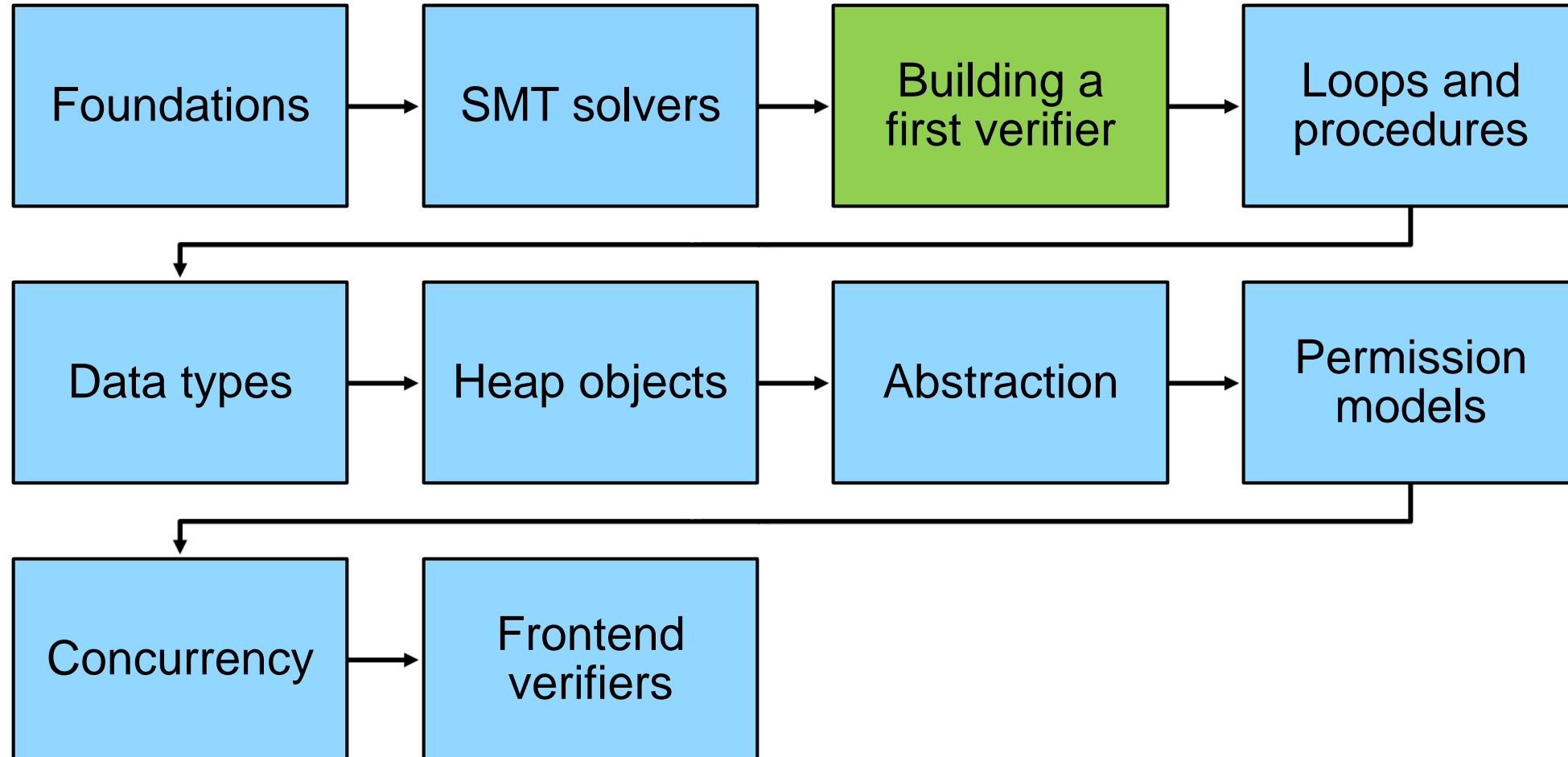


**Peter Müller and Marco Eilers**

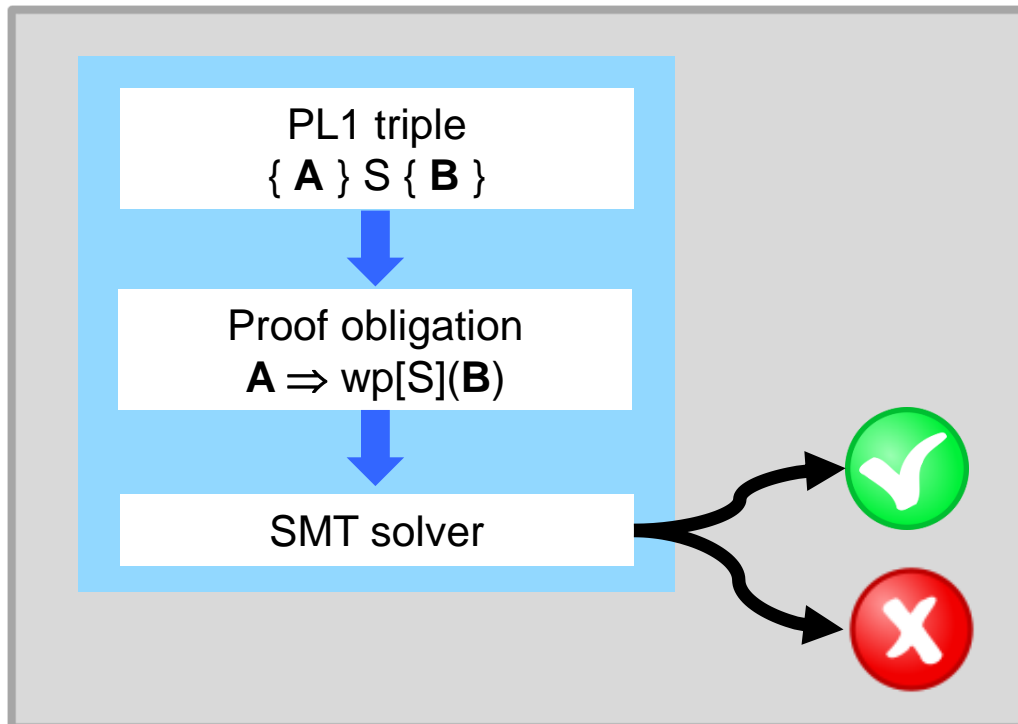
(slides developed in cooperation with Christoph Matheja)

# **PROGRAM VERIFICATION**

# Outline



# A naive first verifier

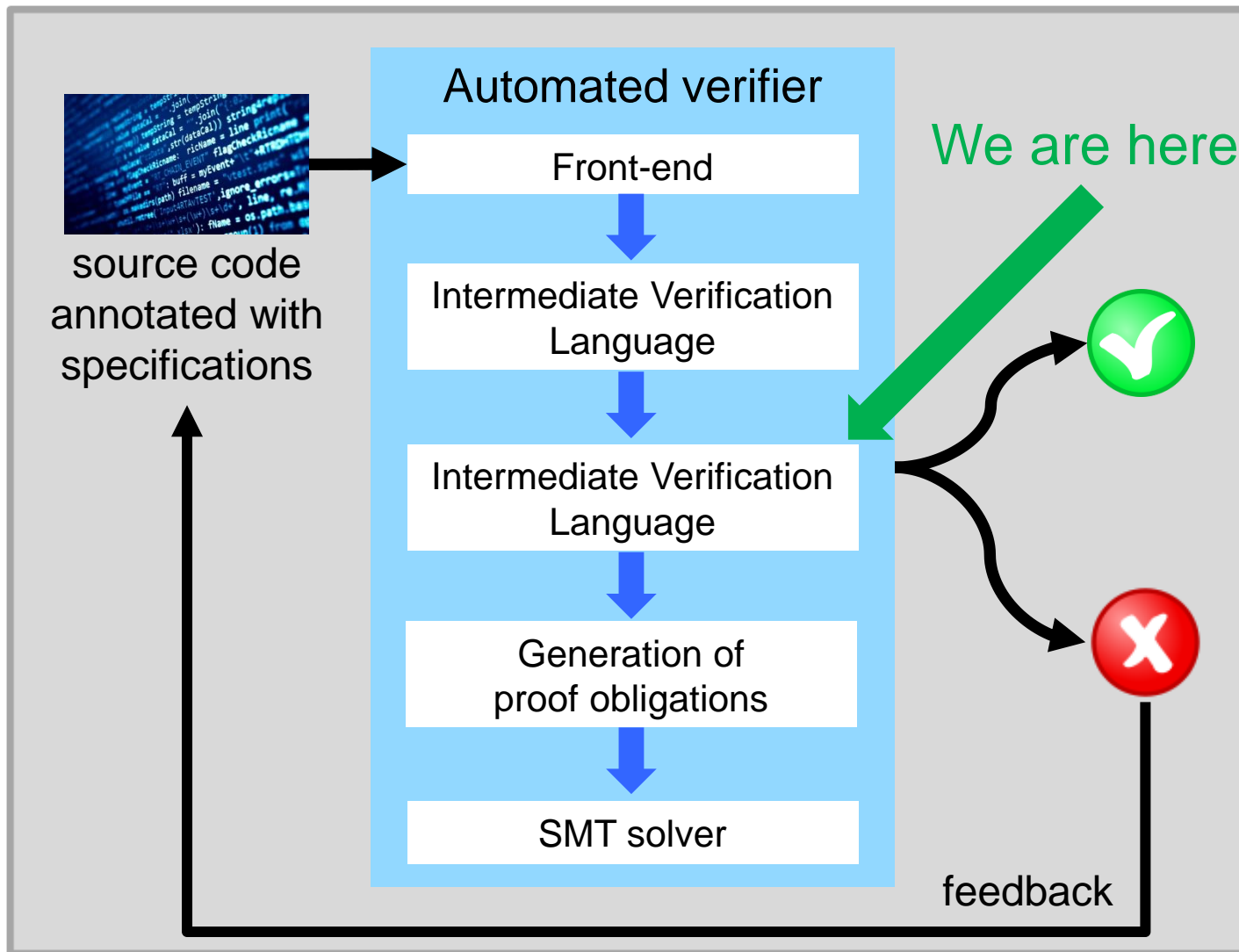


- Is PL1 sufficiently expressive?
  - Is it easy to encode interesting programs?
  - Is it possible to express interesting verification problems?
- How can we provide useful error messages?

# Building a first verifier

1. Two intermediate verification languages
2. Error reporting

# Roadmap



- PL1 allows us to compute proof obligations easily
- To be suitable as low-level IVL, it should also facilitate the encoding of more complex programming concepts
  - Loops
  - Procedures
  - Data structures
  - Concurrency

# How expressive is PL1?

## PL1 statements

```
S ::= skip
    | v := E
    | abort | diverge
    | S; S
    | if (E) { S } else { S }
    | if (*) { S } else { S }
```

- Let's encode some additional statements
  - **assert** E
  - **assume** E

## Expressions

```
E ::= c | x | E + E | E * E | E - E
    | E < E | E ^ E | E v E | ¬E | ...
```

## Types

```
T ::= Bool | Int | Rational | Real
```

# New statement: **assert E**

- Assertions make existing knowledge **explicit** in the program
  - **crash** whenever it violates our knowledge
  - otherwise, do nothing

```
if (x > 0) {  
  x := -x  
} else {  
  skip  
}  
assert x >= 0
```



```
if (x < 0) {  
  x := -x  
} else {  
  skip  
}  
assert x >= 0
```



## Operational semantics

$$\frac{[[E]](\sigma) = \text{true}}{\langle \text{assert } E, \sigma \rangle \rightarrow \langle \text{term}, \sigma \rangle}$$
$$\frac{[[E]](\sigma) = \text{false}}{\langle \text{assert } E, \sigma \rangle \rightarrow \langle \text{err}, \sigma \rangle}$$

What is the weakest precondition of **assert E**?

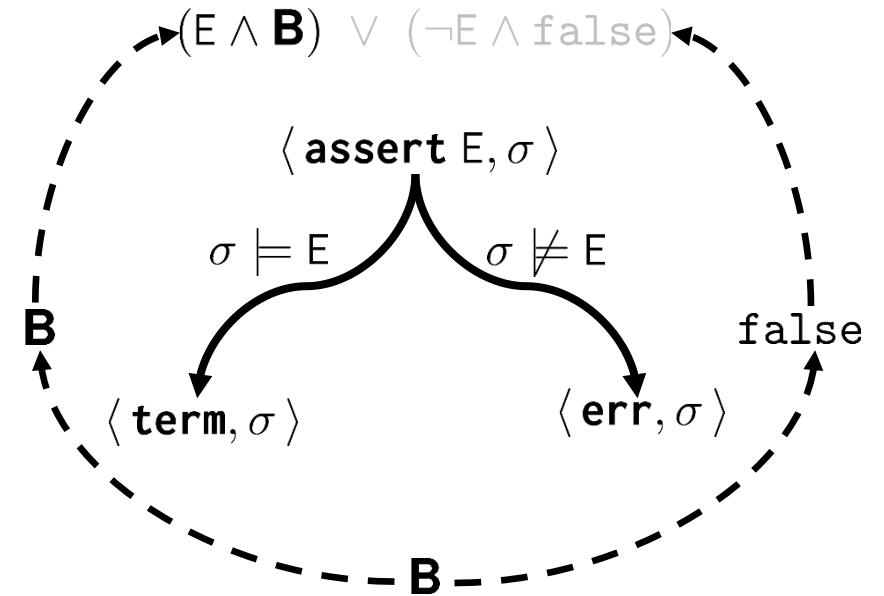
How can we encode **assert E** in PL1?

# Encoding assert statements in PL1

## Operational semantics

$$\frac{\llbracket E \rrbracket(\sigma) = \text{true}}{\langle \text{assert } E, \sigma \rangle \rightarrow \langle \text{term}, \sigma \rangle}$$

$$\frac{\llbracket E \rrbracket(\sigma) = \text{false}}{\langle \text{assert } E, \sigma \rangle \rightarrow \langle \text{err}, \sigma \rangle}$$



$\llbracket \text{assert } E \rrbracket = \text{if } (E) \{ \text{skip} \} \text{ else } \{ \text{abort} \}$

“is encoded as”

$$\begin{aligned} & wp \llbracket \text{assert } E \rrbracket (\mathbf{B}) \\ &= E \wedge \mathbf{B} \\ &= (E \wedge \mathbf{B}) \vee (\neg E \wedge \text{false}) \\ &= (E \wedge wp \llbracket \text{skip} \rrbracket (\mathbf{B})) \vee (\neg E \wedge wp \llbracket \text{abort} \rrbracket (\mathbf{B})) \\ &= wp \llbracket \text{if } (E) \{ \text{skip} \} \text{ else } \{ \text{abort} \} \rrbracket (\mathbf{B}) \end{aligned}$$



# Encoding assert statements in PL1: discussion

```
[[ assert E ]] = if ( E ) { skip } else { abort }
```

- E is an expression of PL1
  - It is often useful to have a more expressive language to express assertions
  - For instance, quantifiers are useful to express properties of arrays (e.g., sortedness)
- The problem cannot be solved (easily) by extending the expression syntax
  - Expressions must be efficiently executable, which is not always the case for quantifiers
  - Procedure calls in expressions cannot be encoded easily into an SMT formula

# New statement: **assume E**

- Assumptions add **unverified** knowledge
  - properties of the execution environment
  - (e.g., about results of system calls)
  - properties that are justified elsewhere (e.g., a mathematical fact or properties guaranteed by a type system)

```
// Fermat's Last theorem
assume 0 < x && 0 < y && 0 < z ==>
        x*x*x + y*y*y != z*z*z
```

## Operational semantics

$$\frac{\llbracket E \rrbracket(\sigma) = \text{true}}{\langle \text{assume } E, \sigma \rangle \rightarrow \langle \text{term}, \sigma \rangle}$$

$$\frac{\llbracket E \rrbracket(\sigma) = \text{false}}{\langle \text{assume } E, \sigma \rangle \rightarrow \langle \text{diverge}, \sigma \rangle}$$

- If E holds, **assume E** is equivalent to **skip**
- Otherwise, **magic** happens

What is the weakest precondition of **assume E**?

How can we encode **assume E** in PL1?

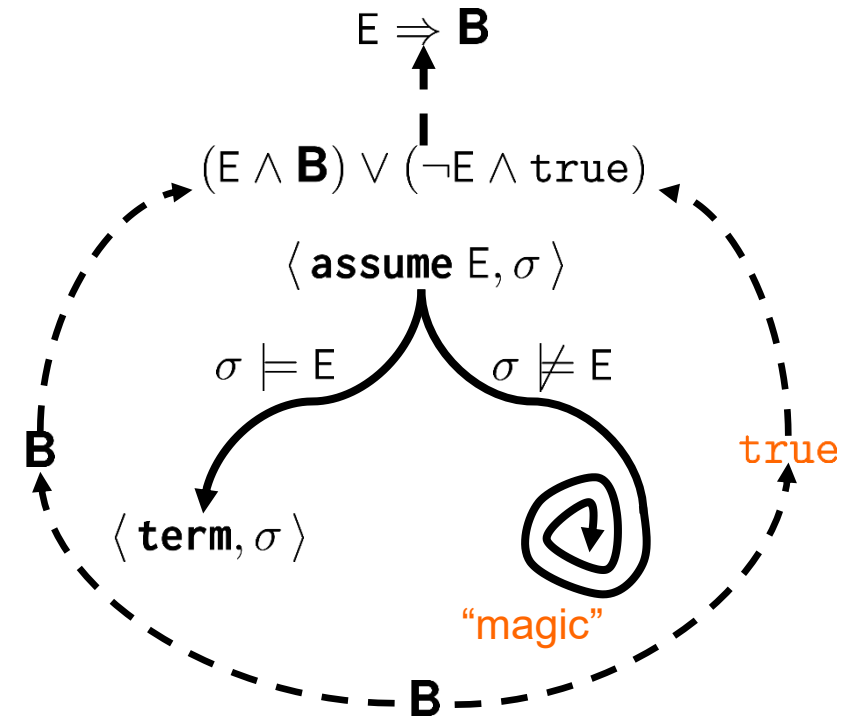
# Encoding assume statements in PL1

## Operational semantics

$$\frac{\llbracket E \rrbracket(\sigma) = \text{true}}{\langle \text{assume } E, \sigma \rangle \rightarrow \langle \text{term}, \sigma \rangle}$$

$$\frac{\llbracket E \rrbracket(\sigma) = \text{false}}{\langle \text{assume } E, \sigma \rangle \rightarrow \langle \text{diverge}, \sigma \rangle}$$

$$\llbracket \text{assume } E \rrbracket = \text{if } (E) \{ \text{skip} \} \text{ else } \{ \text{diverge} \}$$



$$\begin{aligned} & wp \llbracket \text{assume } E \rrbracket (\mathbf{B}) \\ &= E \Rightarrow \mathbf{B} \\ &= (E \wedge \mathbf{B}) \vee (\neg E \wedge \text{true}) \\ &= (E \wedge wp \llbracket \text{skip} \rrbracket (\mathbf{B})) \vee (\neg E \wedge wp \llbracket \text{diverge} \rrbracket (\mathbf{B})) \\ &= wp \llbracket \text{if } (E) \{ \text{skip} \} \text{ else } \{ \text{diverge} \} \rrbracket (\mathbf{B}) \end{aligned}$$

# Encoding assume statements in PL1: discussion

```
[[ assume E ]] = if (E) { skip } else { diverge }
```

- Like for assert statements, it would be useful to have a richer language than PL1 expressions
- Encoding works only as long as we focus on partial correctness
  - Encoding behaves like assert E for total correctness
- Generally, assume statements have to be used with great care to avoid introducing invalid assumptions

```
// Fermat's Last theorem  
assume forall x, y, z ::  
  0 < x && 0 < y && 0 < z ==>  
  x*x*x + y*y*y != z*z*z
```

```
x := 0; y := 0; z := 0  
assume x*x*x + y*y*y != z*z*z  
assert false
```



# Assertions

- To increase expressiveness without sacrificing efficient executability, we distinguish between (executable) expressions and (non-executable) assertions

## Expressions

$E ::= c \mid x \mid E + E \mid E * E \mid E - E \mid E < E \mid E \wedge E \mid E \vee E \mid \neg E \mid \dots$

Like before

## Assertions

$A ::= E \mid \forall x:T :: A \mid \exists x:T :: A \mid A \Rightarrow A \mid \dots$

FO logic over suitable theories

- Expressions are used in all standard statements (assignments, if, while, etc.)
- Assertions are used as pre- and postconditions, and in assert and assume statements
  - As a consequence, *wp* yields an assertion

# Toward a better IVL

- Since `assert` and `assume` statements are very common in verification problems, we support them natively
  - No encoding required
  - Works for partial and total correctness
- This allows us to remove other statements that can now be encoded easily

```
S ::= x := E
    | assert A
    | assume A
    | S ; S
    | if ( * ) { S } else { S }
```

```
[[ skip ]] = assert true
```

```
[[ abort ]] = assert false
```

```
[[ diverge ]] = assume false
```

```
[[ if ( E ) { S1 } else { S2 } ]] = if ( * ) { assume E ; S1 } else { assume ¬E ; S2 }
```

# Exercise: encoding of if-statements

$$\llbracket \mathbf{if} (E) \{ S_1 \} \mathbf{else} \{ S_2 \} \rrbracket = \mathbf{if} (*) \{ \mathbf{assume} E; S_1 \} \mathbf{else} \{ \mathbf{assume} \neg E; S_2 \}$$

Show that the encoding of if-statements preserves the weakest precondition:

$$wp \llbracket \mathbf{if} (E) \{ S_1 \} \mathbf{else} \{ S_2 \} \rrbracket (\mathbf{A}) = wp \llbracket \mathbf{if} (*) \{ \mathbf{assume} E; S_1 \} \mathbf{else} \{ \mathbf{assume} \neg E; S_2 \} \rrbracket (\mathbf{A})$$

# PL2: a consolidated verification language

## Types

$T ::= \text{Bool} \mid \text{Int} \mid \text{Rational} \mid \text{Real}$

We assume that all variables, expressions, assertions, and programs are well-typed.

## Expressions (executable)

$E ::= c \mid x \mid E + E \mid E * E \mid E - E \mid E < E \mid E \wedge E \mid E \vee E \mid \neg E \mid \dots$

## Assertions (FO logic over suitable theories)

$A ::= E \mid \forall x:T :: A \mid \exists x:T :: A \mid A \Rightarrow A \mid \dots$

## PL2 statements

$S ::= x := E$   
| **assert**  $A$   
| **assume**  $A$   
|  $S; S$   
| **if**  $(*) \{ S \}$  **else**  $\{ S \}$



# Verification problem for PL2

- So far, we showed  $\models \{ \mathbf{A} \} s \{ \mathbf{B} \}$  by proving  $\models \mathbf{A} \Rightarrow wp \llbracket s \rrbracket (\mathbf{B})$
- In PL2, we can encode pre- and postconditions into the program

$$\begin{aligned} & wp \llbracket \text{assume } \mathbf{A}; S; \text{assert } \mathbf{B} \rrbracket (\text{true}) \\ = & wp \llbracket \text{assume } \mathbf{A}; S \rrbracket (wp \llbracket \text{assert } \mathbf{B} \rrbracket (\text{true})) \\ = & wp \llbracket \text{assume } \mathbf{A}; S \rrbracket (\mathbf{B}) \\ = & wp \llbracket \text{assume } \mathbf{A} \rrbracket (wp \llbracket S \rrbracket (\mathbf{B})) \\ = & \mathbf{A} \Rightarrow wp \llbracket S \rrbracket (\mathbf{B}) \end{aligned}$$

- Consequently, we do not have to consider pre- and postconditions explicitly

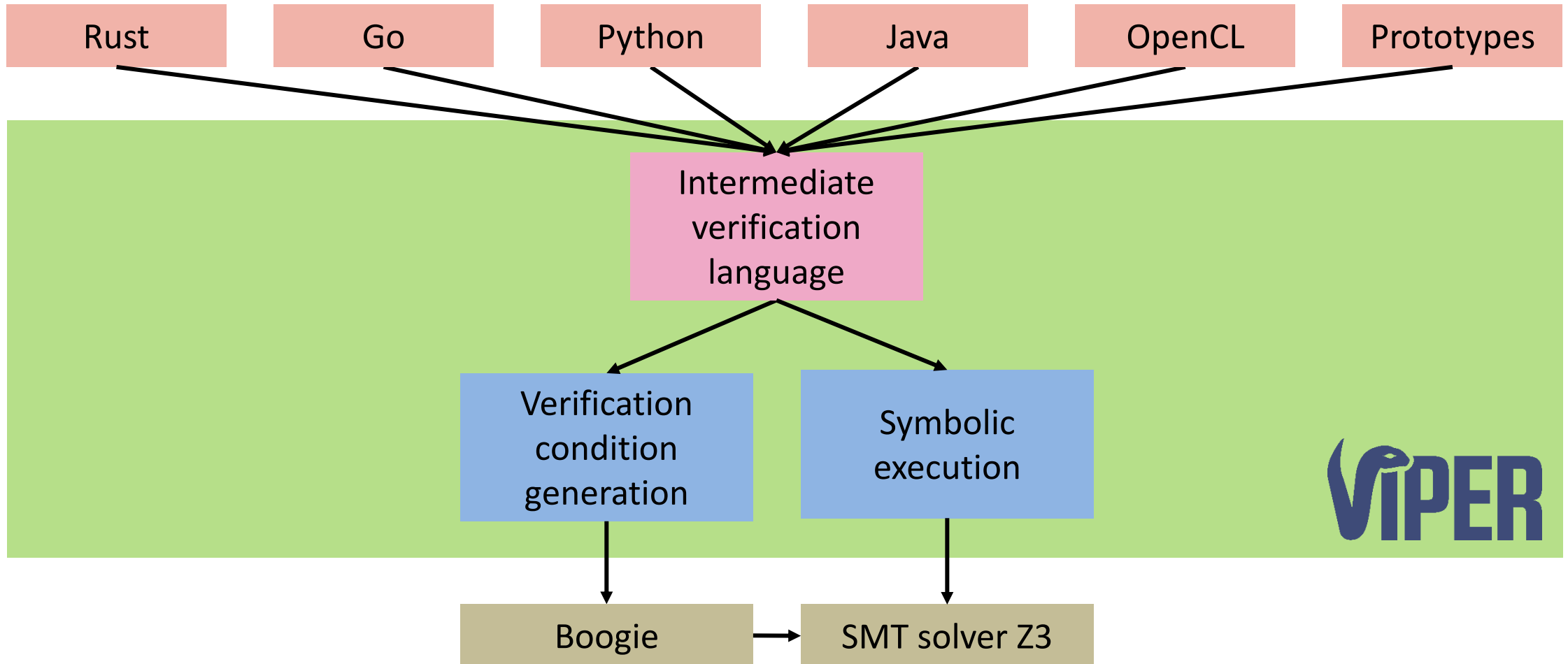
Verification problem for PL2

Given a PL2 program  $S$ , is  $wp \llbracket S \rrbracket (\text{true})$  valid?

# Summary: weakest preconditions for PL2

PL2 program S	$wp \llbracket S \rrbracket (\mathbf{B})$
$x := E$	$\mathbf{B}[x / E]$
<b>assert A</b>	$\mathbf{A} \wedge \mathbf{B}$
<b>assume A</b>	$\mathbf{A} \Rightarrow \mathbf{B}$
$S_1 ; S_2$	$wp \llbracket S_1 \rrbracket (wp \llbracket S_2 \rrbracket (\mathbf{B}))$
<b>if</b> (*) $\{ S_1 \}$ <b>else</b> $\{ S_2 \}$	$wp \llbracket S_1 \rrbracket (\mathbf{B}) \wedge wp \llbracket S_2 \rrbracket (\mathbf{B})$

# The Viper verification infrastructure



# The PL2 fragment of Viper

```
method main() { // name is irrelevant for now

  var x: Int; var y: Int

  assume x == 1; // semicolon is optional

  if (x != y) {
    x := x + y
    y := x - y
    x := x - y
  } else {
    // else-block is optional
  }

  assert y == 1
}
```

Viper statements need to be placed in methods

Preamble for variables in the precondition

Precondition { x == 1 }

Statements, expressions, and assertions include PL2 up to minor syntax changes, in particular, if-statements instead of non-deterministic choice

Postcondition { y == 1 }




# Macros

Viper supports simple parameterized macros

- syntactically inlined if invoked
- not type-checked before inlining
- untyped parameters
- recursion is not allowed


```
// Macro for expressions / assertions
define inc(a) (a + 1)

method main() {
  assert inc(16) == 17
}
```



```
// Macros for program statements
define isPositive(i) {
  assert i > 0
}

method main() {
  var x: Int
  if (x > 17) {
    isPositive(x)
  }
}
```



# Viper examples

```
method main() {  
  var x: Int  
  var y: Int  
  var res: Int  
  
  assume true  
  
  res := x*x + 2*x*y + y*y  
  
  assert res == (x+y) * (x+y)  
}
```



```
define diverge() {  
  assume false  
}  
  
define skip() {  
  assert true  
}  
  
define abort() {  
  assert false  
}  
  
method main() {  
  skip()  
  diverge()  
  abort()  
}
```



# Exercise: first Viper example

Use Viper to expose the overflow issue in the code below for 16-bit integers in two's complement.

Recall: `INT_MAX = +32767`, `INT_MIN = -32768`

```
if (i < 0) {  
  res := -i  
} else {  
  res := i  
}
```

# Global variable declarations

- So far, we assumed **implicitly** that all programs and specifications are correctly typed
- In an implementation of a verifier, we need to make the types **explicit**, especially because SMT solvers require variables to be declared with a sort

- We tacitly assume a preamble of variable declarations

## Global declarations

$D ::= \text{var } x : T \mid D ; D$

All variables in a Hoare triple must be declared in the preamble

- The initial value of all variables is unknown
  - We check the validity of  $\models \mathbf{A} \Rightarrow wp \llbracket S \rrbracket (\mathbf{B})$  for all interpretations that is, **for all initial variable values**

$x = \text{Int}('x')$

```
var x: Int;
var b: Bool
// -----
{ x = 23 }
if (b) {
  x := 42
} else {
  x := x + 17
}
{ x ≥ 40 }
```



# Local variable declarations

```
var x: Int
var y: Int
// ----
{ x ≠ y }
if (x > 0) {
  var t: Int
  t := x
  x := y
  y := t
} else {
  skip
}
{ x ≠ y }
```

- Local variables improve the structure and readability of code
- **var** x: T declares a local variable
- Rules (checked by type checker)
  - All variables must be declared before they are used (local or global)
  - Local variables cannot be used outside the scope that declares them
  - Every variable is declared at most once for every trace
- No implicit initialization: locals start out with arbitrary value of their type

# Reasoning about local variable declarations

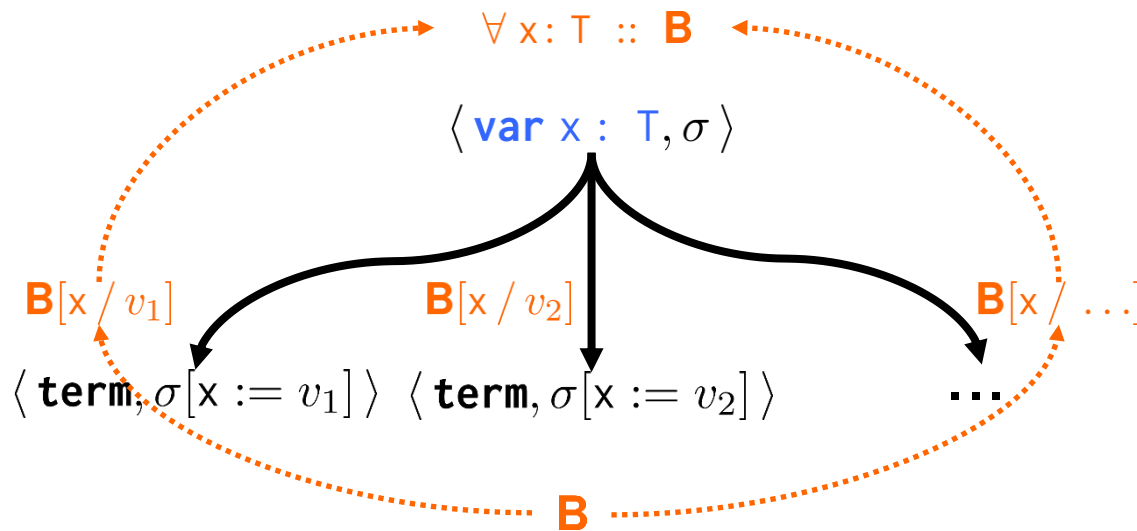
## Operational semantics

$$\frac{v \in \mathbf{Values}(T)}{\langle \mathbf{var} \ x : T, \sigma \rangle \rightarrow \langle \mathbf{term}, \sigma[x := v] \rangle}$$

$$\mathbf{Values}(\mathbf{Int}) = \mathbb{Z}$$

$$\mathbf{Values}(\mathbf{Bool}) = \{\mathbf{true}, \mathbf{false}\}$$

...



“we know nothing about  $x$  before declaration”

“ $x$  has an arbitrary value right after declaration”

## Weakest precondition

$$wp \llbracket \mathbf{var} \ x : T \rrbracket (A) = \forall x : T :: A$$

$$\llbracket \mathbf{var} \ x : T \rrbracket = x := y$$

where  $y$  is a fresh global variable

# PL3: Supporting global and local variables

## Types

$T ::= \text{Bool} \mid \text{Int} \mid \text{Rational} \mid \text{Real}$

## Global declarations

$D ::= \text{var } x : T$

We assume that all variables, expressions, assertions, and programs are well-typed.

## Expressions (executable)

$E ::= c \mid x \mid E + E \mid E * E \mid E - E \mid E < E \mid E \wedge E \mid E \vee E \mid \neg E \mid \dots$

## Assertions (FO logic over suitable theories)

$A ::= E \mid \forall x : T :: A \mid \exists x : T :: A \mid A \Rightarrow A \mid \dots$

## PL3 statements

$S ::= \text{var } x : E$   
|  $x := E$   
| **assert**  $A$   
| **assume**  $A$   
|  $S ; S$   
| **if**  $(E) \{ S \} \text{ else } \{ S \}$

# Exercise: encoding non-deterministic choice

We have seen earlier that we can encode if-statements into non-deterministic choice (using assume statements).

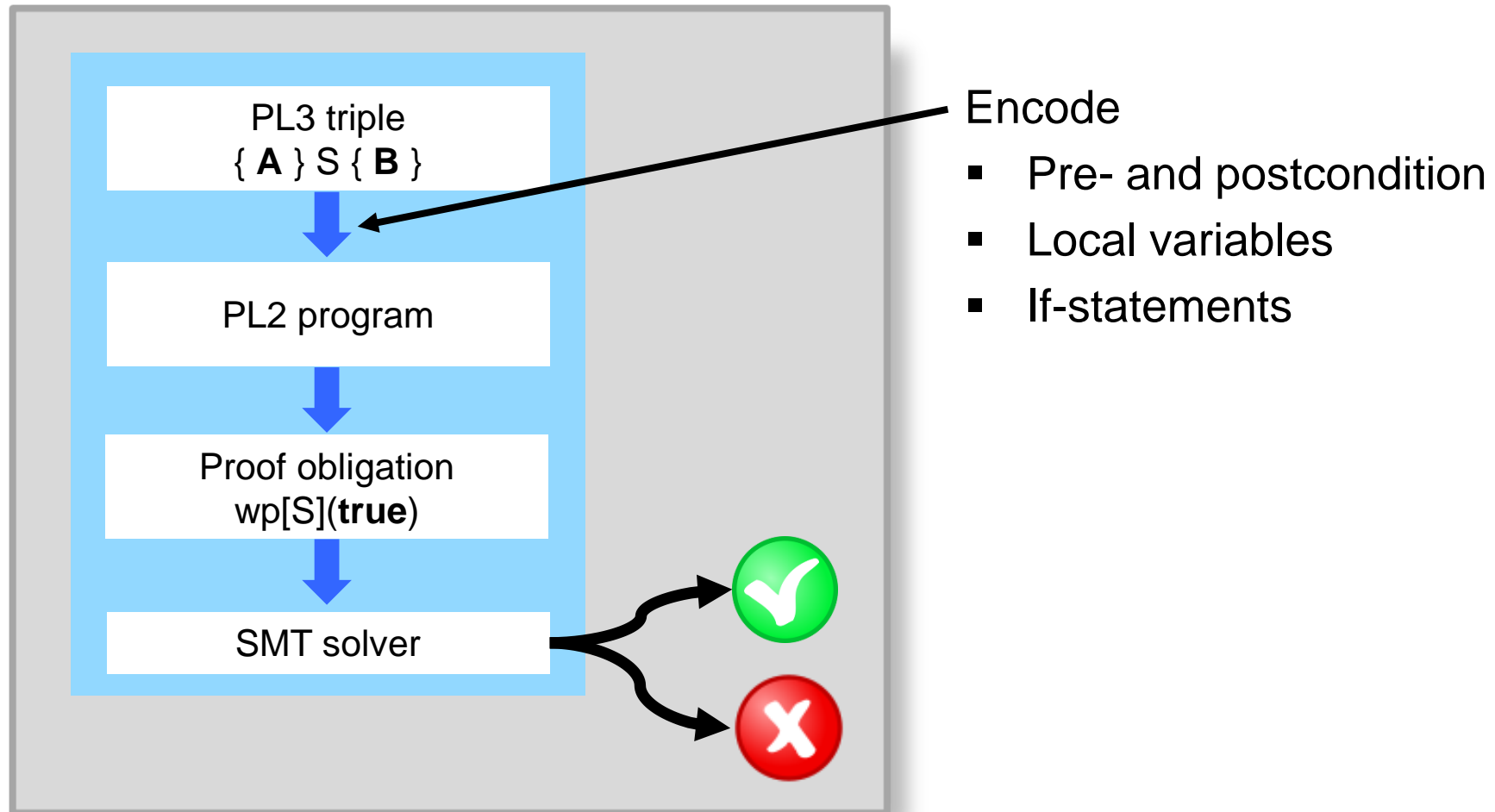
Show that it is also possible to encode non-deterministic choice into PL3.

Apply the encoding to the following program and check the verification results.

```
var x: Int
if (*) {
  x := 42
} else {
  x := 23
}

assert x == 42 || x == 23 // succeeds
assert x == 42           // fails
assert x == 23           // fails
```

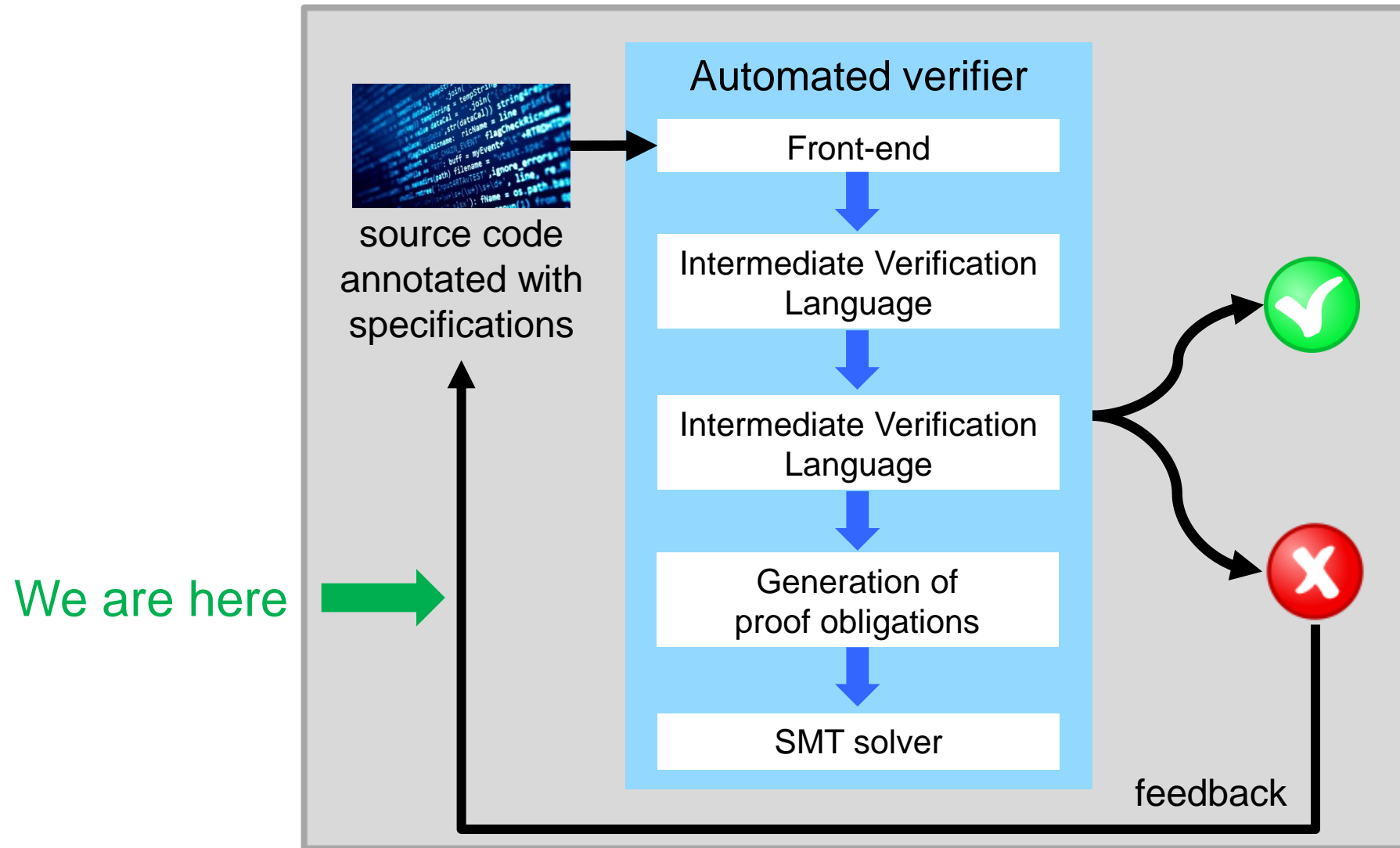
# The tool stack so far



# Building a first verifier

1. Two intermediate verification languages
2. Error reporting

# Roadmap



# Verification debugging

Verification failures may be caused by:

- Errors in the implementation
- Errors in the specification
- Insufficient annotations  
(e.g., missing loop invariants, as we will see later)
- Incompleteness of the verifier  
(spurious errors, false positives)

```
{ 0 ≤ b*b - 4*c }  
discriminant := b*b - 4*a*c;  
x := (-b + √discriminant) / 2  
{ a*x2 + b*x + c = 0 }
```



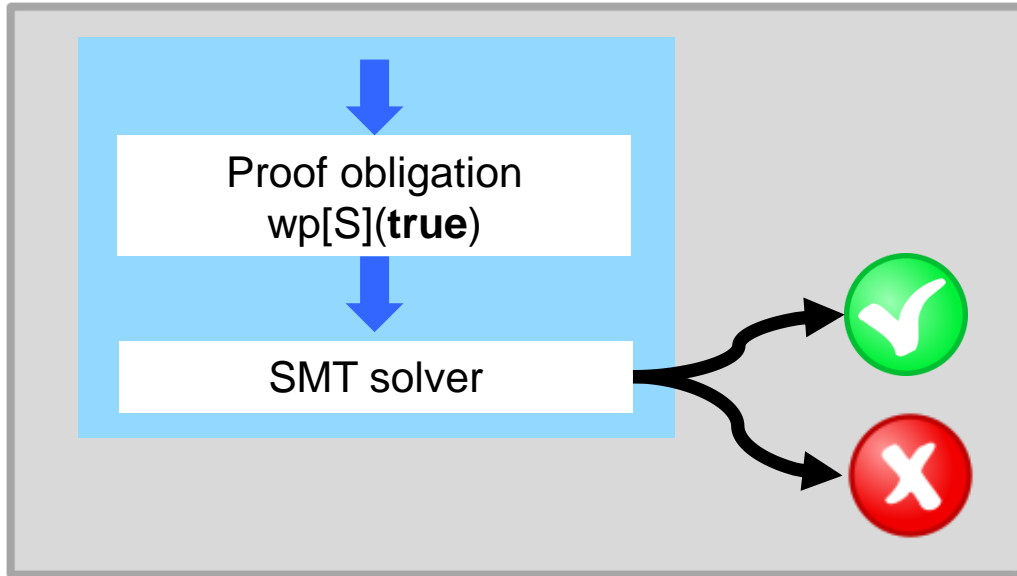
```
// Fermat's Last theorem  
assert 0 < x && 0 < y && 0 < z ==>  
       x*x*x + y*y*y != z*z*z
```



Verifiers should help users to localize and fix verification failures



# Counterexamples



- Models contain a value for each variable, such that the proof obligation is not valid
- They are counterexamples to the correctness of the program
- Viper command line option `--counterexample variables`

## Recall

- Verifier checks validity of  $wp[S](true)$
- SMT solver checks the satisfiability of the negation, that is, of  $\neg wp[S](true)$
- Verification fails if the SMT solver returns sat, together with a model
- If the verifier returns unknown, it typically provides at least a partial model

```
assert x*x > 0
```



```
⊗ ^ Assert might fail. Assertion  $x * x > 0$  might not hold.  
counterexample:  
x -> 0 [2, 10]
```

# Localizing errors

- Realistic programs contain a large number of proof obligations
  - For user-provided specifications such as postconditions
  - For all potential reasons for execution failures, e.g., division by zero, null-pointer dereferencing, out-of-bounds access
  - For other undesirable behaviors, e.g., overflows, data races, deadlocks
- To debug a verification error, it is crucial to know which of these proof obligations failed
- The technique so far checks validity of a single proof obligation  $wp[S](true)$ , but cannot report which part of this proof obligation is invalid

```
assert MIN_INT <= x + y
assert x + y <= MAX_INT
res := x + y

assert MIN_INT <= x - y
assert x - y <= MAX_INT
d := x - y

assert d != 0
res := res / d
```

# Verification failures

PL0 program $S$	$wp \llbracket S \rrbracket (\mathbf{B})$
<b>assert A</b>	$\mathbf{A} \wedge \mathbf{B}$
<b>assume A</b>	$\mathbf{A} \Rightarrow \mathbf{B}$
$S_1 ; S_2$	$wp \llbracket S_1 \rrbracket (wp \llbracket S_2 \rrbracket (\mathbf{B}))$
<b>if</b> $(*)$ <b>{</b> $S_1$ <b>}</b> <b>else</b> <b>{</b> $S_2$ <b>}</b>	$(b \Leftrightarrow \mathbf{B}) \Rightarrow wp \llbracket S_1 \rrbracket (b) \wedge wp \llbracket S_2 \rrbracket (b)$ where $b$ is a fresh Boolean variable

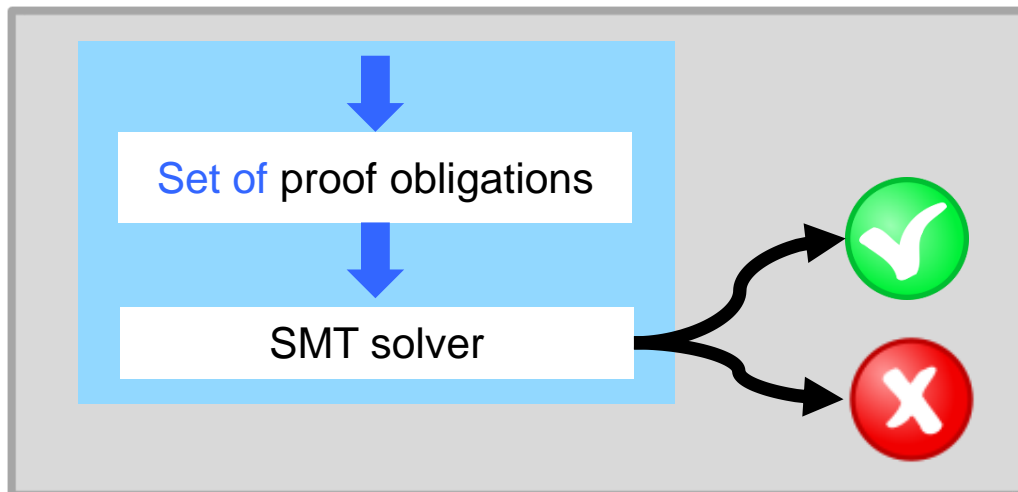
- Each verification error is caused by a **failing assertion**
- Since we check  $wp[S](\text{true})$ , assert statements are the only statements that lead to non-trivial proof obligations

- To determine which assertion to blame, we **split** the program at assertions **into multiple verification problems**

# Computing multiple proof obligations

- *mwp* is a weakest precondition transformer that computes a set  $M$  of proof obligations
- To verify a statement  $S$ , compute  $mwp[S](\emptyset)$

PL0 program $S$	$mwp \llbracket S \rrbracket (M)$
<b>assert</b> $A$	$M \cup \{A\}$
<b>assume</b> $A$	$\{A \Rightarrow B \mid B \in M\}$
$S_1 ; S_2$	$mwp \llbracket S_1 \rrbracket (mwp \llbracket S_2 \rrbracket (M))$
<b>if</b> $(*) \{ S_1 \}$ <b>else</b> $\{ S_2 \}$	$mwp \llbracket S_1 \rrbracket (M) \cup mwp \llbracket S_2 \rrbracket (M)$



- Verification succeeds if all proof obligations are valid
- For each failed proof obligation, report the corresponding assertion

# Exercise: error localization

Compute  $mwp[S](\emptyset)$  for the statement on the right.

Which of the proof obligations are valid?

For each invalid proof obligation, find an initial state such that the corresponding assertion fails

```
if(*) {  
  assert x == 7  
} else {  
  assert x == 2  
  assert x > 0  
}
```

Verify the example on the right in Viper using the Carbon verifier. How many error messages do you get?

Hint: CTRL+L allows you to choose the verifier.

```
method foo(x: Int, b: Bool) {  
  if(b) {  
    assert x == 7  
  } else {  
    assert x == 2  
    assert x > 0  
  }  
}
```

# Avoiding masked verification errors

- Both  $wp$  and  $mwp$  ignore the order of assertions

$$wp \llbracket \text{assert } \mathbf{A}_1 ; \text{assert } \mathbf{A}_2 \rrbracket (\mathbf{B}) = \mathbf{A}_1 \wedge \mathbf{A}_2 \wedge \mathbf{B}$$

$$mwp \llbracket \text{assert } \mathbf{A}_1 ; \text{assert } \mathbf{A}_2 \rrbracket (M) = M \cup \{ \mathbf{A}_1 \} \cup \{ \mathbf{A}_2 \}$$

```
assert x == 2
assert x > 0
```

```
assert x > 0
assert x == 2
```

- We would like to check the second assertion only for executions that may reach it, that is, in which the first assertion holds
- We achieve this by adding an assumption after each assertion

```
assert A
```

```
assert A
assume A
```

# Error reporting in Viper

- Viper has two verification backends
- Carbon
  - Uses weakest preconditions, similarly to the technique taught in this course, but replaces *mwp* by a more efficient approach
  - Counterexamples can be enabled via command line option
  - Reports multiple verification failures
- Silicon
  - Uses symbolic execution
  - Counterexamples can be enabled via command line option
  - Reports only one verification error per method (use command line option to enable multiple errors)
  - Default verifier in the IDE

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