Analyzing multiple groups

in Siena

Why do we want to analyze multiple networks

- Analyzing one network is a case-study
  - it can be argued to represent a larger population
  - but these are substantive arguments, no way to test them (Snijders, 2011, p. 136)
- The analysis of multiple networks can tell us something about a population of networks based on G ≥ 2 (stochastically independent) groups
- Three major ways to do this
  - multigroup analysis
  - multilevel analysis
  - meta-analysis

statistical assumptions

statistical power

Assumptions

- Overall assumption for all three:
  - networks obey the same model
- Multigroup analysis
  - networks are exact replicates of each other
  - parameter values are the same in all groups
- Multilevel analysis
  - parallel networks rather than exact replicates
  - network-level parameters are estimated assuming a normal distribution
- Meta-analysis
  - parallel networks rather than exact replicates
  - two-step process →
  - estimation of network-level parameters does not assume a normal distribution

Aims of this presentation

- There are at least three reasonable ways to analyse multiple groups together
- They have different assumptions and should be applied in different situations
- Their results can be very different from each other – but how different, and under what circumstances?
- A good overview would be extremely useful
    - many different contexts
    - different model complexities
    - different sample sizes
    - different levels of heterogeneity
    - …
- This short presentation show results of a case study
    - same sample and analogous model specifications modelled in three different ways

Overview

- Brief intuitive introduction to the three methods
- Example results
- Investigation: how much can we trust our results?

Substantive topic and model specification

- Peer influence on alcohol consumption in adolescence
- Do drinking behavior of adolescents become (and remain) similar to that of their friends over time?
- Classic Siena problem: separating between selection and influence
- Selection: those similar in alcohol consumption will be more likely to become and stay friends
  - alcohol egoXaltX effect (+ egoX + altX)
- Influence: friends will be more likely to become similar in alcohol consumtion
  - alcohol averageAlter effect (no ego and alter effects for power reasons)

Sample

- 27 classrooms of first year secondary school students (age of 15)
- Sample was collected in Hungary in 2012 by the *MTA TK "Lendület" Research Center for Educational and Network Studies*
- Two waves, 6 months apart
- **Av. group size:** 30.6 students per class (s.d.: 4.7)

Dependent variables

- Friendship
  - **av. densities**: 0.2 (s.d.: 0.05) in wave 1; 0.17 (s.d.: 0.04) in wave 2
  - **av. Jaccard indeces**: 0.4 (s.d. 0.08)
- Drinking behavior: ordinal scale from 1 $\longrightarrow$ 4 (1: never, 4: at least once a week)
  - wave 1: **mean:** 2.3, **median:** 2, **mode:** 3
  - wave 2: **mean:** 2.6, **median:** 3, **mode:** 3

Multigroup analysis

- Main assumption:
  - dynamics are identical in all groups
  - except: basic rate parameter which can vary
- If parameters differ too much: meaningless results
  - this can be tested empirically
  - a way to account for heterogeneity: extra effects can be specified for separate groups (group dummies) and interacted with variables to capture differences
  - but this can quickly lead to a very large model
  - this is similar to analyzing more than two data waves in Siena (“time dummies”)
- A multigroup option can be useful when groups are too small and/or model specifications are too complex

Example results

	Estimate	Standard Error
Network Dynamics		
28. eval outdegree (density)	-1.1958	(0.0558)
29. eval reciprocity	1.8646	(0.0643)
30. eval transitive triplets	0.2528	(0.0157)
31. eval transitive recipr. triplets	-0.0483	(0.0240)
32. eval indegree - popularity	0.0155	(0.0063)
33. eval outdegree - popularity	-0.1699	(0.0090)
34. eval outdegree - activity	-0.0094	(0.0035)
35. eval drink alter	0.0776	(0.0274)
36. eval drink ego	0.0156	(0.0303)
37. eval drink similarity	0.1633	(0.1044)
38. eval sex.F alter	0.0120	(0.0352)
39. eval sex.F ego	-0.0022	(0.0368)
40. eval sex.F ego x sex.F alter	0.3735	(0.0670)
Behavior Dynamics		
68. eval drink linear shape	0.8955	(0.1437)
69. eval drink quadratic shape	-0.8370	(0.1544)
70. eval drink average alter	0.8076	(0.5161)

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Testing heterogeneity

- We assumed that parameters are the same across groups
- Is this empiricallty true?
- sienaTimeTest tests whether group × effect dummy variables are jointly significantly different from 0
- This analysis had to be conducted in two parts, as the collection of effects was too collinear (this often happens), following suggestions of the Manual
- Results:
  - Selection part: chi-squared = 1143.39, d.f. = 260, $p < 0.0001$ strong heterogeneity
  - Influence part: chi-squared = 75.47, d.f. = 78, $p = 0.5602$ no evidence

Handling heterogeneity

- Turns out, our model had the wrong assumptions!
- In this case, parameters are not to be trusted.
- What can we do?
- If overall heterogeneity is due to a small number of heterogeneous effects, we can add group dummies and their interactions with these effects to account for heterogeneity
- Is this the case here?

Testing homogeneity of parameters

	chi-sq.	df	p-value
outdegree (density)	98.56	26	0.000
reciprocity	70.62	26	0.000
transitive triplets	190.60	26	0.000
transitive recipr. triplets	82.75	26	0.000
indegree - popularity	126.58	26	0.000
outdegree - popularity	106.50	26	0.000
outdegree - activity	185.49	26	0.000
drink alter	54.23	26	0.001
drink ego	65.90	26	0.000
drink similarity	52.32	26	0.002
sex.F alter	43.5	26	0.017
sex.F ego	71.11	26	0.000
sex.F ego x sex.F alter	53.55	26	0.001
drink linear shape	13.47	26	0.979
drink quadratic shape	40.08	26	0.038
drink average alter	23.23	26	0.620

all but two parameters are heterogeneous when considered by themselves!

Handling heterogeneity: more ideas

- We now know that heterogeneity is not due to one or few parameters
- What else can we do?
- Overall hetegoreneity may also be due to the inclusion of one or few very different groups!
- Are there very different groups?

Testing homogeneity of groups

	chi-sq.	df	p-value
Period 1	82.96	10	0.000
Period 2	26.94	10	0.003
Period 3	27.49	10	0.002
Period 4	45.40	10	0.000
Period 5	27.32	10	0.002
Period 6	18.63	10	0.045
Period 7	51.87	10	0.000
Period 8	36.23	10	0.000
Period 9	36.21	10	0.000
Period 10	93.44	10	0.000
Period 11	24.04	10	0.007
Period 12	30.13	10	0.001
Period 13	46.92	10	0.000
Period 14	16.03	10	0.099
Period 15	44.51	10	0.000
Period 16	15.43	10	0.117
Period 17	103.06	10	0.000
Period 18	16.77	10	0.080
Period 19	55.54	10	0.000
Period 20	17.97	10	0.055
Period 21	16.48	10	0.087
Period 22	88.74	10	0.000
Period 23	15.39	10	0.118
Period 24	17.66	10	0.061
Period 25	19.51	10	0.034
Period 26	50.05	10	0.000
Period 27	18.53	10	0.047

most groups are significantly different

	chi-sq.	df	p-value
Period 1	6.59	3	0.086
Period 2	9.56	3	0.023
Period 3	4.24	3	0.237
Period 4	2.42	3	0.490
Period 5	3.13	3	0.372
Period 6	0.62	3	0.892
Period 7	0.43	3	0.934
Period 8	4.58	3	0.205
Period 9	1.26	3	0.739
Period 10	1.93	3	0.587
Period 11	1.23	3	0.746
Period 12	0.27	3	0.966
Period 13	3.39	3	0.335
Period 14	2.11	3	0.550
Period 15	1.42	3	0.701
Period 16	0.70	3	0.873
Period 17	7.17	3	0.067
Period 18	3.58	3	0.311
Period 19	0.01	3	1.000
Period 20	4.08	3	0.253
Period 21	2.78	3	0.427
Period 22	0.06	3	0.996
Period 23	9.91	3	0.019
Period 24	1.00	3	0.801
Period 25	1.06	3	0.787
Period 26	1.88	3	0.598
Period 27	1.49	3	0.685

two groups are significantly different

Conclusions

- The multigroup Siena model suggests that those who drink more are somewhat more popular (no influence effect)
- However, heterogeity test finds that our overall assumption of identical groups was wrong
- The sample is highly heterogeneous and no single group or effect is responsible for this

Multilevel analysis

- Main assumptions
  - social mechanisms are similar in all networks, but parameters can be different
  - multilevel framework: network processes take place at the lower (network) level
  - groups, therefore group-level parameters θ^g are independent
  - parameters θ^g for the networks are randomly drawn from a global distribution of parameters
  - this is multivariate normal with a mean μ and covariance matrix Σ
  - if a parameter has a variance 0, that parameter is the same across groups
- The aim is to say something about the global parameters μ and Σ
- If normality is a valid assumption, it helps us estimating network-level parameters

Multilevel analysis

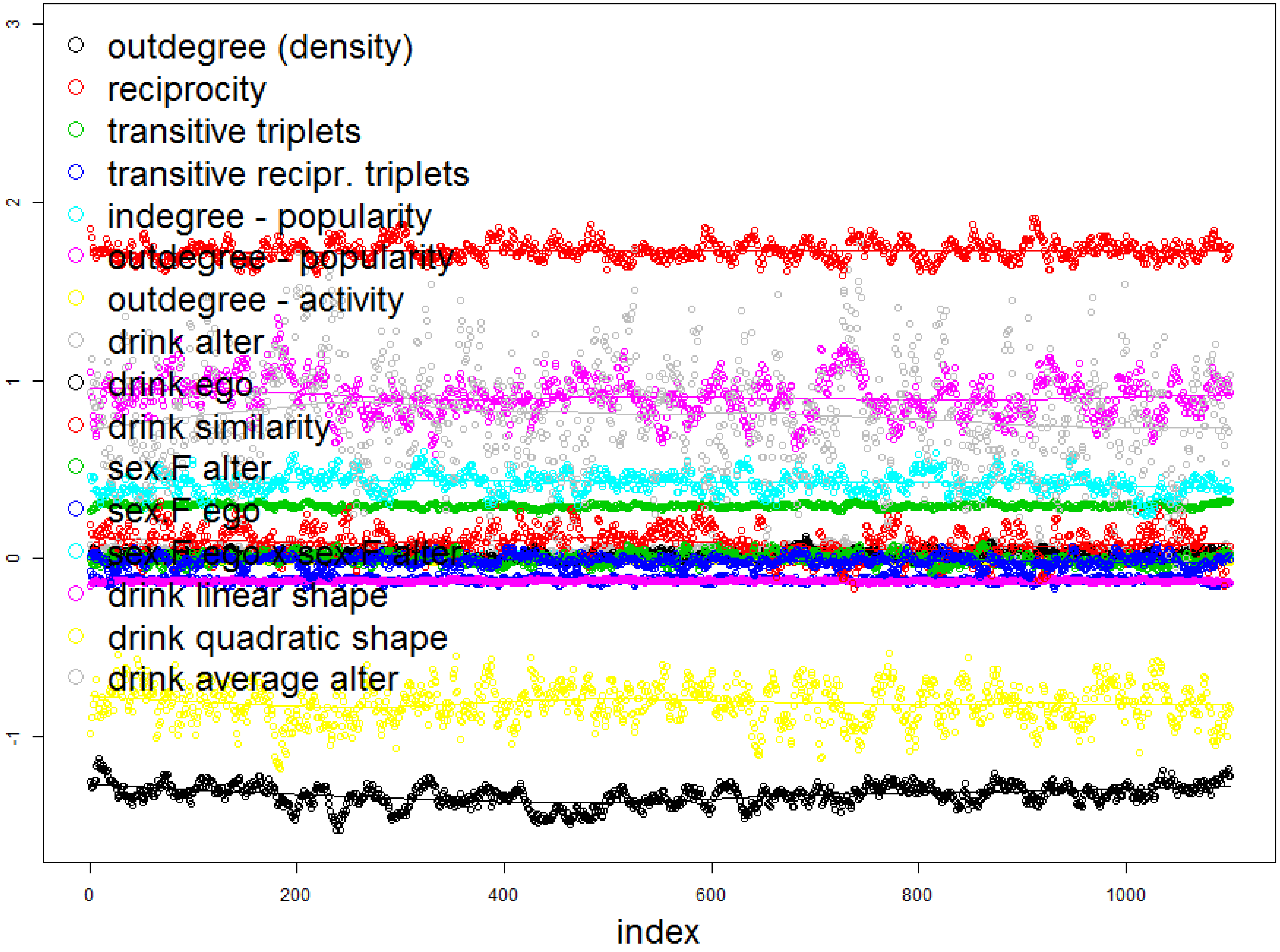
- Bayesian approach
  - parameters are assumed to have a probability distribution
  - prior distribution: based prior beliefs and uncertainties about parameter values of the researcher without having seen the data
  - posterior distribution: estimated based on the observed data
- Parameter values for each group and for the population are both estimated using MCMC
- Estimation is similar to likelihood-based estimation methods for SAOMs

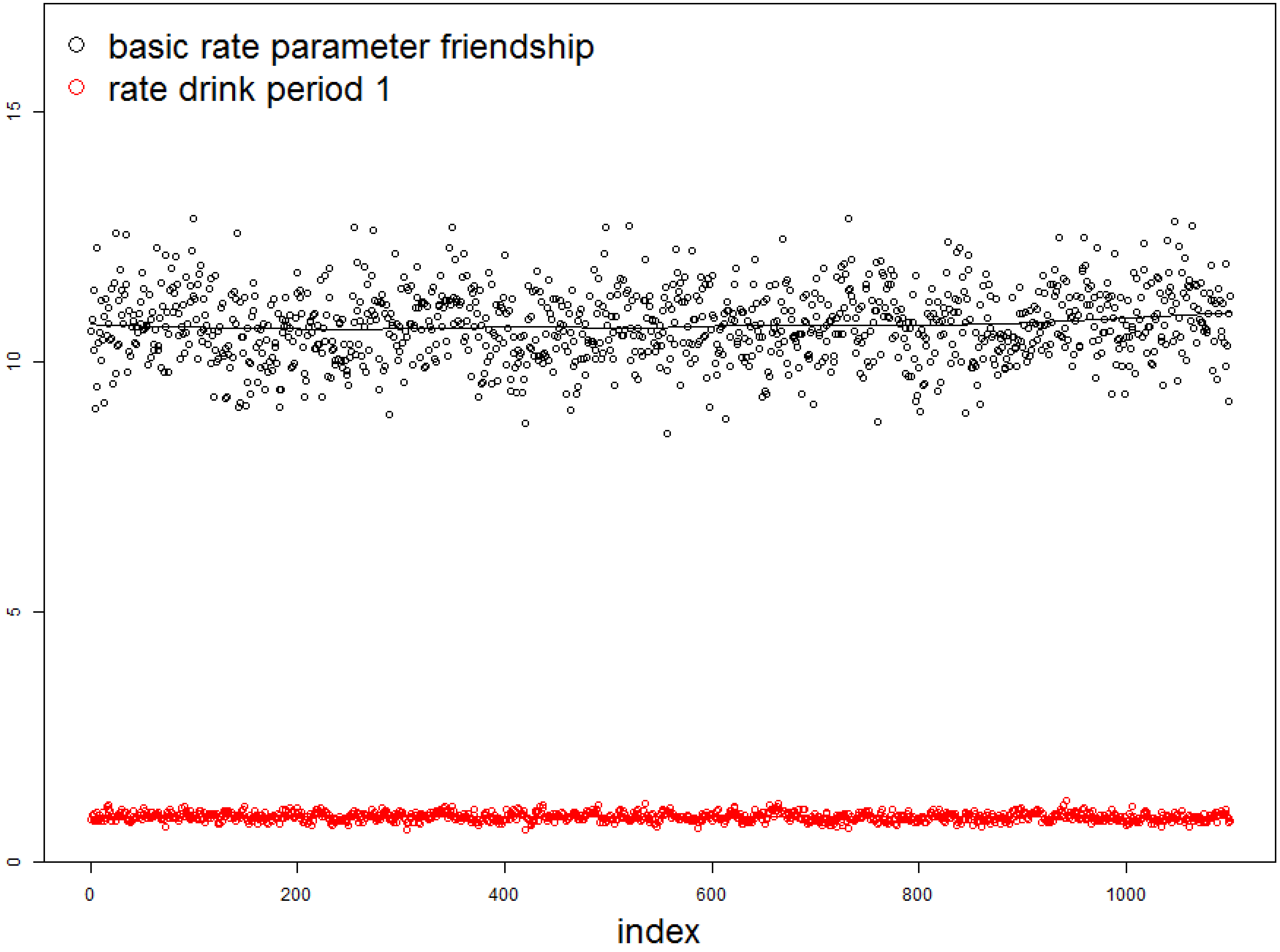
Multilevel analysis

- Prior distrubition of rates:
  - from a multigroup analysis
- Prior distribution of other parameters
  - Mean of prior μ: 0
  - Prior Σ: identity matrix (mean variances: 1, mean covariances: 0)

Example results

	Post. mean	Post. s.d.	cred. from	cred. to	p
Network Dynamics					
28. eval outdegree (density)	-1.2940	(0.1155)	-1.5175	-1.0727	0.00
29. eval reciprocity	1.7307	(0.0519)	1.6387	1.8348	1.00
30. eval transitive triplets	0.2947	(0.0116)	0.2717	0.3181	1.00
31. eval transitive recipr. triplets	-0.1210	(0.0154)	-0.1516	-0.0916	0.00
32. eval indegree - popularity	0.0186	(0.0058)	0.0077	0.0294	1.00
33. eval outdegree - popularity	-0.1279	(0.0055)	-0.1384	-0.1175	0.00
34. eval outdegree - activity	-0.0159	(0.0030)	-0.0218	-0.0093	0.00
35. eval drink alter	0.0408	(0.0221)	-0.0064	0.0821	0.96
36. eval drink ego	0.0235	(0.0235)	-0.0196	0.0719	0.82
37. eval drink similarity	0.0816	(0.0786)	-0.0753	0.2282	0.85
38. eval sex.F alter	-0.0010	(0.0308)	-0.0562	0.0630	0.47
39. eval sex.F ego	-0.0163	(0.0323)	-0.0872	0.0420	0.31
40. eval sex.F ego x sex.F alter	0.4259	(0.0591)	0.3075	0.5412	1.00
Behavior Dynamics					
68. eval drink linear shape	0.9072	(0.1135)	0.6910	1.1341	1.00
69. eval drink quadratic shape	-0.8218	(0.1088)	-1.0437	-0.6290	0.00
70. eval drink average alter	0.8145	(0.3109)	0.2154	1.4521	1.00


outdegree (density)
reciprocity
transitive triplets
transitive recipr. triplets
indegree - popularity
outdegree - popularity
outdegree - activity
drink alter
drink ego
drink similarity
sex.F alter
sex.F ego
sex.F ego x sex.F alter
drink linear shape
drink quadratic shape
drink average alter
3
2
1
0
-1
0
200
400
600
800
1000
index



basic rate parameter friendship
rate drink period 1
15
10
5
0
0
200
400
600
800
1000
index

Conclusion

- The multilevel results show social influence, while selection is not significant anymore
- How much can we trust these results?
- Models converged
- To see how well the model fits the observed data, GOF would be good
- Does not exist yet

Meta-analysis

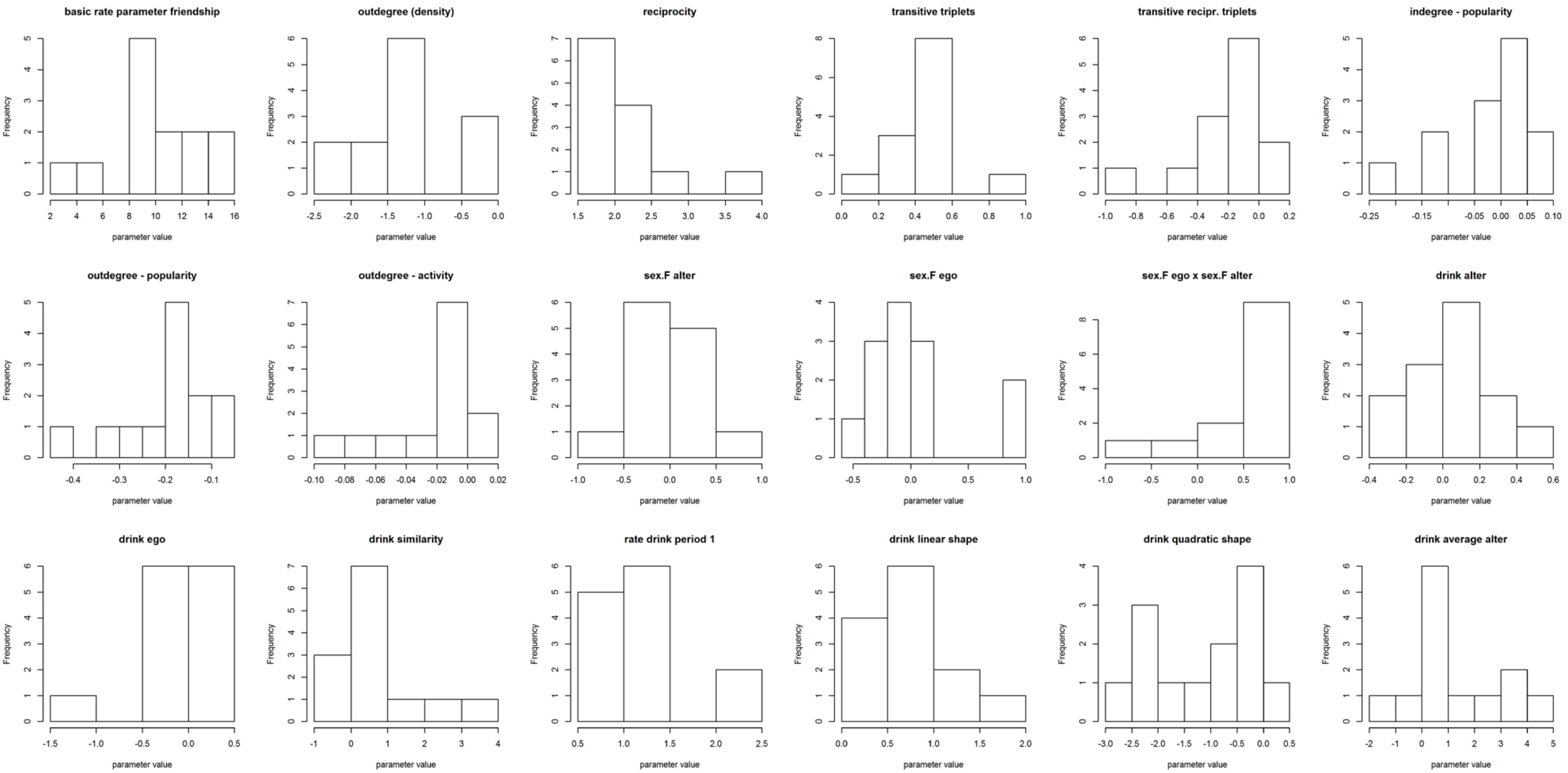
- Well-known two-step procedure
- Has fewer assumptions than the multilevel analysis
- Parameters are estimated for the individual groups first, without having assumptions about their distribution
- In the second step, they are assumed to come from a normal distribution, but this can be tested (or at least looked at)
- Meta-analysis is done for all parameters separately, correlations do not receive attention

Meta-analysis

- Here, the same model specification was applied to all classrooms separately
- However, some classrooms were problematic
  - Convergence was not achieved in 8 cases
  - Standard errors were very large in 5 cases
- This is a common problem of selection-influence models in small groups
- For illustration purposes, the remaining 14 groups were used for meta-analysis

Example results

	Estimate	St. Error.	Confidence interval		Tau2	Q	p
basic rate parameter friendship	8.67	1.000	6.71	10.63	7.29	78.50	0.00
outdegree (density)	-1.34	0.229	-1.78	-0.89	0.22	16.53	0.04
reciprocity	1.92	0.151	1.62	2.21	0.00	7.00	0.54
transitive triplets	0.40	0.050	0.31	0.50	0.01	14.87	0.06
transitive recipr. triplets	-0.16	0.056	-0.26	-0.05	0.00	8.66	0.37
indegree - popularity	0.01	0.020	-0.03	0.05	0.00	12.19	0.14
outdegree - popularity	-0.16	0.018	-0.19	-0.12	0.00	9.39	0.31
outdegree - activity	-0.02	0.009	-0.04	0.00	0.00	8.58	0.38
sex.F alter	-0.06	0.073	-0.21	0.08	0.00	9.34	0.31
sex.F ego	-0.09	0.086	-0.26	0.08	0.01	8.94	0.35
sex.F ego x sex.F alter	0.42	0.162	0.10	0.74	0.03	9.33	0.32
drink alter	0.07	0.075	-0.07	0.22	0.01	9.53	0.30
drink ego	-0.02	0.073	-0.17	0.12	0.01	9.84	0.28
drink similarity	0.46	0.255	-0.04	0.96	0.00	4.99	0.76
rate drink period 1	0.93	0.138	0.66	1.20	0.00	6.73	0.57
drink linear shape	0.43	0.178	0.08	0.78	0.00	2.27	0.97
drink quadratic shape	-0.34	0.173	-0.68	0.00	0.00	6.23	0.62
drink average alter	0.61	0.641	-0.65	1.86	0.00	1.42	0.99


basic rate parameter friendship
outdegree (density)
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indegree - popularity
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outdegree - activity
sex.F alter
sex.F ego
sex.F ego x sex.F alter
drink alter
drink ego
drink similarity
rate drink period 1
drink linear shape
drink quadratic shape
drink average alter
Frequency
parameter value

Conclusion

- We compared three different ways of analyzing multiple groups together
- We chose a type of model specification that is of general interest (selection-influence models), but often not easy to deal with
- Meta-analysis mostly failed due to lack of statistical power
- Multigroup assumptions were false in case of our group
- Multilevel model seemed to work – but it needs more investigation to learn how well exactly
- All in all, multilevel models could potentially be very important for certain kinds of analyses in the future