

# Program Verification

## Exercise Sheet 9: Heap Reasoning and Permissions

### Assignment 1 (Pure Assertions)

An IDF assertion is called *pure* if it doesn't contain any accessibility predicates. For example,  $x.f = 4$  is a pure assertion. In practice, pure assertions are typically expressions, but can technically also include the separating conjunction  $*$ .

1. Prove that, for all states  $H, P, \sigma$  and pure assertions  $A$ , if  $H, P, \sigma \models A$  then  $H, \emptyset, \sigma \models A$  (where  $\emptyset$  represents the empty permissions mask, which maps all field locations to 0).
2. Prove that, for any assertion  $A'$  and any pure assertion  $A$  the assertions  $A * A'$  and  $A \wedge A'$  are logically equivalent (i.e. they are true in the same states)<sup>1</sup>.

### Assignment 2 (Permissions Required by an Assertion)

Recall that permission masks  $P$  (as defined in the lecture) are maps from an (object, field-name) pair to a rational number in the range  $[0, 1]$ . Let  $\text{Perms}(A)_{(H, \sigma)}$  be a function that maps an assertion  $A$  to the (pointwise) *minimal* permission mask  $P_A$  such that: when applying the definition of  $H, P_A, \sigma \models A$  (from slide 213), the *second* case of the definition (that for accessibility predicates) never evaluates to false. In other words,  $\text{Perms}(A)_{(H, \sigma)}$  must contain the minimal possible permissions *necessarily required* due to accessibility predicates occurring in  $A$  (in the given  $H$  and  $\sigma$ ).

1. Why does the result of this operation need to depend on  $H$  and  $\sigma$ ? Hint: think how would you handle an implication  $e \Rightarrow A$ .
2. How will the case for  $\text{Perms}(A_1 \wedge A_2)_{(H, \sigma)}$  differ from the case for  $\text{Perms}(A_1 * A_2)_{(H, \sigma)}$ ?
3. Write down a definition for  $\text{Perms}(A)_{(H, \sigma)}$  for each case in a similar style to the  $\models$  definition on slide 213.

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<sup>1</sup>Note that, as a simple consequence of this latter result, any pure assertion can be equivalently represented as a (boolean-typed) expression.