Homework # 1
due February 25, 13:00

The natural language proof (§2) and inversion exercises (§3) should be done on paper and submitted at the *beginning* of the lecture on Tuesday, February 25th. The SASyLF proofs (§4,5) should be submitted as attachments by email to `scmalte@inf.ethz.ch` *before* 1pm on Tuesday, February 25th. Please use UTF-8 text encoding for your SASyLF files.

1 Reading

Please read Chapters 1–3 in your textbook through the end of 3.3

2 Problems

Please do the following problem from the book:

2.2.7 (transitive closure) full natural language proof required

This is such an elementary result that sometimes it is hard to explicitly go through the steps. To get an idea of how to write such a proof, please read the accompanying handout.

3 Inversion

Suppose we have the following definition of natural numbers:

$n ::= \mathsf{z} \mid \mathsf{s}\, n$

And further, the following definition of addition:

$$\textsc{PlusZero}\ \frac{}{\mathsf{z} + n = n} \qquad \textsc{PlusSucc}\ \frac{n_1 + n_2 = n_3}{\mathsf{s}\, n_1 + n_2 = \mathsf{s}\, n_3}$$

For each of the following possibilities, give the cases that are possible inverting this relation just *once*.

For example:

$A + B = B$

We have two cases:

1. (PlusZero) $A = \mathsf{z}$
2. (PlusSucc) $A = \mathsf{s}\, D$, $B = \mathsf{s}\, E$ and we have the additional relation $D + B = E$. (As it happens, this case is impossible, but one level of inversion is not enough to prove this fact.)

1. $A + B = C$
2. $\mathsf{s}\,\mathsf{z} + B = C$
3. $A + \mathsf{z} = C$.
4. $A + A = C$
5. $A + B = \mathsf{z}$

4 Natural Numbers

Using the following definition of "greater than" on natural numbers as defined in §3.

```
judgment gt: n > n

------- gt-one
s n > n

n1 > n2
--------- gt-more
s n1 > n2
```

Prove the following theorems in SASyLF:

1. For any n, we have $(\mathtt{s}\,n) > 0$.
2. "Greater than" is transitive.
3. If $\mathtt{s}\,n_1 > \mathtt{s}\,n_2$ then $n_1 > n_2$.
4. If $n > n$ then we have a contradiction.
   For this you need to define a contradiction judgment, e.g.:

   ```
   judgment absurd: contradiction
   ```

Put your SASyLF text in a file `gt.slf`. Include your full name in a comment at the start of the file.

5 Terms

Define the boolean term language (`true`, `false` and `if`) and define equality (with only one rule) over the terms and then prove the following theorems:

1. If $\mathtt{if}\ t_0\ \mathtt{then}\ t_1\ \mathtt{else}\ t_2\ \mathtt{==}\ \mathtt{if}\ t_0'\ \mathtt{then}\ t_1'\ \mathtt{else}\ t_2'$, then $t_0\ \mathtt{==}\ t_0'$.
2. If $t_0\ \mathtt{==}\ t_0'$, $t_1\ \mathtt{==}\ t_1'$, and $t_2\ \mathtt{==}\ t_2'$ then $\mathtt{if}\ t_0\ \mathtt{then}\ t_1\ \mathtt{else}\ t_2\ \mathtt{==}\ \mathtt{if}\ t_0'\ \mathtt{then}\ t_1'\ \mathtt{else}\ t_2'$.

Put your solution in a file `term.slf`, again with your full name at the start.