

Formal Methods and Functional Programming current semester

Solution Sheet 5: Type Inference

Assignment 2:

Use the typing rules for mini-Haskell for proving the following statements.

- (a) $\lambda x. (x \text{ 1 } \text{True}, x \text{ 0}) :: (Int \rightarrow Bool \rightarrow a) \rightarrow (a, Bool \rightarrow a)$
- (b) $(\lambda x. \lambda y. (y \text{ (iszero } (y \text{ x}))) \text{ True}) :: (Bool \rightarrow Int) \rightarrow Int$
- (c) $\lambda x. \lambda y. \text{if } y \text{ x then (fst } x) \text{ else (snd (snd } x)) :: (a, (b, a)) \rightarrow ((a, (b, a)) \rightarrow Bool) \rightarrow a$

Solution:

(a)

$$\begin{array}{c}
 \frac{\frac{\frac{\Gamma \vdash x :: Int \rightarrow Bool \rightarrow a}{\Gamma \vdash x \text{ 1} :: Bool \rightarrow a} \quad \frac{\Gamma \vdash 1 :: Int}{\Gamma \vdash \text{True} :: Bool}}{\Gamma \vdash x \text{ 1 True} :: a} \quad \frac{\frac{\Gamma \vdash x :: Int \rightarrow Bool \rightarrow a}{\Gamma \vdash x \text{ 0} :: Bool \rightarrow a} \quad \frac{\Gamma \vdash 0 :: Int}{\Gamma \vdash \text{True} :: Bool}}{\Gamma \vdash (x \text{ 1 True}, x \text{ 0}) :: (a, Bool \rightarrow a)} \\
 \hline
 \Gamma \vdash \lambda x. (x \text{ 1 True}, x \text{ 0}) :: (Int \rightarrow Bool \rightarrow a) \rightarrow (a, Bool \rightarrow a)
 \end{array}$$

with $\Gamma := \{x : Int \rightarrow Bool \rightarrow a\}$

(c)

$$\begin{array}{c}
 \frac{\frac{\frac{\Gamma \vdash y :: (a, (b, a)) \rightarrow Bool}{\Gamma \vdash y x :: Bool} \quad \frac{\Gamma \vdash x :: (a, (b, a))}{\Gamma \vdash \text{fst } x :: a}}{\Gamma \vdash y x \text{ then (fst } x) \text{ else (snd (snd } x)) :: a} \quad \frac{\frac{\Gamma \vdash x :: (a, (b, a))}{\Gamma \vdash \text{snd } x :: (b, a)}}{\Gamma \vdash \text{snd (snd } x) :: a} \\
 \hline
 \frac{x : (a, (b, a)), y : (a, (b, a)) \rightarrow Bool \vdash \text{if } y x \text{ then (fst } x) \text{ else (snd (snd } x)) :: a}{x : (a, (b, a)) \vdash \lambda y. \text{if } y x \text{ then (fst } x) \text{ else (snd (snd } x)) :: ((a, (b, a)) \rightarrow Bool) \rightarrow a} \\
 \hline
 \Gamma \vdash \lambda x. \lambda y. \text{if } y x \text{ then (fst } x) \text{ else (snd (snd } x)) :: (a, (b, a)) \rightarrow ((a, (b, a)) \rightarrow Bool) \rightarrow a \\
 \hline
 \text{with } \Gamma := \{x : (a, (b, a)), y : (a, (b, a)) \rightarrow Bool\}
 \end{array}$$

(b)

$x : Bool, y : Bool \rightarrow Int \vdash y :: Bool \rightarrow Int$	$x : Bool, y : Bool \rightarrow Int \vdash x :: Bool \rightarrow Int \vdash x :: Bool$
$x : Bool, y : Bool \rightarrow Int \vdash y :: Bool \rightarrow Int$	$x : Bool, y : Bool \rightarrow Int \vdash y x :: Int$
$x : Bool, y : Bool \rightarrow Int \vdash y (\mathbf{iszero} (y x)) :: Int$	$x : Bool, y : Bool \rightarrow Int \vdash \mathbf{iszero} (y x) :: Bool$
$x : Bool \vdash \lambda y. (y (\mathbf{iszero} (y x))) :: (Bool \rightarrow Int) \rightarrow Int$	
$\vdash \lambda x. \lambda y. (y (\mathbf{iszero} (y x))) :: Bool \rightarrow (Bool \rightarrow Int) \rightarrow Int$	$\vdash True :: Bool$
$\vdash (\lambda x. \lambda y. (y (\mathbf{iszero} (y x)))) True :: (Bool \rightarrow Int) \rightarrow Int$	