Tropical and Convex Geometry and Feynman integrals
05 – 09 September 2022
List of titles and abstracts

Monday, 05 September 2022

Simon Telen
11:00 – 12:00

Generalized Euler integrals: (co)homology and tropical numerics

Feynman integrals in suitable representations are generalized Euler integrals. We study vector spaces generated by such integrals. Their dimension is the Euler characteristic of a very affine variety, which we explain via twisted (co)homology. We show how to compute such Euler characteristics, as well as linear relations between the generators, using tools from numerical nonlinear algebra. Tropical geometry helps to compute large examples: the moduli space of eight points in three-space in linearly general position has Euler characteristic 5211816.

Francesca Zaffalon
14:00 – 15:00

Computing positroid cells in the Grassmannian of lines, their boundaries and intersections

The study of scattering amplitudes for the N = 4 SYM theory is closely related to the study of the positive Grassmannian, via the amplituhedron. The amplituhedron is a geometric object introduced by Arkani-Hamed and Trnka, defined as the image of the positive Grassmannian under a linear map. The geometry of the positive Grassmannian can be studied using positroids, a family of matroids introduced by Postnikov with a remarkable combinatorial structure. In particular, positroids label a CW decomposition of the positive Grassmannian. The image of these positroid cells are good candidates to decompose the amplituhedron. For this reason, it is interesting to study the dimension of positroid cells and how they intersect.

Postnikov has identified several families of combinatorial objects in bijections with positroids, however they cannot be used to clearly study both the dimension and the relations between different positroids. I will provide yet another characterization of positroids for $Gr_{\geq 0}(2, n)$, the Grassmannians of lines, in terms of certain graphs. This characterization can be used to compute the dimension, the intersection and the boundary of positroid cells. This relies on determining different ways to enlarge a given collection of subsets of $\{1, \dots, n\}$ to represent the dependent sets of a positroid, that is the dependencies among the columns of a matrix with non-negative maximal minors.

René Klausen
15:45 – 16:45

Feynman integrals and their singularities from the A-hypergeometric perspective

In 1968, it was proposed by T. Regge to consider Feynman integrals as a kind of generalized hypergeometric functions. Today, more than 50 years later, we can revisit this proposal and describe Feynman integrals by means of the A-hypergeometric theory initiated by Gelfand, Kapranov and Zelevinsky (GKZ) in the late 1980s. This perspective turns out to have the potential to give us many insights for Feynman integrals. After a short overview about the basic notions of Feynman integrals and GKZ systems, we will focus on the analytic structure of Feynman integrals. The interest in this analytic structure, which is also known as Landau variety, goes back to the 1950s and is of great relevance for many applications. However, many heuristically motivated statements from this time have still lacked a solid mathematical basis. We want to point out in the talk that A-hypergeometric theory will provide the right tools for a mathematically rigorous description of the singular locus of Feynman integrals. Inter alia, using results from GKZ, we will find that the usual decomposition of Landau varieties into leading Landau varieties of subgraphs works only for very simple Feynman graphs. However, a decomposition in the general case can also be provided by means of GKZ.

Tuesday, 06 September 2022

Claudia Fevola
11:00 – 12:00

Generalized Euler integrals: differential operators and convex polytopes

In this talk, I will continue where Simon left off. I will focus on two different vector spaces of generalized Euler integrals. Their dimension is again an Euler characteristic, but it is also expressed as the holonomic rank of a D-module, or the volume of a convex polytope.

I will recall definitions regarding algebras of differential and difference operators, GKZ systems, and Mellin transforms. New insights in connections between our different vector spaces will be illustrated. Feynman integrals provide examples for these theories.

Felix Tellander
14:00 – 15:00

Analytic and Algebraic Structure of Two-point Functions

Today there are many different ways to think about Feynman integrals and in this talk I will review some of them for one of the simplest physical cases; the two-point functions. The analytic structure (i.e. singularities) is dictated by traditional results due to Landau and Cutkosky, now we understand part of these results in terms of the associated GKZ A-hypergeometric system. These systems are connected to the canonical differential equation where the singularities come as symbol letters of some cluster algebra. Moreover, the GKZ system comes with a natural toric geometry which has been shown to not only help with fast numerical evaluation of the integrals but also describing the singularities appearing in renormalization.

Sebastian Mizera
15:45 – 16:45

Feynman Polytopes and the Tropical Geometry of UV and IR Divergences

We introduce a class of polytopes that concisely capture the structure of UV and IR divergences of general Feynman integrals in Schwinger parameter space, treating them in a unified way as worldline segments shrinking and expanding at different relative rates. While these polytopes conventionally arise as convex hulls - via Newton polytopes of Symanzik polynomials - we show that they also have a remarkably simple dual description as cut out by linear inequalities defining the facets. It is this dual definition that makes it possible to transparently understand and efficiently compute leading UV and IR divergences for any Feynman integral. In the case of the UV, this provides a transparent geometric understanding of the familiar nested and overlapping divergences. In the IR, the polytope exposes a new perspective on soft/collinear singularities and their intricate generalizations. Tropical geometry furnishes a simple framework for calculating the leading UV/IR divergences of any Feynman integral, associating them with the volumes of certain dual cones. As concrete applications, we generalize Weinberg's theorem to include a characterization of IR divergences, and classify space-time dimensions in which general IR divergences (logarithmic as well as power-law) can occur. We also compute the leading IR divergence of rectangular fishnet diagrams at all loop orders, which turn out to have a surprisingly simple combinatorial description.

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Wednesday, 07 September 2022

Henrik Munch
10:00 – 11:00

Differential equations for Feynman integrals from GKZ hypergeometric systems

Feynman integrals are known to arise as special cases of Gelfand-Kapranov-Zelevinsky (GKZ) hypergeometric functions. Employing the GKZ framework, together with its connection to D-module theory, we propose an algorithm for deriving Pfaffian systems for Feynman integrals, i.e., systems of 1st order PDEs satisfied by a basis of masters integrals. Using the Pfaffian system, we further show how to obtain integration-by-parts relations for Feynman integrals.

Uli Walther
11:00 – 12:00

On GKZ-systems and matroids attached to Feynman graphs

In Lee--Pomeransky form, Feynman periods are integrals of the powers of a polynomial G_m.

Klausen, Helmer and Tellander suggested to study the properties of the GKZ-system attached to the collection A of support vectors of G_m; this holonomic system of PDEs governs the periods as functions of masses and momenta. Combinatorial properties of A imply good behavior of the solution space to the GKZ-system. Generalizing ideas of Helmer and Tellander, we discuss the aspects of the saturation property of the cone of A, and show it holds in a significant number of interesting cases.

Thursday, 08 September 2022

Florian Loebbert
11:00 – 12:00

Yangian Symmetry, Fishnet Integrals and Geometry

Via the fishnet-limit of AdS/CFT, individual Feynman integrals inherit a conformal Yangian symmetry. This extends to large families of integrals, including graphs with massive propagators on the boundary. We demonstrate how to use the Yangian symmetry for bootstrapping Feynman integrals from scratch. In particular, we consider the example of fishnet integrals in two spacetime dimensions. These allow to systematically understand the role of the Yangian at higher loop orders and reveal a new volume interpretation.

Johannes Schlenk
14:00 – 15:00

Geometric sector decomposition and expansion by regions

The Newton polytope of a Feynman integral is a geometric object that encodes information about the associated integral. I will discuss geometric approaches to sector decomposition and expansion by regions based on the Newton polytope. Sector decomposition is a method for the numerical calculation of Feynman integrals, while expansion by regions is used to approximate integrals containing a large hierarchy of scales. Both strategies are implemented in the publicly available code pySecDec.

Christoph Nega

15:45 – 16:45

Calabi-Yau Geometries and Feynman Integrals

In my talk I will show how techniques and insights from Calabi-Yau varieties and their period integrals can be used to solve different families of Feynman integrals. Here the Calabi-Yau geometries will serve mainly in two different ways. First of all, their period integrals serve as a useful function space relevant for certain Feynman integrals. Secondly, properties of the moduli space of Calabi-Yau varieties will give geometric boundary conditions telling us how the previously defined functions have to be combined to obtain the Feynman integral. I will demonstrate this method first on the so-called banana graphs which are at the moment the best understood example of an infinite family of Feynman graphs where Calabi-Yau techniques are used. As a completely new example I will also talk about the fishnet/traintrack family of Feynman integrals where we could made new achievements by using the above mentioned Calabi-Yau techniques.

Friday, 09 September 2022

Aaron Hillman

11:00 – 12:00

Feynman Polytopes and Subtraction Schemes for Feynman Integrals

We describe a scheme for introducing local subtractions in order to prescribe the finite integrals needed to construct the Laurent expansion of an arbitrary scalar Feynman graph. The prescription is quite general and admits a certain freedom, but is efficient because of the knowledge of the fan associated with an arbitrary scalar Feynman graph, itself coming from the particular simple facet descriptions of Feynman polytopes. As an application, this furnishes new means of computing the Laurent expansions for infrared-divergent Feynman graphs of interest in a variety of physical contexts.

Sumit Banik

14:00 – 15:00

Mellin-Barnes, Conic-Hulls and Triangulations

In my talk, I give an overview of the recent progress made to evaluate N-fold MB integrals which appear in the intermediate steps to evaluate Feynman diagrams. In particular, I focus on the connection between MB integrals and conic hulls, and show how studying the intersection of conic hulls can help in solving MB integrals. As applications of this approach, I discuss the solutions of the non-trivial conformal hexagon and double-box integrals using the conic-hull method. I will end my talk with a brief discussion on my ongoing work in developing an alternative computationally efficient technique based on the triangulation of point configurations to compute MB integrals.

Yao Ma

15:45 – 16:45

The on-shell expansion: from Landau equations to the Feynman polytope

We study the expansion by regions of Feynman integrals with massless propagators, which contribute to off-shell Green's functions in Minkowski spacetime (with non- exceptional momenta) around vanishing external masses. This on-shell expansion allows us to identify all infrared-sensitive regions, in terms of infrared subgraphs in which a subset of the propagators become collinear to external lightlike momenta and others become soft. Each such region can be viewed as a solution to the Landau equations, or equivalently, as a facet in the Feynman polytope constructed from the Symanzik graph polynomials. This identification allows us to study the properties of the graph polynomials associated with infrared regions, as well as to construct a graph-finding algorithm for the on-shell expansion. Furthermore, it provides a criterion for the commutativity of multiple on-shell expansions, and a connection to the infrared forest formula, which may be used to establish the convergence of the expansion.