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Merry Bullock, Gertrude Nunner-Winkler,
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Using a Complex Rule in Different Domains: When Familiar Schemes Do Not Help

Merry Bullock*
Estonian Academy of Sciences, Tartu University
Gertrude Nunner-Winkler
Max Planck Institute for Psychological Research
Elsbeth Stern
Max Planck Institute for Human Development
Alejandro López
University of Hamburg
Albert Ziegler
University of Munich

Children's reasoning within and across content domains has been a core research area in developmental psychology. Theoretically, performance across domains can help articulate the ways in which cognitive development arises from content-independent changes in the constraints on or structure of general problem-solving competencies such as operational structures or memory capacity (Case et al., 1991; Case & Griffin, 1989; Ginsburg & Opper, 1988; Halford, 1989; Kuhn, Langer, Kohlberg, & Haan, 1977); and the ways in which it arises from modular changes in knowledge structure associated with the acquisition of domain-specific expertise (Bastien-Toniazzo, 1997; Hirshfeld & Gelman, 1994; Karmiloff-

*Merry Bullock is presently at the American Psychological Association, Washington, DC.

Smith, 1992). The extent to which cognitive performance is domain general or not has pragmatic implications for education, in particular about the degree to which pedagogy should focus on training general skills or focus on the acquisition of rich domain-specific expertise.

There is ample evidence that domain knowledge influences cognitive performance. A rich knowledge base facilitates within-domain performance in such areas as memory (e.g., Weinert & Schneider, 1995), covariation reasoning (e.g., Richardson, 1992), analogical reasoning (e.g., Gentner, Ratterman, Markman, & Kotovsky, 1995), inferences (e.g., Chi, Hutchinson, & Robin, 1989); mathematics (e.g., Gelman, 1990), and categorization (e.g., Kelemen & Bloom, 1994; see Hirshfeld & Gelman, 1994).

There are several explanations for why content effects occur. One is that general, domain-independent cognitive structures can be applied more easily to familiar content. For example, the Piagetian notion of décalage specifies that children may have more or less difficulty in applying cognitive structures (e.g., hierarchical set logic, logico-mathematical structures) depending on content.

Other explanations refer to how domain-specific knowledge or knowledge structures may facilitate reasoning (in the use of cognitive resources, or in the ease of reasoning) or how motivational differences (in preferences, motivational processes or attitudes) may affect reasoning. We elaborate on each of these here in more detail to form a framework for this chapter.

Domain-Specific Knowledge Affects Cognitive Resources. According to this explanation, reasoning performance depends on the activation of cognitive structures. The context in which a problem is embedded may elicit appropriate or inappropriate mental models or schemata that frame the abstract problem structure (Johnson-Laird, 1983). When the context elicits an appropriate mental model, it can facilitate reasoning performance by, for example, freeing cognitive resources or making the underlying structure of the problem more visible. Alternatively, when the context suggests false or inappropriate solution strategies, even a familiar context can be detrimental. For example, Stern and Mevarech (1996) showed that performance on problems testing the mathematical concept of infinite divisions was worse when embedded in concrete, familiar situational contexts than in sparse contexts, because familiar contexts led subjects to attempt to solve the problem according to less appropriate pragmatic principles.

Domain Structure and Privileged Access. Another explanation is that learning or performance may be more difficult or different in some domains than in others because information in different domains varies in its underlying structure, organization, and accessibility. For example, agency is an integral mechanism of action in the psychological domain, but not in the natural physics domain, leading to different sorts of available and possible inference structures (Leslie, 1994); analogies and inferences may be easily accessible in some domains, but only available after conscious reflection in others (Gentner et al., 1995). Several researchers (Case & Griffin, 1989; Gelman & Wellman, 1992) proposed that different underlying processes or different sorts of inferential skills underlie logico-mathematical, physical, and social domains. Others (e.g., Cosmides & Tooby, 1994) proposed that especially rich and accurate sets of inferential schemata have evolved in some domains (e.g., contexts involving social exchange such as sharing resources or detecting cheating) but not others.

Attitudes, Preferences and Motivation May Alter Reasoning Processes. The influence of preferences or attitudes on perception and thinking has been documented extensively in recent research (Johnson-Laird & Safir, 1994; Wright & Bower, 1992). Clear-cut preferences may influence the ease or difficulty with which new information can be interpreted and used. Analogously, preferences that are invested with personal commitment may be more difficult to change to accommodate new information as it becomes available. The quality of problem solving may also be affected by expectations, attitudes, and interest. Reasoning in a domain in which one has a "hot" emotional or motivational investment or a personal identification with an opinion, attitude or belief may be less flexible; or one may fail to apply appropriate reasoning schemata. Interest and motivation may also affect the acquisition of content-specific expertise or the cognitive effort invested in solving problems within a domain.

Aim of the Chapter

The purpose of the study presented in this chapter was to compare children's reasoning across different content domains to explore some of the several ways by which context differences have been explained. The domains chosen reflected those considered to be separate in the literature: mathematics, physics and socio-moral. We chose a task in the general area of proportional reasoning because our sample (see later) was in the middle childhood range. This area seemed especially well suited to our purposes, because proportional reasoning is considered a good indicator of general transitions in cognitive competencies. However, because simple proportions are usually understood by children at the ages we tested, we looked at reasoning about a more complex version of proportional reasoning, an "overproportional rule." To give an example: In a proportional rule, one must reason about a relation between two values; this relation remains constant even when the absolute sizes of the values change (e.g., sales tax is a constant proportion of the price, regardless of whether the price is $1 or $100). In contrast, in an overproportional rule, the relation between two values changes as absolute size changes (e.g., sales tax increases from a low percentage of the price for low prices to a higher percentage of the price for more expensive items). Thus, applying an overproportional rule requires comparing changes in proportions, a more complex application of a proportional rule. To test reasoning about this rule across content domains, we designed structurally identical "overproportional thinking" problems embedded in mathematics, physics and socio-moral contexts.

Design and Hypotheses

Children were tested in a repeated measures design on three isomorphic tasks tapping three different domains: mathematics, physics, and socio-moral. In each task, problems had to be solved by following an overproportional rule that was embedded within a particular context. The specific domain-contexts in which each task was embedded were as follows:

> *Mathematics (arbitrary game rules)*: A game was to be played in which black chips were assigned to boxes with varying numbers of white chips. The rules of the game specified that the number of black chips to be assigned should increase overproportionally as the number of white chips increased.
>
> *Physics (speed and momentum)*: Policemen were controlling highway safety and had to determine whether cars on crowded highways had enough distance to brake. They were told that safe braking distance can be calculated as a proportion of speed, and that the size of the proportion increases as speed increases (Note: although most driver education instructors tell their students to use a proportional rule of thumb—such as one car length for each 10 mph—an overproportional rule is indeed correct).
>
> *Socio-moral (sharing resources)*: The employees of a small company were planning a staff excursion. One of the employees could not afford the cost of the trip and the other employees wanted to help by paying for this person's trip. They decided that each person's contribution would be a percentage of their salary, determined according to an (overproportional) rule in which the size of the percentage increased as salary size increased (similar to the logic underlying progressive income tax).

Because this was an exploratory study, we made no strong *a priori* predictions about performance differences across the three domains, but we did specify several alternative hypotheses based on the different explanatory frameworks just outlined. They are the following:

No difference across the three domains: Within a strict universal structural framework (e.g., early Piagetian models), the acquisition of specific cognitive structures such as formal operational thinking are both necessary and sufficient for applying the concepts of proportions and overproportionality. Once such underlying cognitive structures are in place, the concepts should be available, and there should be no consistent, domain-related differences in applying them. According to this hypothesis, performance should be the same across the three domains.

Familiarity: Better performance in the socio-moral domain. To the extent that simple familiarity affects the application of reasoning structures, the socio-moral domain should be easier than physics or mathematics. Children are likely to be more familiar with sharing resources than with the physics of speed and momentum or with arbitrary game rules. This would predict better performance in the socio-moral context than in the mathematics or physics contexts.

Cognitive schemata: Better performance in the mathematics domain. Children may have a priori beliefs about events in the physics or socio-moral domains that may impede applying an overproportional rule. For example, even many adults assume that braking distance and speed are related proportionately. Giving up this familiar rule for another may depress performance. Analogously, there are many ways to share costs in socio-moral contexts, and children's beliefs about equity may be inconsistent with an overproportional rule. The relatively sparse mathematics context should not introduce such belief biases.

Concept structure: Better performance in the mathematics and socio-moral than in the physics context. The underlying structure of the concepts in each context may affect performance. Although we made every effort to make the tasks entirely equivalent on a surface level, the concepts vary in their complexity. Specifically, speed is a relational variable (distance–time) that cannot be expressed in terms of simpler elements. In contrast, salary or black–white chip ratios can be expressed as simpler, non–relational, absolute values, namely the amount of money made or the number of black/white chips in a given box. To the extent that concept complexity plays a role, this should depress performance in the physics context.

Preferences and attitudes: Better performance in the mathematics and physics than in the socio-moral context. Children may have preferences for how to share resources that interfere with adopting an overproportional rule, and they may differ in their personal identification with socio-moral as opposed to physics or mathematics rules. Children generally have an equality understanding of justice ("to each the same"), a justice conception that develops early in childhood, and may be resistant to change, especially if this understanding is also invested with personal significance or commitment. In addition, accepting the rules of games (mathematics) and the laws of physics are unlikely to be invested with an emotional commitment, at least within the stories used in our study.

METHOD

Participants

The data reported in this chapter were part of the Munich Longitudinal Study on the Genesis of Individual Competencies (LOGIC; Weinert & Schneider, 1999). This study began in 1984 and followed developmental changes in cognitive, moral, social, and personality factors in a cohort of nearly 200 children, who were tested several times each year from first entrance into preschool at age 4 through the completion of grammar school at ages 12 to 13 and once again at ages 16 to 17. The sample was recruited from the population of all native-(German) speaking children who enrolled during one year in each of 22 preschools, selected across a broad and representative spectrum of neighborhoods. The present data were collected in the ninth year of the study. Subjects participating in this measurement included 186 children (99 boys, 87 girls) who were an average of 12 years (+/- 6 months) old at the time of measurement.

Design and Procedure

Children were tested on isomorphic tasks in mathematics, physics and socio-moral contexts. The content of the tasks and specification of the rules in each domain are outlined in Table 7.1. The same procedure, followed for all stories, is outlined in the left column of Table 7.2, with examples from each domain provided in the other columns.

Table 7.1
Tasks in Proportional Reasoning

	Domain		
Task Components	Physical	Socio-Moral	Mathematical
Cover story	Estimating safe braking distances	Charity: colleague's participation in a trip	Game: distributing chips
Reference variable	Speed (km/hr)	Income ($/hr)	Number (white chips per box)
Critical variable	Braking distance	Contribution amount	Number black chips to be added
Rules	Laws of physics	Distribution rules	Game rules
Constant	Braking distance does not depend on speed	Contribution does not depend on income	Number of black chips to be added does not depend on white chips
Proportional	Braking distance is a constant proportion of speed	Contribution is a constant proportion of income	Number of black chips added is a constant proportion of white chips
Overproportional	Braking distance is an increasing proportion of speed	Contribution is an increasing proportion of income	Number of black chips is an increasing proportion of white chips
Friendship	---	Closer friends should give more	---
Personal discretion	---	Each should decide how much to give	---

Table 7.2
Measures in Proportional Reasoning Tasks

Question Asked	Physics Domain	Socio-Moral Domain	Mathematics Domain
Prior knowledge and preferences			
What factors influence X? and why?	X = braking distance	X = contribution amount	Not asked
Who do you think is Y?	Y = more careful	Y = more generous	---

	driver	speed (km/hr)	distance (m)	employee	wage ($/hr)	contribution ($)
choose A1 or B1	A1	40	10	A1	10.-	5.-
	B1	80	15	B1	20.-	8.-
choose A2 or B2	A2	50	12	A2	10.-	1.-
	B2	100	24	B2	50.-	5.-

Question Asked	Physics Domain	Socio-Moral Domain	Mathematics Domain
Which of the rules is ...?	**most/least likely to be true? Why?**	**best/worst? Why?**	**What do you think of "friendship rule"/ discretion rule"?**
Recognition tasks (three constant, three underproportional, three proportional, three overproportional)			
Concrete example:	The braking distance of car X at speed Y	The contribution of employee X making Y $/hr.	The number of black and white chips in box 1
Problem information:	Braking distance of car X at two different speeds	The contributions and hourly wages of two other employees	The number of black chips put into two other boxes with a given numbers of white chips

Response: Does the information conform	CC = braking distance	CC = contribution	CC = number of black chips added to the rule?
Production tasks (4)			
Concrete example:	Braking distance of car X at speed Y	Contribution of employee X making Y $/hr.	Number of white chips in Box 1
Then you are to follow the overproportional rule to . . .	Check which braking distances are minimally needed at the other speeds given	Check those contributions made by higher income employees	Check the numbers of black chips to be put in other boxes with given numbers of white chips that follow Rule 3

Recognition of similarities

"You've been given different tasks this afternoon—one with cars, one with collecting money, one involving a game with chips. Did you think these tasks were alike in any way? How were they alike?"

The procedure in each domain began with a cover story with accompanying illustrations in which participants were introduced to the general domain context and relevant dimensions. Next, children's spontaneous rule preferences were elicited for the physics and socio-moral domains. To do this, a series of illustrations representing each of the different rules described in Table 7.1 was shown, and children were asked to rank order which rule they believed applied in the context from the "best" to "worst." In the physics domain, the rules relating speed and braking distance included a constant rule, a proportional rule, and an overproportional rule. In the socio-moral domain, the rules relating salary and donation included these rules, as well as a "friendship" rule (people should donate more to their friends) and an "own discretion" rule (people should donate as they see fit). Next, children were asked three control questions testing their ability to apply elementary constant, proportional, and overproportional rules, and feedback was provided if needed (nearly all children were correct on these control questions).

Then participants were instructed that the overproportional rule was the correct rule to use, and were asked to use it for the remaining problems in the task. The central measures consisted of a series of recognition and production problems.

Recognition Problems. These problems required children to choose examples that fit an overproportional rule. The problems were identical in structure for all three domains, with numerical values chosen to be equivalent in difficulty level, but to realistically fit the different contexts. For physics, speeds varied between 20 and 100 km per hour; for socio-moral, wages varied between $10 and $50 per hour (only integers were used); for mathematics, there were no number constraints, so a mixture of values used in the moral and physics domains was chosen. For each problem, children were shown an initial example with a specific numerical relation (e.g., "at 20 km per hour a particular car required 3 meters to brake"). Then they were shown two additional examples of the same car and asked to compare them with the initial example to see whether these followed the overproportional rule (see Fig. 7.1a. for an illustration of one overproportional recognition problem). The 12 recognition problems in each domain included 4 that showed a constant relation, 4 with an underproportional relation, 4 with a proportional relation, and 4 with an overproportional relation. Correct use of the overproportional rule required rejecting the constant, underproportional and proportional examples, and accepting the overproportional examples.

Production/Selection Problems. The production/selection problems were also identical in basic structure across domains. Children

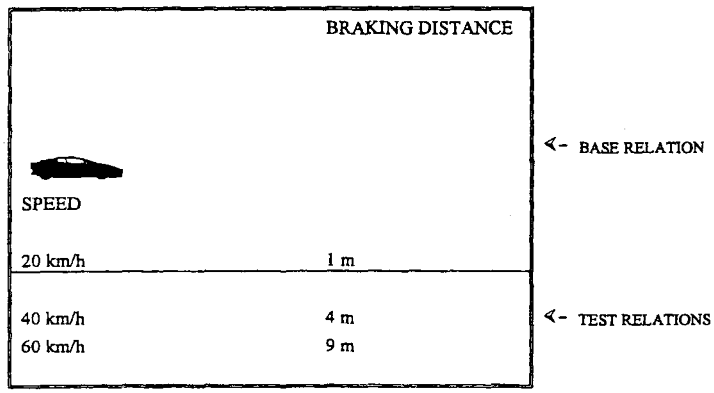

FIG. 7.1a. Recognition Task (overproportional).

were shown an initial example (e.g., at 40 km per hour a car needs 5 meters to brake) and then given two test problems. In each problem, they were given a base amount (e.g., 60 km per hour, 100 km per hour), and asked to indicate which of several values (e.g., braking distances) followed an overproportional rule (see example in Fig. 7.1b). There were four production problems for each domain.

The order of presentation of problems in the three domains was determined as follows: the physics and socio-moral tasks were presented first or last in counterbalanced order, and the mathematics task was always presented in the middle. Presentation of the task in each domain lasted about 7 minutes. To maximize the spacing between tasks across the three domains, testing was interspersed among other activities presented during the same 2 1/2-hour session in the LOGIC longitudinal study. After the last domain had been presented, children were asked whether they had seen any commonalties among the tasks across content areas and what those commonalties might be.

RESULTS

Preliminary analyses of variance showed that there were no effects due to order (physics or socio-moral presented first), and no gender differences across any of the performance measures. These variables are not discussed further.

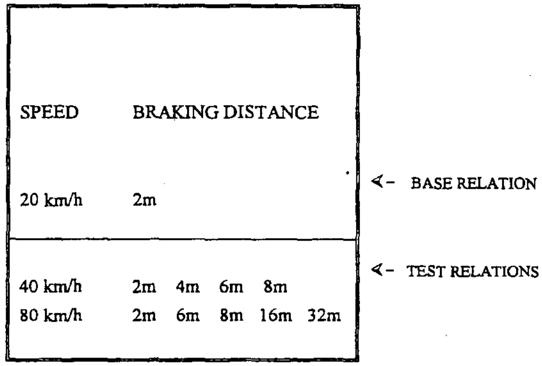

FIG. 7.1b. Production task.

Recognition Task Performance. Recognition tasks in each domain included four tests of each of four relation types (constant, underproportional, proportional, overproportional). Performance was at ceiling on the constant relation problems (correctly rejected: mathematics 95%, physics 98%, socio-moral 97%) and close to ceiling on the underproportional relation problems (correctly rejected: mathematics 85%, physics 86%, socio-moral 82%), with no differences across domains. More errors were evident on the proportional relation problems (correctly rejected: mathematics 63%, physics 58%, socio-moral 64%) and overproportional relation (correctly accepted: mathematics 74%, physics 59%, socio-moral 45%) problems.

Our central question was whether children would use an overproportional rule. To be scored as using an overproportional rule children had to both accept examples following the rule, and also reject the examples of other rules (constant, underproportional, and proportional) as incorrect. To classify rule use, we created a Guttman-type of scale with the categories "No Clear Rule Use," "Proportional Rule Use," and "Overproportional Rule Use," Children classified as "No Clear Rule Use" did not solve either the proportional or overproportional problems. Children classified as "Proportional Rule Use" rejected the constant and underproportional examples, but (incorrectly) accepted the proportional problems as correct. Children classified as "Overproportional Rule Use" solved all problems correctly (rejected the constant, underproportional and proportional examples; accepted the overproportional examples). In scoring each rule type, we used a criterion of at least two of three consistent answers. Table 7.3 shows the distribution of children in each of these three categories.

As Table 7.3 shows, rule use varied as a function of domain. Repeated measures analysis of variance showed domain differences in the use of an overproportional rule, $F(2,368) = 10.59$, $p < .001$; post-hoc tests showed less "Overproportional Rule Use" in the socio-moral domain than in the mathematics and physics domains, which did not differ. "No Clear Rule Use" was highest in the mathematics context, with no differences in the physics or socio-moral contexts, $F(2,368) = 23.28$, $p < .001$). This pattern suggests that those children who were weaker in applying the formal rule (shown by poorer performance in the sparse Mathematics context) used more complex strategies when the problem was embedded in a more everyday context, a facilitation that was not mirrored for those children who could apply the formal rule in the sparse context.

Production Task. Children were given 1 point for each correct answer on the production tasks (for a maximum of 4 points). The mean scores across domains were 2.2 for mathematics, 2.1 for physics, and 1.7 for socio-moral. Repeated measures analysis of variance showed that these scores differed across domain, $F(2,183) = 6.96$, $p < .001$. Similar to the recognition tasks, post-hoc comparisons showed that performance in the socio-moral domain was worse than in the other two domains, which did not differ.

Effects of Prior Beliefs. Table 7.4 depicts children's answers to the initial questions concerning which rule best described the physics (speed and braking distance) and socio-moral (income and charitable donation) events, indicating their beliefs and preferences before hearing the story task and being instructed to use an overproportional rule. Few

Table 7.3
Percentage of Children Classified According to Best Rule Use

	Domain		
	Mathematics	Socio-Moral	Physics
No Clear Rule	35	12	15
Proportional Rule	15	57	42
Overproportional Rule	50	31	43

Table 7.4
Children's Initial Rule Preferences in the Socio-Moral and Physics Domains

Domain	Rule Type	Proportion Preferring Rule
Socio-moral	Constant rule: (contribution from each person should be the same)	15.1
	Proportional rule: (contribution should be a constant proportion of salary)	36.8
	Overproportional rule: (contribution should be an increasing proportion of salary as salary increases)	5.4
	Friendship: (contribution should depend on closeness of friendship)	4.9
	Personal discretion: (contribution should be individually decided)	37.8
Physics	Constant rule: (Braking distance does not depend on speed)	2.1
	Proportional rule: (Braking distance is a constant proportion of speed)	74.5
	Overproportional rule: (Braking distance increases at accelerating rate as speed increases)	23.2

children in either domain spontaneously picked the overproportional rule to describe the relation between variables. The proportional rule was chosen by most children in the physics domain. In the socio-moral domain, there was less consistency in prior preferences, but close to two-thirds of the children chose a rule other than a proportional or overproportional rule. We noted that those children who preferred rules other than proportional or overproportional in the socio-moral domain often justified these choices as an explicit rejection of overproportional rules (e.g., "those who earn more, have to work more, and if they have to pay more, they are exploited"; "one should not be forced to pay so much"). This suggested that children's poorer performance in the structurally similar socio-moral task might have arisen because of a belief that this rule was unjust, not a failure in understanding.

To further explore this possibility, we derived a score to measure explicit rejection of overproportional sharing concepts. A "Rejection of Overproportional Sharing" score was calculated by giving 1 point for each of the following responses (a) selecting the overproportional rule as the worst among the five sharing rules proposed, (b) including evidence of understanding the formal structure of overproportionality (e.g., "those who earn twice as much have to give more than twice as much") in justifications for rejection of the overproportional rule, (c) mentioning "fairness as equality" concerns (e.g., "it is not fair that some should have to give more"; "it would not be fair if they had to give extra more"), and (d) rejecting the friendship rule by referring to strict equality concerns (e.g., "those who are close friends would be disadvantaged if they had to contribute more"). We then asked whether this score predicted performance of those children who failed to use an overproportional rule in the socio-moral domain. To do this, we compared the "Rejection of the Overproportional Rule" score for different performance patterns across the three domains. We summarized children's performance across all three domains as follows:

- Overproportional in all domains, that is, overproportional rule use in the socio-moral, mathematics, and physics domains: (S+ / M+ / P+).
- Overproportional in at least the socio-moral domain, that is, overproportional rule use in the socio-moral domain only or in the socio-moral domain and one of either the mathematics or physics domains: (S+ / M+ or P+) or (S+ / M- and P-).
- Overproportional only in mathematics and physics domains, that is, no overproportional rule use in the socio-moral domain: (S- / M+ / P+).
- Overproportional only in either mathematics or physics, that is, overproportional rule use in one of the mathematics or physics domains, and not in the socio-moral domain: (S- / M-, P+) or (S- / M+, P-).
- Not overproportional, that is, no overproportional rule use: (S- / M- / P-).

The means and standard deviations of the rejection score for each group are depicted in Table 7.5. There was a significant rule group effect, $F(4,180) = 3.99$, $p < .001$. Post-hoc tests showed that the scores of the "Overproportional Only in Mathematics *and* Physics" and the "Overproportional Only in Mathematics *or* Physics" groups were significantly higher than the scores of the other three groups, $p < .05$. Thus, children who explicitly rejected the overproportional sharing rule had difficulties in accessing the appropriate mathematical knowledge when presented with the socio-moral task, although they could use this knowledge for the physics and/or mathematics problems.

Table 7.5
Means and Standard Deviations of the "Rejection of the Overproportional Sharing Rule" Score for Each Performance Group

Group	N	M	s
Overproportional all	30	1.90	.66
Overproportional socio-moral	27	1.96	.85
Overproportional mathematics and physics	21	2.57	1.07
Overproportional mathematics or physics	51	2.21	.90
Not overproportional	56	1.83	.73

Recognition of Similarity Across Domains. After children had been presented tasks in all three domains (note: these tasks were separated over a 1-hour testing period that included other activities), they were reminded about the tasks concerning "sharing," "cars," and "chips" and were asked whether they had noted any similarities among them. If so, they were asked to articulate what these differences might be. In posing this question, we were interested in whether children explicitly recognized that the tasks had a similar underlying structure, and whether they had noted the mathematical similarity. Nearly all children (92%) said that there was some similarity among the tasks. When explaining what this similarity was, 61% referred to superficial similarities (e.g., "there were stories"; "you had to figure things out") and 39% referred to structural or mathematical similarities ("there were always the same rules: first all equal, second regularly increasing, and third always more increasing"). Correlations between the children's explanation of similarity in mathematical terms and overall performance scores (the percentage of correct problems overall) were small but significant for the mathematics and physics domains, $p = .25$ ($p < .01$) and .24 ($p < .01$), and not significant $p = .08$ (ns) for the socio-moral domain.

Correlations Between Reasoning Task Measures and Other Tasks. Because the overproportional reasoning tasks were embedded within the larger LOGIC study, we had available other concurrent information assessing specific and general cognitive skills. Table 7.6 shows correlations between children's overall overproportional performance score and these other skills, which included verbal (Hamburg–Wechsler Scale) and nonverbal (Culture-Fair Picture Test) intelligence, scientific thinking (a composite score reflecting understanding of experimental design), mathematics (word problems), operational thinking (Arlin,

Table 7.6
Correlations Between Rule Use Performance and Concurrent Cognitive Measures

Intelligence—Verbal	.50
Intelligence—Nonverbal	.41
Scientific thinking	.42
Math word problems	.40
Operational thinking	.38
Proportional subtest	.29

1984, a paper-and-pencil test measuring the transition to formal operations), and proportional reasoning (a subtest from the operational thinking task that included four problems testing proportional reasoning), arranged in order of those skills most generally to most specifically related to proportional thinking. Surprisingly, the more general measures better predicted rule use than measures more directly related to the rules tested (e.g., the proportional reasoning subtest from a paper-and-pencil task of formal reasoning). This further supports the notion that the ability to apply complex, formal rules is highly context dependent (the context addressed in the proportional reasoning subtest involved spatial concepts).

DISCUSSION

We designed the overproportional reasoning task to provide information relevant to several hypotheses concerning cross-domain performance differences: (a) no difference across domains, because solving complex problems requires a set of general cognitive capacities; (b) better performance in a more familiar domain (socio-moral); (c) better performance in a neutral context (mathematics game) for which there should be no a priori expectations; (d) better performance in domains with less complex concepts (mathematics and socio-moral) than in domains with more complex concepts (Physics); and (e) better performance in domains with affectively "neutral" concepts (mathematics and physics) than in domains with concepts bound with personal commitment (socio-moral concepts of equality). Overall, the data supported the last hypothesis.

Approximately half the children showed correct overproportional rule use in the mathematics context, a measure we interpret to show the base measure of rule understanding. Although not statistically significant, performance was somewhat worse in the physics domain, and it was significantly depressed in the socio-moral domain. Why was performance worse in the more context-rich domains? We suggest that children's failure to apply an overproportional rule across all domains resulted from the strength of their prior preferences, rather than from cognitive deficits. Specifically, the slightly depressed performance in the physics domain reflected children's difficulty in rejecting the preferred proportional relation rule, and the significantly depressed performance in the socio-moral domain reflected children's difficulties in using a rule that contradicted their conceptions of equality in sharing. This conclusion is also supported by the small but significant correlations between children's descriptions of cross-task similarities in mathematical terms with performance in the mathematics and physics domains but not with the socio-moral domain, and by the higher scores on the "Rejection of Overproportional Sharing" for those who failed to use overproportional rules in the socio-moral but not mathematics or physics domains. Thus, children's performance was strongly affected not only by their preference for different rules, but by the strength of that preference. Those children who had stronger preferences for socio-moral equality were less likely to use a rule that differed with these preferences in this domain.

Overall, the data support the conclusion that children's reasoning performance was based on two factors: understanding the overproportional rule in a mathematical manner, and being able to set aside prior beliefs or preferences about a particular content to use that rule. Our results also underscore the idea that cognitive change is gradual and variable during transition phases. That is, in a transition phase, whether or not a new rule is used is dependent on the problem context. Microgenetic studies on the process of cognitive change show that newly acquired strategies are applied continuously in familiar contexts only after a period of practice, and strategy generalization is even more protracted (Siegler, 1995; Siegler & Stern, in press).

Although most children had less difficulty in applying the newly acquired overproportional rule in the more sparse problem context (mathematics) than in a context that involved established personal attitudes and preferences, context effects for other children showed the opposite pattern. More children were assigned the category of "No Clear Rule Use" in the mathematics domain than in the other contexts, indicating that application of the basic problem structure was not available. These same children often used a proportional rule in the socio-moral and physics contexts. Although this still resulted in a wrong answer, it was at least a systematic solution approach. Therefore, when those children who performed poorly in the mathematics context were presented with a richer problem context, they showed more advanced reasoning than when the problem was embedded in a sparse context. The domain-specific knowledge presented in the richer contexts may have aided children in keeping track of the information when constructing a problem representation (Anderson, Reder, & Simon, 1996).

Overall, the results underscore the importance of task demands in the assessment of children's competence as well as in designing learning environments. Moreover, showing that attitudes, beliefs, and expectations have an impact on reasoning broadens the range of dimensions defined as *task demands* beyond the richness of the context or the complexity of the concepts.

Our results contribute to the current educational debate about "situated learning," a perspective that addresses content-specific effects on the acquisition and the use of knowledge. The findings from many experimental studies that structurally isomorphic problems can differ considerably in difficulty, and the failure of many training studies to show knowledge transfer even on very similar new problems, have led proponents of this perspective to question the effectiveness of conventional instruction (see especially Lave, 1988; Lave & Wenger, 1991). Lave, and other advocates of the situated learning view, criticize learning environments in which students work independently under a teacher's supervision on sparse and formal problems, arguing that the best one learns under these conditions is how to fulfill school requirements, not how to acquire problem-solving skills with real-life relevance. According to this view, learning has to be situated in social environments that provide the opportunity to work on complex problems grounded in authentic everyday experience for effective learning to take place.

The detrimental effects of the sparse mathematical context on poor performers might be interpreted as consistent with this view: Some children develop systematic solution approaches only within richer, relevant problem contexts. In arbitrary contexts, they loose track of the relevant aspects of the situation and apply unsystematic and arbitrary procedures.

Proponents of the situated learning perspective claim that children are unlikely to use or understand complex rules such as the overproportional rule when taught in abstract mathematical contexts (as in the LOGIC study). Rather, they should be given the opportunity to discuss these rules and their application with partners of equal status. However, given the results on the effects of personal attitudes on reasoning, there is good reason to suspect that social environments may not always provide better learning opportunities. In collaborative learning settings, solution strategies often have to be negotiated between group members, who all try to defend their own position against others. However, this strategy may fail when children's reasoning is guided by strong personal preferences, as was the case in our socio-moral task. Children might be reluctant to give up their preferred, but incorrect solution strategies because there is a strong tendency to retain beliefs invested with personal commitment (Chambers, 1995). Individual learning settings might be more suitable for some problems, to ensure that appropriate knowledge rather than personal attitudes guides student reasoning.

According to the situated cognition view, the conventional method of breaking a complex problem down into its single components and practicing these parts separately is resistant to transfer. To prepare learners for later, non-school problem solving, practice has to involve complex and authentic problems. Thus, proponents of this view would argue that when a mathematical story problem is reduced to its formal structure, students learn how to manipulate formulas but not how to deal with analogous complex problems that require the consideration of several dimensions. In contrast, our results suggest that reducing a problem to its formal gist may be particularly important for extending the understanding of situations for which informal solution patterns already exist. If children who passed the mathematical and the physical tasks but failed to solve the socio-moral task were to be taught the overproportional rule of sharing obligations in social settings, explaining the formal analogy between this task and the mathematical or physical task would be the obvious method of instruction. That this might be a promising method is suggested by our finding that many children recognized the formal similarity between the three tasks, and that this recognition was related to improved performance.

CONCLUSIONS

To summarize, this chapter presents information from a study on children's reasoning using a complex rule, tested across three content domains. Analyses of performance within and across domains suggests a range of content-related context effects. Sparse content can inhibit the application of systematic reasoning strategies; content for which children have some specific knowledge can interfere with applying a new rule by competing with preferred strategies, or with prior attitudes or preferences. Overall, our results suggest that the ability to learn and apply a new, complex concept competes with knowledge about familiar, simpler concepts (e.g., proportional rules), and that it is especially difficult to reject these more familiar concepts for new ones if they are in accord with personal attitudes. The pedagogical implications of this are that instruction should not only introduce new concepts but also offer practice in applying those concepts in contexts that are both sparse and rich, and that require flexibility in using rules that do and do not agree with one's personal preferences.

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