

Assignment 9

Exercise 1

You have heard about Rice's theorem several times. A computable partial function f is a partial function that can be implemented by some computer program κ , e.g., the factorial function $x \mapsto x!$. A property of computable partial functions is a predicate P over programs, such that P does not discriminate between programs implementing the same partial function, i.e., if κ_1 and κ_2 implement the same partial function, then $P(\kappa_1) \iff P(\kappa_2)$.

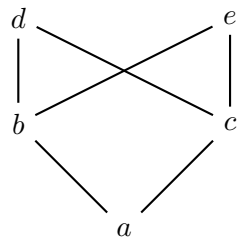
Theorem (Rice). *A property P of computable partial functions (c.p.f.) is decidable iff it is trivial, i.e., either no c.p.f. has P or all c.p.f. have P .*

Why the theorem speaks about a property of functions and not about an arbitrary property of programs? Give an informal proof of Rice's theorem by reduction to the halting problem.

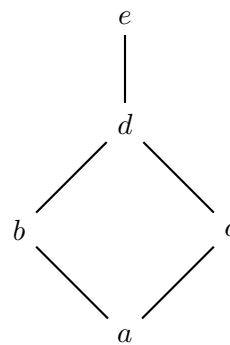
Hint: Show that any algorithm that decides a non-trivial property P can be converted to an algorithm that decides the halting problem, i.e., an algorithm that decides whether a given program halts for a given input.

Exercise 2

Are (a) and (b) complete lattices?



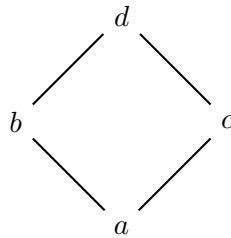
(a)



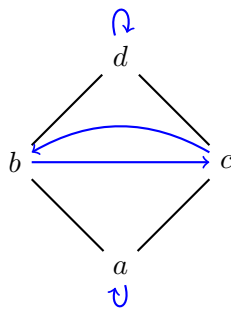
(b)

Exercise 3

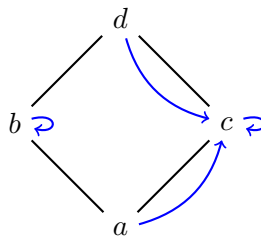
Consider the lattice $L = (A, \sqsubseteq)$, where $A = \{a, b, c, d\}$. The partial order $\sqsubseteq \subseteq A \times A$ is depicted in the Hasse diagram below.



1. List the elements of \sqsubseteq .
2. Consider the following functions $f, g : A \mapsto A$



Function f

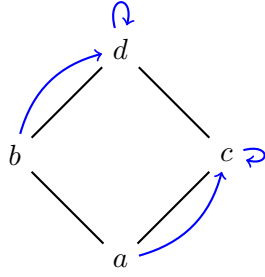


Function g

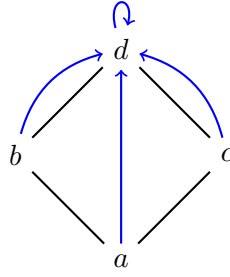
- Is f monotone? Is g monotone?
- List the set $Fix(f)$ of fixpoints of f , and the set $Red(f)$ of post-fixpoints of f .
- List the sets of fixpoints/post-fixpoints of the function g .

Exercise 4

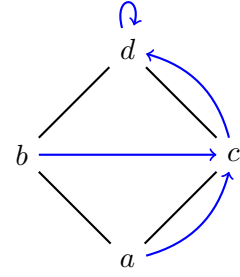
1. Consider the following three functions: $f, g, h : A \mapsto A$, defined below:



Function f



Function g



Function h

- Does g approximate f ?
 - Does h approximate f ?
2. Let $\mathbb{R}^\infty = \mathbb{R} \cup \{-\infty, +\infty\}$ and $\mathbb{Z}^\infty = \mathbb{Z} \cup \{-\infty, +\infty\}$, where \mathbb{R} is the set of rational numbers and \mathbb{Z} is the set of integers.

$(\mathbb{R}^\infty, \leq)$ and $(\mathbb{Z}^\infty, \leq)$ are complete lattices.

Let $\alpha : \mathbb{R}^\infty \mapsto \mathbb{Z}^\infty$ as $\alpha(x) = \lceil x \rceil$. (Here $\lceil x \rceil$ rounds-up x to the nearest greater or equal integer.)

Let $\gamma : \mathbb{Z}^\infty \mapsto \mathbb{R}^\infty$ as $\gamma(x) = x$.

Consider the function $f : \mathbb{R}^\infty \mapsto \mathbb{R}^\infty$ defined as $f(x) = \frac{x^2}{3}$.

- Give two functions $g, h : \mathbb{Z}^\infty \mapsto \mathbb{Z}^\infty$ that approximate f . Which one is more precise?
- Give a function $k : \mathbb{Z}^\infty \mapsto \mathbb{Z}^\infty$ that approximates any function $f : R \mapsto R$.