Formal Methods and Functional Programming

Linear Temporal Logic

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The slides in this section are partly based on the course *Automata-based System Analysis* by Felix Klaedtke

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  - For a non-deterministic, non-terminating program $\mathtt{s}$

$$wc : \text{Stm} \times \text{State} \times \mathbb{N} \rightarrow \text{Bool}$$

$$wc(s, \sigma, n) \Leftrightarrow \sigma(o) = 0 \vee (\text{for all } s', \sigma' : \text{if } \langle s, \sigma \rangle \rightarrow_1 \langle s', \sigma' \rangle \text{ then there exists } m \in \mathbb{N} \text{ such that } m < n \text{ and } wc(s', \sigma', m))$$

$$\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \langle s', \sigma' \rangle \text{ and } \sigma(\mathtt{o}) = 0 \text{ and } \sigma'(\mathtt{o}) = 1 \text{ then there exists } n \in \mathbb{N} \text{ such that } wc(s', \sigma', n)$$

5. Linear Temporal Logic

Transition Systems Revisited

- We use a slightly different definition here
- A finite transition system is a tuple $(\Gamma, \sigma_I, \rightarrow)$
  - Γ: a finite set of configurations
  - σ_I: an initial configuration, $\sigma_I \in \Gamma$
  - $\rightarrow$: a transition relation, $\rightarrow \subseteq \Gamma \times \Gamma$
- We add an initial configuration
  - Transition system models only one system, not all programs of a programming language
- We omit terminal configurations
  - Simplifies theory
  - Termination can be modelled by special sink state

Transition System of a Promela Model

- Configurations: states (see previous section)
  - Global variables, global channels
  - Per active process: local variables, local channels, location counter
- Initial configuration: initial state (see previous section)
- Transition relation: defined by operational semantics of statements
  - We keep semantics informal
- A Promela model has a finite number of states
  - Finite number of active processes (limited to 255)
  - Finite number of variables and channels
  - Finite ranges of variables
  - Finite buffers of channels
- Therefore, it is possible to enumerate all possible states
  - How many states are there?

State Space of Sequential Programs

- Number of states

$$\#\text{program locations} \times \prod_{\text{variable } x} |\, dom(x) \,|$$

  - where $|\, dom(x) \,|$ denotes the number of possible values of variable x

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- Example: sequential program with 10 locations and 3 boolean variables

$$10 \times 2 \times 2 \times 2 = 10 \times 2^3 = 80$$

- Adding two integer variables yields $80 \times 2^{32} \times 2^{32} = 80 \times 2^{64}$

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- Number of states grows exponentially in the number of variables
- State space explosion

State Space of Concurrent Programs

- The number of states of $P \equiv P_1 \| \ldots \| P_N$ is at most

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- Number of states grows exponentially in the number of processes
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State Space of Promela Models

- The number of states of a system with N processes and K channels is at most

$$\prod_{i=1}^{N}(\#\text{program locations}_i \times \prod_{\text{variable } x_i} |\, dom(x_i)\,|) \times \prod_{j=1}^{K} |\, dom(c_j)\,|^{cap(c_j)}$$

  - $|\, dom(c)\,|$ denotes the number of possible messages of channel c
  - $cap(c)$ is the capacity (buffer size) of channel c

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  - $\mid dom(c) \mid$ denotes the number of possible messages of channel c
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- Number of states grows exponentially in the number and capacity of channels
- State space explosion

Computations

- Infinite sequences
  - S^ω is the set of infinite sequences of elements of set S
  - s_i denotes the i-th element of the sequence $s \in S^\omega$
- $\gamma \in \Gamma^\omega$ is a computation of a transition system if:
  - $\gamma_0 = \sigma_I$
  - $\gamma_i \rightarrow \gamma_{i+1}$
- $\mathcal{C}(TS)$ is the set of all computations of a transistion system TS

Linear-Time Properties

- Linear-time properties (LT-properties) specify the admissible computations of a transition system
- A linear-time property over Γ is a subset of Γ^ω
- TS satisfies LT-property P (over Γ)

$$TS \models P \text{ if and only if } \mathcal{C}(TS) \subseteq P$$

  - All computations of TS are admissible

- By contrast: branching-time properties can also express the existence of a computation
  - Example: "It is always possible to return to the initial state"

LT-Properties: Example

- All opened files must be closed eventually

$$P = \{\gamma \in \Gamma^{\omega} \mid \forall i \geq 0 : \gamma_i(o) = 1 \Rightarrow \exists n > 0 : \gamma_{i+n}(o) = 0\}$$

- LT-properties elegantly express properties of computations
  - Non-termination is handled by infinite sequences
  - Non-determinism is handled by considering each computation separately
- Logical formalism needed to simplify specification of LT-properties

From States to Propositions

- For a transition system TS, we specify the set AP of atomic propositions
  - An atomic proposition is a proposition containing no logical connectives
  - Example: $AP = \{open, closed\}$

- We define a labeling function that maps configurations to sets of atomic propositions
  - $L : \Gamma \to \mathcal{P}(AP)$
  - Example: $L(\sigma) = \begin{cases} \{open\} & \text{if } \sigma(o) = 1 \\ \{closed\} & \text{if } \sigma(o) = 0 \\ \{\} & \text{otherwise} \end{cases}$
- We call $L(\sigma)$ an abstract state
- From now on, we consider AP and L to be part of the transition system

Traces

- A trace is an abstraction of a computation
  - Observe only the propositions of each state, not the concrete state itself
  - Infinite sequence of abstract states ($\mathcal{P}(AP)^\omega$)
- $t \in \mathcal{P}(AP)^\omega$ is a trace of a transition system TS if
  $t = L(\gamma_0)L(\gamma_1)L(\gamma_2), \ldots$ and γ is a computation of TS
- $\mathcal{T}(TS)$ is the set of all traces of a transistion system TS
- LT-properties are typically specified on traces

$$P = \{t \in \mathcal{P}(AP)^\omega \mid \forall i \geq 0 : open \in t_i \Rightarrow \exists n > 0 : closed \in t_{i+n}\}$$

Safety Properties

- Intuition
  - "nothing bad ever happens"
  - "if something bad happens then it is irremediable"
- An LT-property P is a safety property if for all traces $t \in \mathcal{P}(AP)^\omega$: if $t \notin P$ then there is a finite prefix $\hat{t}$ of t such that for each trace t' with prefix $\hat{t}$, $t' \notin P$
  - $\hat{t}$ is called a bad prefix
- Safety properties are violated in finite time and cannot be repaired
- Examples
  - State properties, for instance, invariants

$$P = \{t \in \mathcal{P}(AP)^\omega \mid \forall i \geq 0 : \mathit{open} \in t_i \vee \mathit{closed} \in t_i\}$$

  - "Money can be withdrawn only after correct PIN has been entered"

Liveness Properties

- Intuition
  - "something good will happen eventually"
  - "if the good thing has not happened yet, it will happen in the future"
- An LT-property P is a liveness property if every finite sequence $\hat{t} \in \mathcal{P}(AP)^*$ is a prefix of a trace $t \in P$
  - A liveness property does not rule out any prefix
  - Every finite prefix can be extended to a trace that is in P
- Liveness properties are violated in infinite time
- Examples
  - All opened files must be closed eventually

$$P = \{t \in \mathcal{P}(AP)^\omega \mid \forall i \geq 0 : open \in t_i \Rightarrow \exists n > 0 : closed \in t_{i+n}\}$$

  - "The program terminates eventually"

5. Linear Temporal Logic

Linear Temporal Logic

- Linear Temporal Logic (LTL) allows us to formalize LT-properties of traces
- We will discuss syntax and semantics, but not inference rules
- Whether the traces of a finite transition system satisfy an LTL formula is decidable

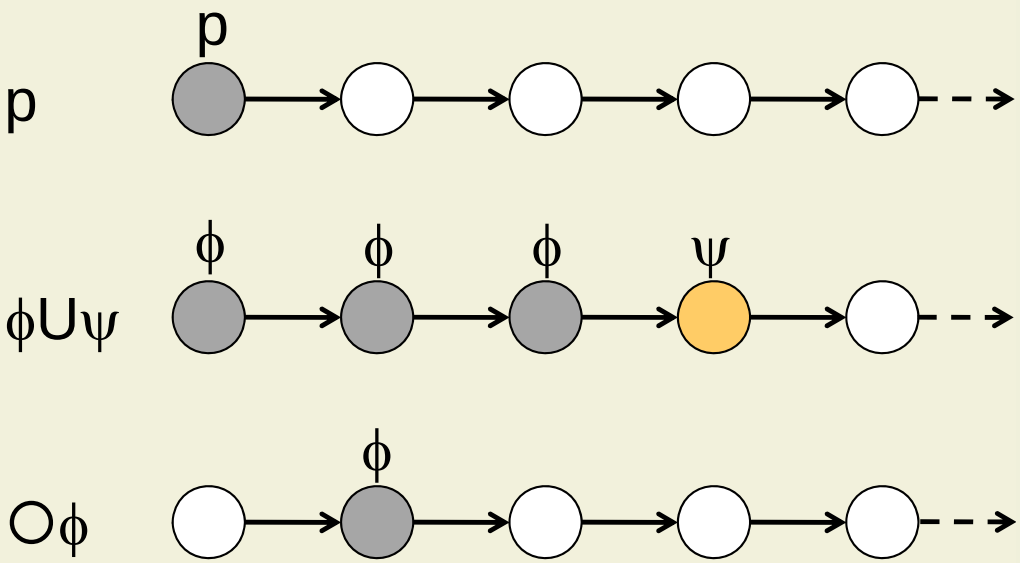
LTL: Basic Operators

- Syntax

$$\phi = p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \,\mathsf{U}\, \phi \mid \bigcirc\phi$$

  - where p is a proposition in $AP \neq \varnothing$

- Intuitive meaning of temporal operators

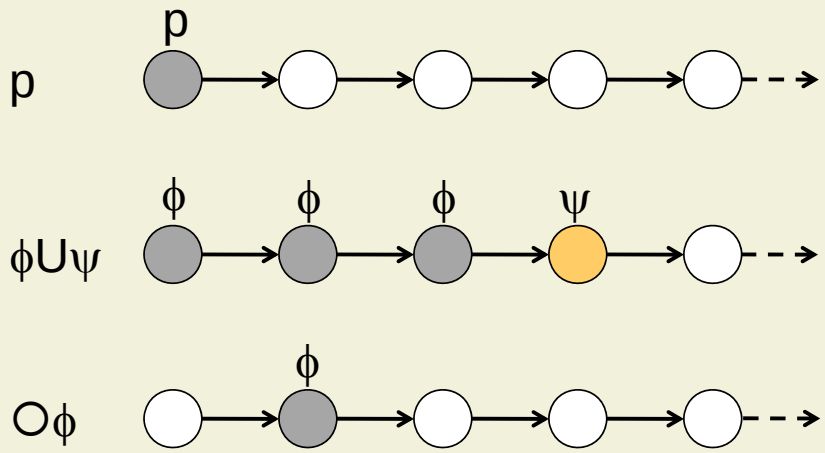


LTL: Semantics

- $t \models \phi$ expresses that trace $t \in \mathcal{P}(AP)^\omega$ satisfies LTL formula ϕ

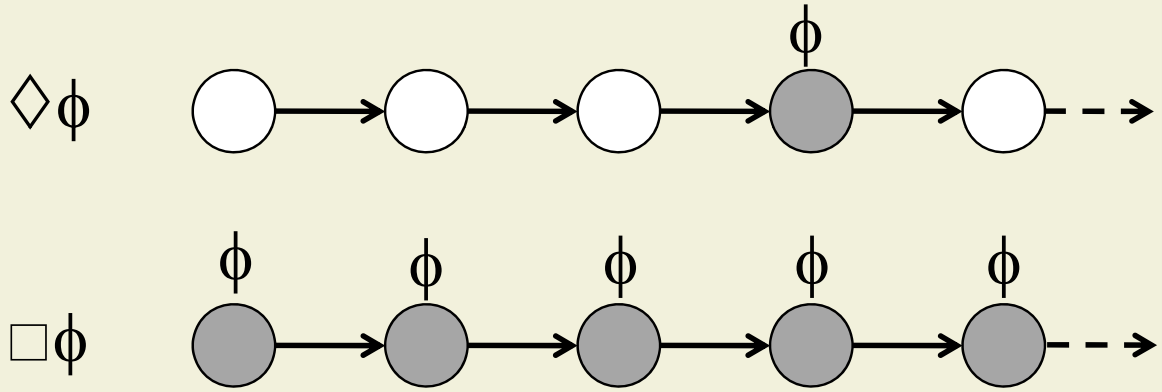
$t \models p$	iff	$p \in t_0$
$t \models \neg\phi$	iff	not $t \models \phi$
$t \models \phi \wedge \psi$	iff	$t \models \phi$ and $t \models \psi$
$t \models \phi \,\mathsf{U}\, \psi$	iff	there is a $k \geq 0$ with $t_{\geq k} \models \psi$ and $t_{\geq j} \models \phi$ for $0 \leq j < k$
$t \models \bigcirc\phi$	iff	$t_{\geq 1} \models \phi$

- where $t_{\geq i}$ is the suffix of t starting at t_i



Derived Operators

- *true*, *false*, $\vee$, $\Rightarrow$ as usual
- Eventually: $\Diamond\phi \equiv \mathit{true}\,\mathsf{U}\,\phi$
- Generally: $\Box\phi \equiv \neg \Diamond \neg\phi$
- Intuitive meaning



Specification Patterns

- Strong invariant
  - $\Box p$: p always holds
  - A file is always open or closed: $\Box(open \vee closed)$
  - Safety property
- Monotone invariant
  - $\Box(p \Rightarrow \Box p)$: once p, always p
  - Once information is public, it can never become secret again (but it may always stay secret): $\Box(public \Rightarrow \Box public)$
  - Safety property
- Establishing an invariant
  - $\Diamond \Box\, p$: eventually p will always hold
  - System initialization starts daemon process: $\Diamond \Box\, daemonRunning$
  - Liveness property

Specification Patterns (cont'd)

- Responsiveness
  - $\Box(p \Rightarrow \Diamond q)$: everytime p holds, q will eventually hold
  - All opened files must be closed eventually: $\Box(\mathit{open} \Rightarrow \Diamond \mathit{closed})$
  - Liveness property
- Fairness
  - $\Box \Diamond p$: p holds infinitely often
  - Producer does not wait infinitely long before entering the critical section: $\Box \Diamond \mathit{crit}$
  - Liveness property

Needham-Schroeder Protocol

- If Alice and Bob have completed their protocol runs then Alice should believe her partner to be Bob if and only if Bob believes to talk to Alice

$$\Box(statusA = 1 \wedge statusB = 1 \Rightarrow (partnerA = agentB \Leftrightarrow partnerB = agentA))$$

- If Alice completed her protocol run with Bob, the intruder should not have learned Alice's nonce

$$\Box(statusA = 1 \wedge partnerA = agentB \Rightarrow knows\_nonceA = 0)$$

- If Bob completed his protocol run with Alice, the intruder should not have learned Bob's nonce

$$\Box(statusB = 1 \wedge partnerB = agentA \Rightarrow knows\_nonceB = 0)$$

5. Linear Temporal Logic

5.1 Linear-Time Properties

5.2 Linear Temporal Logic

5.3 LTL Model Checking

LTL Model Checking Problem

Given a finite transition system TS and an LTL formula ϕ, decide whether $t \vDash \phi$ for all $t \in \mathcal{T}(TS)$

- We need to check inclusion of traces
  - LTL formula ϕ describes set of traces $P(\phi)$
  - We need to determine whether $\mathcal{T}(TS) \subseteq P(\phi)$

A Simpler Problem: Regular Safety Properties

- A safety property is regular if its bad prefixes are described by a regular language
- Every invariant over AP is a regular safety property
  - Invariant $\Box p$
  - Bad prefixes start with $q^* r$ where $p \in q$ and $p \notin r$
- Regular safety property that is not an invariant
  - Traffic light: red is immediately preceeded by yellow
    $\Box(\neg yellow \Rightarrow \bigcirc \neg red)$
  - Bad prefixes start with $(green \mid yellow\ yellow^* (red \mid green))^* red$
- Non-regular safety property
  - Vending machine: at least as many coins inserted as drinks dispensed
  - Cannot be expressed in LTL with $AP = \{pay, drink\}$ ("LTL cannot count")
  - Bad prefixes: regular languages cannot count

Checking Regular Safety Properties

- Safety properties are violated in finite time
  - Look at all finite prefixes $\mathcal{T}_{fin}(TS)$ of the traces $\mathcal{T}(TS)$ of a transition system TS
  - Check whether $\mathcal{T}_{fin}(TS)$ contains a bad prefix

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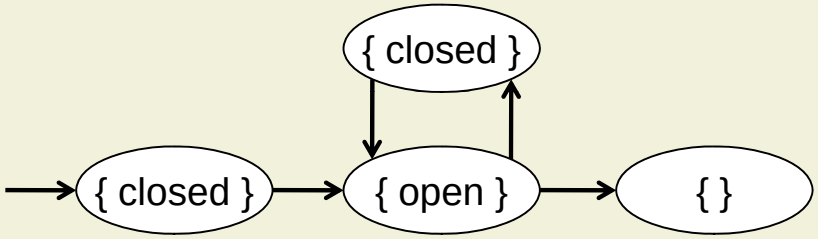
  3. Construct finite automaton for intersection of $\mathcal{FA}_{TS}$ and $\mathcal{FA}_{\bar{P}}$
  4. Check whether intersection is empty
     - If intersection is non-empty, property P is violated
     - Each word in the intersection is a counterexample

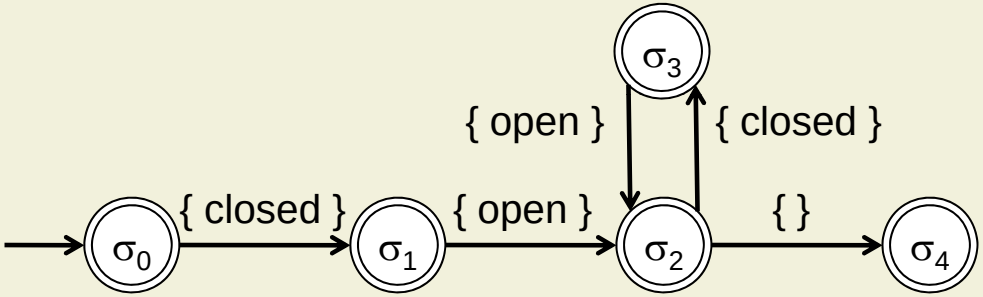
Reminder: Finite Automata

- A finite automaton (FA) is a tuple $(Q, \Sigma, Q, \delta, q_0, F)$
  - Q: a finite set of states
  - Σ: a finite alphabet
  - δ: a transition relation, $\delta \subseteq Q \times \Sigma \times Q$
  - q_0: an initial state
  - $F \subseteq Q$: a set of accepting states

Step 1: Finite Automaton for Finite Prefixes

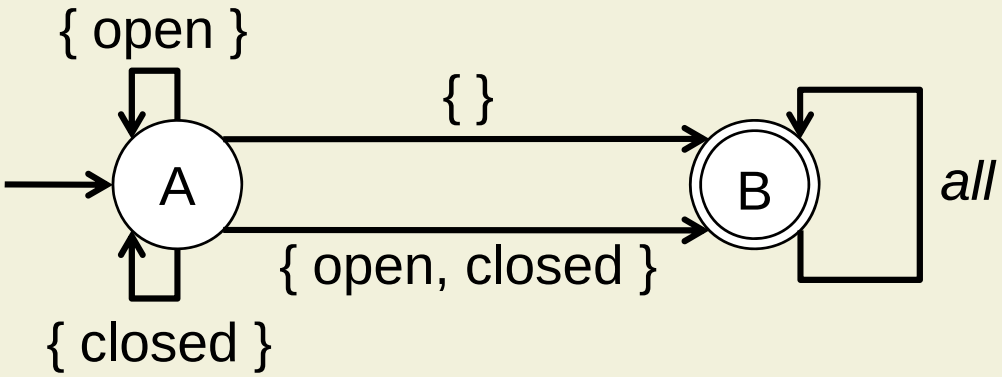
- Given a transition system $TS = (\Gamma, \sigma_I, \rightarrow)$, propositions AP, and labeling function L
- The automaton $\mathcal{FA}_{TS} = (Q, \Sigma, \delta, q_0, F)$ accepts $\mathcal{T}_{fin}(TS)$
  - $Q = \Gamma \cup \{\sigma_0\}$, where $\sigma_0 \notin \Gamma$
  - $\Sigma = \mathcal{P}(AP)$
  - $\delta = \{(\sigma, p, \sigma') \mid \sigma \rightarrow \sigma' \text{ and } p \in L(\sigma')\} \cup \{(\sigma_o, p, \sigma_I) \mid p \in L(\sigma_I)\}$
  - $q_0 = \sigma_0$
  - $F = Q$
- Example: `o:=o+1; while * do o:=o-1; o:=o+1 end; o:=o+1`





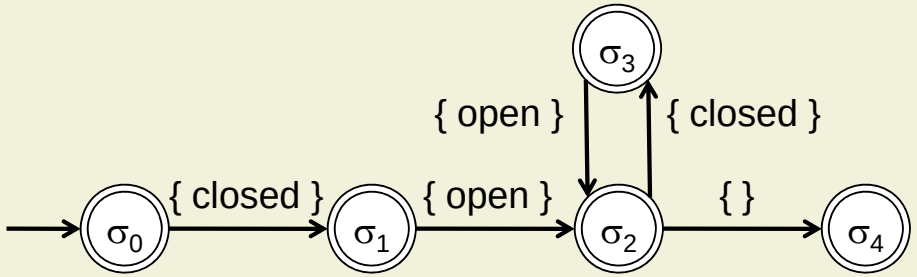
Step 2: Finite Automaton for Bad Prefixes

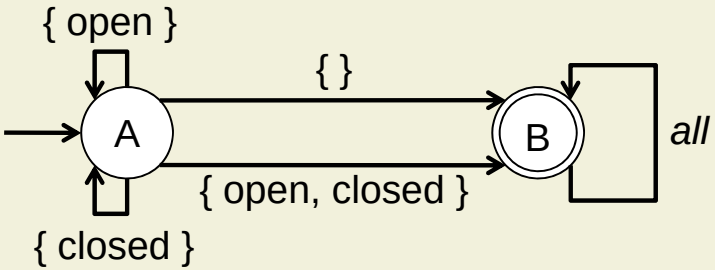
- By definition, bad prefixes are described by a regular language
- Apply standard construction to obtain FA $\mathcal{FA}_{\bar{P}}$ from regular expression
- Example: $\Box(open \vee closed)$
  - Bad prefixes start with
    $(\{open\} \mid \{closed\})^*(\{\} \mid \{open, closed\})$

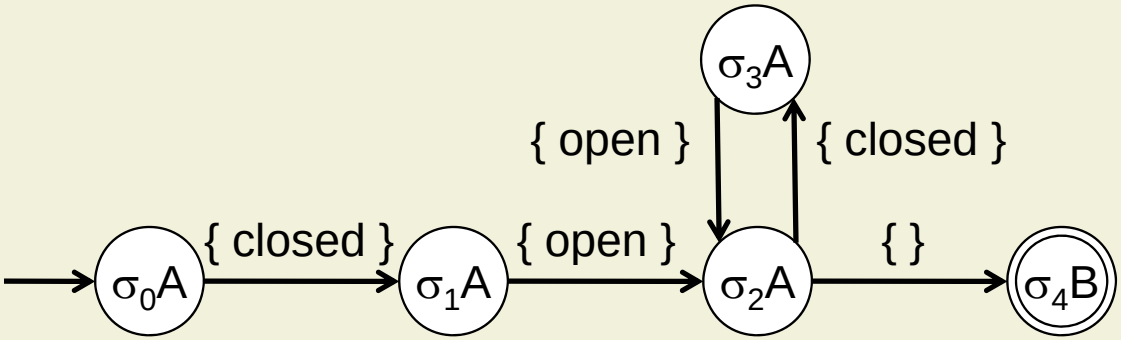


Step 3: Finite Automaton for Intersection

- Construct FA $\mathcal{FA}_{TS \cap \bar{P}}$ that accepts the intersection of the languages accepted by $\mathcal{FA}_{TS}$ and $\mathcal{FA}_{\bar{P}}$
- Apply standard construction for product of two FA
- Example

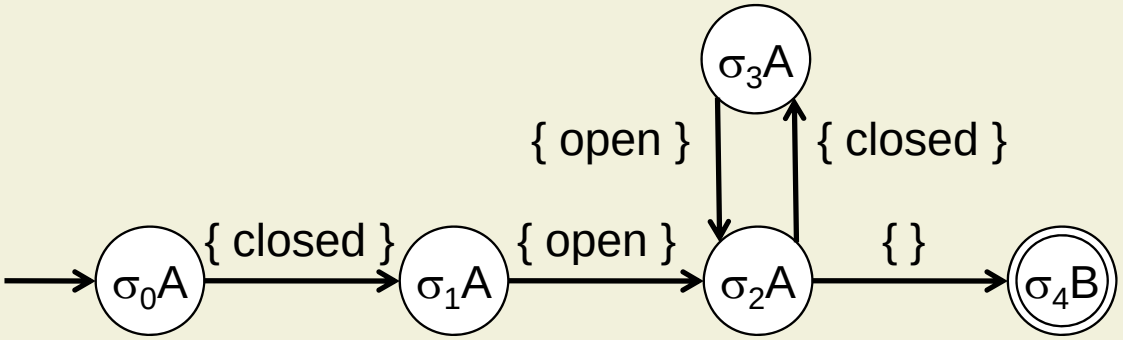






Step 4: Check Emptiness

- If $\mathcal{FA}_{TS \cap \bar{P}}$ accepts a word w then
  - $w \in \mathcal{T}_{fin}(TS)$ because it is accepted by $\mathcal{FA}_{TS}$ and
  - w is a bad prefix because it is accepted by $\mathcal{FA}_{\bar{P}}$
  - Therefore, P is not satisfied, and w is a counterexample
- Apply standard algorithm to check emptiness of FA
- Example



  - Accepts $\{closed\}\{open\}(\{closed\} \mid \{open\})^*\{\}$
  - Smallest counterexample: $\{closed\}\{open\}\{\}$
  - Counterexample can be mapped back to transition system

Büchi Automata

- Büchi automata are similar to finite automata, but accept infinite words
- A Büchi automaton (BA) is a tuple $(Q, \Sigma, Q, \delta, q_0, F)$
  - Q: a finite set of states
  - Σ: a finite alphabet
  - δ: a transition relation, $\delta \subseteq Q \times \Sigma \times Q$
  - q_0: an initial state
  - $F \subseteq Q$: a set of accepting states
- A run of a BA accepts its input if it passes infinitely often through an accepting state
- Büchi automata enjoy many of the properties of finite automata
  - We can construct the product of two BA
  - Emptiness is decidable

LTL Model Checking: Approach

1. Describe traces $\mathcal{T}(TS)$ by Büchi automaton $\mathcal{BA}_{TS}$
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   - If intersection is non-empty, property ϕ is violated
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Complexity Results

For a finite transition system TS and an LTL formula ϕ, the model checking problem $TS \models \phi$ is solvable in $\mathcal{O}(\mid TS \mid \times 2^{|\phi|})$

- $\mid TS \mid$ is the size of the transition system (which grows exponentially in the number of variables, processes, and channels)
- $\mid \phi \mid$ is the size of ϕ; exponential complexity comes from the construction of $\mathcal{BA}_{\neg\phi}$

Advanced Model Checking Techniques

- On-the-fly model checking
  - Often violation of a property can be detected without checking all possible states or traces (for instance, $\Box p$)
  - Generate transition system and check property step-by-step
  - Implemented in Spin

- Partial order reduction
  - Remove redundancy from different interleavings of concurrent executions
  - Code segments that operate only on local state are not affected by interleaving
  - Implemented in Spin

Advanced Model Checking Techniques (cont'd)

- Bounded model checking
  - Check only prefixes of traces up to a certain length
  - Closer to testing than verification
  - Very effective in practice

- Symbolic model checking
  - Uses sets of states rather than individual states
  - Sets of states are represented through boolean functions
  - Very efficient data structure: binary decision diagram (BDDs)
  - Typically used to check branching-time properties
  - Can deal with larger models

Conclusions

- Variety of approaches
  - Best method depends on application area
- Tool support is essential
  - Proofs are tedious and error-prone
  - Some tools have reached maturity for industrial applications