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Spontaneous Use of Conceptual Mathematical Knowledge in Elementary School Children

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Recent research has shown that most elementary school children under 10 fail to use a shortcut strategy to solve inversion problems such as "a + b − b", but solve them in the same way they solve problems such as "a + b − c" (Bisanz & LeFevre, 1990). There is no reason to believe that children older than 8 years lack the necessary conceptual knowledge for discovering the shortcut strategy. This study addresses the question of whether children in fact are already able to discover the shortcut strategy but do not use it because they prefer to use more familiar computing strategies. By providing supporting conditions and sufficient practice it was asked whether children will discover and use the shortcut strategy. In a first phase, half of the children (n = 44) were presented one block of a + b − c problems and one block of a + b − b problems (Supporting Condition), whereas for the other half (n = 44) the problems were randomly mixed (Control Condition). In a second phase, all subjects were presented a + b − b problems and a + b − c problems in a mixed order. In this phase, there were significantly more shortcut strategy users under the Supporting Condition than under the Control Condition. The results showed that most of the children younger than 10 are able to use the shortcut strategy, but only do so when it does not have to compete with a more familiar strategy.

Mathematical knowledge can be represented in at least three ways: procedural knowledge referring to computation skills, factual knowledge consisting of memorized information about relations among numbers, and conceptual knowledge involving understanding mathematical principles (Bisanz & LeFevre, 1990). One way to find out whether someone has conceptual knowledge is to determine whether s/he selectively uses a concept to discover and use a *strategy* when it is appropriate and helpful to do so. According to Bisanz and LeFevre (1990) a strategy is defined as something "invoked in a flexible, goal oriented manner and that influences the selection and implementation of subsequent procedures" (p. 236). While a mathematical procedure is characterized by the effective execution of a determined operation and does not necessarily presuppose conceptual knowledge, using a strategy implies looking for facilitatory

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methods instead of running long winded and time consuming procedures. The conditions under which children are able to develop conceptually based strategies constitute one of the main interests in research on the acquisition of mathematical knowledge.

Bisanz and LeFevre (1990) investigated elementary school children's use of conceptually based *shortcut strategies* in *inversion-problems* such as "a + b − b." Merely solving an inversion problem does not presuppose access to the shortcut strategy but only requires addition and subtraction procedures. However, solving these problems without computation is possible if one recognizes that "b − b = 0" and the result is always "a." Therefore, recognizing that inversion problems need no computation is clearly *strategic* behavior in the sense used in Bisanz and LeFevre (1990). A *selection* process is necessary because a subject has to choose between using computing procedures and conceptually based rules such as "if zero is added to a number, this number does not change." The selection process is *goal oriented* because using the shortcut strategy is the best way to a successful and quick outcome. The solution process is also *flexible* because the shortcut strategy can be used in different stages of the solution process: a computing procedure can be interrupted, when it is recognized that the two numbers compensate each other. Therefore, a subject who has already added "b" on "a" in the problem "a + b − b" can always interrupt the computation process and run the shortcut strategy.

Bisanz and LeFevre (1990) have investigated inversion problems with subjects aged 6, 7, 9, 11, and 20. They presented *standard problems* (such as "a + b − c") and *inversion problems*, each problem type half with small numbers and half with large numbers, on a computer screen and measured solution times. Bisanz and LeFevre (1990) reasoned that if the shortcut strategy was used, number size should have an effect on the solution time for standard problems but not for inversion problems. Their results showed no difference in the percentage of children who used the shortcut strategy between 6 and 9 years of age, although there was a clear difference among these groups in their ability to solve the standard problems. The use of the shortcut strategy was frequent only in children older than 10. Stern (1992) found similar results with German children. This result is surprising because only three mathematical concepts which children have usually acquired by second grade (ages 7–8) are required: (1) commutativity (a + b − b = b − b + a), (2) inverse element (b − b = 0), and (3) neutral element (a + 0 = a).

This paper addresses the question of why children younger than 10 do not use their knowledge to build the shortcut strategy. One possible explanation is that children's lack of *general strategy knowledge* (Pressley, Forrest-Pressley, Elliot-Faust, & Miller, 1985) prevents them from using their domain-specific knowledge. Subjects possessing general strategy knowledge are characterized by carefully looking for facilitation before a procedure is run. Considering that Schneider and Pressley (1989) have shown that children about the age of 10 use a variety of strategies in different memory tasks, one might conclude that 10-year-old children also possess general strategy knowledge that allows them to use domain-specific knowledge for building new strategies in mathematics. However, one need not resort to a lack of general strategy knowledge in order to explain the results of Bisanz and Lefevre (1990) and Stern (1992). Both studies show only that children younger than 10 do not spontaneously use the shortcut strategy, but they do not show that children are not capable of using the strategy. From the extensive work by Robert Siegler on strategy use and strategy discovery (for an overview, see Siegler and Crowley, 1991) it is well known that, although they have smart strategies available, subjects often fail to use them because they prefer to use other more familiar strategies.

It has been shown in different areas of mathematics (Siegler & Shrager, 1984; Siegler & Campbell, 1989; Siegler & Jenkins, 1989; Siegler & Crowley, 1991), and also in other domains such as spelling and vocabulary learning in a second language (Siegler, 1989) and time telling (Siegler and McGilley, 1989), that subjects of all age groups have several different strategies available that compete with each other. The associative strength between the problems and the different strategies can vary considerably. In general, the associative strength of a strategy increases, and the associative strength of all other competing strategies decreases, with every use. Therefore, the more a strategy has been used in the past, the higher its associative strength and the greater its chance to be used in the future. However, which strategy is activated to solve a certain problem at a certain time also depends on such factors as the particular characteristics of the problem and the performance level of the subject. Inefficient but familiar strategies are often chosen for difficult problems, because in this case smart strategies often are prone to error. Younger children have fewer strategies available and the associative strength for long winded strategies is higher than for older children. Siegler and Jenkins (1989) investigated the conditions of discovering and using the min-strategy in addition (count on the larger addend) with preschoolers. One of their main conclusions was that strategy *discovery* does not automatically lead to strategy *use*. None of the children in Siegler and Jenkins's (1989) study who had once discovered the min-strategy used this strategy continuously after discovery. Rather they continued to use less efficient but highly familiar strategies such as counting from one. The authors' explanation for this result is that the associative strength of the recently discovered strategy was too low and therefore it could not compete with other more frequently used and thus stronger strategies. To raise the associative strength of a new strategy, one has to elicit its use, for example, by presenting challenging problems, such as "3 + 22 = ." With development, changes in strategy preferences occur: People prefer to use smart strategies instead of long winded ones.

If one has to solve arithmetic problems with more than two numbers, two general strategies are possible: (1) One can calculate successively one number after the other, that is, use the "start from the first number" strategy; or (2) One can use knowledge based on mathematical understanding to find out whether starting from numbers in the middle of the problem facilitates the solution process, such as the shortcut strategy. In the beginning of elementary school, the familiar "start from the first number" strategy has a high associative strength and especially for subtraction problems which are evaluated as difficult, children may prefer familiar strategies instead of looking for facilitation.

It is claimed that the kind of item presentation in Bisanz and LeFevre (1990) and Stern (1992) prevents children from discovering and using the shortcut strategy, even if the knowledge base allowed strategy discovery. In both studies, half of the problems were inversion problems and half of the problems were standard problems, presented in a random order (*mixed series*). Even if a child has a diffuse idea that some computation facilitation is possible for problems such as "a + b − b," a mixed series does not support the implementation of this idea, because the next problem presented may be a standard problem, preventing children from verifying his/her ideas. Siegler and Jenkins (1989) noted about the process of strategy discovery: "Only as people employ new concepts and strategies, and observe their consequences in the world, do they achieve a deep understanding of the strategies, advantages, disadvantages and conditions of applicability. Use may be especially critical to children's understanding. Children may generally be less able than adults to anticipate the properties of a novel approach. This may doom them to 'rediscover' the same idea a number of times before they understand it sufficiently to use it appropriately" (p. 112).

However, even if children *discovered* the shortcut strategy in a mixed series, its chances of being used are low because it has to compete with the familiar "start computing from the first number" strategy. This strategy is the adequate strategy for solving standard problems and therefore, in a mixed series, its associative strength increases continuously. As the "start computing from the first number" also leads to a correct result for inversion problems, the shortcut strategy, which in any case is less familiar to the children, is put at a disadvantage with every presented standard problem.

Hence, to give the shortcut strategy a chance to be discovered and used

one has to prevent the associative strength of the "start computing from the first number" from being increased. One way to do this might be to present only inversion problems for several subsequent runs (*blocked series*). It is known from research in inductive reasoning (Gick and Holyoak, 1987; Spiro, Feltovich, Jacobson, & Coulson, 1991) that the construction of abstract rules is supported by presenting several problems with the same structure. Proceeding on the assumption that the majority of the children have access to the knowledge underlying the shortcut strategy, presenting a blocked series of shortcut problems should lead to the discovery of the strategy. Once the shortcut strategy has been discovered in a blocked series, it has a greater chance of being used immediately under this condition than in mixed series because no other strategies have to be activated and children have a chance to achieve a deep understanding of the strategy and its conditions of applicability. Only if a strategy has been frequently used under such supporting conditions will its associative strength be raised; it will have a real chance of being used later under less supportive conditions if, for instance, standard problems and inversion problems are mixed in later trials.

The assumption tested in the present study is that children younger than 10 will use the shortcut strategy in mixed series after some experiences in blocked series.

METHOD

Subjects

Ninety-five elementary school children from Munich after school care centers participated in the study (35 second graders, mean age: 8 years, 3 months; 37 third graders, mean age: 9 years, 1 month; 23 fourth graders, mean age 10 years, 4 months). The children were tested individually.

Procedure

The children were randomly assigned to two different conditions. A schema of the procedure is depicted in Fig. 1. In the *Supporting Condition*, phase 1, the children were first presented 20 problems of the type a + b − c. Then, they received 20 problems of the type a + b − b. Children in the *Control Condition* were presented the same problems, but the problems were randomly mixed. Phase 1 ended with a break of 15 min. The second phase, which was the same for both groups, was then conducted. The children were presented 20 a + b − b problems and 20 a + b − c problems in random order. Half of the problems of each type included numbers less than 20, and half included numbers greater than 30.

To measure the time children need to solve the problems, we used a computer based procedure. The computer program was written in Turbo-Pascal and run in a Compaq Portable type 286 AT. A problem (for example, 16 + 2 − 2 =) was shown in the middle of the screen. In each lower corner a number was presented. One number was the correct answer, the other was a wrong answer. The subjects were instructed to find out as quickly as possible in which corner the correct result was presented and to push the corresponding button on the right or left side of the keyboard. The position of the left or right corner was randomly varied. The computer program recorded whether the subject pushed the correct

	Control condition	Support condition
First run	40 problems 20 problems: a + b − c 20 problems: a + b − b The order of the problems was mixed randomly	20 problems a + b − c 20 problems a + b − b
15-min. pause		
Second run	40 problems 10 problems: a + b − b 10 problems: a + b − b 10 problems: a + b − c 10 problems: a + b − c The order was mixed randomly	 numbers < 20 numbers > 30 numbers < 20 numbers > 30

FIG. 1. Design of the study.

button and the measured time to answer in units of 20 ms. The next problem was presented after a pause of 3 s. If a child did not push a button within 15 s, the problem disappeared from the screen and the next problem was presented. Children were offered a break of variable length after each set of 10 problems to avoid fatigue.

In both runs, the wrong answers were determined according to the following rules: For seven of the 20 a + b − b problems, the false answer was b, for seven problems it was a + 1, and for six problems it was a random number ranging between the correct answer −8 and the correct answer +8. For seven of the a + b − c problems, the false answer was b, for seven problems it was a, and for six problems it was the correct answer + 1.

Children were told that it was important to solve each problem as quickly as possible, but that it was even more important to make no mistakes.

RESULTS

Statistical analyses were conducted on the 40 problems from the second phase including only those subjects who made no more than five mistakes in the second run. Three second graders, three third graders, and one fourth grader did not fulfill this criterion and were excluded. The mean number of mistakes was 2.7 (SD = 1.3) for the subjects who were included in the analyses.

Mean reaction times were calculated for each of the four kinds of problems for each subject including only the correctly answered problems. The means and standard deviations for reaction times for each experimental condition and grade are given in Table 1.

For a first manipulation check, a 2 (condition: control, supporting) × 3 (grade: 2, 3, 4) × 2 (problem type: standard, inversion) × 2 (number size of the problems: large, small) analysis of variance was conducted with repeated measures on the latter two variables. For all results reported below, $p < .05$. This analysis revealed significant main effects for condi-

TABLE 1
MEAN SOLUTION TIMES (STANDARD DEVIATIONS) FOR EACH CONDITION

	Supporting condition			
	Inversion problems		Standard problems	
Grade	Small number	Large number	Small number	Large number
2	3.82 (.96)	4.17 (1.13)	5.71 (1.05)	6.85 (1.59)
3	3.04 (.93)	3.58 (1.11)	5.09 (1.00)	5.89 (1.17)
4	2.52 (.95)	2.65 (.98)	3.70 (1.93)	4.38 (1.49)
Total	3.19 (1.06)	3.56 (1.21)	4.97 (1.50)	5.86 (1.68)
	Control condition			
	Inversion problems		Standard problems	
Grade	Small number	Large number	Small number	Large number
2	5.19 (1.06)	5.59 (2.16)	5.77 (.99)	7.33 (1.61)
3	3.83 (1.00)	4.29 (.96)	4.49 (1.28)	5.06 (1.22)
4	4.14 (1.63)	4.12 (2.09)	4.20 (1.95)	3.62 (3.06)
Total	4.40 (1.32)	4.72 (1.85)	4.88 (1.52)	5.52 (2.42)

tion, $F(1,82) = 5.75$, $MS_e = 3.98$, grade $F(2,82) = 24.81$, $MS_e = 3.98$, problem type: $F(1,82) = 98.33$, $MS_e = 1.38$, and number size, $F(1,82) = 11.84$, $MS_e = 1.80$, qualified by significant interactions for condition by problem type, $F(1,82) = 31.24$, $MS_e = 1.38$, and grade by problem type: $F(2,82) = 6.06$, $MS_e = 1.38$. The significant interaction between condition and problem type occurred because it took less time to solve inversion problems under the supporting condition than under the control condition, whereas the solution time for standard problems was equivalent across conditions. This result suggests that the shortcut strategy was used more often under the supporting condition than under the control condition. The significant interaction between grade and problem type arose because solution time for inversion problems is short for all strategy users independent of grade, whereas solution time for standard problems varied considerably by grade. Thus, this first manipulation check indicates that strategy use can be encouraged under the supporting condition.

In a further analysis, the number of strategy users under each condition was determined. A subject was defined to be a strategy user if the following conditions were fulfilled: (a) The difference in the mean solution time between inversion problems with large numbers and inversion problems with small numbers was less than 1 s; and (b) The mean solution time for inversion problems with large numbers was less than 3 s. The number and percentage of subjects who did and did not fulfill these conditions are given in Table 2. Fisher's Exact Test revealed that the number of strategy

TABLE 2
PERCENTAGE AND NUMBER OF STRATEGY USERS

	Supporting condition Grade				Control condition Grade			
	2	3	4	All	2	3	4	All
Strategy user								
%	56	59	73	61	13	35	45	30
n	9	10	8	27	2	6	5	13
Nonuser								
%	44	41	27	39	88	65	55	70
n	7	7	3	17	14	11	6	31
Total								
n	16	17	11	44	16	17	11	44

users was greater in the supporting than in the control condition, $\chi^2(1) = 7.7, p < .01$.

The mean solution times for all problem types for strategy users and nonusers are given in Table 3. An analysis of variance, with strategy use and condition as grouping variables and problem type and number size as repeated variables, was conducted to ascertain whether strategy users differ from nonusers not only in the solution time for inversion problems but also for standard problems, and whether the solution times of strategy users in the supporting condition differ from those of strategy users in the control condition. For all results reported below, $p < .05$. This analysis revealed significant main effects for strategy user, $F(1,84) = 24.19$, $MS_e = 4.91$, problem type, $F(1,84) = 100.2$, $MS_e = 1.49$, and number size, $F(1,84) = 10.15$, $MS_e = 1.55$, qualified by significant interactions for condition by problem type, $F(1,84) = 19.58$, $MS_e = 1.49$, condition by number size, $F(1,84) = 4.0$, $MS_e = 1.55$, strategy use by number size,

TABLE 3
MEAN SOLUTION TIMES (STANDARD DEVIATIONS) FOR STRATEGY USERS AND NONUSERS FOR EACH CONDITION

	Supporting condition			
	Inversion problems		Standard problems	
	Small number	Large number	Small number	Large number
Strategy user	2.95 (.69)	2.97 (.79)	4.86 (1.44)	5.42 (1.12)
Nonuser	3.59 (1.40)	4.50 (1.19)	5.14 (1.62)	6.56 (2.17)
	Control condition			
Strategy	3.74 (1.51)	2.72 (1.06)	4.40 (1.83)	4.23 (2.67)
Nonuser	4.68 (1.15)	5.56 (1.41)	5.09 (1.35)	6.07 (2.13)

$F(1,84) = 18.37$, $MS_e = 1.55$, and number size by problem type, $F(2,84) = 4.99$, $MS_e = .96$. The lack of interaction between strategy use and condition shows that strategy users under the supporting condition do not differ from strategy users under the control condition in their solution times. The lack of interaction between strategy use and problem type indicates that strategy users took also less time to solve standard problems than did nonusers.

The data from phase 1 were analyzed on a descriptive level to explore some aspects of strategy discovery and strategy use.

Supporting Condition

When the block of inversion problems was presented, for 37 (of 44) subjects the solution time decreased within the first 10 problems to less than 2 s and remained under this value for the whole block, indicating strategy discovery. The other subjects under the supporting condition did not show this decrease in solution time. All subjects identified as strategy users in phase 2 (see above), showed a clear decrease in solution time within the first 10 problems in phase 1. However, there were 10 subjects (six second graders, three third graders, one fourth grader) in the supporting condition for whom a decrease in solution time in phase 1 indicated strategy discovery and use, who did not show strategy use in phase 2, when inversion and standard problems were mixed. This result indicates that, especially for young children, the presentation of 20 successive inversion problems may not be sufficient to build and strengthen the shortcut strategy so that it can compete with the "start computing from the first number" strategy in the test condition of mixed problem presentation. A comparison of the solution times of strategy discoverers in phase 1 and phase 2 indicates that strategy discoverers needed more time to solve an inversion problem in phase 2. After strategy discovery in phase 1 the solution time was less than 2 s for all students ($M = 1.53$, $SD = .32$) whereas the mean for the same students was 2.9 s in phase 2. The longer solution times in phase 2 might be due to the time that was necessary to find out whether the problem presented is a shortcut or a standard problem under the blocked series presentation while in phase 1, the children did not expect problems other than inversion problems.

Control Condition

In contrast to the supporting condition, the distribution of the reaction times in phase 1 of the control condition was not a clear indicator of strategy discovery. Assuming that a shorter solution time for shortcut problems indicates strategy discovery and strategy use, no child consistently used the shortcut strategy after s/he had used it the first time (operationalized by a decline in solution time of more than 3 s in comparison to the solution time for solving the preceding inversion problem). However, the data regarding those children who were identified as strategy users in phase 2 indicated that the strategy had been *discovered* in phase 1: A comparison of the mean solution times of the first five and the last five shortcut problems showed that the reaction time in phase 1 declined from 4.3 to 2.4 s for strategy users in phase 2 and from 5.0 to 4.6 s for nonusers. A 2 (strategy user, nonuser) × 2 (first five problems, last five problems) analysis of variance with repeated measures on the latter factor revealed significant main effects (strategy user vs nonuser: $F(1,42) = 8.6$, $p < .01$, first vs last problems: $F(1,42) = 8.8$, $p < .01$) and a significant interaction, $F(1,42) = 2.98$, $p < .05$).

DISCUSSION

The results indicate that children younger than 10 are able to discover and use the shortcut strategy under supporting conditions. Therefore, the results of Bisanz and LeFevre (1990) and Stern (1990) do not indicate that children younger than 10 are not able to discover the shortcut strategy, but only that they do not use this strategy spontaneously when inversion problems and standard problems are mixed.

Why was it easier for children to discover and use the shortcut strategy when standard problems and shortcut problems were presented in separate blocks rather than when they were randomly mixed? Under both conditions, children had to solve the same number and the same kinds of standard and shortcut problems. Thus, with the exception of the order of item presentation, learning experience was the same under both conditions. We speculate that the blocked problem presentation helped to build the shortcut strategy, because children need time and confirmation to become aware of their own ideas. When an inversion problem is seen for the first time, a child may recognize that there is something special about the problem, but not be able to explain what it is exactly. The diffuse idea of "something" special can only elicit a strategy if the child soon has another opportunity to confirm his/her conjecture; otherwise, it will be forgotten. There is support for the claim that elementary school children need many confirmations before they are able to build a strategy that has a chance to compete with existing strategies: In phase 1 several children in the supporting condition used the shortcut strategy when inversion problems were presented but failed to use it when inversion problem and standard problems were mixed. For these children, although the presentation of 20 inversion problems was enough to discover the shortcut strategy, it was not enough to raise its associative strength in a way that it could compete with other strategies in mixed series.

The results of this study highlight the need to go beyond all-or-none assessments of concept understanding and use of conceptual knowledge.

Of course, one can only use a strategy that has been discovered, and one can only discover a strategy if the knowledge underlying this strategy is accessible. However, none of these preconditions necessarily leads to strategy discovery and strategy use because a single strategy is not an isolated knowledge structure that is automatically activated when a problem demands it. Rather, whether or not a new strategy has the chance to be discovered and used depends on the associative strength of alternative strategies. To give the new strategy a chance requires decreasing the associative strength of the old strategies.

In this study, solution time was the only indicator of strategy use, and we do not know anything about the role of conscious activities in the discovering of the shortcut strategy. Most of the children spontaneously made remarks like "Oh, I do not have to compute because I get zero" before their solution times decreased. However, more systematic questioning is necessary to learn about how children discover the strategy and to begin to know what information those children, who are not able to discover the strategy lack. Stern (1992) showed that some second graders who do not discover the shortcut strategy do not know that the law of commutativity is valid for the whole number system. These children answered "yes" when they were asked "Is the result of 8 + 7 and 7 + 8 the same," but answered the question "Is the result of 85 + 78 and 78 + 85 the same" with "we did not compute with such large numbers in school." The children were not able to generalize their knowledge about commutativity acquired by computing with small numbers to large numbers with which they were unfamiliar. The reason for this may be that the children have not represented the number space as a unity concept and therefore they do not know that rules valid for small numbers also are valid for large numbers. Such kinds of deficits can of course prevent children from strategy discovery.

The use of shortcut strategies in inversion problems is a method that simplifies mathematical equations. Such simplification skills are needed in algebra. As yet, little is known about the long-term consequences of using or not using shortcut strategies. However, there is some evidence that a lack of flexibility in solving arithmetic problems may cause later difficulties in the acquisition of skills in algebra as Herscovics (1989) and Filloy and Rajano (1989) have shown. Therefore, considering the use of strategies in such inversion problems in elementary school instruction may help build a bridge between arithmetic and algebra.

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