Assignment 9

Exercise 1

Recall (see slide 19 from the lecture) that an interval transformer for an *action* has the type:

$$[\![action]\!]_i : (Var \mapsto L^i) \mapsto (Var \mapsto L^i)$$

where L^i are the elements of the interval domain ($L^i = \{[x, y] \mid x, y \in \mathbb{Z}^\infty, x \leq y\} \cup \{\bot_i\}$).

1. Consider the interval maps:

   $m_1 = x \mapsto [-3, 8], y \mapsto [0, 5]$

   $m_2 = x \mapsto [-3, 8], y \mapsto \bot_i$

   The interval transformer for $\leq$ is defined on slide 31. Apply the transformer to compute the result of:

   - $[\![x \leq y]\!](m_1)$
   - $[\![3 \leq 5]\!](m_1)$
   - $[\![5 \leq 3]\!](m_1)$
   - $[\![x \leq y]\!](m_2)$
   - $[\![3 \leq 5]\!](m_2)$
   - $[\![5 \leq 3]\!](m_2)$

2. Define the interval transformer for assignment.

3. Define the multiplication expression for interval elements.

4. Define the interval transformer for equality.

Exercise 2

Consider the following program:

```
  foo (int x) {
1:      y := 2
2:      if (x <= y)
3:          z := 3 * x
        else
4:          z := y
5:      z := y * z
6: }
```

- Give two concrete traces t_1 and t_2 of the program.
- Apply the interval abstraction function α^i given on slides 14-15 from the lecture on the set $\{t_1, t_2\}$.
- Compute the least fixpoint $\mathsf{lfp}F^i$ of the program using the interval domain abstraction.
- Give a concrete trace $t \in \gamma^i(\mathsf{lfp}F^i)$ that is not a valid trace. Here γ^i is the concretization function; see slides 14-15 from the lecture.

Exercise 3

Consider the following program:

```
  foo (int x) {
1:      y := 0
2:      while (x > 0 && x <= 100)
3:          y := y + x
4:          x := x - 1
        end
5: }
```

- Compute the least fixpoint $\mathsf{lfp}F^i$ of the program using the interval domain abstraction and the widening operator given on slide 57.
- Does there exist a concrete trace such that x has value 10 at label 5? What are the possible values for x at label 5?