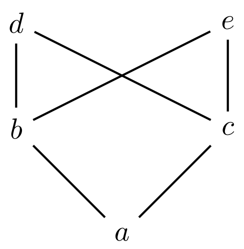
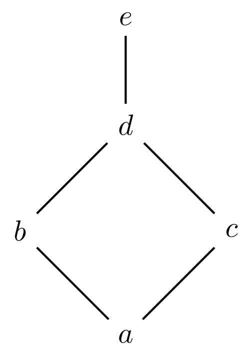
Exercise 10

Exercise 1

Are **(a)** and **(b)** complete lattices?



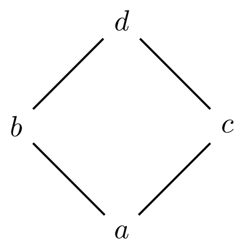
(a)



(b)

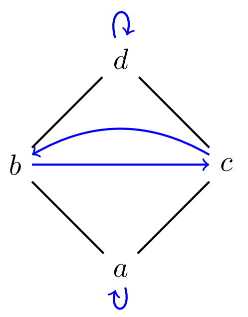
Exercise 2

Consider the lattice $L = (A, \sqsubseteq)$, where $A = \{a, b, c, d\}$. The partial order $\sqsubseteq \subseteq A \times A$ is depicted in the Hasse diagram below.

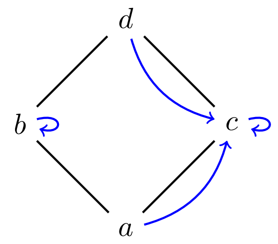


1. List the elements of $\sqsubseteq$.

2. Consider the following functions $f, g : A \mapsto A$



Function f

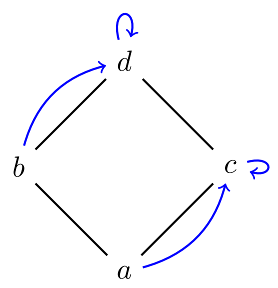


Function g

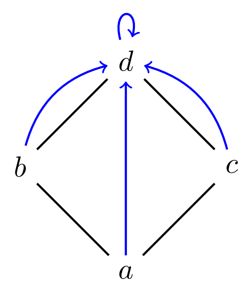
- Is f monotone? Is g monotone?
- List the set $Fix(f)$ of fixpoints of f, and the set $Red(f)$ of post-fixpoints of f.
- List the sets of fixpoints/post-fixpoints of the function g.

Exercise 3

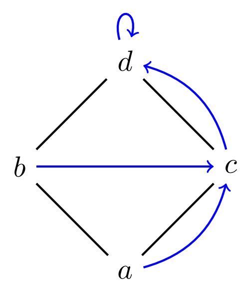
1. Consider the following three functions: $f, g, h : A \mapsto A$, defined below:



Function f



Function g



Function h

   - Does g approximate f?
   - Does h approximate f?

2. Let $\mathbb{R}^\infty = \mathbb{R} \cup \{-\infty, +\infty\}$ and $\mathbb{Z}^\infty = \mathbb{Z} \cup \{-\infty, +\infty\}$, where $\mathbb{R}$ is the set of rational numbers and $\mathbb{Z}$ is the set of integers.

   $(\mathbb{R}^\infty, \leq)$ and $(\mathbb{Z}^\infty, \leq)$ are complete lattices.

   Let $\alpha : \mathbb{R}^\infty \mapsto \mathbb{Z}^\infty$ as $\alpha(x) = \lceil x \rceil$. (Here $\lceil x \rceil$ rounds-up x to the nearest integer.)

   Let $\gamma : \mathbb{Z}^\infty \mapsto \mathbb{R}^\infty$ as $\gamma(x) = x$.

   Consider the function $f : \mathbb{R}^\infty \mapsto \mathbb{R}^\infty$ defined as $f(x) = x^2$.

- Give two functions $g, h : \mathbb{Z}^\infty \mapsto \mathbb{Z}^\infty$ that approximate f. Which one is more precise?
- Give a function $k : \mathbb{Z}^\infty \mapsto \mathbb{Z}^\infty$ that approximates any function $f : R \mapsto R$.

Exercise 4

Recall that an interval transformer for an action a is defined as:

$$[\![a]\!]_i : (\mathit{Var} \mapsto L^i) \mapsto (\mathit{Var} \mapsto L^i)$$

where $L^i = \{[x, y] \mid x, y \in \mathbb{Z}^\infty, x \leq y\} \cup \{\bot_i\}$ are the interval domain's elements (see slide 19 from the lecture).

1. Consider the interval maps:

   $m_1 = x \mapsto [-3, 8], y \mapsto [0, 5]$

   $m_2 = x \mapsto [-3, 8], y \mapsto \bot_i$

   The interval transformer for $\leq$ is defined on slide 36. Apply the transformer to compute the result of:

$[\![x \leq y]\!](m_1) =$	$[\![x \leq y]\!](m_2) =$
$[\![3 \leq 5]\!](m_1) =$	$[\![3 \leq 5]\!](m_2) =$
$[\![5 \leq 3]\!](m_1) =$	$[\![5 \leq 3]\!](m_2) =$

2. Define the interval transformer for assignment:

   $[\![x := a]\!](m) =$

3. Define the multiplication expression for interval elements:

   $\langle a_1 * a_2, m \rangle \Downarrow_i$?

4. Define the interval transformer for equality:

   $[\![x = y]\!](m) =$

Exercise 5

Consider the following program:

```
foo (int x) {
1       y := 2
2       if (x <= y)
3           z := 3 * x
else
4           z := y
5       z := y * z
6 }
```

- Give two concrete traces t_1 and t_2 of the program.
- Apply the interval abstraction function α^i given on slide 21 from the lecture on the set $\{t_1, t_2\}$.
- Compute the least fixpoint $\mathsf{lfp}F^i$ of the program using the interval domain abstraction.
- Give a concrete trace $t \in \gamma^i(\mathsf{lfp}F^i)$ that is not a valid trace. Here γ^i is the concretization function; see slides 21-22 from the lecture.