# 10 The Development of Mathematical Competencies

## *Sources of Individual Differences and Their Developmental Trajectories*

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## 10.1 INTRODUCTION

Children's ease in learning to count in the low-number range and to model changes in set size by addition and subtraction contrasts sharply with the well-documented difficulties many children have when learning mathematics in school. What are the reasons for this discrepancy?

Results from infant research have provided abundant evidence for the modularized bases of mathematical competencies. Transfer of mathematical language to situations in the perceivable world is facilitated by intuitive, universal mathematical knowledge. A set of objects or events may be quantified by counting, and an increase or decrease in set size can be modeled by addition and subtraction. In the course of cultural development, complex mathematical concepts that have been crucial for technological and scientific progress have been elaborated from simple mathematical principles, often combined with certain means of visual-graphical representation. Despite the overwhelming importance of mathematics to modern society, the vast majority of people nonetheless find it challenging to acquire mathematical competencies. At all age levels, it is a particular challenge for students to solve complex problems embedded in new contexts, for which they have no ready solution.

In this chapter, I will describe individual differences in mathematical competencies from immediately before the LOGIC participants entered school (age 6) to adulthood (age 23). In the first part of the chapter, I will address the kind of competencies children acquire before they eventually approach an advanced level of mathematical understanding based on problems presented to the participants in the LOGIC study. The second part of the chapter will address the development of individual differences. Questions will include the age at which individual differences become stable and the impact of sex and general cognitive capabilities on mathematics achievement.

## 10.2 UNIVERSAL TRANSITIONS IN MATHEMATICS: FROM INTUITIVE QUANTIFICATION TO SYMBOL-BASED REASONING

If young children are shown two sets of objects, one with three objects and one with four, they already know which set contains more objects. Similarly, when young children are asked to compare a set with 10 objects to one with 40 objects, it will be obvious to them which set contains more objects. On the other hand, if asked to compare a randomly organized set of 103 objects with a randomly organized set of 104 objects, even adults are unlikely to spontaneously tell which set is larger. Mastering this task requires access to culturally based counting tools, whereas the first two tasks can be solved on a perceptual basis available to all normally developing human beings (Baroody & Lai, 2007). Results from infant research have suggested competencies both for quantitatively comparing small quantities and for estimating large quantities. There is evidence that the small-set and large-set quantitative systems initially work independently of each other, but become integrated as a uniform system of quantitative reasoning through the use of number symbols (Feigenson, Dehaene, & Spelke, 2004).

All cultures, even preliterate ones, have number words. However, having specific symbols for numbers is not present among all cultures, even those with script. The Arabic place-value number system, which is now common in most parts of the world, was developed only about 1,000 years ago. Our present decimal system was possible only after the number zero had made its way from India via the Arabic countries to Europe. Even the most intelligent inhabitant of the Roman world would not have been able to solve the problem CIV / XXVI = __, whereas today an average elementary school child will easily find an answer for 104 / 26 = ____. The Roman number system, although appropriate for quantification, only allowed restricted computation. It was superseded by the Arabic system centuries ago. It was the Arabic number system that opened the pathway to advanced academic mathematics. The core contents of the mathematics curriculum in secondary higher education, such as calculus, were developed less than 3 centuries ago. It thus comes as no surprise that teachers are challenged by the fact that young people in modern schools have to acquire knowledge within a few years that has developed over centuries by genius minds with tremendous effort.

There is a long tradition in mathematics education that proscribes practicing problems followed by feedback as the most effective way of learning mathematics. Only recently, this approach has been complemented by debates about what kinds of problems are most helpful at what age levels, how practice problems should be organized, and what degree of teacher support should be provided at what level of competence. Following constructivist models of learning, it is now



widely accepted that students benefit from working on problems for which they have no ready solution, but for which they have to construct a new strategy out of already available elements of knowledge. Even if students do not ultimately solve a problem, trying to find their own solution in working on the problem allows them to activate already available knowledge and to build on it. Teachers thus have an opportunity to involve students in deliberate cognitive activities that help them to extend and restructure their already available knowledge.

The findings from the LOGIC study support this constructivist approach. Staub and Stern (2002) showed that there were higher achievement gains in word problem solving for students who attended classrooms with teachers who held a more constructivist view of learning mathematics. Teacher's views of learning were assessed with a questionnaire from Fennema, Carpenter, and Loef (1990) that contained 48 statements about children's best way of learning word problems. Teachers expressed their degree of agreement with the statement made in the item on a Likert scale. Half of the items expressed a more constructivist view, such as "Children learn math best by figuring out for themselves the ways to find answers to simple word problems" or "Children should have many informal experiences solving simple word problems before they are expected to memorize number facts." The other half of the items expressed a more direct transmission view, such as "An effective teacher demonstrates the right way to do a word problem" or "Time should be spent practicing computational procedures before children are expected to understand the procedures." Hierarchical linear model analyses showed a remarkable impact of teachers on students' achievement trajectories—teachers' attitudes accounted for more than 25% of the between-classroom variance in achievement growth in mathematics.

During childhood, mathematical cognition undergoes a number of transitions before students are ready to understand advanced mathematical concepts. Most importantly, children must become aware of the dual function of mathematical language: Mathematical symbols can be used as instruments of reasoning for describing real-world situations (signifier function), and they can be used as objects of reasoning because they possess a meaning in themselves. For instance, in its signifier function, the number 9 is a symbol that refers to a set of nine objects. In its signifier function, 9 may mean *root of 81* or *square of 3.* This distinction is compatible with the distinction made by Geary (2005) between primary and secondary cognitive abilities. Related to the field of mathematics, *primary cognitive abilities* refer to the counting function of numbers. Learning to count is biologically prepared and does not need professional support. In contrast, secondary cognitive abilities required for academic learning are based on cultural transformations and therefore require schooling. Learning the signifier function of numbers (as tools) is based on primary abilities, whereas understanding the signifier function is based on secondary cognitive abilities. Thus, mathematical symbols both represent parts of the perceivable world and themselves become objects of reasoning, allowing the construction of concepts that have no direct relation to the perceivable world. For example, *infinity* is a genuine mathematical concept that contrasts our experience with the material world, where everything is limited. Despite its high degree of abstractness, elementary school children already understand the concept of infinity because they can reenact that numbers can be indefinitely increased by just adding 1 to a newly generated number. Because data sampling for the LOGIC study started before the participants entered school, the transition from primary to secondary mathematical abilities took place within this time period. A question of interest is the extent to which individual differences in academic mathematical competencies can be traced back to basic aspects of mathematical understanding measured before children entered school. In this chapter, I will ask how measures of mathematical competence collected over the course of preschool and school mathematics tasks predict later mathematical competencies at age 18 and at age 23.

## 10.3 DEVELOPMENT TESTS OF MATHEMATICAL COMPETENCIES BETWEEN AGE 6 AND AGE 12

LOGIC participants were presented with tasks measuring mathematical competencies during the entire period of data collection. All tasks measured more general mathematical reasoning rather than direct outcomes of school instruction. We chose one measure to represent children's mathematical knowledge at each of four ages during the LOGIC study—6, 8, 10, and 12 years—to compare with later mathematical competencies in late adolescence and adulthood. They are described in the following sections.

### 10.3.1 Number Sense at the Age of 6

Around their fourth birthday, children increasingly focus on the exact number of objects or events when faced with quantitative problems. Hannula and Lehtinen (2005) showed that an early spontaneous focus on numerosity was a good predictor for mathematical skills in early school grades. Two tasks given to children in the LOGIC study before entering school measured similar abilities: a number conservation task and an estimation of quantities task. The first task requires the children to understand that the quantity of objects rather than their spatial spread is decisive for answering questions referring to a quantitative comparison ("more" or "less" questions). The second task requires children to estimate the quantity of sets of objects without counting. Estimating quantities, therefore, might be a first step toward an advanced number concept. Each task is illustrated in Figure 10.1. The tasks were presented when LOGIC participants were 5 to 6 years old, which was in the year before they entered school. At this time, 56% of the children mastered the number invariance task reliably. The mean number of estimated objects was 5.2.

### 10.3.2 Understanding Quantitative Comparisons at Age 8

Although most children are able to add and to subtract numbers before they enter school, their limited arithmetic understanding becomes apparent when they are



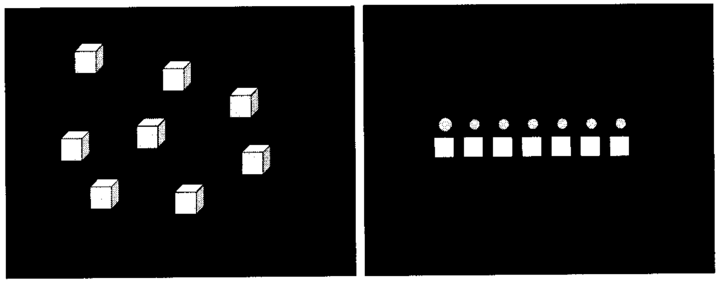

**FIGURE 10.1** Material for measuring early number sense: the number estimation task (left) and number invariance task (right).

asked to solve arithmetic word problems. Whereas about 90% of 6-year-old children solve exchange problems such as "Mary had 8 marbles. Then she gave John 3 marbles. How many marbles does Mary have now?" only 20% of the same children can solve the comparison problem "Mary has 8 marbles. She has 3 more marbles than John. How many marbles does John have?" (Stern & Lehrndorfer, 1992). The discrepancy between solution rates for problems with an isomorphic structure is highlighted by the following example. Eighty percent of preschoolers solve the following problem: "5 birds are hungry. They find 3 worms. How many birds won't get a worm?" However, if the problem ends with the comparison question "How many more birds than worms are there?" the solution rate drops below 30% even for children through grade 3.

Children's inability to solve problems dealing with comparisons of sets is because they require a more sophisticated understanding of numbers that goes beyond their counting function (Stern, 1993). In the sentence "John has 5 more marbles than Peter," the information does not refer to a concrete, existing set but rather describes the *relation* between two sets. To solve comparison problems, children need to construct a mental representation abstracted from the concrete objects of the situation described. If they are unable to associate the number 5 with anything but five objects, they will fail to understand the sentence. However, if they can represent 5 as a section of a number line representing the relation between two numbers (e.g., between 0 and 5, between 2 and 7, or between 4 and 9), they will be able to understand comparison tasks.

There is wide variation in the extent to which children are exposed to such problems in school, as shown by comparisons between mathematics textbooks for elementary school children of different nations. Although quantitative comparison problems such as "John and Peter have 12 marbles altogether. John has 2 marbles less than Peter. How many marbles does Peter have?" appear quite frequently in Japanese textbooks, they are rarely found in U.S. textbooks (Fuson, Stigler, & Bartsch, 1988). Comparisons between Eastern European countries (the former Soviet Union, Slovakia, and the former East Germany [GDR]) and the former West Germany revealed similar results (Stern, 2005). Fewer than 4% of the word problems in the textbooks used in the classrooms of the LOGIC participants dealt with quantitative comparisons. Classroom observation revealed that a large amount of time was spent on training number facts in multiplication and on written subtraction and division. LOGIC participants did not have many opportunities in school for extending their number understanding through practicing problems based on quantitative comparisons. Many children were nevertheless able to solve such kinds of problems, even complex ones.

At the age of 8, when most of the LOGIC participants were in grade 2, they were presented 10 comparison word problems that varied in difficulty. The easiest problems were simple comparisons, such as "John has 5 rabbits. Peter has 3 rabbits more than John. How many rabbits does Peter have?" The most difficult problem was a multistep problem: "John has 5 rabbits. He has 3 rabbits more than Peter. Susanne has 6 rabbits. She has 2 rabbits less than Cordula. How many rabbits do Peter and Cordula have altogether?" There was broad individual variation: Solution rates ranged from .06 to .82, with a mean rate of .52 ($SD$ = .23).

### 10.3.3 Beyond Dividing Cakes and Repeated Addition: Advanced Understanding of Division and Multiplication at Age 10

Similar to word problems requiring addition and subtraction of numbers, word problems that require multiplication or division can be based on different situational models (Greer, 1992). Several studies have shown that it is relatively easy to understand multiplication as repeated addition ("3 boys had 4 marbles each. How many marbles did they have altogether?") and to understand division as equal sharing ("12 cookies were divided equally among 3 girls. How many did each get?"). In contrast, the literature shows that situations that are based on quantitative measures ("In a photograph, a car is 4 cm long. If the photograph is enlarged by a factor of 2, how long will the car be?") or that require an understanding of the Cartesian product (e.g., "Mary has 4 pairs of trousers and 3 T-shirts. How many outfits can she create?") present severe difficulties. Solution rates for word problems with the same underlying equation vary between about 90% for problems that can be understood as repeated addition and 10% for mathematically isomorphic problems that require an understanding of the Cartesian product (for 10 year olds; Greer, 1992).

At the age of 10, when most of the LOGIC participants were in grade 4, they were presented with 10 written word problems dealing with the multiplication and division of two or more numbers. Three of these problems dealt with the Cartesian product. The mean solution rate was .43 ($SD$ = .19).

### 10.3.4 A New View to Numbers: Proportional Reasoning and Understanding Fractions at Age 12

Understanding rational numbers such as fractions requires moving beyond principles that underlie the understanding of natural numbers. Although every natural number has a next larger successor, this is not true for rational numbers. For example, although there is a referent for *the next number after one*, there is no referent for *the next number after one half.* There is a smallest natural number but no smallest rational number, and although all natural numbers that lie between two numbers can be enumerated, this does not hold for rational numbers. Natural numbers get their meaning by denominating sets of objects, whereas rational numbers get their meaning by purely symbolic manipulations. Adolescents' difficulties with understanding proportions, decimal numbers, and fractions are well documented. The most frequent mistake made is that larger numerals are considered to refer to larger values than smaller numerals. Thus, because nine is larger than eight, children have difficulties with understanding that 7/8 is larger than 7/9.

LOGIC participants were presented with classical proportional reasoning tasks, among them mixing juice. They had to compare which of two beverages made from glasses of raspberry juice and glasses of water would taste more intensive. Depending on the numbers chosen, children can use different kinds of strategies to find the right answer (Stern, 1999). At the age of 12, when most of the children were in grade 6, they used three strategies: A *bidimensional comparison strategy* is based on a comparison between water and juice in each beverage. These children chose the glass where the amount of raspberry juice exceeded the amount of water. If this was the case for both glasses, they guessed. The *bidimensional strategy with quantification* was to subtract the amount of juice from the amount of water for each glass. These children chose the glass with the smaller difference as the one with the more intensive taste. The *ratio strategy* based the decision on division. The inappropriate bidimensional strategy with quantification often supplies the correct results. Only the use of particular numbers makes evident whether a child uses the inappropriate subtraction method or whether he or she derived an answer from the ratio strategy. For instance, having one glass with three parts of raspberry and four parts of water, and having a glass with seven parts of raspberry and nine parts of water, would lead to the wrong answer if the child chooses the one with the smaller difference.

Children were presented with 12 proportional reasoning problems that were embedded in different context stories, such as testing the taste of raspberry juice, estimating the weight of pieces of cheese differing in size, or collecting money for a good purpose. For half of the problems, subtracting the numbers would result in the wrong answer. Thus, the test should indicate whether children use a ratio strategy.

Results revealed a mean solution rate of .53 ($SD$ = .24). Only 4% of the students performed 100% correct, and 30% were correct at least in one of the six problems that required the ratio strategy. These results suggest that most of the students were not fully capable of proportional reasoning. Rather, the bidimensional strategy with quantification was the prevailing strategy.

## 10.4 PREDICTING LATER MATHEMATICAL PERFORMANCE

### 10.4.1 THE IMPACT OF INTELLIGENCE AND PRIOR KNOWLEDGE ON ADVANCED MATHEMATICAL COMPETENCIES AT THE AGE OF 18

In the follow-up study conducted with LOGIC participants when they were about 18 years old, mathematical competencies were tested by tasks from the Third International Mathematics and Science Study (TIMSS; Baumert et al., 1997) solved under time pressure. Though these tasks mainly target content usually taught in grade 8, solutions are not trivial, even for participants with more sophisticated mathematical competencies. This is illustrated by the following example: "Which $x$ value fulfills the equation $x^2 - 14x + 49 = 0$: A) 7 and 0, B) 7, C) –14, D) 7 and –7, or E) 14 and 0?" The number of tasks presented was so large that even mathematics experts could not have solved them within the time limit. The mean number of problems solved was 4.5, with a standard deviation of 2.1.

We used the LOGIC data to help ask how these achievement differences in algebra might be explained. For the analyses that follow, we used children's performance in the LOGIC mathematics tasks described above as well as IQ scores collected annually throughout the LOGIC study. A subsample of the LOGIC sample—58 participants, all of them students in upper secondary school (Gymnasium)—was considered. We may assume that if correlations for this subsample are significant in spite of its restricted variance, they indicate substantial influences.

Table 10.1 shows the Spearman correlations between mathematics performance at age 18 with intelligence and mathematics performance at earlier age levels. The highest coefficient turned out to be the one between achievement in comparison problems at age 8 and mathematics achievement at age 18, whereas there is no correlation between intelligence in the earlier age and mathematics achievement at age 18 (for the complete sample, the correlation is $r = .38, p < .01$).

**TABLE 10.1**
**Spearman Correlations Between Achievement in Mathematics Measured at Age 18 (Grade 11) and Measures of Intelligence and Prior Knowledge in Mathematics at Different Ages**

| Age | Mathematics | Intelligence |
|---|---|---|
| 6 | .32** | .03 |
| 8 | .62** | .04 |
| 10 | .41** | .40** |
| 12 | .50** | .41** |
| 18 | | .38** |

** p < .01.



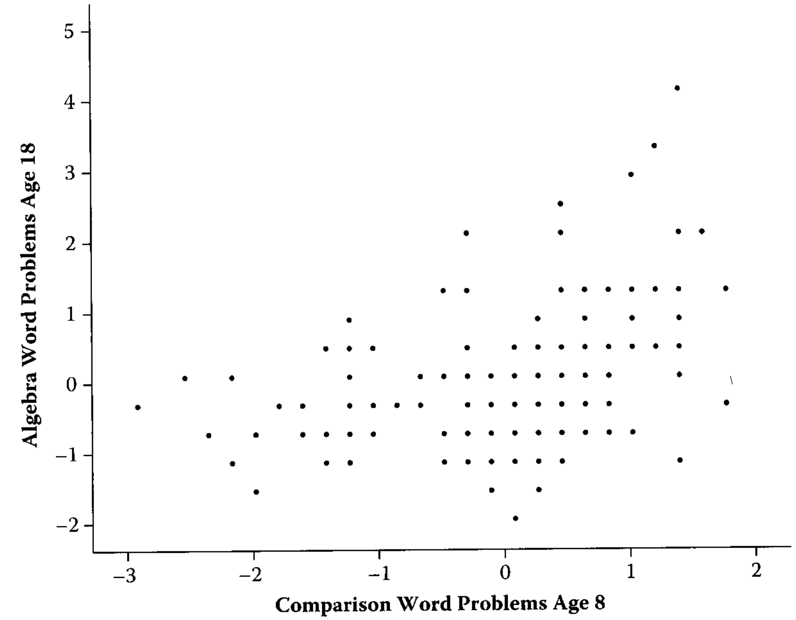


**FIGURE 10.2** Stability of mathematical achievement ($z$ scores at both measurement points) during a period of 10 years.

Moreover, mathematics achievement at age 18 and the IQ at the same age level are not as highly correlated as mathematics achievement at age 18 and achievement in solving word problems at age 8. The scatterplot depicted in Figure 10.2 shows that this correlation is not based on outliers. At the same time, it also indicates that all participants who do not contribute to stability show a similar development—a finding of high importance in understanding the development of mathematical competencies. The upper left half of the scatterplot is empty, which means that no child who failed to show average or above-average achievement in solving comparison problems at age 8 was good or excellent in mathematics performance at age 18. The lower left half, on the contrary, is occupied by several subjects whose achievement had been above average at age 8, when most participants were in grade 2, but had in later years dropped to average or even below average. The data show that early mathematical understanding as shown by the ability to solve demanding word problems is a necessary but by no means sufficient prerequisite for later mathematical competencies.

The results show that even in a domain like mathematics that is closely related to intelligence, high achievement crucially depends on prior knowledge. Knowing, as early as at age 8 (which is grade 2), that numbers can be used not only for modeling the size and transformation of sets but also for representing relations between sets seems to be a necessary, though not sufficient, prerequisite for higher mathematics achievement in upper secondary school. As yet, however, German curricula for elementary school hardly ever include demanding word problems that involve a comparison of sets.

### 10.4.2 The Explanation of Achievement Differences in Mathematical Reasoning at the Age of 23

At the final LOGIC measurement point, when participants were 23 years of age, all participants had been out of high school for at least 4 years. Some of these were working and some were in university, but most had not had subsequent mathematics instruction. Under these conditions, it would not have made sense to administer achievement tests focusing on academic mathematics. Rather, we presented participants with a standardized test of mathematical reasoning. Six numerical subtests of the Berliner Intelligenz Struktur Test (Jäger, Süß, & Beauducel, 1997) were administered as speed tests (time limits between 1 and 4 minutes). These subtests were selected because the mathematical knowledge required for solving the problems was taught in all types of schools and all participants would have been exposed to it during their secondary school education. The test consisted of six subtests:

Word Problems: Participants had to solve five word problems with different degrees of difficulty. One example was "Which number has to be added to the number 12 for the proportion between the sum of these two numbers and 15 to be the same as the proportion between 30 and 25?"

Number Series: In these tasks, nine rows with 6 to 7 numbers were presented, requiring participants to supplement the subsequent number according to an underlying principle they had to figure out. For example, "5 10 20 10 20 40 30 __?"

Computing with Large Numbers: A computing task with large numbers was presented. The participants had to select the appropriate answer in a multiple-choice format. For example:
56324186 – 52418218 = ?
a) 3643074 b) 4094032 c) 3905968 d) 4742404 e) 3126068

Computing with Small Numbers: A list of 130 one- or two-digit numbers was presented. The task for the participant was to cross out all numbers that were bigger by three than the preceding number. The achievement score was determined by subtracting the number of correctly crossed-out numbers from the number of incorrectly crossed-out numbers.

Memorizing Numbers: Participants were given one minute to memorize nine 4-digit numbers. Afterward, they were presented with 66 numbers, among them the nine numbers that had to be memorized. These numbers had to be crossed out. The achievement score was determined by subtracting the number of correctly crossed-out numbers from the number of incorrectly crossed-out numbers.



Number Combinations: Within a given time limit, participants had to find out as many numbers as possible that would fit the following equation: ______ × ______ + ______ = 60.

The standardized test allowed a comparison between the LOGIC sample scores and test norms. The overall score of the LOGIC sample was slightly higher ($d = .18$) than the norm sample mean. This result corresponds to other findings that have shown that the LOGIC sample was modestly positively skewed.

The test data showed good internal consistency: With one exception, the correlations among subtests were significant and varied between $r = .32$ and $r = .58$, indicating that mathematical reasoning at the age of 23 can be seen as a unified construct. The largest correlation was between number series and word problems ($r = .58$). There were no significant correlations between Memorizing Numbers and the other tests.

Table 10.2 shows the correlations between mathematical reasoning at age 23 and measures of mathematical competencies at earlier ages without and with partialing out intelligence.

The correlation of $r = .43$ between early number sense at age 6 and mathematical reasoning at age 23 suggests that individual differences in mathematics are, to a considerable extent, determined by factors that took effect before children entered school. These relationships are further depicted in Figures 10.3a and 10.3b, which show scatterplots for performance at age 23 and ages 6 and 12. The void upper left field is common to each. Almost none of the students who scored below average at age 6 or at age 12 caught up later on. On the other hand, there were several children who scored above average on mathematical achievement measures at age 6, but dropped off afterward, as shown in Figure 10.3a. As shown in Figure 10.3b, with increasing age there were fewer students who were above average at an earlier age but dropped off later.

**TABLE 10.2**
**Correlation Between Mathematical Reasoning at Age 23 and Earlier Measures**

| Measure of Mathematical Ability and Age Level of Administering | Without Partialing Out Intelligence | Intelligence Partialed Out |
|---|---|---|
| Number sense (age 6) | .43** | .34** |
| Comparison word problems (age 8) | .44** | .32** |
| Mathematical word problems (age 10) | .49** | .35** |
| Proportional reasoning age (age 12) | .51** | .32** |
| Algebra problems (age 18) | .56** | .41** |

** $p < .01$.

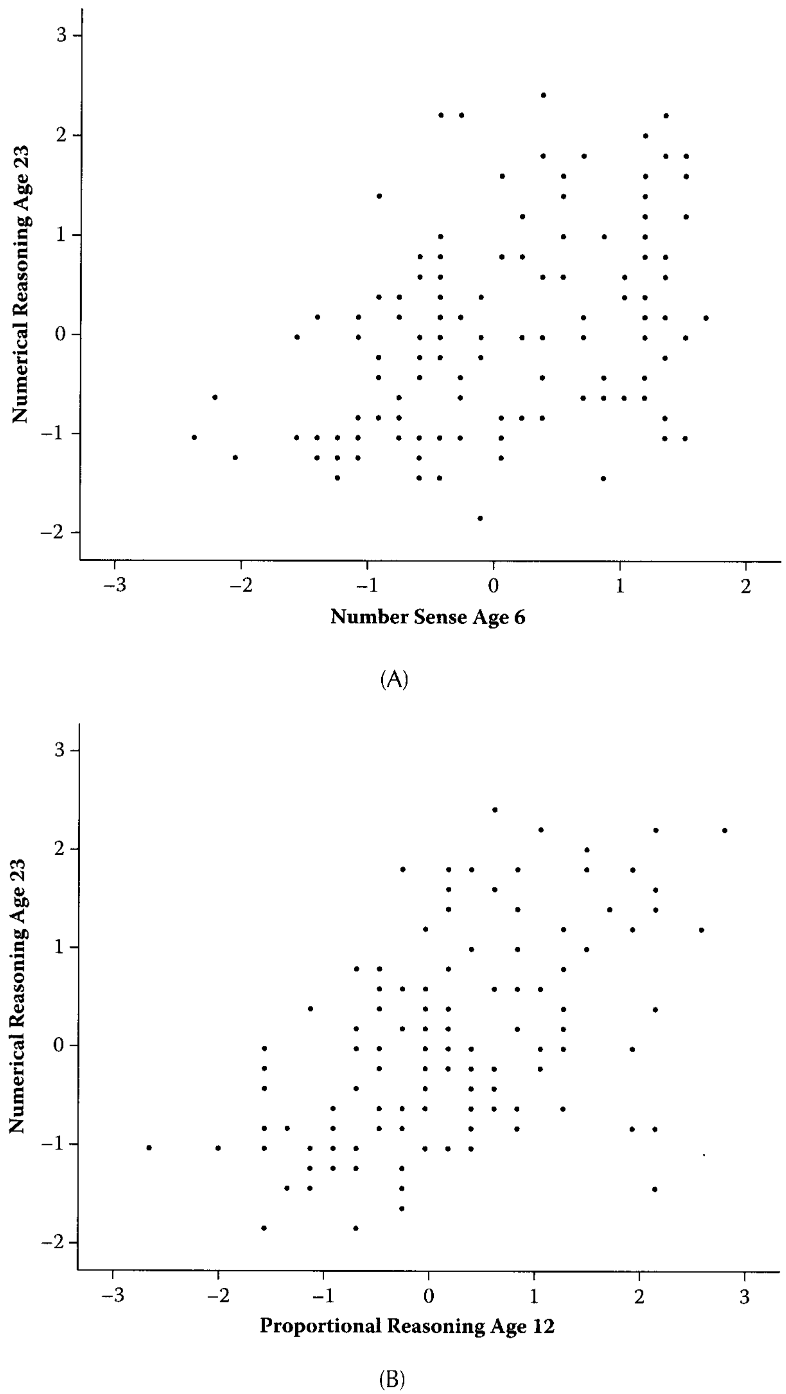


**FIGURE 10.3** Scatterplots of the relationship between mathematical reasoning at the age of 23 and earlier mathematical competencies ($z$ scores at all measurement points).



It is most remarkable that all correlations between mathematical reasoning at the age of 23 and mathematical achievement measures from earlier ages remained significant after the intelligence score from the respective age was partialed out. This result clearly suggests that the source of individual differences in mathematics goes beyond the general intelligence level and already emerges before the start of systematic instruction.

### 10.4.3 Mathematical Competencies in Female Students: The Leaky Pipeline

A large number of studies have shown that males outperform females in many aspects of mathematics (see Hyde [2005] for a review). Females are particularly underrepresented in the upper quartile of mathematical performance. Earlier analyses of the LOGIC data confirmed these findings (Stern, 1998). Significant sex differences in favor of males were found at all age levels, with an effect size varying between $eta^2 = .03$ and .09. Many studies also report that females are particularly underrepresented in the upper achievement levels, a trend also found in the LOGIC data, where females were underrepresented in the upper achievement quarter at all ages; this trend increased with age. There was not an analogous bias for general intelligence scores.

Table 10.3 depicts the percentage of females in the top quartile of the achievement distribution. Were there equal numbers of females and males in this category? Even before children entered school, females were underrepresented in the top quartile of mathematics achievement. Around the age of 10, there are twice as many male top achievers as female. Comparisons between females who stayed in the top quartile and those who decreased in performance did not reveal any differences with respect to general intelligence. Analogously, there were no differences between male and female top achievers with respect to general intelligence or with regard to visual spatial abilities.

Despite the preponderance of males at the higher achievement level, there was at least one female among the top 5% at every age level. These results are in accord with other findings suggesting that there are talented female students at

**TABLE 10.3**
**Adjusted Percentage of Female Students in the Top Quartile of the Achievement Distribution**

| Measure | % |
|---|---|
| Number sense (age 6) | 43 |
| Comparison word problems (age 8) | 39 |
| Mathematical word problems (age 10) | 32 |
| Proportional reasoning age (12) | 33 |
| Algebra problems (age 18) | 23 |
| Mathematical reasoning (age 23) | 27 |

all age levels (Lubinski & Benbow, 2006) and correspond to discussions of gender imbalances in the fields of science and mathematics (Spelke, 2005): There is no evidence for sex differences in the intrinsic aptitudes necessary for advanced competencies in these fields. This does not, of course, necessarily imply that the lower proportion of females can be entirely traced back to environmental differences in gender-specific socialization. Rather, Spelke concluded, "We must look beyond cognitive ability to other aspects of human biology and society for insights into this phenomenon" (p. 956).

### 10.4.4 Development of Mathematical Competencies Within a Tracking System: The Interaction Between Preconditions and Learning Environment

After finishing 4 years of elementary school (e.g., around age 10), German students are assigned to different school types based on their grades in the core elementary school subjects. At the time of the LOGIC study, the state of Bavaria offered two main school tracks for students (special education tracks will not be discussed here): Gymnasium as the upper track and Hauptschule or Realschule as the lower track. Gymnasium lasts for the next 9 years and is considered as preuniversity education. The Hauptschule or Realschule track splits further after grade 6 into a middle track (Realschule) and the lowest track (Hauptschule). After grade 4, about 48% of the LOGIC participants were recommended for the Gymnasium, which is above the average for the state.

Table 10.4 depicts the amount of achievement variance accounted for by the two school tracks students started to attend around the age of 10. Long before children joined the separate school tracks, they differed in their mean achievement level. Nonetheless, the differences were relatively small at all age levels, given that the tracking system is entirely based on the idea of different types of giftedness. At the age of 23, only 15% of achievement differences in mathematics can be traced back to the school tracks. The highest amount of explained variance by school track was found when children were about 9 years old, which was some months before the final decision was made. Nonetheless, even at this age level, there was a remarkable achievement overlap between students who were supposed to attend the Gymnasium as compared to those who were not. Figure 10.4 depicts the distribution of achievement in mathematical word problem solving at the age of 10 for those students subsequently assigned to the Gymnasium and those who were not. By comparison, the variance found in IQ tests traced to school tracking was 28% on average, clearly higher than for mathematics. Although there is no evidence for an increase in the IQ gap as a result of tracking (Schneider & Stefanek, 2004), intelligence seems to determine selection processes to a larger extent than achievement in mathematics does.

**TABLE 10.4**
**Effect Sizes *d* and Percentage of Explained Variance by School Track (Gymnasium Versus No Gymnasium)**

| Mathematical Achievement Measures | D | $Eta^2 \times 100$ |
|---|---|---|
| Number sense (age 6) | 48 | 10 |
| Comparison word problems (age 8) | 68 | 14 |
| Mathematical word problems (age 10) | 49 | 10 |
| Proportional reasoning (age 12) | 78 | 23 |
| Algebra problems (age 18) | 76 | 18 |
| Mathematical reasoning (age 23) | 64 | 15 |



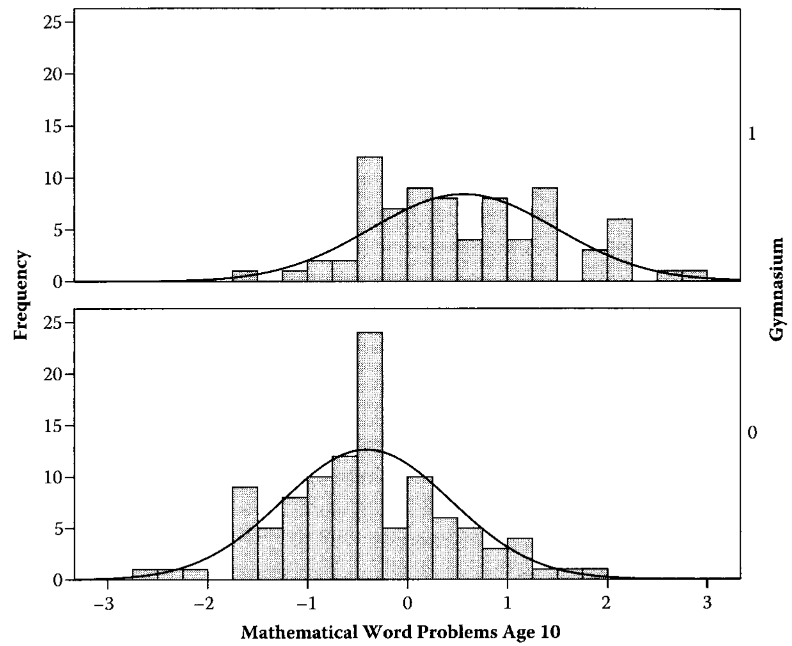


**FIGURE 10.4** Distribution of the achievement in mathematical word problem solving ($z$ values) immediately before the separation into the school tracks (gymnasium or no gymnasium) was realized.

## 10.5 CONCLUSIONS

The most remarkable result of the longitudinal analyses was the high stability of interindividual differences from the very beginning of data collection. This was found not only for mathematics but also for other content areas investigated in the LOGIC sample. The results presented in this chapter reveal different trajectories of mathematical competencies starting to emerge even before children enter school. These differences are hardly affected in the subsequent years of formal instruction at school. It goes without saying that there is universal growth in mathematical competencies from professional classroom instruction. However, at the same time, the results of the LOGIC study reveal that differential effects of schooling seem to be relatively temporary and instable. Even quite strong "interventions" such as assignment to different school tracks have a relatively modest impact on the development of individual differences. A reason for this may be the quite uniform and restricted curriculum and classroom practice usual in Germany in the time period the LOGIC data were collected. The content as well as the presentation and practice in mathematics were generally specified, and even creative teachers had only few opportunities to translate innovative ideas of teaching into classroom practice. This was also evident in the study by Staub and Stern (2002), which demonstrated significant but limited effects of constructivist beliefs held by the teachers on students' achievement gains. Since the late 1980s and the 1990s, the time period the LOGIC participants had been in school, the German educational system has undergone tremendous challenges, particularly after the results of international comparison studies like PISA or TIMMS (Baumert, Artelt, Klieme, & Stanat, 2002) showed quite dramatic deficits in German school education. Since that time, innovative and creative ideas of designing learning environments, particularly in mathematics, have had a much better chance of implementation in the classroom. This may also affect trajectories of individual differences.

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