Assignment 9

Exercise 1

Recall (see slide 19 from the lecture) that an interval transformer for an *action* has the type:

$$[\![action]\!]_i : (\mathit{Var} \mapsto L^i) \mapsto (\mathit{Var} \mapsto L^i)$$

where L^i are the elements of the interval domain ($L^i = \{[x, y] \mid x, y \in \mathbb{Z}^\infty, x \leq y\} \cup \{\perp_i\}$).

1. Consider the interval maps:

   $m_1 = x \mapsto [-3, 8], y \mapsto [0, 5]$

   $m_2 = x \mapsto [-3, 8], y \mapsto \perp_i$

   The interval transformer for $\leq$ is defined on slide 28. Apply the transformer to compute the result of:

$[\![x \leq y]\!](m_1) =$	$[\![x \leq y]\!](m_2) =$
$[\![3 \leq 5]\!](m_1) =$	$[\![3 \leq 5]\!](m_2) =$
$[\![5 \leq 3]\!](m_1) =$	$[\![5 \leq 3]\!](m_2) =$

2. Define the interval transformer for assignment:

   $[\![x := a]\!]_i(m) =$

3. Define the multiplication expression for interval elements:

   $\langle a_1 * a_2, m \rangle \Downarrow_i$?

Exercise 2

Consider the following program:

```
   foo (int x) {
1:      y := 2
2:      if (x <= y)
3:          z := 3 * x
        else
4:          z := y
5:      z := y * z
6: }
```

1. Give two concrete traces t_1 and t_2 of the program.
2. Apply the interval abstraction function α^i (similarly to slide 15) on the set $\{t_1, t_2\}$.
3. Compute the least fixpoint $\mathsf{lfp}F^i$ of the program using the interval domain abstraction.
4. Give a concrete trace $t \in \gamma^i(\mathsf{lfp}F^i)$ that is not a valid trace. Here γ^i is the concretization function.

Exercise 3

What are the correct transformers for $-, /$ in the interval domain?

Exercise 4

For the interval domain, what is $pair_<$ and $pair_=$?

Exercise 5

Show 2 programs that are equivalent (for the same input state, they always produce the same output state) in the concrete but under the Interval domain, the invariants produced are not equivalent.