Assignment 7

Exercise 1

The syntax of arithmetic and boolean expressions for the SPL language is:

$$a ::= v \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \qquad \text{(AExp)}$$
$$b ::= \mathsf{true} \mid \mathsf{false} \mid a_1 = a_2 \mid a_1 \leq a_2 \mid \neg b \mid b_1 \vee b_2 \mid b_1 \wedge b_2 \qquad \text{(BExp)}$$

where $v \in \mathbb{Z}$ and $x \in \mathit{Var}$. See slide 1 of the exercise slides for the full syntax.

1. Define the evaluation rules for v, $a_1 - a_2$, and $a_1 * a_2$. (For inspiration, see slide 34 for the semantics of $a_1 + a_2$ and x.)
2. Define the evaluation rules for true, false, $\neg b$ and $b_1 \vee b_2$. (For inspiration, see slide 36 for the semantics of $\leq$, $=$, and $\wedge$.)
3. Show that $\langle (x{+}1 \leq y) \wedge (\neg \mathsf{false}), \sigma \rangle \Downarrow_b \mathsf{true}$, where $\sigma = \{x \mapsto 1, y \mapsto 2\}$.

Exercise 2

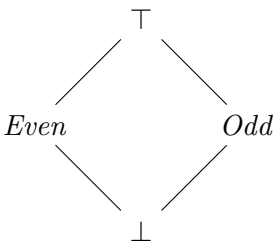
1. Write a simple SPL program P that computes $x := |x|$. Show that the program can be derived using the grammar.
2. Derive the longest finite sequence from the starting configuration

   $$c_0 \equiv \langle P, \{x \mapsto -1\} \rangle$$

   and justify each step.

Exercise 3

In the lecture you have seen two abstract domains: *Sign* and *Interval*. For this exercise we use another domain called *Parity*:



where *Even* represents all even numbers, including zero, and *Odd* represents all other numbers.

An (abstract) parity state is a mapping $PC \mapsto Vars \mapsto \{\bot, \top, odd, even\}$.

You are given the following program P:

```
   function (int x, int y)
   {
0:    x := y * 2
1:    while x >= 0
2:       if x = 1
3:          x := x + 1
          else
4:          y := x - 1
5:          x := y - 1
6:    end
   }
```

The initial parity state of P is:

pc	x	y
0	$\bot$	$\bot$
1	$\bot$	$\bot$
2	$\bot$	$\bot$
3	$\bot$	$\bot$
4	$\bot$	$\bot$
5	$\bot$	$\bot$
6	$\bot$	$\bot$

Iterate over P's states, starting from the initial parity state, until you reach a fixed-point. An intuitive overview of the iterations is given in slide set 6 of the lecture. What is the fixed-point state?

Exercise 4

Are the domains Sign, Parity, and Interval pair-wise comparable?

1. For those that are comparable, which one is more precise?

2. For those that are incomparable, give an example program that can be verified only with each domain.