Assignment 8

Exercise 1

Recall the syntax of arithmetic and boolean expressions for the SPL language:

$$a ::= v \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \qquad \text{(AExp)}$$
$$b ::= \mathsf{true} \mid \mathsf{false} \mid a_1 = a_2 \mid a_1 \leq a_2 \mid \neg b \mid b_1 \vee b_2 \mid b_1 \wedge b_2 \qquad \text{(BExp)}$$

where $v \in \mathbb{Z}$ and $x \in \mathit{Var}$.

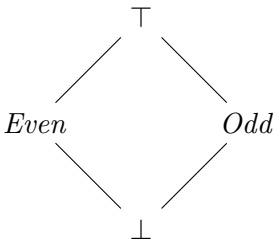
1. Define the evaluation rules for v, $a_1 - a_2$, and $a_1 * a_2$. (See slide 36 for the semantics of x and $a_1 + a_2$.)
2. Define the evaluation rules for true, false, $\neg b$ and $b_1 \vee b_2$. (See slide 37 for the semantics of the remaining boolean expressions.)
3. Define the semantics of conjunction $b_1 \wedge b_2$ without using short-circuiting.
4. Show that $\langle (x{+}1 \leq y) \wedge (\neg \mathsf{false}), \sigma \rangle \Downarrow_b \mathsf{true}$, where $\sigma = \{x \mapsto 1, y \mapsto 2\}$.

Exercise 2

1. Write a simple SPL program P that computes $x = |x|$. SPL's syntax is given on slide 12.
2. Derive the longest trace from the starting configuration $c_0 \equiv \langle P, \{x \mapsto -1\} \rangle$ and justify each step.
3. Give the trace semantics $[\![P]\!]$ of the program P if the initial state is $\sigma = \{x \mapsto -1\}$.

Exercise 3

In the lecture you have seen two abstract domains: *Sign* and *Interval*. For this exercise we use another domain called *Parity*:



where *Even* represents all even numbers, including zero, and *Odd* represents all other numbers.

An (abstract) parity state is a mapping $PC \mapsto Vars \mapsto \{\bot, \top, odd, even\}$.

You are given the following program P:

```
0: int x := 5
1: int y := 7
2: while (i >= 0) do
3:    y := y + 1
4:    i := i - 1
5: end
```

The initial parity state of P is:

pc	x	y	i
0	$\bot$	$\bot$	$\bot$
1	$\bot$	$\bot$	$\bot$
2	$\bot$	$\bot$	$\bot$
3	$\bot$	$\bot$	$\bot$
4	$\bot$	$\bot$	$\bot$
5	$\bot$	$\bot$	$\bot$

Iterate over P's states, starting from the initial parity state, until you reach a fixed-point. An intuitive overview of the iterations is given on slides 39-54. What is the fixed-point state?

Exercise 4

Are the domains Sign, Parity, and Interval pair-wise comparable?

1. For those that are comparable, which one is more precise?
2. For those that are incomparable, give an example program that can be verified only with each domain.