Formal Methods and Functional Programming

Structural Operational Semantics

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SOS of IMP

- Skip

$$\frac{}{\langle \texttt{skip}, \sigma\rangle \to_1 \sigma}\ (\textsc{Skip}_{sos})$$

- Assignment

$$\frac{}{\langle x := e, \sigma\rangle \to_1 \sigma[x \mapsto \mathcal{A}[\![e]\!]\sigma]}\ (\textsc{Ass}_{sos})$$

- Sequential Composition

$$\frac{\langle s, \sigma\rangle \to_1 \sigma'}{\langle s; s', \sigma\rangle \to_1 \langle s', \sigma'\rangle}\ (\textsc{Seq1}_{sos})$$

$$\frac{\langle s, \sigma\rangle \to_1 \langle s'', \sigma'\rangle}{\langle s; s', \sigma\rangle \to_1 \langle s''; s', \sigma'\rangle}\ (\textsc{Seq2}_{sos})$$

SOS of IMP (cont'd)

- Conditional

$$\frac{}{\langle \texttt{if } b \texttt{ then } s \texttt{ else } s' \texttt{ end}, \sigma\rangle \to_1 \langle s, \sigma\rangle}\ (\textsc{IfT}_{sos}) \quad \text{if } \mathcal{B}[\![b]\!]\sigma = tt$$

$$\frac{}{\langle \texttt{if } b \texttt{ then } s \texttt{ else } s' \texttt{ end}, \sigma\rangle \to_1 \langle s', \sigma\rangle}\ (\textsc{IfF}_{sos}) \quad \text{if } \mathcal{B}[\![b]\!]\sigma = ff$$

- Loop

$$\frac{}{\langle \texttt{while } b \texttt{ do } s \texttt{ end}, \sigma\rangle \to_1 \langle \texttt{if } b \texttt{ then } s\texttt{;while } b \texttt{ do } s \texttt{ end else skip end}, \sigma\rangle}\ (\textsc{While}_{sos})$$