Formal Methods and Functional Programming

Modeling

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The slides in this section are partly based on the course *Automata-based System Analysis* by Felix Klaedtke

Example 1: Protocol Verification

- Protocol for resource access with primitives `open`, `close`, `write`
- Task: verify that a program obeys the following (informal) rules:
  A. All opened resources must be closed eventually
  B. An opened resource must be closed before the next open and vice versa

  For simplicity, we assume there is only one resource (a file), which is initially closed
- Problem is typical for verification of protocols
  - Locking (acquire, access, release)
  - Authentication (authenticate, access)

Example 1: Encoding in IMP

- File is represented by variable `f`
  - Write is encoded by assignment to `f`
  - Variable `o` counts how often file was opened/closed

- Encoding of primitives:
  - `open`: $\texttt{o} := \texttt{o} + 1$
  - `close`: $\texttt{o} := \texttt{o} - 1$
  - `write`: $\texttt{f} := e$

- Informal rules:

  A. After `o` has been set to one, it must eventually be re-set to zero
  B. In all execution states, `o` is zero or one

  Variable `o` is initially zero

Example 1: Specification in NS and Hoare Logic

A. For a terminating program `s`, `o` must be zero in the terminal state

$$\boxed{\text{If } \vdash \langle \mathtt{s}, \sigma \rangle \to \sigma' \text{ and } \sigma(\mathtt{o}) = 0 \text{ then } \sigma'(\mathtt{o}) = 0}$$

$$\boxed{\vdash \{\ \mathtt{o} = 0\ \}\ \mathtt{s}\ \{\ \mathtt{o} = 0\ \}}$$

Property cannot be expressed for non-terminating programs

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Property cannot be expressed for non-terminating programs

B. In all execution states, `o` is zero or one

- Natural semantics and Hoare logic can express properties of initial and terminal states, but not of intermediate states

Example 1: Specification in SOS (A)

- A: After o has been set to one, it must eventually be re-set to zero
  - For a terminating program s

$$\boxed{\text{If } \langle s, \sigma \rangle \rightarrow_1^* \sigma' \text{ and } \sigma(o) = 0 \text{ then } \sigma'(o) = 0}$$

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$$\text{If } \langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma' \text{ and } \sigma(\mathtt{o}) = 0 \text{ then } \sigma'(\mathtt{o}) = 0$$

  - For a deterministic, non-terminating program `s`

$$\text{If } \langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \langle s', \sigma' \rangle \text{ and } \sigma(\mathtt{o}) = 0 \text{ and } \sigma'(\mathtt{o}) = 1 \text{ then there exist } s'', \sigma'' \text{ such that } \langle s', \sigma' \rangle \rightarrow_1^* \langle s'', \sigma'' \rangle \text{ and } \sigma''(\mathtt{o}) = 0$$

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  - For a non-deterministic, non-terminating program `s`

$$\begin{aligned}
&wc : \text{Stm} \times \text{State} \times \mathbb{N} \rightarrow \text{Bool} \\
&wc(s, \sigma, n) \Leftrightarrow \sigma(o) = 0 \vee \\
&\qquad (\text{for all } s', \sigma' : \text{if } \langle s, \sigma \rangle \rightarrow_1 \langle s', \sigma' \rangle \text{ then there exists} \\
&\qquad m \in \mathbb{N} \text{ such that } m < n \text{ and } wc(s', \sigma', m))
\end{aligned}$$

$$\text{If } \langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \langle s', \sigma' \rangle \text{ and } \sigma(\mathtt{o}) = 0 \text{ and } \sigma'(\mathtt{o}) = 1 \text{ then there exists } n \in \mathbb{N} \text{ such that } wc(s', \sigma', n)$$

Example 1: Specification in SOS (B)

- B: In all execution states, $\mathtt{o}$ is zero or one

If $\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \langle s', \sigma' \rangle$ and $\sigma(\mathtt{o}) = 0$ then $\sigma'(\mathtt{o}) = 0$ or $\sigma'(\mathtt{o}) = 1$

Example 1: Verification

A. For a terminating program $\mathtt{s}$

If $\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma'$ and $\sigma(\mathtt{o}) = 0$ then $\sigma'(\mathtt{o}) = 0$

- Proof needs to consider all possible derivation sequences $\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma'$ to find all possible terminal states
- Problematic in the presence of non-determinism or parallelism

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$$\text{If } \langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma' \text{ and } \sigma(\mathtt{o}) = 0 \text{ then } \sigma'(\mathtt{o}) = 0$$

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For a deterministic, non-terminating program $\mathtt{s}$

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- Proof about unbounded/infinite derivation sequence requires invariant, which cannot be found automatically

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- Proof about unbounded/infinite derivation sequence requires invariant, which cannot be found automatically

B. In all execution states, $\mathtt{o}$ is zero or one

$$\text{If } \langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \langle s', \sigma' \rangle \text{ and } \sigma(\mathtt{o}) = 0 \text{ then } \sigma'(\mathtt{o}) = 0 \text{ or } \sigma'(\mathtt{o}) = 1$$

- Proof needs to consider all possible multi-step executions

Example 2: Verification of Parallel Programs

- A (simplified) Java program

```
class Cell {
  int x = 0;

  static void main(...) {
    Cell c = new Cell();
    Thread t1 = new Even(c);
    Thread t2 = new Even(c);
    t1.start(); t2.start();
    t1.join();  t2.join();
    System.out.println(c.x);
  }
}
```

```
class Even extends Thread {
  Cell c;

  Even(Cell c) {
    this.c = c;
  }

  void run() {
    c.x = c.x + 1;
    c.x = c.x + 1;
  }
}
```

Example 2: Verification of Parallel Programs

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    c.x = c.x + 1;
    c.x = c.x + 1;
  }
}
```

```
3
```

Example 2: Encoding in IMP

- The following program `s` represents the core of the Java program
  - `x` represents shared variable `c.x`
  - `y` and `z` represent thread-local state

```
   (y := x;  y := y + 1;  x := y;
    y := x;  y := y + 1;  x := y)
par
   (z := x;  z := z + 1;  x := z;
    z := x;  z := z + 1;  x := z)
```

- Desired property:
  If `x` is zero in the initial state then `x` is even in the terminal state
  - NS and Hoare logic cannot handle parallelism
  - SOS specification:

$$\boxed{\text{If } \langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma' \text{ and } \sigma(\mathtt{x}) = 0 \text{ then } \sigma'(\mathtt{x}) \ \mathbf{mod}\ 2 = 0}$$

  (this is not true for the code above)

Example 2: Verification

- In this case, spotting the counterexample is easy, but how to attempt a formal proof?
- Induction does not work because there is no suitable induction hypothesis
  - Observation also holds for corrected example
- Proof strategy: enumerate all possible derivations of $\langle \mathtt{s}, \sigma \rangle \rightarrow_1^* \sigma'$ and inspect terminal state σ'
  - Number of derivations grows exponentially in number of executed statements
  - Here, $\frac{12!}{6! \times 6!} = 924$ possible derivations!
  - Manual enumeration not feasible, especially for programs with loops

Examples: Observations

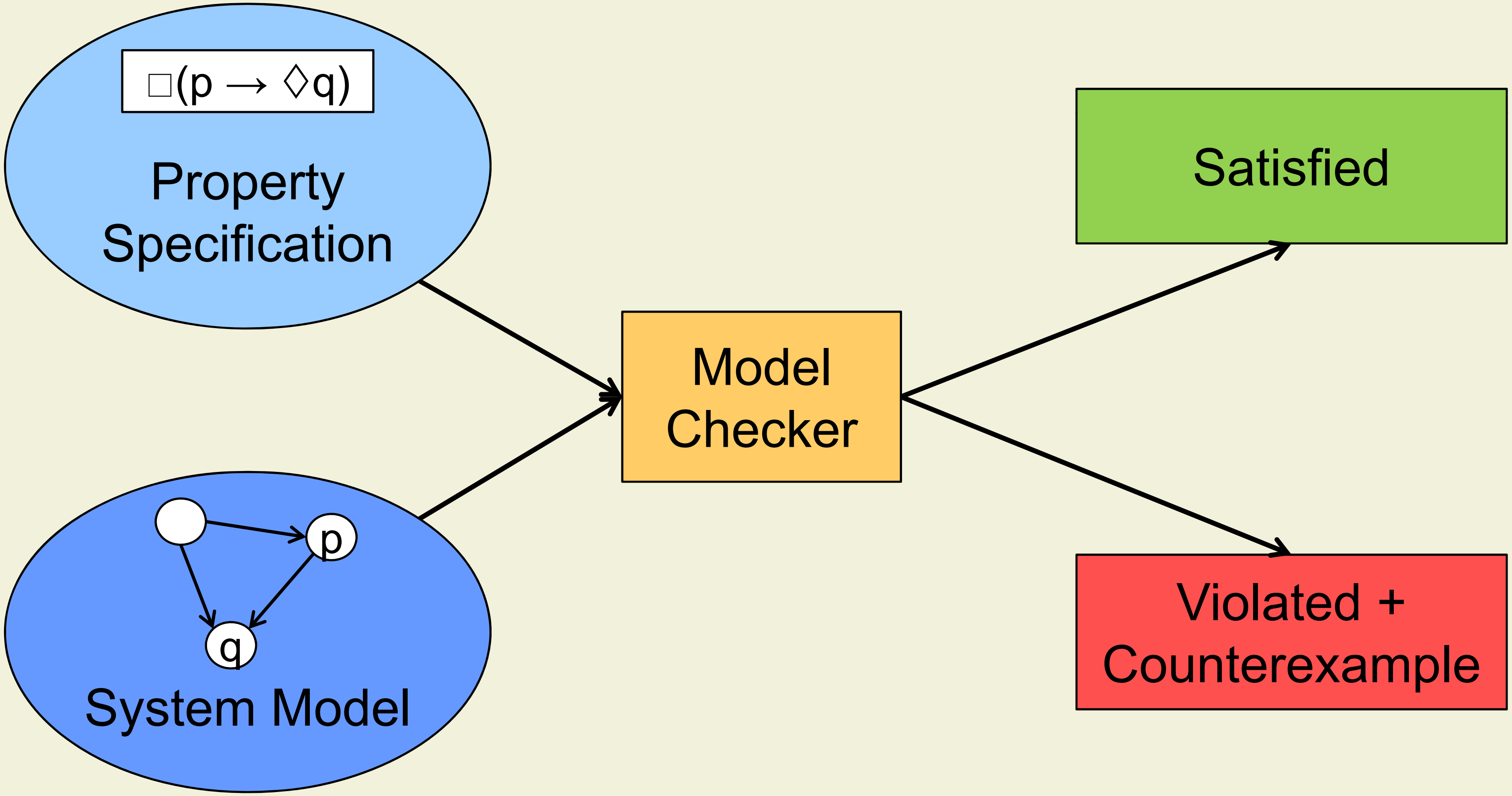
- Specification challenge
  - How to specify properties of sequences of states concisely
- Verification challenges
  - Concurrent systems: How to prove properties of all possible program executions
  - Reactive systems: How to automatically prove properties of infinite derivation sequences

Model Checking

> Model checking is an automated technique that, given a finite-state model of a system and a formal property, systematically checks whether this property holds for (a given state in) that model. [Baier and Katoen]

- Model checkers enumerate all possible states of a system:
  - Explicit state model checking:
    represent state explicitly through concrete values
  - Symbolic model checking:
    represent state through (boolean) formulas
- We focus on explicit state model checking

Model Checking



$\Box(p \rightarrow \Diamond q)$

Property Specification

System Model

Model Checker

Satisfied

Violated + Counterexample

Model Checking Process

- Modeling phase
  - Model the system under consideration using the description language of your model checker (possibly a programming language)
  - Formalize the properties to be checked
- Running phase
  - Run the model checker to check the validity of the property in the system model
- Analysis phase
  - If property is satisfied, celebrate and move on to next property
  - If property is violated, analyze counterexample
  - If out of memory, reduce model and try again

Main Purposes of Model Checking

- Model checking is mainly used to analyze system designs (as opposed to implementations)
- Typical properties to be analyzed include
    - Deadlocks
    - Reachability of undesired states
    - Protocol violations

Modeling Concurrent Systems

- Systems are modeled as finite transition systems

- We model systems as communicating sequential processes (agents)
  - Finite number of processes
  - Interleaved process execution

- Processes can communicate via:
  - Shared variables
  - Synchronous message passing
  - Asynchronous message passing

Protocol Meta Language Promela

- Input language of the Spin model checker
- Main objects are processes, channels, and variables
- C-like syntax

```
init {
  printf("Hello World!\n")
}
```

- Spin can “execute” (simulate) models
- References
    - Quick reference: `www.spinroot.com/spin/Man/Quick.html`
    - Further references: `www.spinroot.com/spin/Man/index.html`

Promela Programs

- Constant declarations

```
#define N 5
mtype = { ack, req };
```

- Structure declarations

```
typedef vector { int x; int y };
```

- Global channel declarations

```
chan buf = [2] of { int };
```

- Global variable declarations

```
byte counter;
```

- Process declarations

```
proctype myProc(int p) { ... }
```

Promela Process Declarations

- Simple form

```
proctype myProc(int p) { ... }
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  - Body consists of a sequence of variable declarations, channel declarations, and statements
  - No arrays as parameters

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```
active [N] proctype myProc(...) { ... }
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  - Start `N` instances of `myProc` in the initial state

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  - Start `N` instances of `myProc` in the initial state

- `init` process is started in the initial state

Promela Types

- Primitive types

Type	Value range
`bit` or `bool`	$0 \ldots 1$
`byte`	$0 \ldots 255$
`short`	$-2^{15} \ldots 2^{15} - 1$
`int`	$-2^{31} \ldots 2^{31} - 1$

  - No floats or mathematical (unbounded) integers

- User-defined types
  - Arrays: `int name[4]`
  - Structures
  - Type of symbolic constants: `mtype`

- Channel type: `chan`

Promela Variable and Channel Declarations

- Variable declarations

```
byte a, b = 5, c;
int d[3], e[4] = 3;
mtype msg = ack;
vector v;
```

  - Variables are initialized to zero-equivalent values

Promela Variable and Channel Declarations

- Variable declarations

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int d[3], e[4] = 3;
mtype msg = ack;
vector v;
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- Channel declarations

```
chan c1 = [2] of { mtype, bit, chan };
chan c2 = [0] of { int };
chan c3;
```

  - `c1` can store up to two messages
    Messages sent via `c1` consist of three parts (triples)
  - `c2` models rendez-vous communication (no message buffer)
  - `c3` is uninitialized; must be assigned an initialized channel before usage

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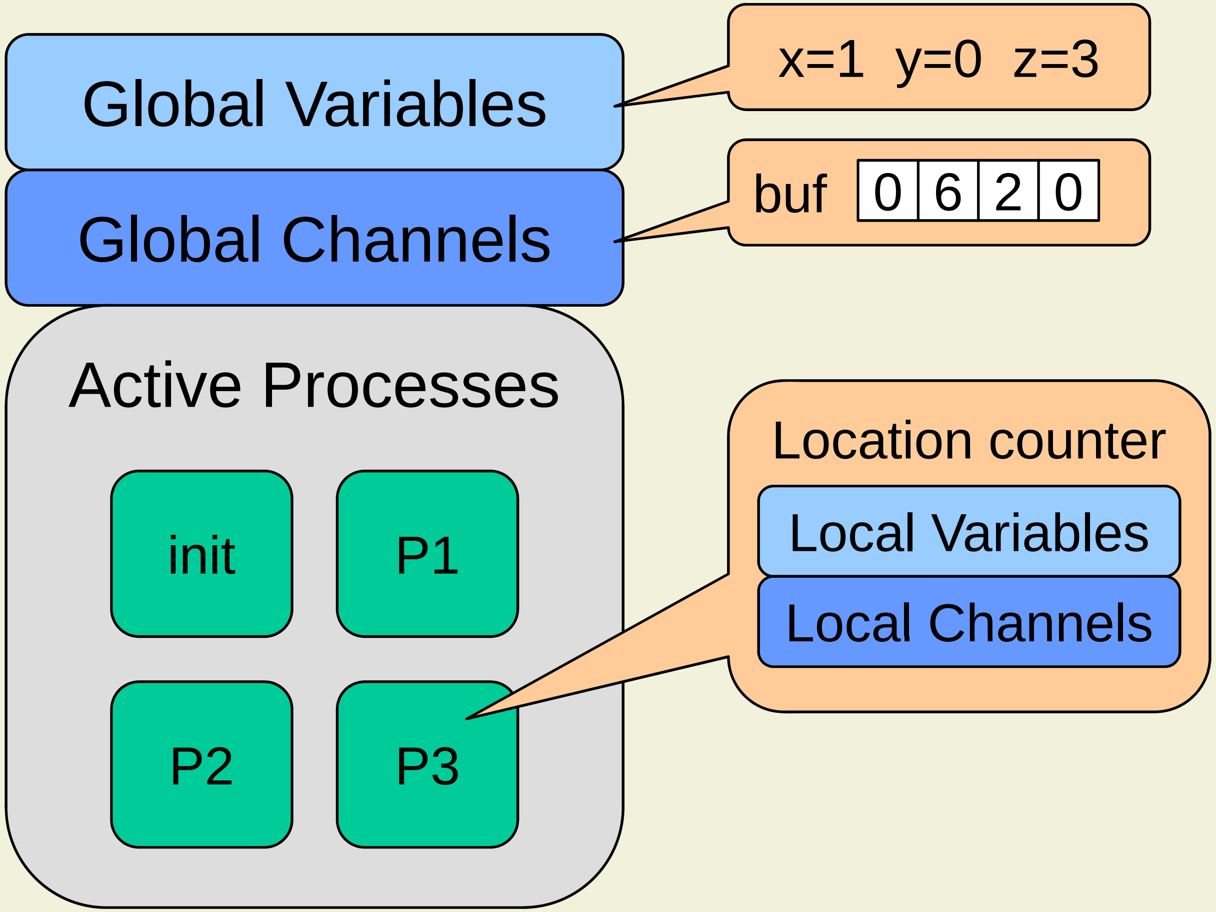
- Variable and channel declarations are local to a process or global

State Space of a Promela System



State Space of Sequential Programs

- Number of states

$$\#\text{program locations} \times \prod_{\text{variable } x} |\, dom(x) \,|$$

  - where $|\, dom(x) \,|$ denotes the number of possible values of variable x

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- Example: sequential program with 10 locations and 3 boolean variables

$$10 \times 2 \times 2 \times 2 = 10 \times 2^3 = 80$$

- Adding two integer variables yields $80 \times 2^{32} \times 2^{32} = 80 \times 2^{64}$

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- Number of states grows exponentially in the number of variables

- State space explosion

State Space of Concurrent Programs

- The number of states of $P \equiv P_1 \| \ldots \| P_N$ is at most

$$\#\text{states of } P_1 \times \ldots \times \#\text{states of } P_N =$$

$$\prod_{i=1}^{N} (\#\text{program locations}_i \times \prod_{\text{variable } x_i} |\, dom(x_i) \,|)$$

State Space of Concurrent Programs

- The number of states of $P \equiv P_1 \| \ldots \| P_N$ is at most

$$\#\text{states of } P_1 \ \times\ \ldots\ \times\ \#\text{states of } P_N =$$

$$\prod_{i=1}^{N}(\#\text{program locations}_i \ \times \prod_{\text{variable } x_i} |\, dom(x_i)\, |)$$

- Number of states grows exponentially in the number of processes

- State space explosion

State Space of Promela Models

- The number of states of a system with N processes and K channels is at most

$$\prod_{i=1}^{N}(\#\text{program locations}_i \times \prod_{\text{variable } x_i} \mid dom(x_i) \mid) \times \prod_{j=1}^{K} \mid dom(c_j) \mid^{cap(c_j)}$$

  - $\mid dom(c) \mid$ denotes the number of possible messages of channel c
  - $cap(c)$ is the capacity (buffer size) of channel c

State Space of Promela Models

- The number of states of a system with N processes and K channels is at most

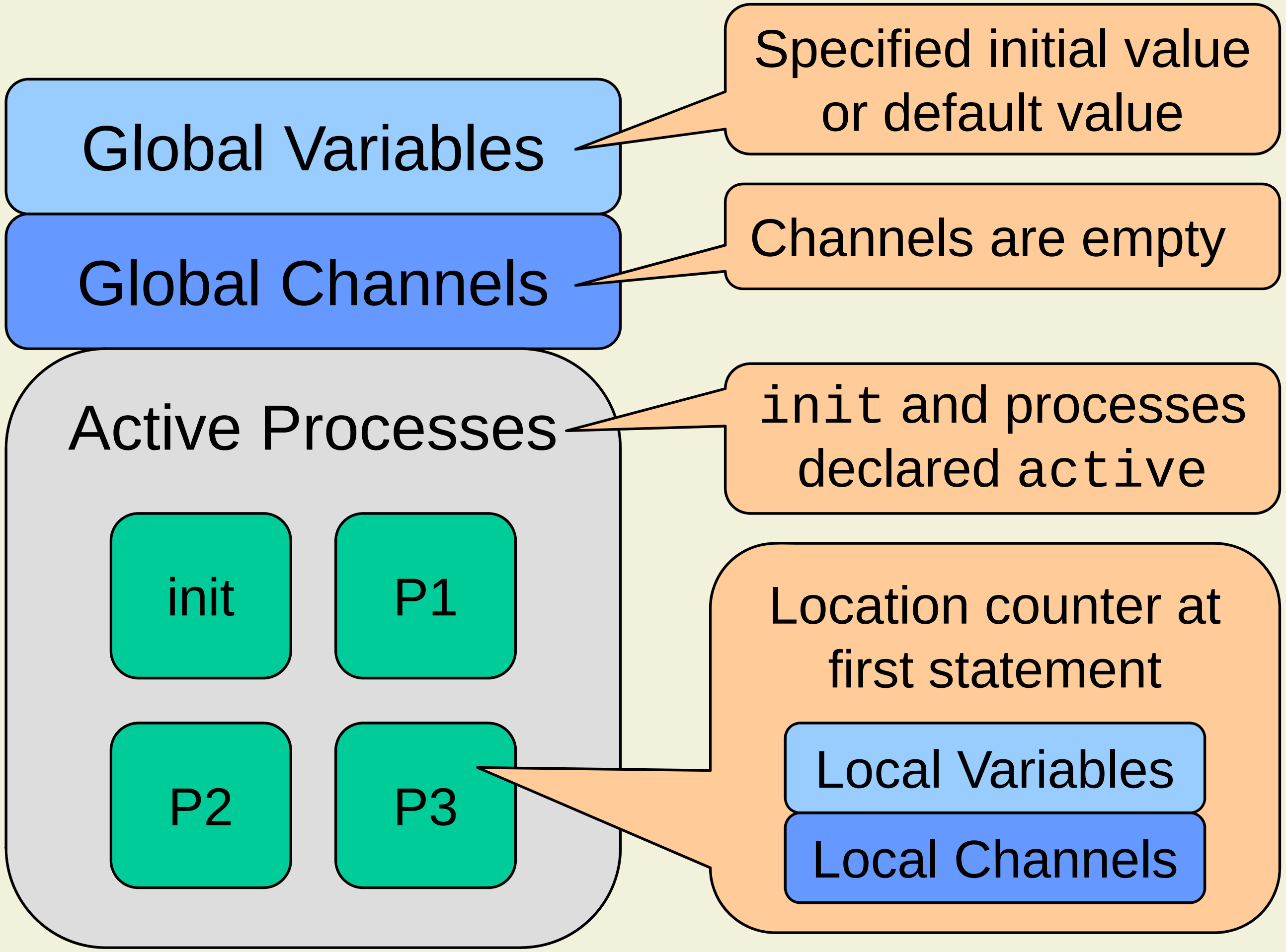
$$\prod_{i=1}^{N}(\#\text{program locations}_i \times \prod_{\text{variable } x_i} \mid dom(x_i) \mid) \times \prod_{j=1}^{K} \mid dom(c_j) \mid^{cap(c_j)}$$

  - $\mid dom(c) \mid$ denotes the number of possible messages of channel c
  - $cap(c)$ is the capacity (buffer size) of channel c

- Number of states grows exponentially in the number and capacity of channels

- State space explosion

Initial State



Global Variables

Specified initial value or default value

Global Channels

Channels are empty

Active Processes

`init` and processes declared `active`

Location counter at first statement

Local Variables

Local Channels

State Transitions

- A statement can be executable or blocked
    - Send is blocked if channel is full
    - `s1;s2` is blocked if `s1` is blocked
    - `timeout` is executable if all other statements are blocked

A transition is made in three steps:

- Determine all executable statements of all active processes
    - If no executable statement exists, transition system gets stuck

- Choose non-deterministically one of the executable statements
    - Non-determinism models concurrency through interleaving

- Change the state according to the chosen statement

Promela Expressions

- Variables, constants, and literals
- Structure and array accesses
- Unary and binary expressions with operators

```
+    -    *     /    %    >
>=   <    <=    ==   !=   !
&    ||   &&    |    ~    >>
<<   ^    ++    --
```

- Function applications

```
len()   empty()   nempty()    nfull()     full()
run     eval()    enabled()   pcvalue()
```

- Conditional expressions: `(E1 -> E2 : E3)`

Promela Statements

- `skip`
  - Does not change the state (except the location counter)
  - Always executable
- `timeout`
  - Does not change the state (except the location counter)
  - Executable if all other statements in the system are blocked
- `assert(E)`
  - Aborts execution if expression `E` evaluates to zero; otherwise equivalent to `skip`
  - Always executable
- Assignment
  - `x = E` assigns the value of `E` to variable `x`
  - `a[n] = E` assigns the value of `E` to array element `a[n]`
  - Always executable

Promela Statements (cont'd)

- Sequential composition
  - `s1;s2` is executable if `s1` is executable
- Expression statement
  - Evaluates expression `E`
  - Executable if `E` evaluates to value different from zero
  - `E` must not change state (no side effects)
  - Examples:

```
run myProcess;
x > 0;
```

Motivation: Verification of Parallel Programs

- A (simplified) Java program

```
class Cell {
  int x = 0;

  static void main(...) {
    Cell c = new Cell();
    Thread t1 = new Even(c);
    Thread t2 = new Even(c);
    t1.start(); t2.start();
    t1.join();  t2.join();
    System.out.println(c.x);
  }
}
```

```
class Even extends Thread {
  Cell c;

  Even(Cell c) {
    this.c = c;
  }

  void run() {
    c.x = c.x + 1;
    c.x = c.x + 1;
  }
}
```

```
3
```

Example: Modeling Even.run

```
class Even extends Thread {
  Cell c;

  Even(Cell c) {
    this.c = c;
  }

  void run() {
    c.x = c.x + 1;
    c.x = c.x + 1;
  }
}
```

```
int x;

proctype EvenRun() {
  x = x + 1;
  x = x + 1;
}
```

```
int x;

proctype EvenRun() {
  int y = x;
  y = y + 1;
  x = y;
  y = x;
  y = y + 1;
  x = y;
}
```

Example: Modeling Cell.main

```
class Cell {
  int x = 0;

  static void main(...) {
    Cell c = new Cell();
    Thread t1 = new Even(c);
    Thread t2 = new Even(c);
    t1.start(); t2.start();
    t1.join();  t2.join();
    System.out.println(c.x);
  }
}
```

```
init {
  x = 0;

  run EvenRun();
  run EvenRun();

  /* wait for termination */
  _nr_pr == 1;

  printf("x: %d\n", x);
  assert x % 2 == 0;
}
```

- `_nr_pr` is a predefined global variable that yields the number of active processes
- Simulation in Spin shows the possible outcomes 2, 3, and 4 (like Java program)

Promela Statements: Selection

```
if
:: s1   /* option 1 */
:: ...
:: sn   /* option n */
fi
```

- Executable if at least one of its options it executable
- Chooses an option non-deterministically and executes it

```
if  /* Move a sprite */
:: x < maxX -> x = x + 1;
:: x > minX -> x = x - 1;
:: y < maxY -> y = y + 1;
:: y > minY -> y = y - 1;
:: color = color + 1;
fi
```

- Statement `else` is executable if no other option is executable (may occur at most in one option)

Promela Statements: Repetition

```
do
:: s1   /* option 1 */
:: ...
:: sn   /* option n */
od
```

- Executable if at least one of its options it executable
- Chooses repeatedly an option non-deterministically and executes it
- Terminates when a `break` or `goto` is executed

```
/* compute factorial of n */

int r = 1;

do
:: n > 1 -> r = r*n; n = n-1;
:: else -> break
od
```

```
/* deadlock detection */
active proctype watchDog() {
  do
  :: timeout ->
     /* reset the state */
  od
}
```

Promela Statements: Atomic

- Basic statements are executed atomically
  - No interleaving during execution of statement
  - `skip`, `timeout`, `assert`, assignment, expression statement

- `atomic { s }` executes `s` atomically
  - Executable if the first statement of `s` is executable
  - If any other statement within `s` blocks once the execution of `s` has started, atomicity is lost

- Example: Binary semaphores (locks)

```
bit locked; /* global */
```

```
/* lock */
locked == 0;
locked = 1;


/* critical section */
locked = 0; /* unlock */
```

```
/* lock */
atomic {
  locked == 0;
  locked = 1;
}
/* critical section */
locked = 0; /* unlock */
```

Promela Macros

- Promela does not contain procedures
- Effect can often be achieved using macros

```
inline lock() {
  atomic {
    locked == 0;
    locked = 1
  }
}
```

```
inline swap(a, b) {
  int tmp;
  tmp = a;
  a = b;
  b = tmp
}
```

- A macro just defines a replacement text for a symbolic name, possibly with parameters
  - The inline call `lock()` is replaced by the body of the definition
  - No new variable scope
  - No recursion
  - No return values
- Define macro globally before its first use

Motivation: Deadlock

- Threads are synchronized via locks
- Interleaved execution of `a.transfer(b,n)` and `b.transfer(a,m)` might deadlock
- Multi-threaded programs are extremely hard to test

```
class Account {
  int balance;

  void transfer(Account to, int amount) {
    acquire this;
    acquire to;
    this.balance -= amount;
    to.balance += amount;
    release this;
    release to;
  }
}
```

Promela Model: Account

- We need to model accounts and clients
  - General approach: omit all irrelevant details to reduce complexity
- Account
  - Balance is not relevant for potential deadlocks
  - Only model the locks of accounts

```
#define N 5

bit Account_locks[N];

inline lock(n) {
  atomic {
    Account_locks[n] == 0;
    Account_locks[n] = 1;
  }
}
```

Promela Model: Client

- Idea: model the most generic client and run several instances in parallel
  - Pick two arbitrary accounts non-deterministically
  - Lock both accounts
  - Unlock both accounts

- Choosing accounts

```
inline choose(a, l, u) {
  a = l;
  do
  :: (a < u) -> a++
  :: break
  od
}
```

```
inline chooseAccounts(f, t) {
  do
  :: (f != t) -> break
  :: (f == t) -> choose(f, 0, N-1);
                 choose(t, 0, N-1)
  od
}
```

Promela Model: Client Process

```
active [C] proctype transfer() {
  byte from, to;

  /* choose accounts non-deterministically */
  chooseAccounts(from, to);

  /* acquire locks */
  lock(from);
  lock(to);

  /* actual transfer omitted */

  /* release locks */
  Account_locks[from] = 0;
  Account_locks[to] = 0;
}
```

Alternative Account Selection

- Idea: instead of looping until two different accounts are found, restrict range for second choice

```
do
:: (f != t) -> break
:: (f == t) -> choose(f, 0, N-1);
               choose(t, 0, N-1)
od
```

```
choose(f, 0,   N-2);
choose(t, f+1, N-1)
```

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choose(t, f+1, N-1)
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- Alternative model is less general
  - It guarantees `from` < `to`
  - So locks are acquired in order and deadlock is prevented!

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- Alternative model is less general
  - It guarantees `from` < `to`
  - So locks are acquired in order and deadlock is prevented!
- General strategy
  - Start with most general model
  - If model contains errors that cannot occur in real system (spurious errors), revise model

Promela Channels

- `chan ch = [d] of { t1, ..., tn }` declares a channel
- Channel can buffer up to `d` messages
  - `d` > 0: buffered channel (FIFO)
  - `d` $= 0$: unbuffered channel (rendez-vous)
- Each message is a tuple whose elements have types `t1, ..., tn`
- Example

```
mtype = { req, ack, err };

chan ch = [5] of { mtype, int }
```

Send and Receive: Buffered Channels

```
chan ch = [5] of { mtype, int }
```

- `ch ! e1, ..., en` sends message
  - Type of `ei` must correspond to `ti` in channel declaration
  - Send is executable iff buffer is not full
- `ch ? a1, ..., an` receives message
  - `ai` is a variable or constant of type `ti`
  - Receive is executable iff buffer is not empty and the oldest message in the buffer matches the constants `ai`
  - Variables `ai` are assigned values of the message

```
ch ! req, 7;
ch ! ack, 1
```

```
int n;
ch ? req, n;
printf("Received: %d\n", n);
ch ? req, n;
printf("Received: %d\n", n);
```

Send and Receive: Unbuffered Channels

```
chan ch = [0] of { int };
```

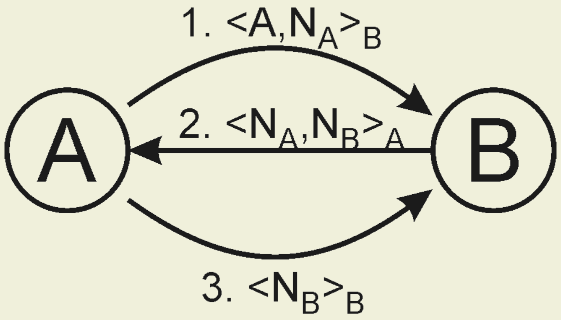
- `ch ! e1, ..., en` sends message
  - Send is executable if there is a receive operation that can be executed simultaneously

- `ch ? a1, ..., an` receives message
  - Receive is executable if there is a send operation that can be executed simultaneously

- Unbuffered channels model synchronous communication (rendez-vous)

Motivation: Needham-Schroeder Protocol

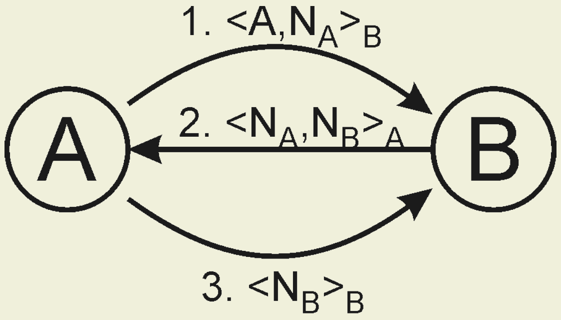
- Establish a common secret over an insecure channel
  1. Alice sends random number N_A to Bob, encrypted with Bob's public key: $\langle A, N_A \rangle_B$
  2. Bob sends random number N_B to Alive, encrypted with Alice's public key: $\langle N_A, N_B \rangle_A$
  3. Alice responds with $\langle N_B \rangle_B$



- Intruders may:
  - Intercept, store, and replay messages
  - Initiate or participate in runs of the protocol
  - Decrypt messages only if encrypted with intruder's public key

- Error: intruder can pretend to be another party

Promela Model: Network



- We model the protocol for two agents plus intruder
- Agents communicate synchronously

```
chan network = [0] of {
  mtype, /* tag:                    msg1, msg2, msg3       */
  mtype, /* intended receiver: agentA, agentB, agentI */
  Crypt  /* message                                         */
};
```

- We use enumeration type `mtype` for all constants
  - Spin treats `mtype` constants as symbols, not values
  - Speeds up model checking

Promela Model: Messages

- Message consists of key and up to two contents

```
typedef Crypt {
  mtype key,        /* public key used to encrypt */
        content1,   /* agent or nonce             */
        content2    /* nonce or don't care        */
};
```

- We model encryption by putting the public key into the message
  - Agent *a* will only look at message content if message key is *a*'s public key
  - No need to model private keys and encryption
- Constants for message tag, public keys, agents, and nonces

```
mtype = { msg1, msg2, msg3,
          keyA, keyB, keyI,
          agentA, agentB, agentI,
          nonceA, nonceB, nonceI };
```

Promela Model: Alice

- Alice starts a protocol run

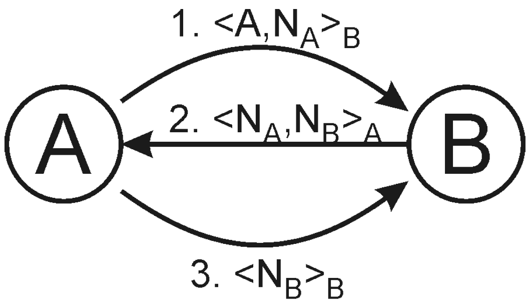
```
mtype partnerA;
bit   statusA; /* 1 = success */

active proctype Alice() {
  mtype pkey;      /* the partner’s public key            */
  mtype pnonce;    /* nonce that we receive from partner */
  Crypt message;   /* Alice’s message to the partner      */
  Crypt data;      /* received message                    */

  if /* choose a partner for this run */
  :: partnerA = agentB; pkey = keyB;
  :: partnerA = agentI; pkey = keyI;
  fi;

  /* Protocol run below */
  statusA = 1; /* Success */
}
```

Promela Model: Alice's Protocol Run



```
/* Prepare and send first message */
build(message, pkey, agentA, nonceA);
network ! msg1, partnerA, message;

/* Wait for answer */
network ? (msg2, agentA, data);

/* Proceed only if the key matches keyA and the
   nonce is the one that we have sent earlier */
(data.key == keyA) && (data.content1 == nonceA);

/* Obtain partner's nonce */
pnonce = data.content2;

/* Prepare and send the last message */
build(message, pkey, pnonce, 0);
network ! msg3, partnerA, message;
```

Intruder

- Intruders may:
  - Intercept messages
  - Store one message
  - Replay messages
  - Initiate or participate in runs of the protocol
  - Decrypt messages only if encrypted with intruder's public key
- How can we model the most powerful attack using these capabilities?
- Solution: Model intruder fully non-deterministically
  - Intruder has no intellegence whatsoever
  - Model checker will explore all possible behaviors of intruder

Promela Model: Intruder

```
bool knows_nonceA, knows_nonceB;

active proctype Intruder() {
  mtype tag;          /* message tag                      */
  mtype recpt;        /* recipient for Intruder's message */
  Crypt data          /* received message                 */
  Crypt intercepted;  /* stored message                   */

  do
  :: /* Receive and learn */

  :: /* Replay or send    */
  od
}
```

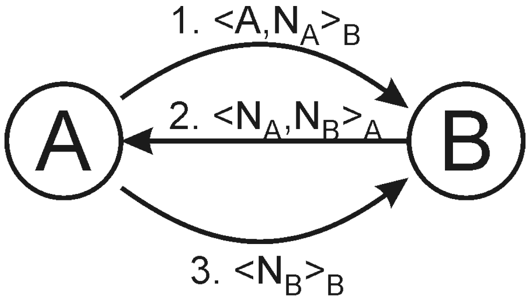
Promela Model: Intruder Receives

```
do
:: network ? (tag, _, data) ->
   if /* perhaps store the message */
   :: copy(data, intercepted);
   :: skip;
   fi;
   if /* record newly learnt nonces */
   :: (data.key == keyI) ->
      knows_nonceA = knows_nonceA ||
                     (data.content1 == nonceA) ||
                     (data.content2 == nonceA);
      knows_nonceB = knows_nonceB ||
                     (data.content1 == nonceB) ||
                     (data.content2 == nonceB);
   :: else -> skip;
   fi;
:: /* Replay or send    */
od
```

Promela Model: Intruder Sends

```
do
:: /* Receive and learn */
:: /* Replay or send    */
   if /* choose message type */
   :: tag = msg1;
   :: tag = msg2;
   :: tag = msg3;
   fi;
   if /* choose recipient */
   :: recpt = agentA;
   :: recpt = agentB;
   fi;
   if /* replay intercepted message or assemble it */
   :: copy(intercepted, data);
   :: /* assemble new message */
   fi;
   network ! tag, recpt, data;
od
```

Promela Model: Intruder Sends (cont'd)



```
:: /* assemble new message */
   if
   :: data.key = keyA;
   :: data.key = keyB;
   fi;
   if
   ::                 data.content1 = agentA;
   ::                 data.content1 = agentB;
   ::                 data.content1 = agentI;
   :: knows_nonceA -> data.content1 = nonceA;
   :: knows_nonceB -> data.content1 = nonceB;
   ::                 data.content1 = nonceI;
   fi;
   if
   :: knows_nonceA -> data.content2 = nonceA;
   :: knows_nonceB -> data.content2 = nonceB;
   ::                 data.content2 = nonceI;
   fi;
```

Summary

- Models are abstractions of the real world
- Omit irrelevant details to reduce complexity
  - Example: balance in account example
- Keep model small to avoid state space explosion
  - As few processes as possible
  - As little data as possible
- Non-determinism is a powerful modeling tool
  - Let model checker explore all options
- Typical sources of non-determinism are:
  - Abstraction
  - Modeling of the environment