Early training: who, what, when, why, and how?

Elsbeth Stern
Erziehungswissenschaftliche Fakultät, Pädagogische Psychologie,
Universität Leipzig

1 Early mathematical training

Mathematics is known to be hard to teach and difficult to learn. It is well known that many students never get real insight into important aspects of the subject. What kind of instructional support should be given to children and at what age level systematic instruction should start is discussed most controversially. In the Piagetian framework of cognitive development, minimal demands are made on systematic instruction and social support. It is widely believed that unless children do not grow up under extremely deprived conditions, their environment offers what they need to undergo the stages of cognitive development and thereby develop cognitive structures which are preconditions for abstract reasoning such as required in mathematics. Some researchers even question the purpose of structured mathematical instruction in elementary school (Kamii, 1985).

Hans van Luit and Bernadette van de Rijt (vL&vdR) clearly take an opposite view. They claim that starting systematic instruction in mathematics at the regular elementary school at the age of 6 may be too late at least for some of the children. Therefore the authors developed the impressive AEM program and presented it to the bottom third of a representative sample of five-year old children. The reported results clearly indicated positive training effects: the trained group performed better than an untrained group with a similar initial performance level. In fact, despite their poor initial performance level, the subjects of the trained group reached the performance level of the untrained upper two third group.

However, the difference between the control group and the training group declined in the follow-up test. The control group caught up, and the experimental group approached the ceiling of the test. When evaluating a training program one has to address the question of whether the trained group only is ahead of the control group for a certain period of time. To justify the costs of the training one has to prove that the training group outperforms the control group also in the long run. Only additional follow up-studies will allow further conclusions concerning the success of the AEM training program. Moreover, one has to prove that the superiority of the training group is due to specific components of the training rather than to general practice effects. In further evaluation studies subjects of the control group should be presented with an unspecific training program. Although currently only preliminary conclusions concerning the impact of the AEM program are possible, the paper of vL&vdR clearly provides an interesting basis for discussing principle questions concerning the effects of early training programs in mathematics.

The AEM program aims at compensating individual differences in cognitive preparedness which may be responsible for the huge variance in mathematical performance already observed when children enter school. Some children can hardly count to 10 while others already have acquired basic computing skills. In the following school years, tremendous achievement differences occur despite of rather homogenous learning environments. In order to justify an early applied training program, it is necessary, although not sufficient to prove stability of interindividual differences over time. Only if the children who had performed poorly at an early age level are still disadvantaged at a later age level, training programs such as the AEM can be considered as useful instruments to improve mathematical performance for children with disadvantageous prognosis. An appropriate application of training programs presupposes knowing in advance who will have particular difficulties with the acquiring mathematical competencies later on. From research on acquiring literacy we know that one can identify children at risk as early as preschool age. Children with underdeveloped phonological awareness can be expected to have particular difficulties with acquiring reading and writing later on (Schneider, in press). Moreover, offering these children exercises such as clapping syllables or recognizing rhymes already in preschool time facilitates later acquisition of reading and writing (Bradley and Bryant, 1985). However, in case of short resources it is only useful to train children who show symptoms of dyslexia, because the great majority of children can be expected to acquire reading and writing skills at school without particular difficulties. What dyslexia is in literacy is dyscalculia in mathematics. A small percentage of children can be expected to have particular difficulties with figuring out even simple calculation problems and with developing a factual network (Lorenz, 1992). The AEM, however, does not particularly focus on children who suffer from dyscalculia. Rather, AEM was applied to the bottom third of a representative sample and therefore not only aims at improving the performance of a small group of extremely disadvantaged children. Moreover, there are principle differences between the domains of mathematics and literacy. The main aim of literacy acquisition is skill-automatization, while the aim of learning mathematics is the acquisition of advanced concepts that can be used as tools of reasoning. Automatization required in reading and writing is acquired by deliberate practicing, and despite large individual differences in learning time, all learners who do not suffer from dyslexia become experts in automatized use of letters. Acquiring automatization, however, is only a subordinate goal of teaching mathematics. Running efficient computing procedures and developing numerical networks is necessary, but in no way sufficient for acquiring expertise in mathematics. The main purpose of elementary school mathematics is to prepare students for understanding advanced concepts such as fractions or decimals.

Infancy research suggests that humans are biologically prepared for understanding numbering and addition and subtraction when faced with small sets of elements (Gelman, 1991). With a minimum of instruction, these conceptual primitives guide activities based on the cardinal function of numbers, such as counting and modeling the exchange of sets by addition and subtraction. While humans are biologically privileged in the use of cardinal numbers, advanced mathematical reasoning is based on concepts which are the result of a long-lasting cultural development. Modeling static relationships between sets such as it is the case in quantitative comparison and measurement situations or the use of non-integers requires people to give up principles that guide the use of numbers as counting instruments (Staub and Stern, in press). Children's difficulties with modeling static relationships become apparent when they are faced with arithmetical word problems dealing with the quantitative comparison (Stern and Lehrndorfer, 1992; Stern, 1993). At the latest when faced with problems dealing with algebra, fractions, or decimals one has to overcome the idea that counting is the only function of numbers and that mathematical operations always correspond to concrete actions (Stern and Mevarech, 1996).

The main focus of this paper will be on the question of how children can be supported in extending their concepts of numbers and mathematical operations in the described sense. Number conservation, undoubtedly, is an important step in developing extended mathematical competencies because children have to understand that an obvious activity of changing the spatial arrangement elements has no effect on the more abstract dimension of quantity. In this sense, number conservation is a precondition for understanding the quantitative comparison. Quantitative comparison and number conservation are among the components to be trained in the program developed by Van Luit and Van de Rijt (1996). Therefore AEM can be expected to support an extended mathematical understanding already at an early age. The longitudinal studies to be discussed in the following investigate the impact of number conservation in preschool time on elementary school children's competencies in dealing with the quantitative comparison, and moreover, the effects and knowledge and the impact of elementary school knowledge on middle grade knowledge has been researched.

2 Longitudinal development of mathematical competencies

In order to research social, motivational, and cognitive development, the longitudinal studies *logic* and *scholastic* were run at the Max-Planck-Institute for Psychological Research in Munich from 1983 to 1993 (Weinert and Schneider, in press). Among other variables not discussed here children were presented with measures of numerical and mathematical competencies and general intelligence. The 186 children of the *logic*-study entered the sample at age 3-4 and were tested in individual sessions three times a year until they reached age 12-13. In 1988, when the *logic* children entered second grade of elementary school, the scholastic-longitudinal study started. In this study about 1200 elementary school children were presented with group tests in their classrooms four times a year from grade 2 to 4. 92 children of the *scholastic* sample also participated in the logic sample. 201 children of the *scholastic*-sample were also tested in fifth and sixth grade. These children were not part of the logic sample.

2.1 The impact of preschool performance on later mathematical competencies

The following analyses present data from the 95 children who participated in the logic sample as well as in the *scholastic* sample by considering the following measures:

Number conservation: Mastering the number-conservation task means to understand that verbal expressions such as 'more than' and 'less than' refer to the number of elements of a set rather than to the spatial expansion of the elements. Thus, the number conservation task might be an indicator of early quantitative reasoning rather than of a general cognitive level. At the age of 3-4 and 5-6, children were presented with number conservation problems.

Estimation of quantities: Another measure of early quantitative abilities was the test of estimating quantities, which is part of a German test of school readiness developed by Kern (1971). Children were presented with a set of 3-9 small cubes and had to tell the size of the quantity without counting. Although this test was developed long before sophisticated theories of knowledge representation had been developed, a post hoc theoretical explanation might be that the test measures the efficiency in transforming visual information into mathematical symbols.

Word problem solving: In the scholastic sample, children were presented two times a school year with mathematical word problems differing in complexity and in the underlying situational model. *Addition and subtraction* problems were presented in grades 2-4. The one-step problems were taken from the 14 standard problems mentioned in Riley, Greeno and Heller (1983). The multiple step problems were constructed from these problems. An example of a multiple-step comparison problem is:

John has 5 marbles.

He has 3 fewer marbles than Peter has.

Peter has 2 more marbles than Susan has.

How many marbles does Susan have?

The results reported in this paper are based on scores developed for each school year by considering the following problem-types:

- six one-step and multiple-step problems dealing with the *exchange* of sets;
- four one-step and multiple-step problems dealing with the *combination* of sets;
- six one-step and multiple-step problems dealing with the *comparison* of sets;
- six one-step and multiple-step *multiplicative* word problems in grades 3-4, which either required the multiplication or the division of numbers. Some of these problems were based on advanced understanding of multiplication and division, such as the cartesian product and multiplicative comparison.

The structure of the problems and the numbers were kept constant at all measurement points, while superficial features such as names and objects were changed. The problems were presented in a booklet with four problems on each side and the children were given sufficient time to work on all problems.

3 Results

For each school year the sum score of correctly solved word problems was developed. The following analyses were conducted: *Stability of word problem solving during elementary school time.*

The results proved high stability of performance in word problem solving during preschool time (correlation between second and third grade: $r = .64$, $p < .001$; correlation between second and fourth grade: $r = 62, p < .001$; correlation between third and fourth grade: $r = .75, p < .001$). These substantial correlations indicate that the sources of individual differences in word problem solving are already established in second grade. The results suggest that already in second stable individual differences in word problem solving are observed. Therefore, the following only presents results regarding the prediction of performance on word problem solving in second grade.

3.1 The impact of mastering number conservation on age level 3-4 on word problem solving

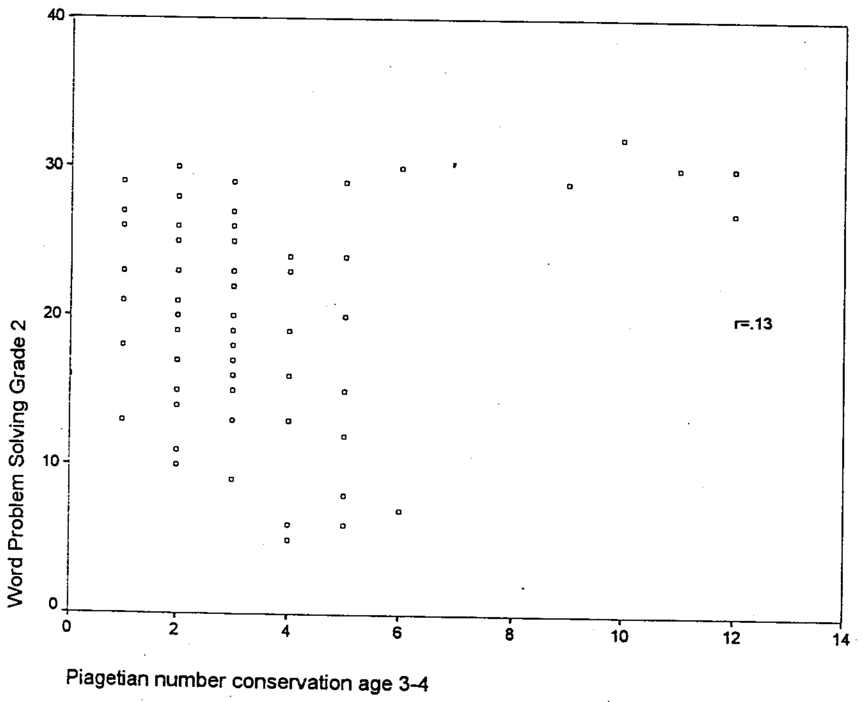

figure 1: the relationship between performance in the Piagetian Number Conservation Test at age 3-4 and word problem solving in grade 2

Figure 1 depicts the correlation coefficient and the scatter-plot between performance in word problem solving in second grade and number conservation at the age-level 3-4. The results suggest that at the age of 3-4 mastering the number conservation task is a sufficient although not a necessary precondition for high performance in word problem solving. The 5 children who had already mastered the number conservation task at this age level belonged to the group of the best word problem solvers and were ahead of their classmates during the whole elementary school time.

3.2 The impact of numerical competencies at age level 5-6 on word problem solving

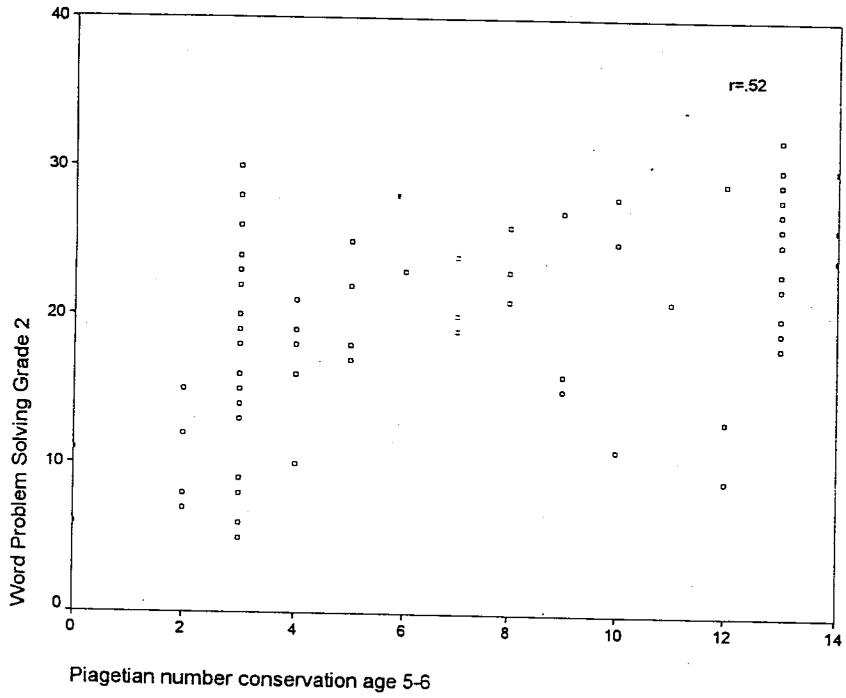

figure 2: the relationship between performance in the Piagetian number conservation test at age 5-6 and word problem solving in grade 2

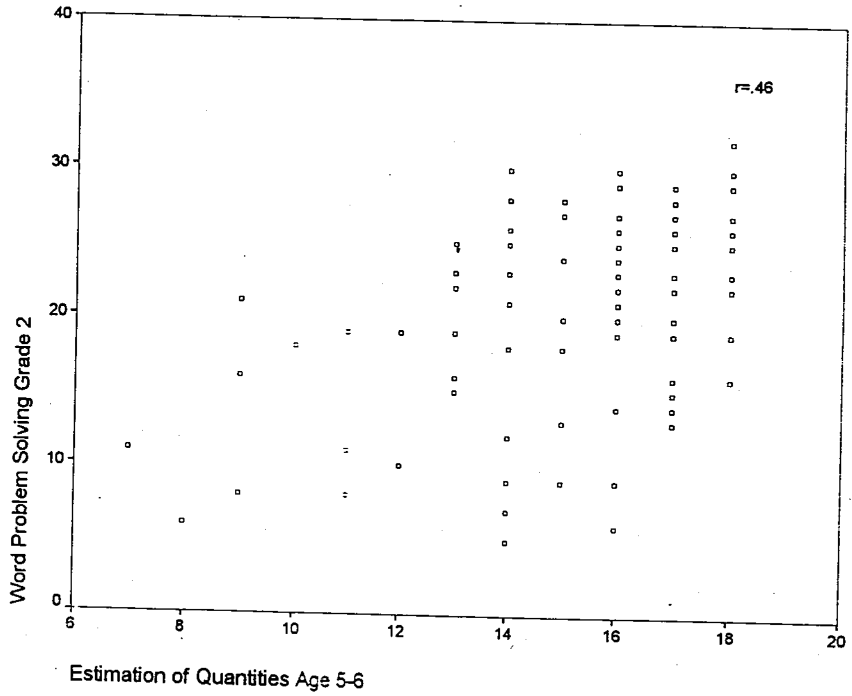

figure 3: the relationship between performance in the estimation of quantities test at age 5-6 and word problem solving in grade 2

Figure 2 and 3 depict substantial correlations between indicators of mathematical competencies at the age of 5-6 and word problem solving in second grade. However, as the plots also demonstrate, that there are many outliers. High numerical competencies do not guarantee high performance in word problem solving and many children who performed poorly in preschool measures showed above-average performance in word problem solving. None of the two measures can be considered as an appropriate indicator of identifying children at risk. However, as the correlation between the two measures is only moderate ($r = .32$, $p < .05$), combining both measures might allow to predict children at risk.

The multiple correlation between the preschool indicators of numerical competencies and word problem solving in second grade was $R = .60, p < .001$. A more detailed analysis showed that 86% of the children who were beyond average in **both** preschool measures also were beyond average in word problem solving in second grade. On the other side, 75% of the children of were above average in both preschool measures also were above average in word problem solving in second grade. The results suggest that before children enter school, individual differences of mathematical competencies are already quite stable. Elsewhere (Stern, in press) is has been shown that the high stability cannot be explained with the stability of measures of general intelligence, which were also administered in the longitudinal sample.

3.3 The impact of preschool numerical competencies of different types of word problem solving

Additional analyses were conducted to find out whether certain word problems are particularly affected by early numerical competencies. Addition and subtraction word problems dealing with the exchange, the combination and the comparison of sets presented in grade 2 and 3 were considered. To fulfil statistical preconditions, for each school year and each problem type the three problems closest to the solution rate of .50 were selected (defined criterium was .45-.55). Table 1 depicts the correlations between number conservation and estimation of quantities at age 5-6 and the three word problem types.

word problem type	number competencies	
	number conservation	estimation of quantities
comparison	.54**	.44*
exchange	.34*	.34*
combination	.36*	.29*

table 1: correlation between scores of word problem types in grade 2 and 3 and number competencies at age 5-6
**p <.001,*p <.05

Significance tests revealed that the correlation between comparison problems and number conservation was higher than the other correlations. The results suggest that performance in solving comparison problems is more affected by early number competencies than are combination and exchange problems.

Altogether the hitherto reported results suggest that early understanding of number conservation facilitates the acquisition of extended mathematical competencies in elementary school time, based on understanding mathematical symbols as instruments for representing static set relations. Children who lack basic numerical skills in preschool time can be expected to have difficulties with school mathematics.

4 The impact of mathematical competencies in elementary school on understanding advanced mathematical concepts in middle grades

The question to be addressed next concerns the stability of interindividual differences during school time. The results reported in the previous section suggest that the sources of individual differences in advanced mathematical understanding in elementary school time go back to preschool time. This paragraph analyzes the impact of performance in elementary school mathematics on extended mathematical understanding in middle grades. Students have to understand that numbers are not only used for counting but also to describe the relations between sets at the latest by middle grades. Understanding rational numbers requires giving up several principles that guided the understanding and use of natural numbers:

- While every natural number has a successor, this is not true for rational numbers. For natural numbers, there is a referent for the phrase 'the next number after one'. However, there is no referent for the phrase 'the next number after one half'.
- There is a smallest natural number but no smallest rational number.
- All natural numbers but not all rational numbers lying between two numbers can be enumerated.

From literature we know that in dealing with decimal numbers and fractions, children are particularly prone to errors and bugs (Hiebert and Wearne, 1986). By relying generally on the counting function of numbers, children conclude that larger numbers always refer to larger quantities and vice verse. Such results reflect children's difficulties with restructuring simple mathematical concepts into more advanced ones. Students who have attended mathematics instructions for years and who have acquired complex computing procedures and strategies have very restricted conceptual mathematical understanding because they have not overcome the cardinal function of numbers. However, long before being presented with problems containing fractions and decimals, children are faced with problem-situations based on number-concepts that go beyond counting. Understanding the quantitative comparison might be a first step in understanding that numbers are not only used as cardinal numbers but also as relational numbers. Therefore, word problems dealing with the comparison of sets might bridge the gap between understanding natural and non-natural numbers.

The following analysis intends to explain variance in conceptual understanding of non-natural numbers. Given that an early understanding of the quantitative comparison helps children to overcome the view that counting is the only purpose of numbers, high achievement in solving comparison problems at the beginning of elementary school is expected to be a valid predictor of later understanding fractions. To test the specific impact of knowledge genesis, measures of general intelligence presented were included in the analysis. To ensure that understanding the specific principles of quantitative comparison is not only an indicator of general mathematical abilities but does especially effect the later understanding of fractions, additional mathematical competencies were considered.

4.1 Subjects

Mathematical achievement measured in fifth grade was predicted by measures gained in second, third, and fourth grade. Two hundred and one children who entered the previously mentioned scholastic longitudinal study at the beginning of elementary school and participated until the end of sixth' grade.

4.2 Measures used as predictors

A test of non-verbal intelligence based on the culture free test of Cattell which was presented in second and fourth grade (Weiß and Osterland, 1979). The arithmetic word problems discussed in the previous section were presented. In addition, speed tests of arithmetic abilities were presented in grades 2-4. The subjects were presented with 20 problems presented on one page and were given one minute to solve as many problems as possible. The tests in grade 2 contained four pages with addition and subtraction problems with numbers up to 20, and the test presented in grade 3 and 4 contained four pages with multiplication and division problems with multiplicators and divisors smaller than 10, and addition and subtraction problems with numbers up to 100. The problems had either to be calculated or subjects had to mark whether given solutions were correct or not. By considering mathematical principles such as commutativity, performance could be improved dramatically in some problems.

4.3 Measures used as criteria

Fraction Understanding Test: This test was used to measure fifth graders' understanding of fractions. At this age level subjects had been taught some formal principles of fractional notation. The children were presented with two fractional numbers and had to choose the larger of the two (e.g. 6/7 or 6/8). Altogether, seven problems were presented and the children were allowed to work on the test for three minutes.

Multidigit Arithmetic Test: To examine the specificity of the predictors, an arithmetic test developed by Halford (1992) was presented in fifth grade. This test requires inserting the signs into numerical equations, such as '5 _ 8 _ 4 = 9'.

In order to pass this test, a rich numerical network is required that allows for the retrieval of the arithmetical relations between numbers. The children were presented with 13 problems and were given three minutes.

5 Results and discussion

Separate regression analyses were performed on the Fractions Understanding Test and on the Multidigit Arithmetic Test. The internal consistency of the predictors varied between .76 and .83. The mean solution rate as well as the variance of the Fraction Understanding Test ($M = .41$, $s = .26$) and the Multidigit Arithmetic Test ($M = .46$, $s = .20$) were alike. The purpose of the regression analysis was to the impact of general of the regression analysis are depicted in Table 2.

	Task	
Predictors	*Fraction*	*Arithmetic*
Intelligence		
Grade 2	n.s.	n.s.
Grade 4	2	n.s.
Arithmetic Tasks		
Grade 2	n.s.	25
Grade 3	n.s.	6
Grade 4	n.s.	n.s.
Word Problems Add. and Subtr.		
Exchange		
Grade 2	n.s.	2
Grade 3	n.s.	n.s.
Grade 4	n.s.	n.s.
Combination		
Grade 2	n.s.	n.s.
Grade 3	n.s.	n.s.
Grade 4	n.s.	n.s.
Comparison		
Grade 2	34	n.s.
Grade 3	9	n.s.
Grade 4	n.s.	2
Word Problems Mult. and div.		
Grade 3	n.s.	n.s.
Grade 4	4	n.s.

table 2: results of the regression analysis: percent of explained incremental variance ($p < .05$) for each predictor

In fact, the best predictor of the Fractions Understanding Test in grade 5 was the ability to solve comparison problems in grade 2. Fluid intelligence, although measured at the same time the criterium was measured, did not explain more variance than specific knowledge effects measured two years ago. The specificity of knowledge effects is supported because when predicting performance in the Multidigit Arithmetic Test, performance on comparison problems only played a minor role. Thus, understanding of comparison problems was not a general predictor of mathematical achievement, but rather was specifically related to the understanding of fractions. The results are in line with the claim that the understanding of fractions is guided by similar principles as the understanding of quantitative comparison problems. Therefore, early understanding of situations in which the function of numbers goes beyond counting facilitates later understanding of more advanced numerical concepts. It is a remarkable result that performance in comparison problems in grade 2 was a better predictor than performance on these problems in grades 3 and 4. This result suggests that children who extend their knowledge about numbers from cardinal use to relational use at an early age have a better chance to redescribe their number knowledge in a way that allows an understanding of rational numbers.

6 Final conclusions

What conclusions do the reported longitudinal results allow concerning the training program developed by Van Luit and Van de Rijt? The reported data contribute to the question of *why* it might be useful to train already preschool children in mathematical competencies. What the authors of the AEM program presuppose has been proved in the longitudinal results: before children receive structured mathematical instruction in regular first grade they already differ considerably in mathematical competencies, and these differences are amazingly stable. The results suggest that children's particular difficulties with mathematics in middle grade goes back at least partly to deficits in preschool time. Training programs that aim at compensating for individual differences at an early age level can be expected to facilitate the acquisition of school mathematics.

The reported longitudinal data revealed that number-conservation, which was a component of the training program, also was a good predictor of word problem solving in elementary school. This, of course, cannot be interpreted as a proof that training number conservation in preschool time guarantees better performance in word problem solving later on. The significant correlation between performance in two problems might go back to a common ability which itself might be rather unaffected by environmental factors. A significant interindividual stability over time is necessary although not sufficient for justifying a training program. The reported longitudinal results encourage to run additional training studies for further clarification.

Given that early training programs have long term effects the question arises of *what* to train. Van Luit and Van de Rijt have chosen eight components, some of them focussing more on the counting function of numbers, while others may support an extended understanding of numbers and mathematical operations. Further research is needed to find out, what training components are particularly helpful for raising mathematical achievement. This question cannot be addressed independent from the question of *who* needs an early training. Given most children's difficulties with mathematical problems that require going beyond the counting function of numbers and the action-based understanding of addition and subtraction, the bottom third of preschool children may not be the only ones who may profit from an early training program. While only few children might need help to master the counting function of numbers, the majority of children might gain from a training program that helps to overcome the view that counting is the only function of numbers. As in the Netherlands the majority of children enter preschool classes around the age of four, early training programs could be broadly applied. Broad application of a training program however, may be incompatible with the goal of compensating for individual differences because of the well known Matthew Effect of training programs. The Matthew Effect means that as a result of a training program variance increases because the higher the initial achievement level of a learner is the more s/he gains from a program. Therefore, when particularly aiming at the compensation for individual differences, training programs should give a start to those children who are expected to have particular difficulties with elementary school mathematics. This concerns the question of *when* - i.e. at what age level - to start with an early training program. In line with other findings, the reported longitudinal data suggest that poor performers in elementary school mathematics cannot be predicted before the age of five. Therefore, the application of compensatory training programs at an earlier age level makes no sense. However, given the reported longitudinal result according to which all children who mastered the number conservation task at a very early age level showed high achievement in elementary school mathematics, one could think about applying more number games to already to young children. In this case children who do not make appropriate progress could be identified more reliably and get a compensatory training.

The authors have developed two training programs which correspond regarding the content of the problems to be trained, but vary with respect to the method of teaching. Results revealed that the way of instruction had no effect on the mean performance rate. The question of *how* problems are presented seems to be subordinate. The authors discuss plausible reasons for why guided and structured instruction did not reveal different effects. The routine teachers have in running their own instruction style prevents them from following the instruction given by the scientists. It may be the content of the problems rather than the way of instruction that is crucial for improving mathematical competencies.

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