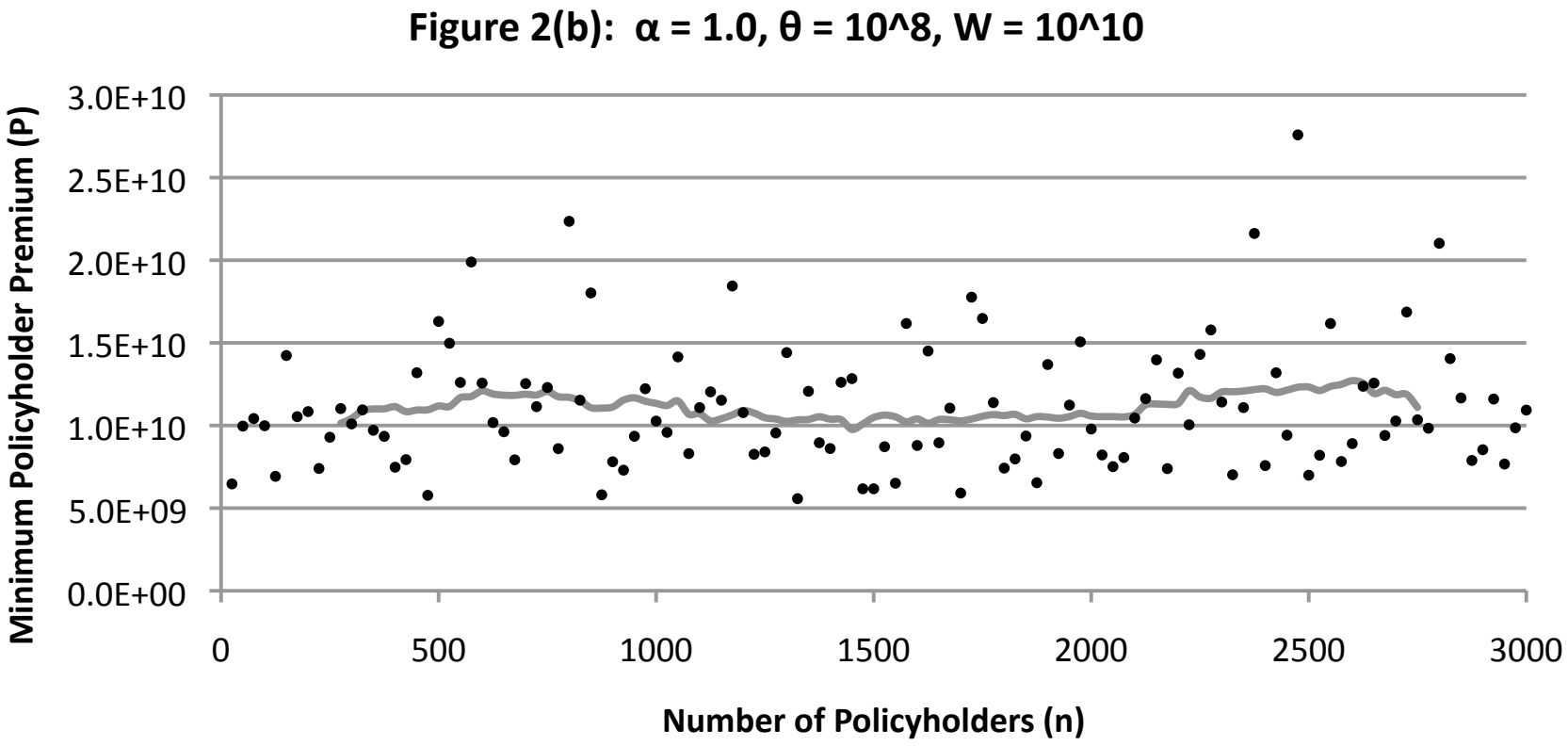
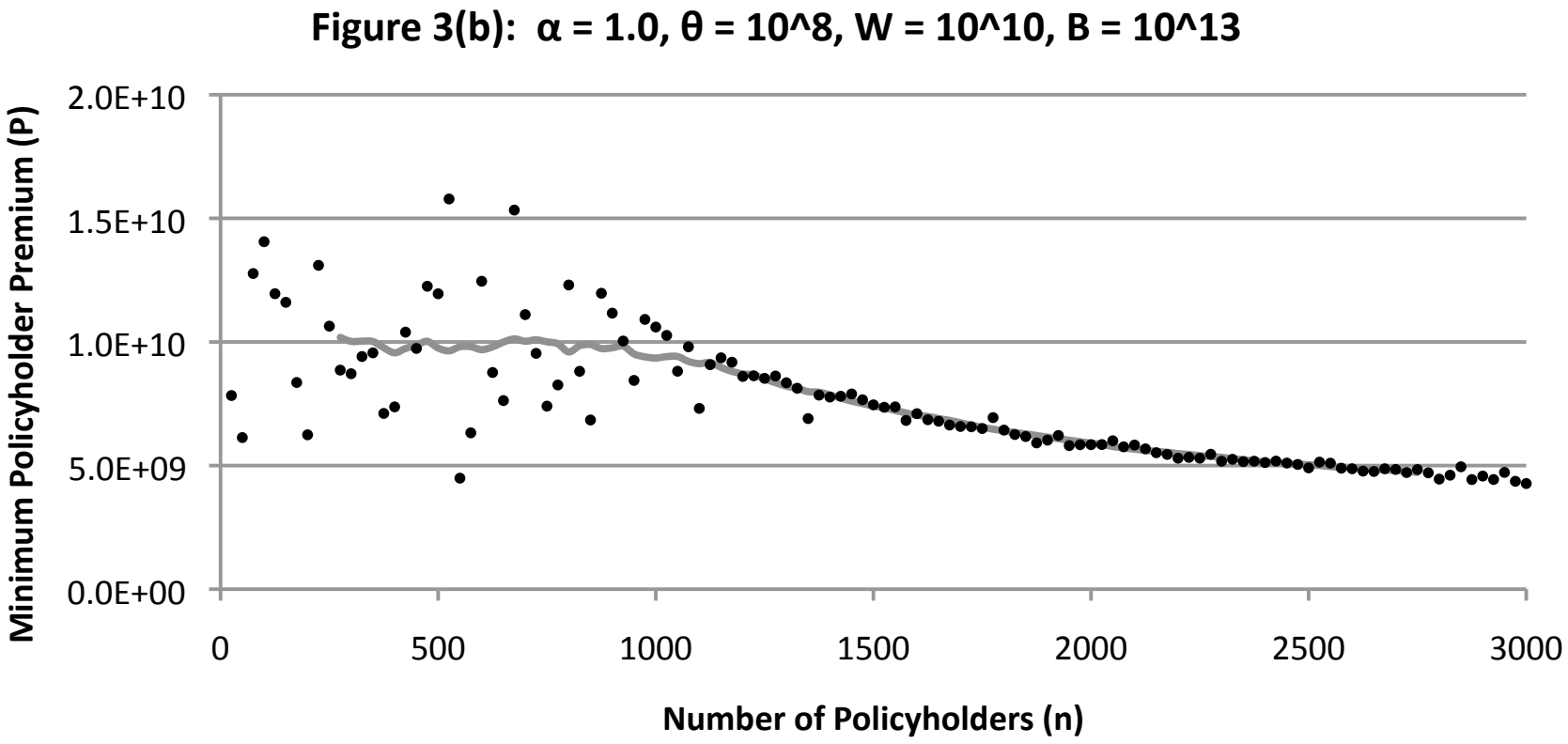


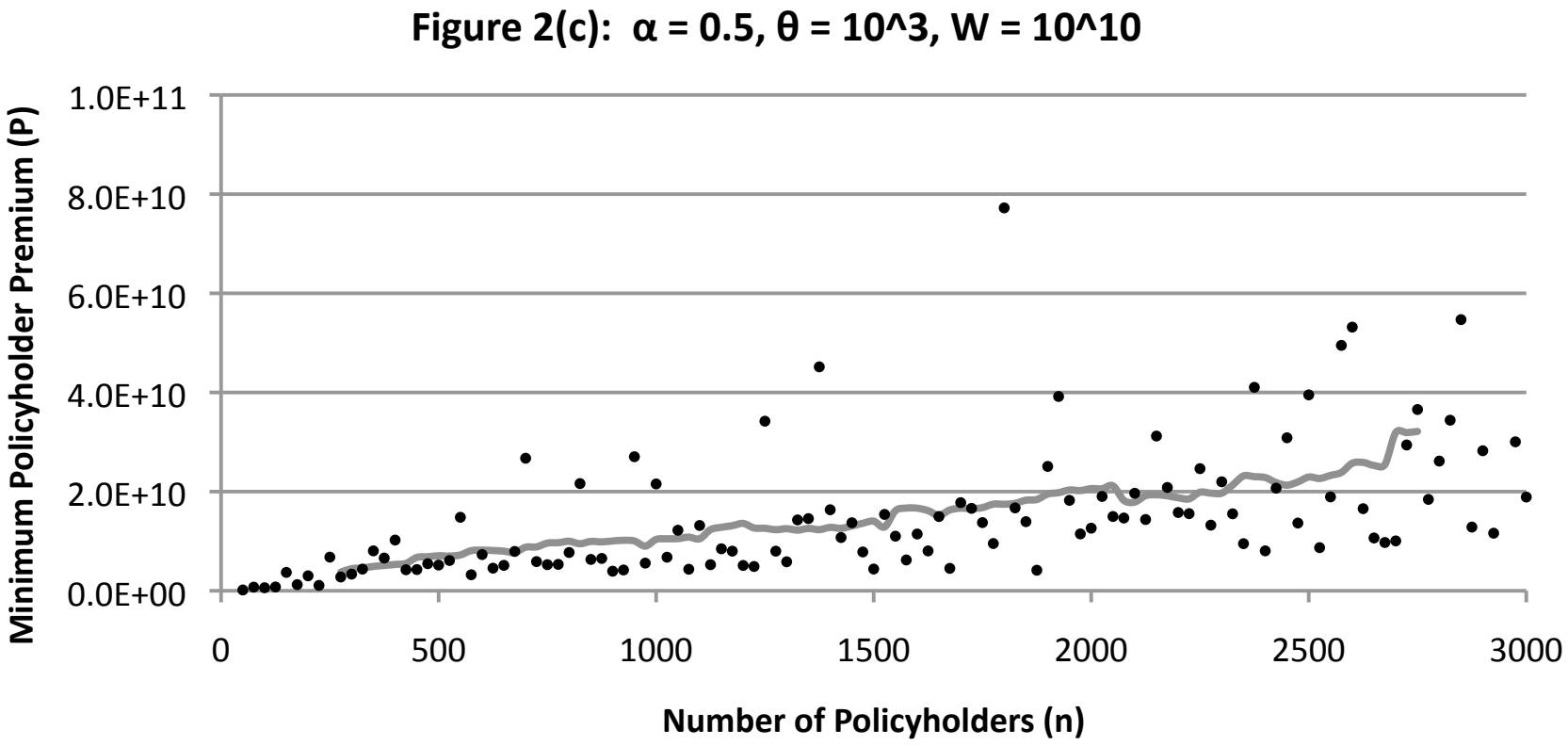
Case of $\alpha = 1.0$, $B = \infty$



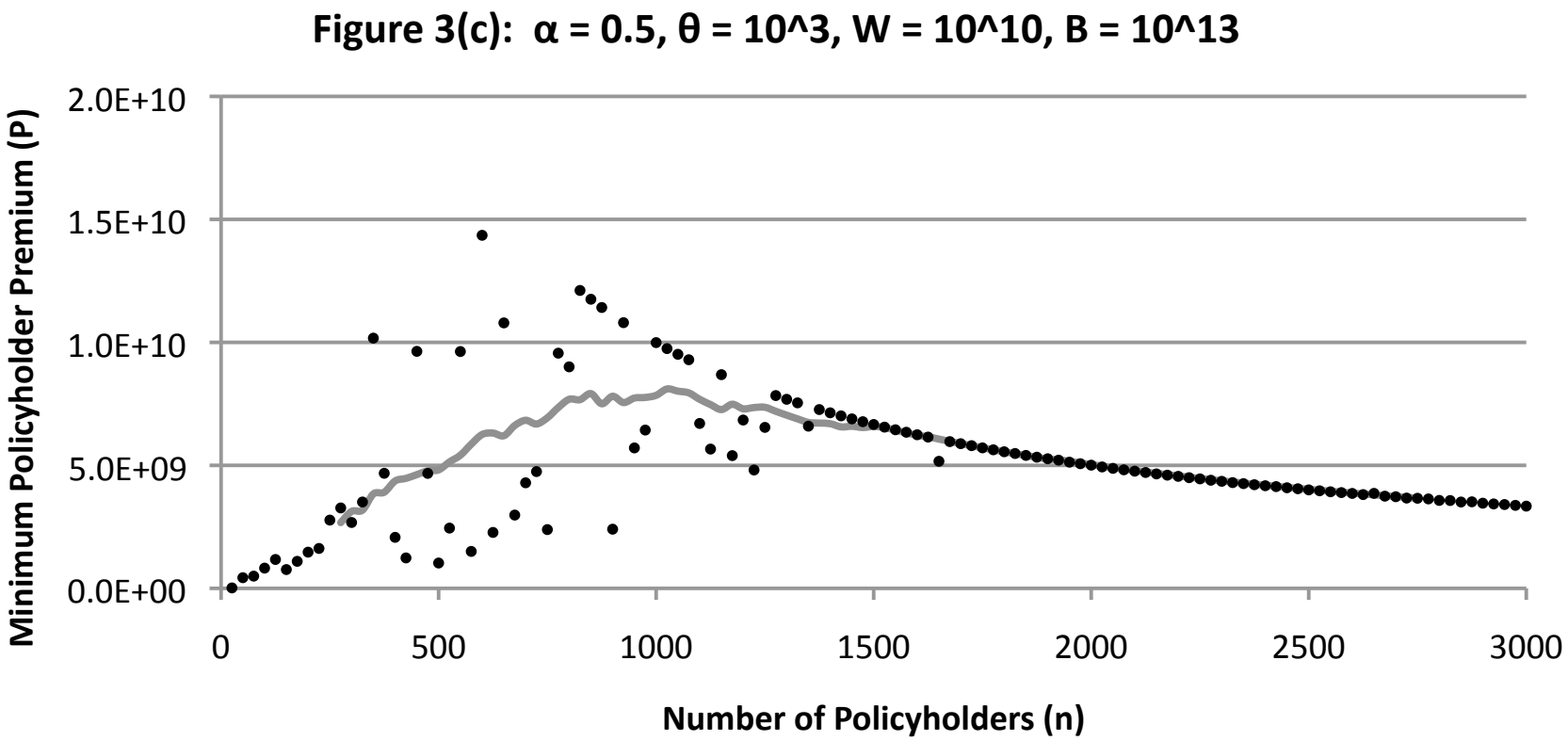
Case of $\alpha = 1.0$, B = 10^13



Case of $\alpha = 0.5$, $B = \infty$



Case of $\alpha = 0.5$, B = 10^13



Conclusions

Infinite mean simply modeling convenience

Similar to using Gaussian distribution for (bounded) human heights, or Poisson distribution for (bounded) number of automobile accidents in given period

"Infinite mean": Expected value of loss vastly greater than expected value of random variable with similar median but light tail