

Formal Methods and Functional Programming

Exercise Sheet 11: Big-Step Semantics

Submission deadline: May 16/17, 2023

Assignment 1 (Reversing Loop-Unrolling)

Task. Prove the following statement:

$$\forall \sigma, \sigma', b, s. (\vdash \langle \text{if } b \text{ then } s; \text{ while } b \text{ do } s \text{ end else skip end}, \sigma \rangle \rightarrow \sigma' \implies \vdash \langle \text{while } b \text{ do } s \text{ end}, \sigma \rangle \rightarrow \sigma')$$

Note: In the lectures, you have seen the proof of this result in the other direction.

Assignment 2 (Execution only Affects Free Variables)

Task. Prove that

$$\forall s, \sigma, \sigma', x. (\vdash \langle s, \sigma \rangle \rightarrow \sigma' \wedge x \notin FV(s) \implies \sigma'(x) = \sigma(x)).$$

Hint: The statement to prove is equivalent to

$$\forall T, s, \sigma, \sigma', x. ((\text{root}(T) \equiv \langle s, \sigma \rangle \rightarrow \sigma') \wedge x \notin FV(s) \implies \sigma'(x) = \sigma(x)),$$

where T ranges over derivation trees.