

Exercise 2

Types and Subtyping

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Task 1

Show:

- A program that is rejected by a statically typed language but is executed without typing errors in a dynamically typed language.

— solution —

Consider

```
if (x==x) y=1; else y=true;
y = y+1;
```

A usual static type system would reject this program, while the program would not cause typing problems.

- A program that is rejected by a statically typed language and runs into a type error when executed in a dynamically typed language.

— solution —

The static type system would reject the following program which would generate a runtime type error:

```
void f(x) { return x+1 }
print( f(true) )
```

Task 2

Suppose that we have a language with structural subtyping, contravariant parameter types and covariant return types. Consider the following types:

```
class A { int m(int x) {...}; }
class B { int m(int x) {...}; int n(int x) {...}; }
class C { int n(int y) {...}; int m(int x) {...}; }
class D { C m(A a) {...}; }
class E { C m(B b) {...}; }
class F { A m(B e) {...}; }
class G { B m(C e) {...}; }
class H { G m(D d, E e) {...}; }
class I { F m(E e, D d) {...}; }
class J { A a; }
class K { B b; }
```

Find all the subtyping relations among them. Assume that `int` has no subtype other than itself.

— solution —

$B=C <: A$ and $D <: E = G <: F$

No other subtyping relations exist, except the reflexive and transitive closure of the above.

Task 3

Consider the following Java program:

```
class B {  
    protected int get () {...}  
}  
  
class A extends B {  
    private int get () {...}  
}  
  
class C extends B {  
    public int get () {...}  
}
```

When we compile it, we obtain the following error:

```
get() in A cannot override get() in B; attempting to  
assign weaker access privileges; was protected  
    private int get () {...}  
                ^
```

Explain why this is the behavior of the Java compiler.

— solution —

Class A restricts the accessibility of method `get()`, since it is protected in B and private in A. This means that class A allows fewer behaviors than B, so it cannot be a subtype of B. On the other hand, class C relaxes the accessibility level of method `get()`, so it allows more behaviors than B, and this is allowed by the Java compiler.

In general, a class can be a subtype of another class if it assigns “weaker” accessibility permissions than the ones of the superclass.

In Java, there are four different types of access modifiers for fields and methods:

- **public**: every class can access the element
- **protected**: only subclasses and classes in the same package can access the element
- **default**: only classes in the same package can access the element
- **private**: only this class can access the element

We can state that

$\text{public} <: \text{protected} <: \text{default} <: \text{private}$

where $a <: b$ means that the accessibility level a is weaker than b , and that a subclass can relax the accessibility level b with a .

Task 4

In C++ object aliasing is achieved using pointers and it is possible to have a pointer to a pointer. Here is an example

```

class X {};

class Initializer {
public:
    void init(X** x) {
        *x = new X();
    }
};

class Value {
private:
    X* x = nullptr;
public:
    Value(Initializer* i) {
        i->init(&x); // The initializer object will set the value of x
    }
};

```

How does the substitution principle apply to values of type pointer to pointer? Is it safe to call methods that have the signature of `init` with a value of type pointer to pointer to a subtype/supertype of `X`? Why?

— solution —

It is not safe to call methods with the signature of `init` with anything but a pointer to pointer to `X`. A pointer to a pointer can be thought of as an array with one object. As we know statically safe arrays are invariant. The code below illustrates what might go wrong if the actual argument's type were allowed to vary.

```

class SuperX {};
class X : public SuperX{public: int a;};
class SubX : public X{public: int b;};

class Initializer{
public:
    virtual void init(X** x) {
        *x = new X();
    }
};

class Initializer2 : public Initializer{
public:
    virtual void init(X** x) {
        (*x)->a=5; // run time error if called from i->init(&super_x)
    }
};

class Value {
private:
    X* x = nullptr;
    SuperX* super_x = new SuperX();
    SubX* sub_x = nullptr;
public:
    Value(Initializer* i) {
        i->init(&x); // ok
        i->init(&super_x); // wrong, if i is of type *Initializer2
        i->init(&sub_x); // wrong, sub_x might get a value of type
                        X
        sub_x->b = 5; // and so cause here a run time error
    }
};

```

Task 5 Union Types

Assume a language with nominal subtyping, covariant return types and contravariant arguments that allows types to be defined as a disjunction of other types, as in the following declarations:

```
String || Number get();
void set(String || Number newValue);
```

Such a type is called a *union type* and the different types that form the disjunction are its *components*. Classes can be thought of as union types with just one component.

A type Sub is a subtype of another type Super , i.e. $\text{Sub} <: \text{Super}$, if for each component C_{sub} of Sub there exists a component C_{sup} of Super such that $C_{\text{sub}} <: C_{\text{sup}}$. The usual nominal subtyping rules apply for classes.

A) Consider the signatures of the four methods below, assuming that $C <: B <: A$ (A , B , and C are regular class types)

```
m1: B          foo (B b)
m2: A          foo (A || B ab)
m3: B || C     foo (A a)
m4: A || B || C foo (C c)
```

Your task is to complete the table below. For each row and column, write 'yes', if the method at the left of the row could override the method at the top of the column. Otherwise write 'no'.

	m1	m2	m3	m4
m1	yes			
m2		yes		
m3			yes	
m4				yes

— solution —

Note that here $A || B || C$, $A || B$, and A can be considered to be identical types, because $C <: B <: A$. Similarly $B || C$ can also be considered identical to B .

	m1	m2	m3	m4
m1	yes	no	no	yes
m2	no	yes	no	yes
m3	yes	yes	yes	yes
m4	no	no	no	yes

B) Assume that A , B , and Q are classes such that $B <: A$ and Q is unrelated to A and B . Consider this code fragment:

```
void foo(A || Q arg) { arg.bar(42); }
```

(i) Assume that the type checker admits method `foo` if all components of `arg`'s static type have a method `bar(int)`. Do we need any run-time checks in order to avoid run-time errors? If so, what are they? Under what conditions could they be omitted?

— solution —

No run-time checks are necessary.

(ii) Assume that the type checker admits method `foo` if at least one component of `arg`'s static type has a method `bar(int)`. Do we need any run-time checks in order to avoid run-time errors? If so, what are they? Under what conditions could they be omitted?

— solution —

We could do an additional static check that all components of the type of `arg` have a method `bar(int)` and, if it fails, we need a dynamic check that the run-time type of `arg` has `bar(int)`.

Answer the questions from (i) and (ii) for the code fragment below.

```
void foo(A || B arg) { arg.bar(42); }
```

— solution —

- (i) No run-time checks are necessary.
- (ii) We can do a static check that `A` has `bar(int)`. If it does, we do not need a run-time check. Otherwise at run-time we need a check that the type of `arg` is `B`.

Task 6

As you have seen in the lectures, arrays are covariant in Java and C#. Because of this, each array update requires a run-time type check. Another approach would have been to adopt contravariant arrays. Does this solution require run-time type checks? If this is the case, explain in which cases you need these run-time type checks and provide an example in which a check would fail.

— solution —

Yes, contravariant arrays would require run-time type checks when reading values from the array.

By the definition of contravariance, we have that $S <: T$ implies $T[] <: S[]$.

Therefore `Object[] <: String[]` since `String <: Object`. So we can pass an array of type `Object[]` to a method that requires a `String[]` argument.

```
class C {  
    String foo(String[] a) {  
        return a[0];  
    }  
}  
  
void client(C c) {  
    Object[] arr = new Object[1];  
    arr[0] = new Object();  
    String s = c.foo(arr);  
}
```