Formal Methods and Functional Programming

Modelling

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The slides in this section are partly based on the course *Automata-based System Analysis* by Felix Klaedtke

Example 1: Protocol Verification

- Protocol for file access with primitives `open`, `close`, and `write`

- Task: verify that a program obeys the following (informal) rules:

  A. All opened files must be closed eventually
  B. An opened file must be closed before the next open and vice versa
  C. All files must be opened before executing a write operation

  There is only one file, which is initially closed

- Problem is typical for verification of protocols
  - Locking (acquire, access, release)
  - Authentication (authenticate, access)

Example 1: Encoding in IMP

- File is represented by variable `f`
  - Write is encoded by assignment to `f`
  - Variable `o` counts how often file was opened/closed

- Encoding of primitives:
  - `open`: `o:=o` $+ 1$
  - `close`: `o:=o` $- 1$
  - `write`: `f:=`e

- Informal rules:

A. After `o` has been set to one, it must eventually be re-set to zero
B. In all execution states, `o` is zero or one
C. When `f` is being assigned to, `o` must be greater than zero

Variable `o` is initially zero

Example 1: Specification in NS and Hoare Logic

A. For a terminating program $\mathtt{s}$, $\mathtt{o}$ must be zero in the terminal state

$$\boxed{\langle \mathtt{s}, \sigma \rangle \to \sigma' \text{ and } \sigma(\mathtt{o}) = 0 \text{ then } \sigma'(\mathtt{o}) = 0}$$

$$\boxed{\{\ \mathtt{o} = 0\ \}\ \mathtt{s}\ \{\ \mathtt{o} = 0\ \}}$$

Property cannot be expressed for non-terminating programs

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- Natural semantics and Hoare logic can express properties of initial and terminal states, but not of intermediate states

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- Natural semantics and Hoare logic can express properties of initial and terminal states, but not of intermediate states

Example 1: Specification in SOS (A)

- A: After $\mathtt{o}$ has been set to one, it must eventually be re-set to zero
    - For a terminating program $\mathtt{s}$

$$\boxed{\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma' \text{ and } \sigma(\mathtt{o}) = 0 \text{ then } \sigma'(\mathtt{o}) = 0}$$

Example 1: Specification in SOS (A)

- A: After `o` has been set to one, it must eventually be re-set to zero
  - For a terminating program `s`

    $\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma'$ and $\sigma(\mathtt{o}) = 0$ then $\sigma'(\mathtt{o}) = 0$

  - For a deterministic, non-terminating program `s`

    $\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \langle s', \sigma' \rangle$ and $\sigma(\mathtt{o}) = 0$ and $\sigma'(\mathtt{o}) = 1$ then there exist s'', σ'' such that $\langle s', \sigma' \rangle \rightarrow_1^* \langle s'', \sigma'' \rangle$ and $\sigma''(\mathtt{o}) = 0$

Example 1: Specification in SOS (A)

- A: After $\mathtt{o}$ has been set to one, it must eventually be re-set to zero
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  - For a non-deterministic, non-terminating program $\mathtt{s}$

$$\begin{aligned} & wc : \mathrm{Stm} \times \mathrm{State} \times \mathbb{N} \rightarrow \mathrm{Bool} \\ & wc(s, \sigma, n) \Leftrightarrow \sigma(o) = 0 \vee \\ & \qquad (\text{for all } s', \sigma' : \text{if } \langle s, \sigma \rangle \rightarrow_1 \langle s', \sigma' \rangle \text{ then there exists} \\ & \qquad m \in \mathbb{N} \text{ such that } m < n \text{ and } wc(s', \sigma', m)) \end{aligned}$$

$$\begin{aligned} & \langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \langle s', \sigma' \rangle \text{ and } \sigma(\mathtt{o}) = 0 \text{ and } \sigma'(\mathtt{o}) = 1 \text{ then} \\ & \text{there exists } n \in \mathbb{N} \text{ such that } wc(s', \sigma', n) \end{aligned}$$

Example 1: Specification in SOS (B and C)

- B: In all execution states, o is zero or one

$$\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \langle s', \sigma' \rangle \text{ and } \sigma(\mathtt{o}) = 0 \text{ then } \sigma'(\mathtt{o}) = 0 \text{ or } \sigma'(\mathtt{o}) = 1$$

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- C: When `f` is being assigned to, `o` must be greater than zero
  - SOS cannot express properties of occurrences of statements
  - Work-around: Enrich state and make sure `write` changes the state

Example 1: Verification

A. For a terminating program $\mathtt{s}$

$$\boxed{\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma' \text{ and } \sigma(\mathtt{o}) = 0 \text{ then } \sigma'(\mathtt{o}) = 0}$$

- Proof needs to consider all possible derivations of $\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma'$ to find all possible terminal states
- Problematic in the presence of non-determinism or parallelism

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- Proof about infinite derivation sequence requires invariant, which cannot be found automatically

B. In all execution states, $\mathtt{o}$ is zero or one

$$\boxed{\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \langle s', \sigma' \rangle \text{ and } \sigma(\mathtt{o}) = 0 \text{ then } \sigma'(\mathtt{o}) = 0 \text{ or } \sigma'(\mathtt{o}) = 1}$$

- Prove needs to consider all possible derivations

Example 2: Verification of Parallel Programs

- A (simplified) Java program

```
class Cell {
  int x = 0;

  static void main(...) {
    Cell c = new Cell();
    Thread t1 = new Even(c);
    Thread t2 = new Even(c);
    t1.start(); t2.start();
    t1.join();  t2.join();
    System.out.println(c.x);
  }
}
```

```
class Even extends Thread {
  Cell c;

  Even(Cell c) {
    this.c = c;
  }

  void run() {
    c.x = c.x + 1;
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  }
}
```

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  }
}
```

```
3
```

Example 2: Encoding in IMP

- The following program `s` represents the core of the Java program
  - `x` represents shared variable `c.x`
  - `y` and `z` represent thread-local state

```
  (y := x;  y := y + 1;  x := y;
   y := x;  y := y + 1;  x := y)
par
  (z := x;  z := z + 1;  x := z;
   z := x;  z := z + 1;  x := z)
```

- Desired property:
  If `x` is zero in the initial state then `x` is even in the terminal state
  - NS and Hoare logic cannot handle parallelism
  - SOS specification:

$$\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma' \text{ and } \sigma(\mathtt{x}) = 0 \text{ then } \sigma'(\mathtt{x}) \textbf{ mod } 2 = 0$$

Example 2: Verification

- In this case, spotting the counterexample is easy, but how to attempt a formal proof?
- Induction does not work because there is no suitable induction hypothesis
  - Observation also holds for corrected example
- Proof strategy: enumerate all possible derivations of $\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma'$ and inspect terminal state σ'
  - Number of derivations grows exponentially in number of executed statements
  - Here, $\frac{12!}{6! \times 6!} = 924$ possible derivations!
  - Manual enumeration not feasible, especially for programs with loops

Examples: Observations

- Specification challenge
  - How to specify properties of sequences of states concisely

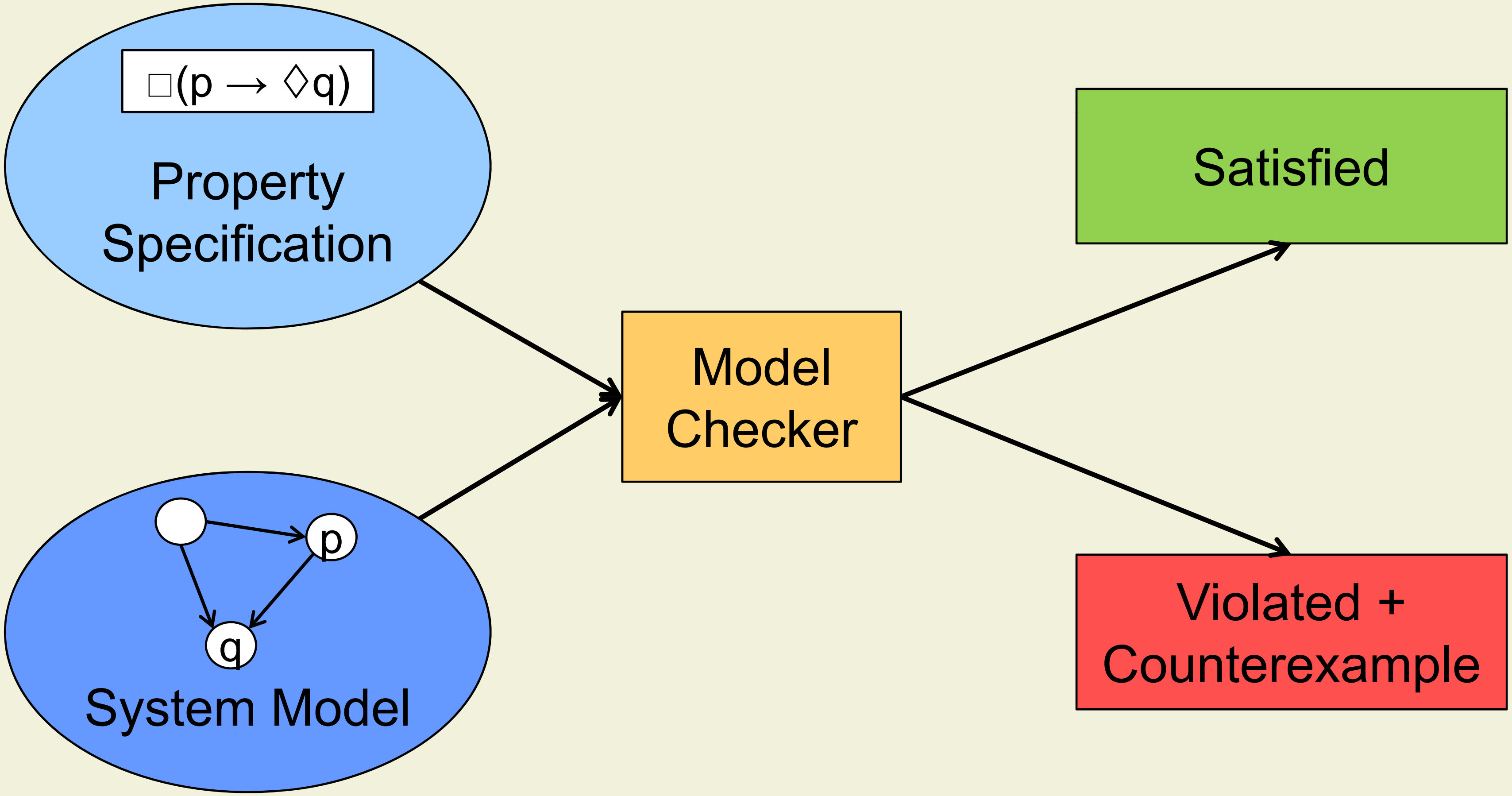
- Verification challenges
  - Concurrent systems: How to prove properties of all possible program executions
  - Reactive systems: How to prove automatically properties of infinite derivation sequences

Model Checking

> Model checking is an automated technique that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for (a given state in) that model. [Baier and Katoen]

- Model checkers enumerate all possible states of a system:
  - Explicit state model checking: represent state explicitly through concrete values
  - Symbolic model checking: represent state through (boolean) formulas
- We focus on explicit state model checking

Model Checking



$\Box(p \rightarrow \Diamond q)$

Property Specification

System Model

Model Checker

Satisfied

Violated + Counterexample

Model Checking Process

- Modelling phase
  - Model the system under consideration using the description language of your model checker (possibly a programming language)
  - Formalize the properties to be checked

- Running phase
  - Run the model checker to check the validity of the property in the system model

- Analysis phase
  - If property is satisfied, celebrate and move on to next property
  - If property is violated, analyze counterexample
  - If out of memory, reduce model and try again

Main Purposes of Model Checking

- Model checking is mainly used to analyze system designs (as opposed to implementations)
- Typical properties to be analyzed include
    - Deadlocks
    - Reachability of undesired states
    - Protocol violations

Modelling Concurrent Systems

- Systems are modelled as finite transition systems

- We model systems as communicating sequential processes (agents)
  - Finite number of processes
  - Interleaved process execution

- Processes can communicate via:
  - Shared variables
  - Synchronous message passing
  - Asynchronous message passing

Protocol Meta Language Promala

- Input language of the Spin model checker
- Main objects are processes, channels, and variables
- C-like syntax

```
init {
  printf("Hello World!\n")
}
```

- Spin can “execute” (simulate) models
- References
  - Quick reference: `www.spinroot.com/spin/Man/Quick.html`
  - Further references: `www.spinroot.com/spin/Man/index.html`

Promela Programs

- Constant declarations

```
#define N 5
mtype = { ack, req };
```

- Structure declarations

```
typedef vector { int x; int y };
```

- Global channel declarations

```
chan buf = [2] of { int };
```

- Global variable declarations

```
byte counter;
```

- Process declarations

```
proctype myProc(int p) { ... }
```

Promela Process Declarations

- Simple form

```
proctype myProc(int p) { ... }
```

  - Body consists of a sequence of variable declarations, channel declarations, and statements
  - No arrays as parameters

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- General form

```
active [N] proctype myProc(...) provided(E) priority M { ... }
```

  - `active`: Start `N` instances of `myProc` in the initial state
  - `provided`: `E` is an enabling condition, evaluated in the inital state
  - `priority`: `M` indicates probability during random simulation ($\mathtt{M} \geq 1$)

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- Deterministic processes lead to extra check during model analysis

```
D_proctype myProc(int p) { ... }
```

Promela Types

- Primitive types

Type	Value range
`bit` or `bool`	$0 \ldots 1$
`byte`	$0 \ldots 255$
`short`	$-2^{15} \ldots 2^{15} - 1$
`int`	$-2^{31} \ldots 2^{31} - 1$

  - No floats and mathematical integers
- Used-defined types
  - Arrays: `int name[4]`
  - Structures
  - Type of symbolic constants: `mtype`
- Channel type: `chan`

Promela Variable and Channel Declarations

- Variable declarations

```
byte a, b = 5, c;
int d[3], e[4] = 3;
mtype msg = ack;
vector v;
```

  - Variables are initialized to zero-equivalent values

Promela Variable and Channel Declarations

- Variable declarations

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byte a, b = 5, c;
int d[3], e[4] = 3;
mtype msg = ack;
vector v;
```

  - Variables are initialized to zero-equivalent values

- Channel declarations

```
chan c1 = [2] of { mtype, bit, chan };
chan c2 = [0] of { int };
chan c3;
```

  - c1 can store up to two messages
    Messages sent via c1 consist of three parts (triples)
  - c2 models rendez-vous communication
  - c3 is uninitialized; must be assigned an initialized channel before usage

Promela Variable and Channel Declarations

- Variable declarations

```
byte a, b = 5, c;
int d[3], e[4] = 3;
mtype msg = ack;
vector v;
```

  - Variables are initialized to zero-equivalent values

- Channel declarations

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chan c1 = [2] of { mtype, bit, chan };
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  - c1 can store up to two messages
    Messages sent via c1 consist of three parts (triples)
  - c2 models rendez-vous communication
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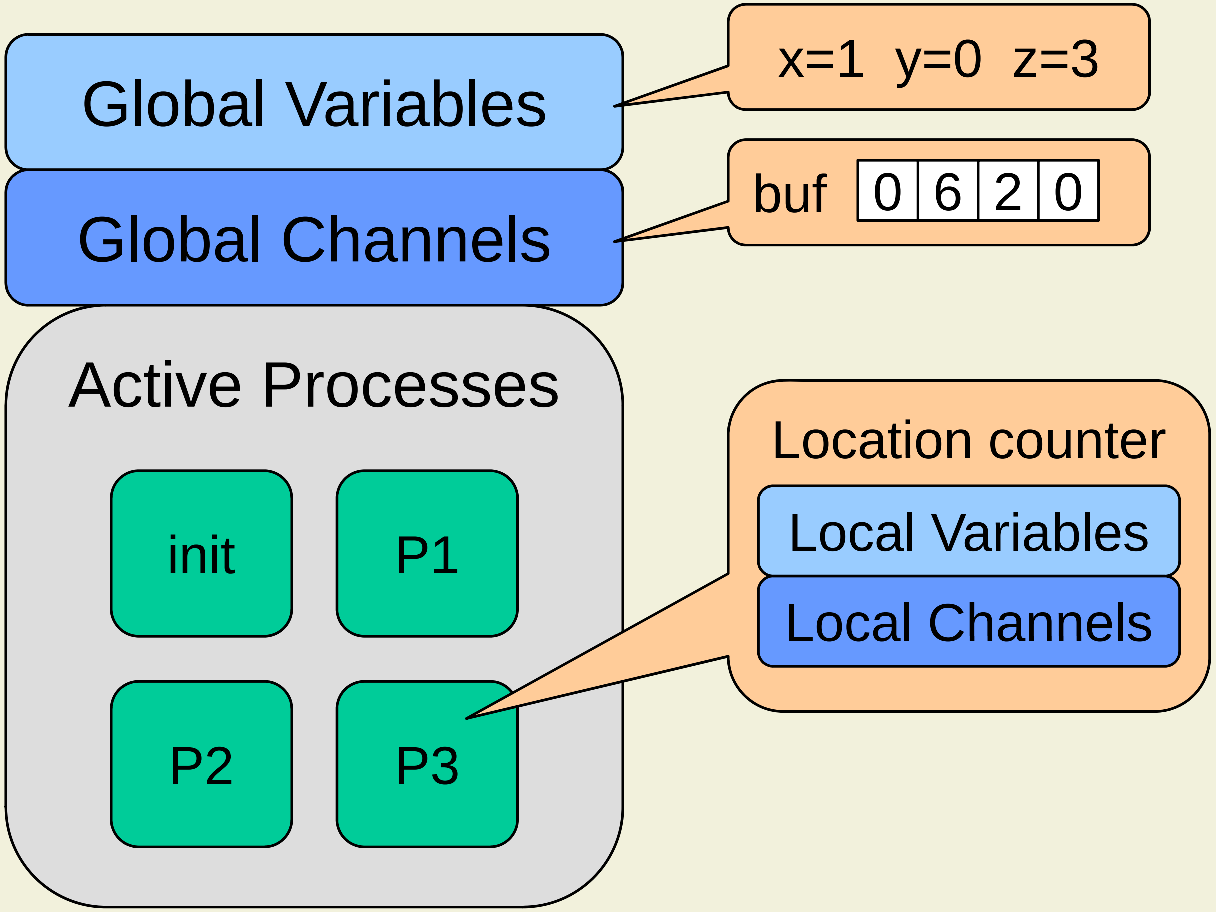
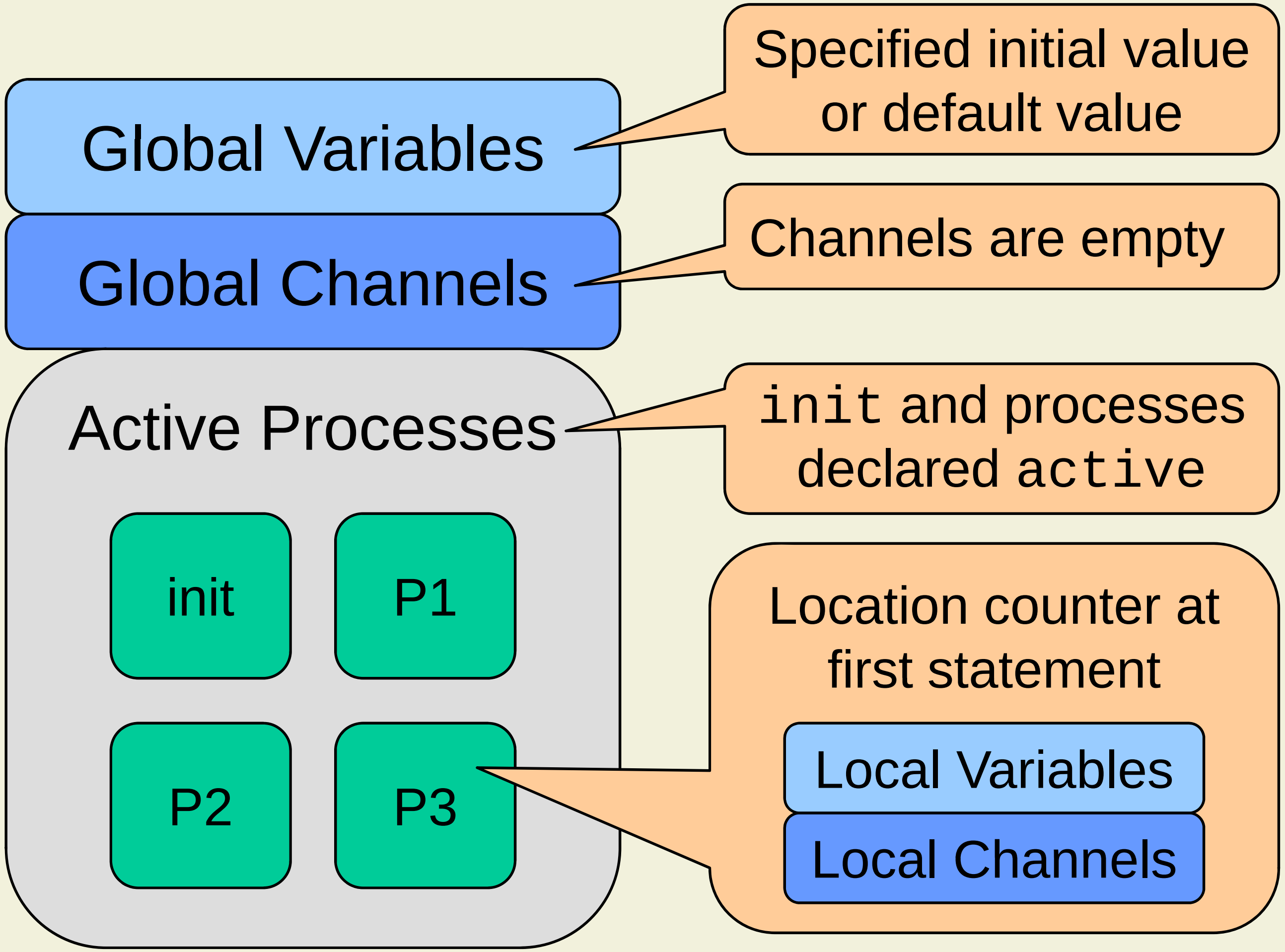
- Variable and channel declarations are local to a process or global

State Space of a Promela System



Initial State



Specified initial value or default value

Channels are empty

`init` and processes declared `active`

Location counter at first statement

State Transitions

- A statement can be executable or blocked
  - Send is blocked if channel is full
  - `s1;s2` is blocked if `s1` is blocked
  - `timeout` is executable if all other statements are blocked

A transition is made in three steps:

- Determine all executable statements of all active processes
  - If no executable statement exists, transition system gets stuck
- Choose non-deterministically one of the executable statements
  - Non-determinism models concurrency through interleaving
- Change the state according to the chosen statement

Promela Expressions

- Variables, constants, and literals
- Structure and array accesses
- Unary and binary expressions with operators

  `+` `-` `*` `/` `%` `>`

  `>=` `<` `<=` `==` `!=` `!`

  `&` `||` `&&` `|` `~` `>>`

  `<<` `^` `++` `--`

- Function applications

  `len()` `empty()` `nempty()` `nfull()` `full()`

  `run` `eval()` `enabled()` `pcvalue()`

- Conditional expressions: `(E1 -> E2 : E3)`

Promela Statements

- `skip`
  - Does not change the state
  - Always executable
- `timeout`
  - Does not change the state
  - Executable if all other statements in the system are blocked
- `assert(E)`
  - Aborts execution if expression `E` evaluates to zero; otherwise equivalent to `skip`
  - Always executable
- Assignment
  - `x = E` assigns the value of `E` to variable `x`
  - `a[n] = E` assigns the value of `E` to array element `a[n]`
  - Always executable

Promela Statements (cont'd)

- Sequential composition
  - `s1;s2` is executable if `s1` is executable
- Expression statement
  - Evaluates expression `E`
  - Executable if `E` evaluates to value different from zero
  - `E` must not change state (no side effects)
  - Common case: function applications
  - Examples:

```
printf("Hello World!\n");
run myProcess;
x > 0;
```

Motivation: Verification of Parallel Programs

- A (simplified) Java program

```
class Cell {
  int x = 0;

  static void main(...) {
    Cell c = new Cell();
    Thread t1 = new Even(c);
    Thread t2 = new Even(c);
    t1.start(); t2.start();
    t1.join();  t2.join();
    System.out.println(c.x);
  }
}
```

```
class Even extends Thread {
  Cell c;

  Even(Cell c) {
    this.c = c;
  }

  void run() {
    c.x = c.x + 1;
    c.x = c.x + 1;
  }
}
```

```
3
```

Example: Modelling Even.run

```
class Even extends Thread {
  Cell c;

  Even(Cell c) {
    this.c = c;
  }

  void run() {
    c.x = c.x + 1;
    c.x = c.x + 1;
  }
}
```

```
int x;

proctype EvenRun() {
  x = x + 1;
  x = x + 1;
}
```

```
int x;

proctype EvenRun() {
  int y = x;
  y = y + 1;
  x = y;
  y = x;
  y = y + 1;
  x = y;
}
```

Example: Modelling Cell.main

```
class Cell {
  int x = 0;

  static void main(...) {
    Cell c = new Cell();
    Thread t1 = new Even(c);
    Thread t2 = new Even(c);
    t1.start(); t2.start();
    t1.join();  t2.join();
    System.out.println(c.x);
  }
}
```

```
init {
  x = 0;

  run EvenRun();
  run EvenRun();

  /* wait for termination */
  _nr_pr == 1;

  printf("x: %d\n", x);
  assert x % 2 == 0;
}
```

- `_nr_pr` is a predefined global variable that yields the number of active processes
- Simulation in Spin shows the possible outcomes 2, 3, and 4 (like Java program)

Promela Statements: Selection

```
if
:: s1   /* option 1 */
:: ...
:: sn   /* option n */
fi
```

- Executable if at least one of its options it executable
- Chooses an option non-deterministically and executes it

```
if  /* Move a sprite */
:: x < maxX -> x = x + 1;
:: x > minX -> x = x - 1;
:: y < maxY -> y = y + 1;
:: y > minY -> y = y - 1;
:: color = color + 1;
fi
```

- Statement `else` is executable if no other option is executable (may occur at most in one option)

Promela Statements: Repetition

```
do
:: s1   /* option 1 */
:: ...
:: sn   /* option n */
od
```

- Executable if at least one of its options it executable
- Chooses repeatedly an option non-deterministically and executes it
- Terminates when a `break` or `goto` is executed

```
/* compute factorial of n */

int r = 1;

do
:: n > 1 -> r = r*n; n = n-1;
:: else -> break
od
```

```
/* deadlock detection */
active proctype watchDog() {
  do
  :: timeout ->
     /* reset the state */
  od
}
```

Promela Statements: Atomic

- Basic statements are executed atomically
  - No interleaving during execution of statement
  - `skip`, `timeout`, `assert`, assignment, expression statement
- `atomic { s }` executes `s` atomically
  - If any statement within `s` blocks, atomicity is lost, and other processes are then allowed to execute statements
- Example: Binary semaphores (locks)

```
bit locked; /* global */
```

```
/* lock */
locked == 0;
locked = 1;


/* critical section */
locked = 0; /* unlock */
```

```
/* lock */
atomic {
  locked == 0;
  locked = 1;
}
/* critical section */
locked = 0; /* unlock */
```

Promela Macros

- Promela does not contain procedures
- Effect can often be achieved using macros

```
inline lock() {
  atomic {
    locked == 0;
    locked = 1
  }
}
```

```
inline swap(a, b) {
  int tmp;
  tmp = a;
  a = b;
  b = tmp
}
```

- A macro just defines a replacement text for a symbolic name, possibly with parameters
  - The inline call `lock()` is replaced by the body of the definition
  - No new variable scope
  - No recursion
  - No return values
- Define macro globally before its first use

Motivation: Deadlock

- Threads are synchronized via locks
- Interleaved execution of `a.transfer(b,n)` and `b.transfer(a,m)` might deadlock
- Multi-threaded programs are extremely hard to test

```
class Account {
  int balance;

  void transfer(Account to, int amount) {
    acquire this;
    acquire to;
    this.balance -= amount;
    to.balance += amount;
    release this;
    release to;
  }
}
```

Promela Model: Account

- We need to model accounts and clients
    - General approach: omit all irrelevant details to reduce complexity

- Account
    - Balance is not relevant for potential deadlocks
    - Only model the locks of accounts

```
#define N 5

bit Account_locks[N];

inline lock(n) {
  atomic {
    Account_locks[n] == 0;
    Account_locks[n] = 1;
  }
}
```

Promela Model: Client

- Idea: model the most generic client and run several instances in parallel
  - Pick two arbitrary accounts non-deterministically
  - Lock both accounts
  - Unlock both accounts

- Choosing accounts

```
inline choose(a, l, u) {
  a = l;
  do
  :: (a < u) -> a++
  :: break
  od
}
```

```
inline chooseAccounts(f, t) {
  do
  :: (f != t) -> break
  :: (f == t) -> choose(f, 0, N-1);
                 choose(t, 0, N-1)
  od
}
```

Promela Model: Client Process

```
active [C] proctype transfer() {
  byte from, to;

  /* choose accounts non-deterministically */
  chooseAccounts(from, to);

  /* acquire locks */
  lock(from);
  lock(to);

  /* actual transfer omitted */

  /* release locks */
  Account_locks[from] = 0;
  Account_locks[to] = 0;
}
```

Alternative Account Selection

- Idea: instead of looping until two different accounts are found, restrict range for second choice

```
do
:: (f != t) -> break
:: (f == t) -> choose(f, 0, N-1);
               choose(t, 0, N-1)
od
```

```
choose(f, 0,   N-2);
choose(t, f+1, N-1)
```

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choose(t, f+1, N-1)
```

- Alternative model is less general
  - It guarantees `from` < `to`
  - So locks are acquired in order and deadlock is prevented!

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```

- Alternative model is less general
  - It guarantees `from` < `to`
  - So locks are acquired in order and deadlock is prevented!
- General strategy
  - Start with most general model
  - If model contains errors that cannot occur in real system (spurious error), revise model

Promela Channels

- `chan ch = [d] of { t1, ..., tn }` declares a channel
- Channel can buffer up to `d` messages
    - `d` > 0: buffered channel (FIFO)
    - `d` $= 0$: unbuffered channel (rendez-vous)
- Each message is a tuple whose elements have types `t1, ..., tn`
- Example

```
mtype = { req, ack, err };

chan ch = [5] of { mtype, int }
```

Send and Receive: Buffered Channels

```
chan ch = [5] of { mtype, int }
```

- `ch ! e1, ..., en` sends message
  - Type of `ei` must correspond to `ti` in channel declaration
  - Send is executable iff buffer is not full
- `ch ? a1, ..., an` receives message
  - `ai` is a variable or constant of type `ti`
  - Receive is executable iff buffer is not empty and the oldest message in the buffer matches the constants `ai`
  - Variables `ai` are assigned values of the message

```
ch ! req, 7;
ch ! ack, 1
```

```
int n;
ch ? req, n;
printf("Received: %d\n", n);
ch ? req, n;
printf("Received: %d\n", n);
```

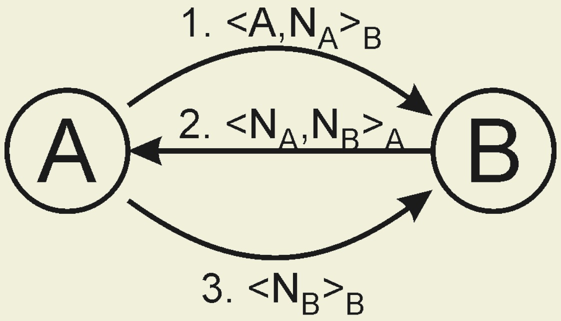
Send and Receive: Unbuffered Channels

```
chan ch = [0] of { int };
```

- `ch ! e1, ..., en` sends message
  - Send is executable if there is a receive operation that can be executed simultaneously
- `ch ? a1, ..., an` receives message
  - Receive is executable if there is a send operation that can be executed simultaneously
- Unbuffered channels model synchronous communication (rendez-vous)

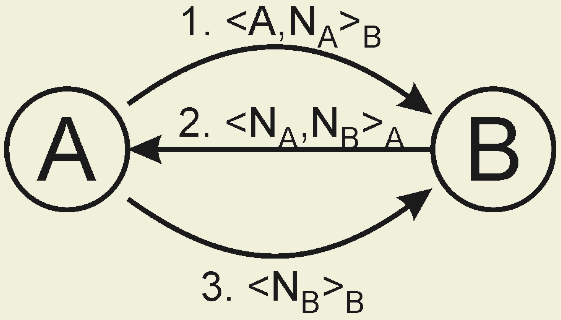
Motivation: Needham-Schroeder Protocol

- Establish a common secret over an insecure channel
  1. Alice sends random number N_A to Bob, encrypted with Bob's public key: $\langle A, N_A \rangle_B$
  2. Bob sends random number N_B to Alive, encrypted with Alice's public key: $\langle N_A, N_B \rangle_A$
  3. Alice responds with $\langle N_B \rangle_B$



- Intruders may:
  - Intercept, store, and replay messages
  - Initiate or participate in runs of the protocol
  - Decrypt messages only if encrypted with intruder's public key
- Error: intruder can pretend to be another party

Promela Model: Network



- We model the protocol for two agents plus intruder
- Agents communicate synchronously

```
chan network = [0] of {
  mtype, /* tag:                msg1, msg2, msg3      */
  mtype, /* intended receiver: agentA, agentB, agentI */
  Crypt  /* message                                    */
};
```

- We use enumeration type `mtype` for all constants
  - Spin treats `mtype` constants as symbols, not values
  - Speeds up model checking

Promela Model: Messages

- Message consists of key and up to two contents

```
typedef Crypt {
  mtype key,         /* public key used to encrypt */
        content1,    /* agent or nonce             */
        content2     /* nonce or don't care        */
};
```

- We model encryption by putting the public key into the message
  - Agent *a* will only look at message content if message key is *a*'s public key
  - No need to model private keys and encryption
- Constants for message tag, public keys, agents, and nonces

```
mtype = { msg1, msg2, msg3,
          keyA, keyB, keyI,
          agentA, agentB, agentI,
          nonceA, nonceB, nonceI };
```

Promela Model: Alice

- Protocol runs are always started by Alice

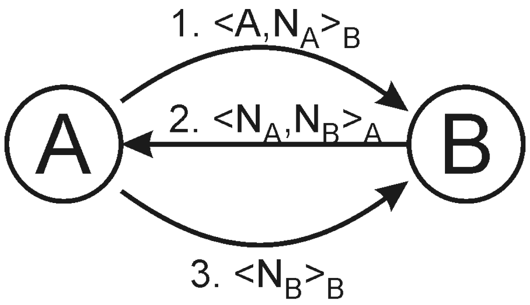
```
mtype partnerA;
bit   statusA; /* 1 = success */

active proctype Alice() {
  mtype pkey;       /* the partner's public key           */
  mtype pnonce;     /* nonce that we receive from partner */
  Crypt message;    /* our message to the partner         */
  Crypt data;       /* received message                   */

  if /* choose a partner for this run */
  :: partnerA = agentB; pkey = keyB;
  :: partnerA = agentI; pkey = keyI;
  fi;

  /* Protocol run below */
  statusA = 1; /* Success */
}
```

Promela Model: Alice’s Protocol Run



```
/* Prepare and send first message */
build(message, pkey, agentA, nonceA);
network ! msg1, partnerA, message;

/* Wait for answer */
network ? (msg2, agentA, data);

/* Proceed only if the key matches keyA and the
   nonce is the one that we have sent earlier */
(data.key == keyA) && (data.content1 == nonceA);

/* Obtain partner’s nonce */
pnonce = data.content2;

/* Prepare and send the last message */
build(message, pkey, pnonce, 0);
network ! msg3, partnerA, message;
```

Intruder

- Intruders may:
  - Intercept messages
  - Store one message
  - Replay messages
  - Initiate or participate in runs of the protocol
  - Decrypt messages only if encrypted with intruder's public key
- How can we model the most powerful attack using these capabilities?
- Solution: Model intruder fully non-deterministically
  - Intruder has no intellegence whatsoever
  - Model checker will explore all possible behaviors of intruder

Promela Model: Intruder

```
bool knows_nonceA, knows_nonceB;

active proctype Intruder() {
  mtype tag;         /* message tag               */
  mtype recpt;       /* recipient for our message */
  Crypt data         /* received message          */
  Crypt intercepted; /* stored message            */

  do
  :: /* Receive and learn */

  :: /* Replay or send    */
  od
}
```

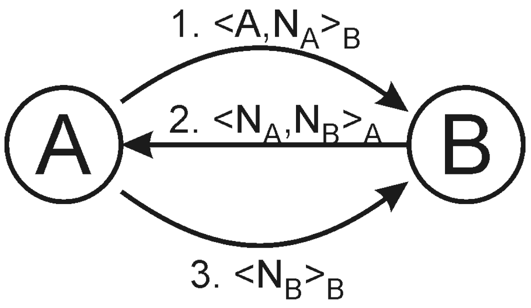
Promela Model: Intruder Receives

```
do
:: network ? (tag, _, data) ->
   if /* perhaps store the message */
   :: copy(data, intercepted);
   :: skip;
   fi;
   if /* record newly learnt nonces */
   :: (data.key == keyI) ->
      knows_nonceA = knows_nonceA ||
                     (data.content1 == nonceA) ||
                     (data.content2 == nonceA);
      knows_nonceB = knows_nonceB ||
                     (data.content1 == nonceB) ||
                     (data.content2 == nonceB);
   :: else -> skip;
   fi;
:: /* Replay or send    */
od
```

Promela Model: Intruder Sends

```
do
:: /* Receive and learn */
:: /* Replay or send    */
   if /* choose message type */
   :: tag = msg1;
   :: tag = msg2;
   :: tag = msg3;
   fi;
   if /* choose recipient */
   :: recpt = agentA;
   :: recpt = agentB;
   fi;
   if /* replay intercepted message or assemble it */
   :: copy(intercepted, data);
   :: /* assemble new message */
   fi;
   network ! tag, recpt, data;
od
```

Promela Model: Intruder Sends (cont'd)



```
:: /* assemble new message */
   if
   :: data.key = keyA;
   :: data.key = keyB;
   fi;
   if
   ::                 data.content1 = agentA;
   ::                 data.content1 = agentB;
   ::                 data.content1 = agentI;
   :: knows_nonceA -> data.content1 = nonceA;
   :: knows_nonceB -> data.content1 = nonceB;
   ::                 data.content1 = nonceI;
   fi;
   if
   :: knows_nonceA -> data.content2 = nonceA;
   :: knows_nonceB -> data.content2 = nonceB;
   ::                 data.content2 = nonceI;
   fi;
```

Summary

- Models are abstractions of the real world
- Omit irrelevant details to reduce complexity
  - Example: balance in account example
- Keep model small to avoid state space explosion
  - As few process as possible
  - As little data as possible
- Non-determinism is a powerful modelling tool
  - Let model checker explore all options
- Typical sources of non-determinism are:
  - Abstraction
  - Modelling of the environment