

Colored Tiles

This problem is based on Project Euler problem #114.

Alice and Bob are decorating their kitchen, and they want to add a single row of fifty tiles on the edge of the kitchen counter. Tiles can be either red or black, and for aesthetic reasons, Alice and Bob insist that red tiles come by blocks of at least three consecutive tiles. Before starting, they wish to know how many ways there are of doing this. They come up with the following algorithm:

```
var count[51] // count[i] is the number of valid rows of size i
count[0] := 1 // []
count[1] := 1 // [B] - cannot have a single red tile
count[2] := 1 // [BB] - cannot have one or two red tiles
count[3] := 2 // [BBB] or [RRR]
for n = 4 to 50 do
    count[n] := count[n-1] // either the row starts with a black tile
    for k = 3 to n-1 do // or it starts with a block of k red tiles
        count[n] := count[n] + count[n-k-1] // followed by a black one
    end-for
    count[n] := count[n]+1 // or the entire row is red
end-for
```

Verification tasks. You should verify that at the end, count[50] will contain the right number.

Hint: Since the algorithm works by enumerating the valid colorings, we expect you to give a nice specification of a valid coloring and to prove the following properties:

1. Each coloring counted by the algorithm is valid.
2. No coloring is counted twice.
3. No valid coloring is missed.