Assignment 7: Solution

Exercise 1

1. Inference rules for v, $a_1 - a_2$, and $a_1 * a_2$:

$$\frac{}{\langle v, \sigma\rangle \Downarrow_a v} \qquad \frac{\langle a_1, \sigma\rangle \Downarrow_a v_1 \qquad \langle a_2, \sigma\rangle \Downarrow_a v_2}{\langle a_1 - a_2, \sigma\rangle \Downarrow_a v_1 - v_2} \qquad \frac{\langle a_1, \sigma\rangle \Downarrow_a v_1 \qquad \langle a_2, \sigma\rangle \Downarrow_a v_2}{\langle a_1 * a_2, \sigma\rangle \Downarrow_a v_1 * v_2}$$

2. Inference rules for true, false, $\neg b$ and $b_1 \vee b_2$:

$$\frac{}{\langle \mathsf{true}, \sigma\rangle \Downarrow_b \mathsf{true}} \qquad \frac{}{\langle \mathsf{false}, \sigma\rangle \Downarrow_b \mathsf{false}} \qquad \frac{\langle b, \sigma\rangle \Downarrow_b \mathsf{true}}{\langle \neg b, \sigma\rangle \Downarrow_b \mathsf{false}} \qquad \frac{\langle b, \sigma\rangle \Downarrow_b \mathsf{false}}{\langle \neg b, \sigma\rangle \Downarrow_b \mathsf{true}}$$

$$\frac{\langle b_1, \sigma\rangle \Downarrow_b \mathsf{true}}{\langle b_1 \vee b_2, \sigma\rangle \Downarrow_b \mathsf{true}} \qquad \frac{\langle b_2, \sigma\rangle \Downarrow_b \mathsf{true}}{\langle b_1 \vee b_2, \sigma\rangle \Downarrow_b \mathsf{true}} \qquad \frac{\langle b_1, \sigma\rangle \Downarrow_b \mathsf{false} \qquad \langle b_2, \sigma\rangle \Downarrow_b \mathsf{false}}{\langle b_1 \vee b_2, \sigma\rangle \Downarrow_b \mathsf{false}}$$

3. Show that $\langle (x-2 \leq y) \wedge (\neg \mathsf{false}), \sigma\rangle \Downarrow_b \mathsf{true}$, where $\sigma = \{x \mapsto 1, y \mapsto 2\}$:

$$\frac{\dfrac{\dfrac{\langle x, \sigma\rangle \Downarrow_a 1 \qquad \langle 1, \sigma\rangle \Downarrow_a 1}{\langle x+1, \sigma\rangle \Downarrow_a 2} \qquad \langle y, \sigma\rangle \Downarrow_a 2}{\langle x+1 \leq y, \sigma\rangle \Downarrow_b \mathsf{true}} \qquad \dfrac{\langle \mathsf{false}, \sigma\rangle \Downarrow_b \mathsf{false}}{\langle \neg \mathsf{false}, \sigma\rangle \Downarrow_b \mathsf{true}}}{\langle (x+1 \leq y) \wedge (\neg \mathsf{false}), \sigma\rangle \Downarrow_b \mathsf{true}}$$

Exercise 2

1. The program P can be defined as:
   if $x \leq 0$ **then** $x := x * (-1)$ **else skip**

2. The derivation is:

$$
\begin{aligned}
S \rightarrow\ & \textbf{if } B \textbf{ then } S \textbf{ else } S \rightarrow \\
& \textbf{if } B \textbf{ then } S \textbf{ else skip} \rightarrow \\
& \textbf{if } A \leq A \textbf{ then } S \textbf{ else skip} \rightarrow \\
& \textbf{if } Var \leq A \textbf{ then } S \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq A \textbf{ then } S \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq Z \textbf{ then } S \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq 0 \textbf{ then } S \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq 0 \textbf{ then } Var := A \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq 0 \textbf{ then } x := A \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq 0 \textbf{ then } x := A * A \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq 0 \textbf{ then } x := Var * A \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq 0 \textbf{ then } x := x * A \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq 0 \textbf{ then } x := x * Z \textbf{ else skip} \rightarrow \\
& \textbf{if } x \leq 0 \textbf{ then } x := x * (-1) \textbf{ else skip}
\end{aligned}
$$

3. The longest trace is:

$c_0 \equiv \langle \textbf{if } x \leq 0 \textbf{ then } x := x * (-1) \textbf{ else skip}, \{x \mapsto -1\}\rangle$

$\rightarrow c_1 \equiv \langle \textbf{if } \mathsf{true} \textbf{ then } x := x * (-1) \textbf{ else skip}, \{x \mapsto -1\}\rangle$

$\rightarrow c_2 \equiv \langle x := x * (-1), \{x \mapsto -1\}\rangle$

$\rightarrow c_3 \equiv \langle x := 1, \{x \mapsto -1\}\rangle$

$\rightarrow c_4 \equiv \langle \{x \mapsto 1\}\rangle$

For the steps $c_1 \rightarrow c_2$ and $c_3 \rightarrow c_4$ we do not need any hypotheses. For the other two steps the hypotheses are:

$$
\frac{\langle x \leq 0, \{x \mapsto -1\}\rangle \Downarrow_b \mathsf{true}}{c_0 \rightarrow c_1} \qquad \frac{\langle x * (-1), \{x \mapsto -1\}\rangle \Downarrow_a 1}{c_2 \rightarrow c_3}
$$