Exercise 10

Exercise 1

You have heard about Rice's theorem several times. A computable partial function f is a partial function that can be implemented by some computer program κ, e.g., the factorial function $x \mapsto x!$. A property of computable partial functions is a predicate P over programs, such that P does not discirminate between programs implementing the same partial function, i.e., if κ_1 and κ_2 implement the same partial function, then $P(\kappa_1) \iff P(\kappa_2)$.

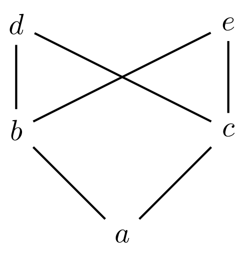
Theorem (Rice)**.** *A property P of computable partial functions (c.p.f.) is decidable iff it is* trivial*, i.e., either no c.p.f. has P or all c.p.f. have P.*

Why the theorem speaks about a property of functions and not about an arbitrary property of programs? Give an informal proof of Rice's theorem by reducing the halting problem to deciding a property.

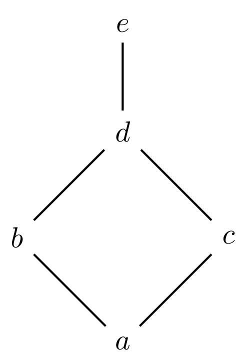
Hint: Show that any algorithm that decides a non-trivial property P can be converted to an algorithm that decides the halting problem, i.e., an algorithm that decides whether a given program halts for a given input.

Exercise 2

Are **(a)** and **(b)** complete lattices?



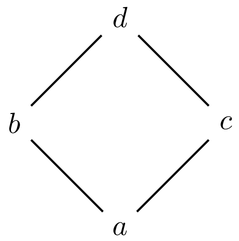
(a)



(b)

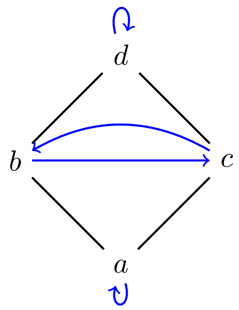
Exercise 3

Consider the lattice $L = (A, \sqsubseteq)$, where $A = \{a, b, c, d\}$. The partial order $\sqsubseteq \subseteq A \times A$ is depicted in the Hasse diagram below.

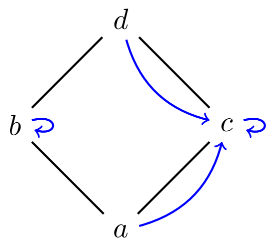


1. List the elements of $\sqsubseteq$.

2. Consider the following functions $f, g : A \mapsto A$



Function f

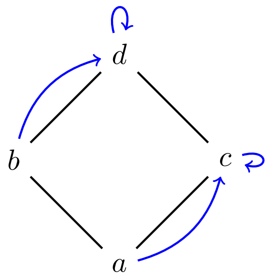


Function g

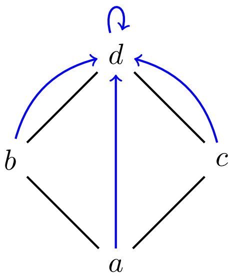
- Is f monotone? Is g monotone?
- List the set $Fix(f)$ of fixpoints of f, and the set $Red(f)$ of post-fixpoints of f.
- List the sets of fixpoints/post-fixpoints of the function g.

Exercise 4

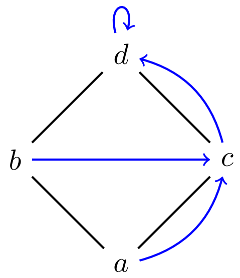
1. Consider the following three functions: $f, g, h : A \mapsto A$, defined below:



Function f



Function g



Function h

- Does g approximate f?
- Does h approximate f?

2. Let $\mathbb{R}^\infty = \mathbb{R} \cup \{-\infty, +\infty\}$ and $\mathbb{Z}^\infty = \mathbb{Z} \cup \{-\infty, +\infty\}$, where $\mathbb{R}$ is the set of rational numbers and $\mathbb{Z}$ is the set of integers.

   $(\mathbb{R}^\infty, \leq)$ and $(\mathbb{Z}^\infty, \leq)$ are complete lattices.

   Let $\alpha : \mathbb{R}^\infty \mapsto \mathbb{Z}^\infty$ as $\alpha(x) = \lceil x \rceil$. (Here $\lceil x \rceil$ rounds-up x to the nearest integer.)

   Let $\gamma : \mathbb{Z}^\infty \mapsto \mathbb{R}^\infty$ as $\gamma(x) = x$.

   Consider the function $f : \mathbb{R}^\infty \mapsto \mathbb{R}^\infty$ defined as $f(x) = x^2$.

   - Give two functions $g, h : \mathbb{Z}^\infty \mapsto \mathbb{Z}^\infty$ that approximate f. Which one is more precise?
   - Give a function $k : \mathbb{Z}^\infty \mapsto \mathbb{Z}^\infty$ that approximates any function $f : R \mapsto R$.