ETH zürich

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Formal Methods and Functional Programming

Session Sheet 11: Big-Step Semantics

Assignment 1 (Substituting for Absent Variables)

Task: Prove that substituting absent variables has no effect, i.e., prove that

$$\forall e, e', x \cdot \big(x \notin \mathit{FV}(e) \implies e[x \mapsto e'] \equiv e\big).$$

Assignment 2 (`do-times` statement)

Consider the statement

$$\texttt{do } e \texttt{ times } s \texttt{ end}$$

where s is a statement and e is an arithmetic expression. The intuitive semantics of this statement is to execute s for e times.

Task. Give rules for the natural semantics that capture the semantics of this loop construct

Note: There is more than one possible solution.

Assignment 3 (`time` statement)

Task. Extend the natural semantics of **IMP** to support the statement $x := \texttt{time } s$, where $\texttt{time } s$ returns an integer that counts the number of assignments executed during the execution of the statement s.

Assignment 4 (`repeat-until` and `while` loops)

Consider the extension of the programming language **IMP** with the statement

$$\texttt{repeat}\ s\ \texttt{until}\ b$$

where s is a statement and b is a Boolean expression. In the natural semantics, the semantics of this new statement is captured by the following two rules:

$$\frac{\langle s, \sigma\rangle \to \sigma'}{\langle \texttt{repeat}\ s\ \texttt{until}\ b, \sigma\rangle \to \sigma'}\ (\textsc{RepT})\ \text{if}\ \mathcal{B}[\![b]\!]\sigma' = \mathrm{tt}$$

$$\frac{\langle s, \sigma\rangle \to \sigma'' \quad \langle \texttt{repeat}\ s\ \texttt{until}\ b, \sigma''\rangle \to \sigma'}{\langle \texttt{repeat}\ s\ \texttt{until}\ b, \sigma\rangle \to \sigma'}\ (\textsc{RepF})\ \text{if}\ \mathcal{B}[\![b]\!]\sigma'' = \mathrm{ff}$$

Task. Prove that, for all σ, σ', b, s, if

$$\vdash \langle \texttt{repeat}\ s\ \texttt{until}\ b, \sigma\rangle \to \sigma'$$

then

$$\vdash \langle s; \texttt{while not}\ b\ \texttt{do}\ s\ \texttt{end}, \sigma\rangle \to \sigma'.$$