





M. Besta, T. Hoefler

WITH N. BLACH, A. KUBICEK, R. GERSTENBERGER, AND MANY OTHERS

Graph of Thoughts: Solving Elaborate Problems with Large Language Models





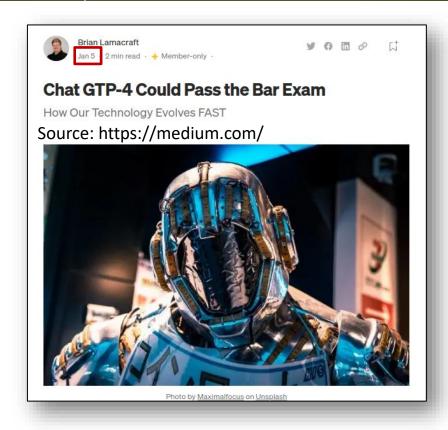




















Chat GTP-4 Could Pass the Bar Exam

How Our Technology Evolves FAST

Source: https://medium.com/

AI chatbot's MBA exam pass poses test for business schools

ChatGPT earned a solid grade and outperformed some humans on a Wharton course



A professor at the University of Pennsylvania's Wharton School said ChatGPT earned a 'B to B-' on his operations management test © Bloomberg

Source: https://www.ft.com/

Andrew Jack in New York JANUARY 21 2023

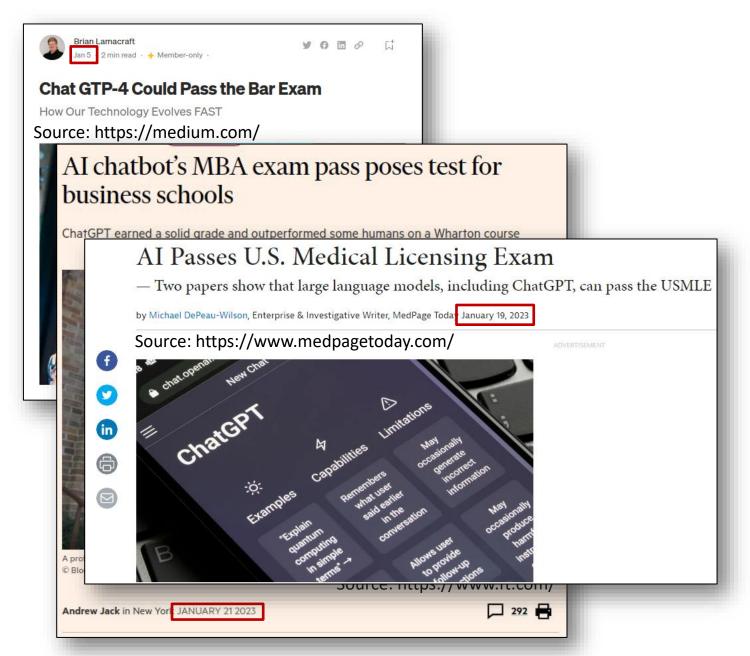








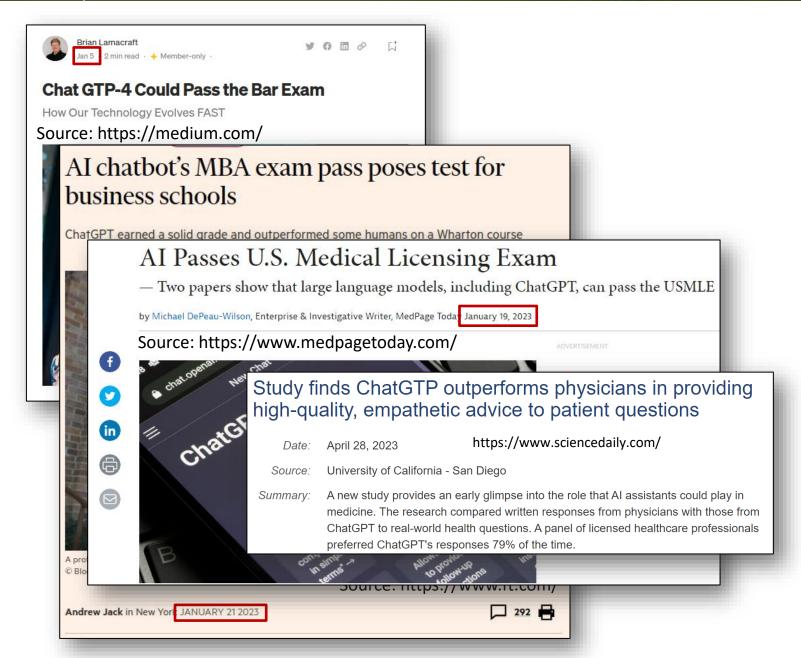








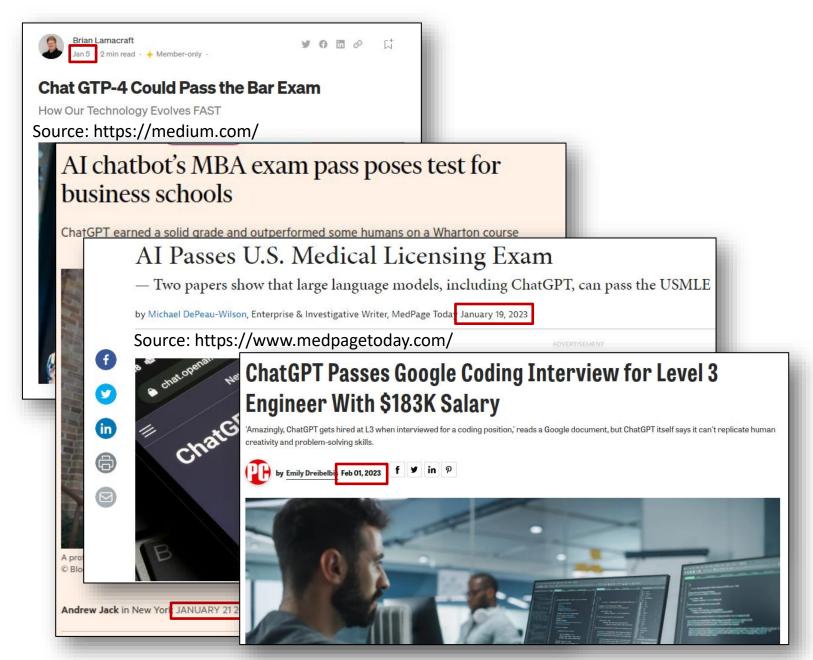








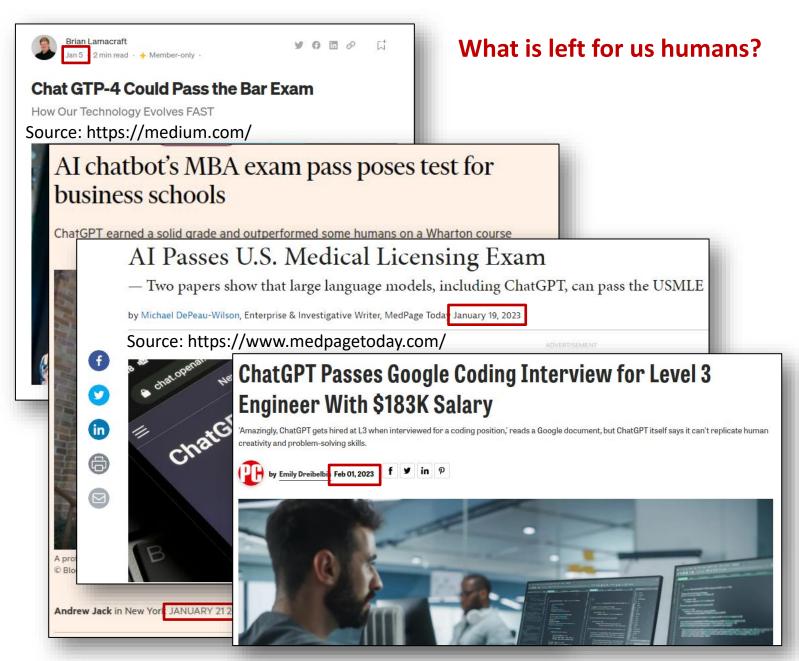








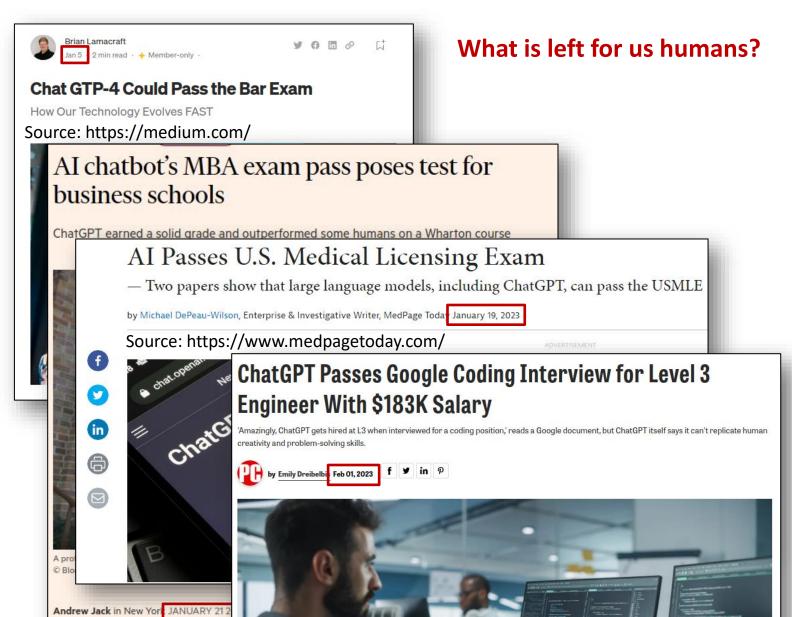












SCIENTIFIC AMERICAN

BEHAVIOR

What Is the Special Something That Makes the Human Mind Unique?

The capacity to engage in shared tasks such as hunting large game and building cities may be what separated modern humans from our primate cousins

By Gary Stix on October 1, 2016

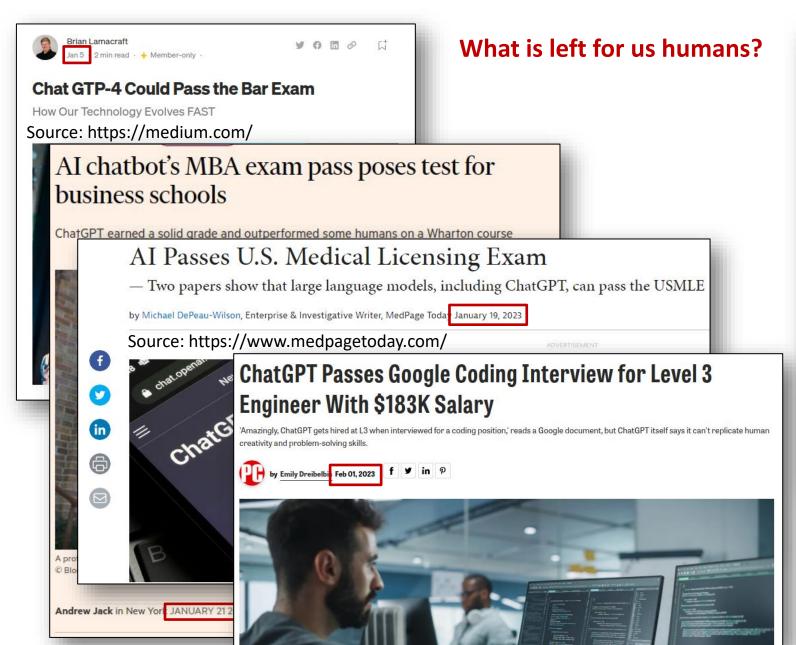
IN BRIEF

- Humans—it was once thought—differed from other animals by their use of tools
 and their overall superiority in a range of cognitive abilities. Close observation of
 the behaviors of chimpanzees and other great apes has proved these ideas to be
 wrong.
- Chimpanzees score as highly as young children on tests of general reasoning abilities but lack many of the social skills that come naturally to their human cousins. Unlike humans, chimps do not collaborate in the large groups needed to build complex societies.
- Comparison of human and chimp psychology reveals that an essential source of the differences in humans may be the evolution of the ability to intuit what another person is thinking so that both can work toward a shared goal.







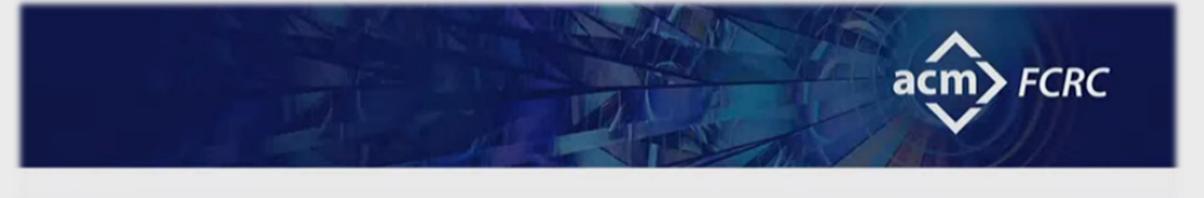












2018 ACM A.M. Turing Lecture

June 23, 2019 5:15pm MST







Geoffrey Hinton

Yann LeCun







"Really the deciding factor [for the Al revolution] was the increase in compute power" (26:50)



2018 ACM A.M. Turing Lecture

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Yann LeCun







"Really the deciding factor [for the AI revolution] was the increase in compute power" (26:50)

"I think a lot of the credit for deep learning goes to [... others ...] and the people who made the computers go fast." (27:00)

2018 ACM A.M. Turing Lecture

June 23, 2019 5:15pm MST







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Yann LeCun







How do we "Make Computers go Fast"?







How do we "Make Computers go Fast"?

2021 Turing award – Jack Dongarra The Take Away

- HPC Hardware is Constantly Changing
 - Scalar
 - Vector
 - Distributed
 - Accelerated
 - · Mixed precision
- Three computer revolutions
 - High performance computing
 - Deep learning
 - Edge & AI
- Algorithm / Software advances follows hardware
 - · And there is "plenty of room at the top"

"There's plenty of room at the Top: What will drive computer performance after Moore's law?"

Leiserson et al., Science 368, 1079 (2020) 5 June 2020

The Top

Software Algorithms Hardware architecture

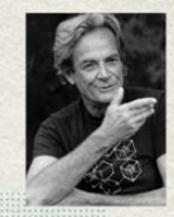
Software Algorithms Hardware streamlining engineering Removing software to hardware features

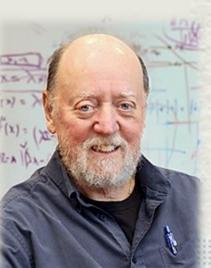
New machine models Comain specialization Comain specialization

The Bottom

r example, semiconductor technology

Leiserson et al., Science 368, 1079 (2020) 5 June 2







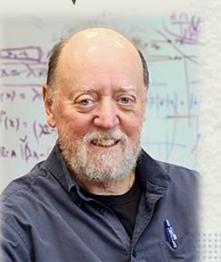




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Supercomputers are very (>70%) efficient at dense linear algebra!

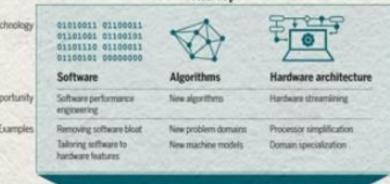


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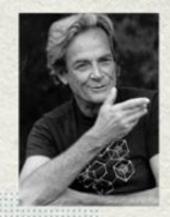
The Top



The Bottom

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source: ft.com





FINANCIAL TIMES

Artificial intelligence

+ Add to myFT

The billion-dollar bet to reach human-level AI

OpenAI believes that huge computing power is key driver

In the race to build a machine with human-level intelligence, it seems, size really matters.

"We think the most benefits will go to whoever has the biggest computer," said Greg Brockman, chairman and chief technology officer of OpenAI.

The San Francisco-based AI research group, set up four years ago by tech industry luminaries including Elon Musk, Peter Thiel and Reid Hoffman, has just thrown down a challenge to the rest of the AI world.















Facebook parent Meta creates powerful AI supercomputer

Facebook's parent company Meta says it has created what it believes is among the fastest artificial intelligence supercomputers running today

By The Associated Press January 24, 2022, 10:33 PM

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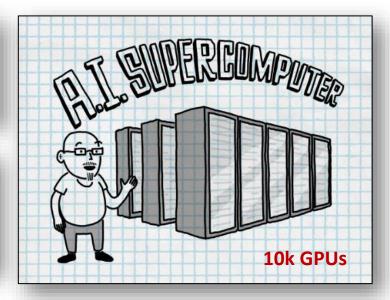
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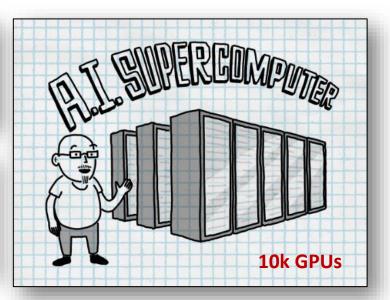
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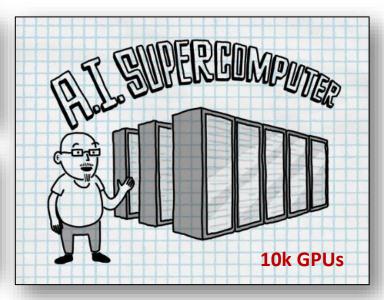
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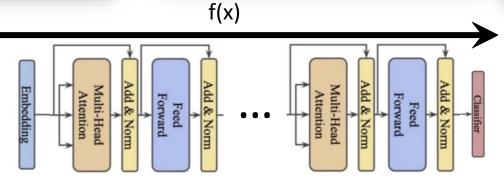
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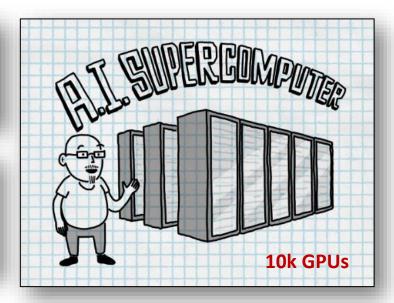
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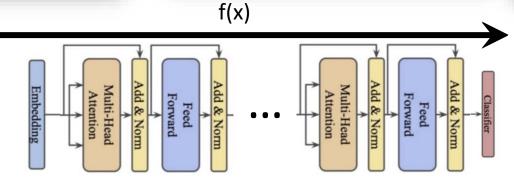
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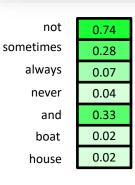






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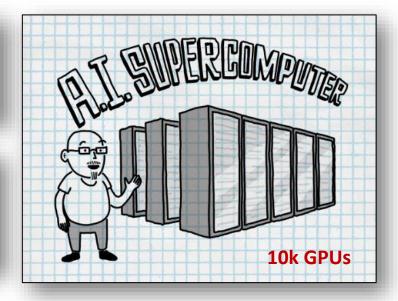
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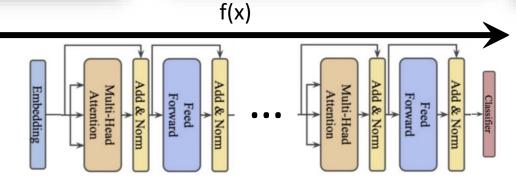
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sometimes 0.28
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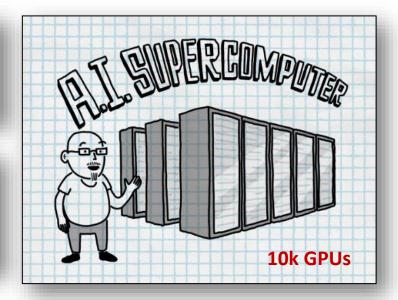
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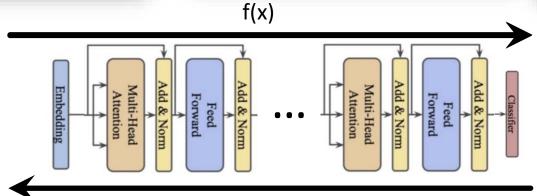
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layer-wise weight update

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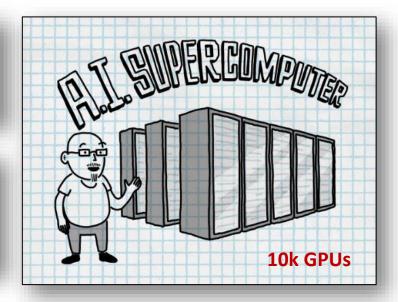
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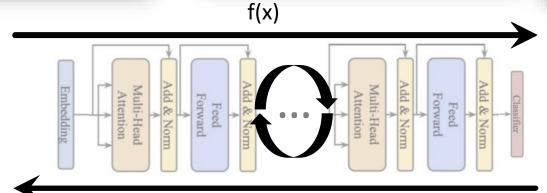
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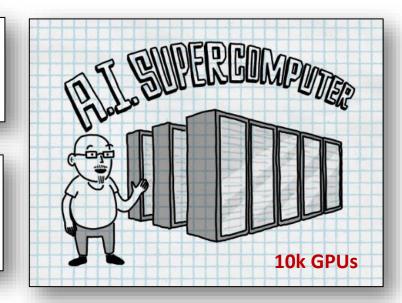
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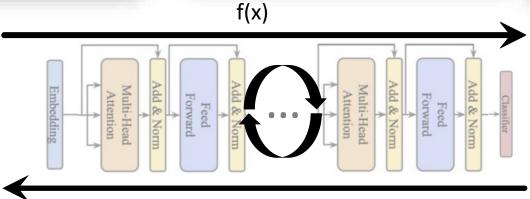
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■ ImageNet (22k): A few TB

Soon: the whole internet!







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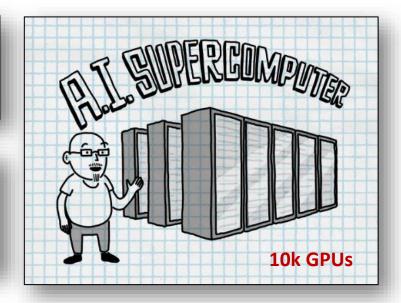
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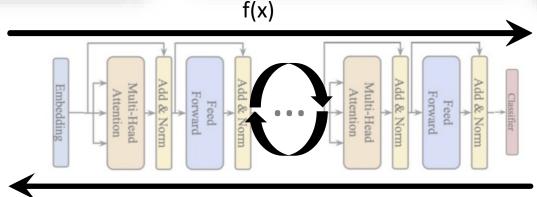
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reddit



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GPT-3: 96 (complex) layers
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2048-token "sentences"

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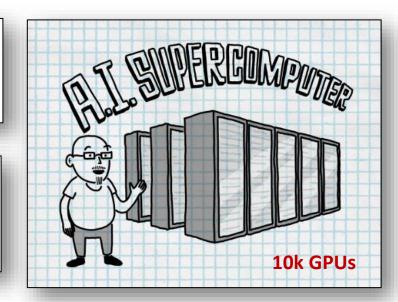
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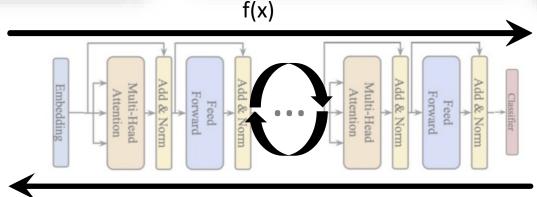
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- GPT-3: 30-50k dictionaries
- takes weeks to train







Large-Scale AI is the Future



Large-Scale AI is the Future

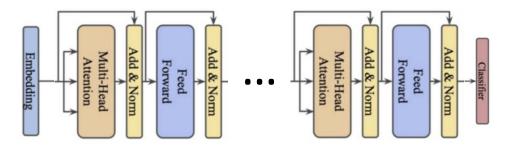
We need a Principled Approach to it







Three Systems Dimensions in Large-scale Super-learning ...



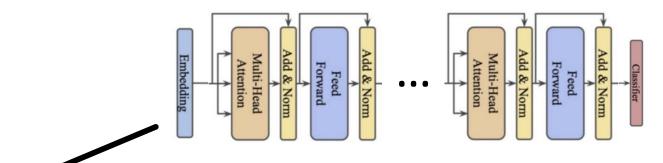








Three Systems Dimensions in Large-scale Super-learning ...





High-Performance I/O

- Quickly growing data volumes
 - Scientific computing!
- Use the specifics of machine learning workloads
 - E.g., intelligent prefetching

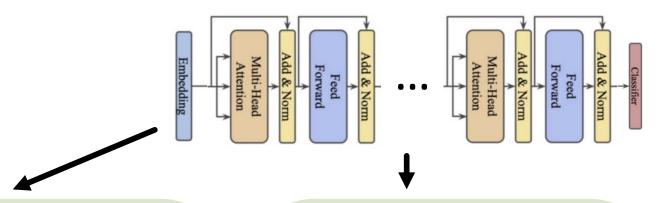
CLAIRVOYANT PREFETCHING FOR DISTRIBUTED MACHINE LEARNING I/O Roman Böhringer¹ Nikoli Dryden¹ Tal Ben-Nun¹ Torsten Hoefler¹ ABSTRACT I/O is emerging as a major bottleneck for machine learning training, especially in distributed environments such as clouds and supercomputers. Optimal data ingestion pipelines differ between systems, and increasing efficiency requires a delicate balance between access to local storage, external filesystems, and remote workers; yet existing frameworks fail to efficiently utilize such resources. We observe that, given the seed generating the random access pattern for training with SGD, we have clairvoyance and can exactly predict when a given sample will be accessed. We combine this with a theoretical analysis of access patterns in training and performance models produce a novel machine learning I/O middleware, HDMLP, to tackle the I/O bottleneck. HDMLP provides an easy-to-use, flexible, and scalable solution that delivers better performance than state-of-the-art approaches while requiring very few changes to existing codebases and supporting a broad range of environments.

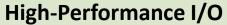






Three Systems Dimensions in Large-scale Super-learning ...





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High-Performance Compute

- Deep learning is HPC
 - Data movement!
- Quantization, Sparsification
 - Drives modern accelerators!

Data Movement Is All You Need: A Case Study on **Optimizing Transformers**

Andrei Ivanov*, Nikoli Dryden*, Tal Ben-Nun, Shigang Li, Torsten Hoefler ETH Zürich $\begin{tabular}{ll} {\tt firstname.lastname@inf.ethz.ch} \\ {\tt * Equal contribution} \\ \end{tabular}$

modeling and sequence learning tasks, and are one of the most

ery compute-intensive task, often taking days or weeks, and Despite this, existing implementations do not efficiently utilize GPUs. We find that data movement is the key bottleneck when te performance, training has now become memory-bound Further, existing frameworks use suboptimal data layouts, Using to 22.91% and overall achieve a 1.30× performance imp ment over state-of-the-art frameworks when training BERT.

improving transformer performance has been in the focus of

Significant attention has been given to optimizing transform structured sparsity [33], learned sparsity [34]-[36], and other algorithmic techniques [19], [37] improve the performance of transformers. Major hardware efforts, such as Tensor Cores and TPUs [38] have accelerated tensor operations like matrix matrix multiplication (MMM), a core transformer operation Despite this, existing implementations do not efficiently utilize GPUs. Even optimized implementations such as Megatron [18] report achieving only 30% of peak GPU flop/s.

We find that the key bottleneck when training transform

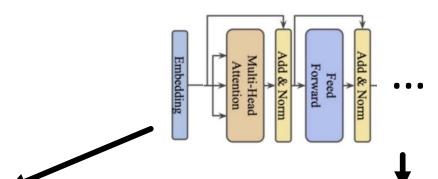


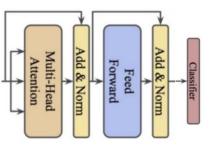






Three Systems Dimensions in Large-scale Super-learning ...







High-Performance I/O

- Quickly growing data volumes
 - Scientific computing!
- Use the specifics of machine learning workloads
 - E.g., intelligent prefetching

CLAIRVOYANT PREFETCHING FOR DISTRIBUTED MACHINE LEARNING I/O

Roman Böhringer 1 Nikoli Dryden 1 Tal Ben-Nun 1 Torsten Hoefler 1

ABSTRACT

I/O is emerging as a major bottleneck for machine learning training, especially in distributed environments such as clouds and supercomputers. Optimal data ingestion pipelines differ between systems, and increasing efficiency requires a delicate balance between access to local storage, external flesystems, and remote workers; yet existing frameworks fail to efficiently utilize such resources. We observe that, given the seed generating the random access pattern for training with SGD, we have clairvoyance and can exactly predict when a given sample will be accessed. We combine this with a theoretical analysis of access patterns in training and performance modeling to produce a novel machine learning I/O middleware, HDMLP provides an easy-to-use, flexible, and scalable solution that delivers better performance than state-of-the-art approaches while requiring very few changes to existing codebases and supporting a broad range of environments.

High-Performance Compute

- Deep learning is HPC
 - Data movement!
- Quantization, Sparsification
 - Drives modern accelerators!

Data Movement Is All You Need: A Case Study on Optimizing Transformers

Andrei Ivanov*, Nikoli Dryden*, Tal Ben-Nun, Shigang Li, Torsten Hoefler ETH Zürich firstname.lastname@inf.ethz.ch

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* Equal contribution

Abstract—Iransformers have become widely used for inaguage modeling and sequence learning tasks, and are one of the most very compute-intensive task, often taking days or weeks, and significant attention has been given to optimizing transformers. Despite this, existing implementations do not efficiently utilize GPUs. We find that data movement is the key bottleneck when training. Due to Amdahl's Law and massive improvements in compute performance, training has now become memory-bound. Further, existing frameworks the subject of the computer of the comp

ost improving transformer performance has been in the focus of a numerous research and industrial groups.

Significant attention has been given to optimizing transformers: local and fixed-window attention [28]–32], more general structured sparsity [133], tearned sparsity [134], 36], and other algorithmic techniques [19], [37] improve the performance of transformers. Major hardware efforts, such as Tensor Cores and TPUs [38] have accelerated tensor operations like matrix-matrix multiplication (MMM), a core transformer operation. Despite this, existing implementations do not efficiently utilize GPUs. Even optimized implementations such as Megatron [18] report achieving only 30% of peak GPU flop/s.

We find that the key bottleneck when training transform-

High-Performance Communication

- Use larger clusters (10k+ GPUs)
- Model parallelism
 - Complex pipeline schemes
- Optimized networks

Distribution and Parallelism

Data | Pipeline | Operator







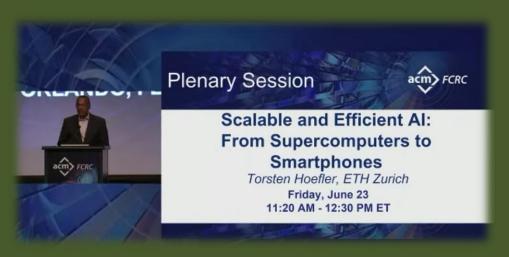






This is the past – LLMs work today!

(you can find that full talk on youtube)



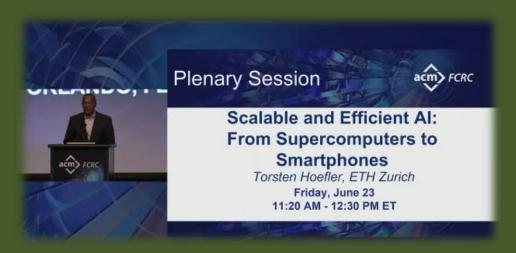






This is the past – LLMs work today!

(you can find that full talk on youtube)



We know training – but do we understand using (prompting)?













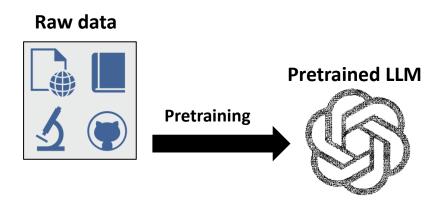
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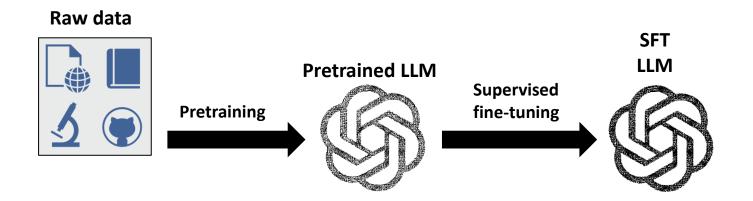








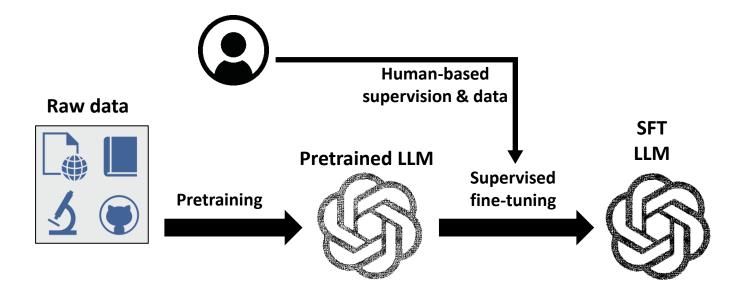








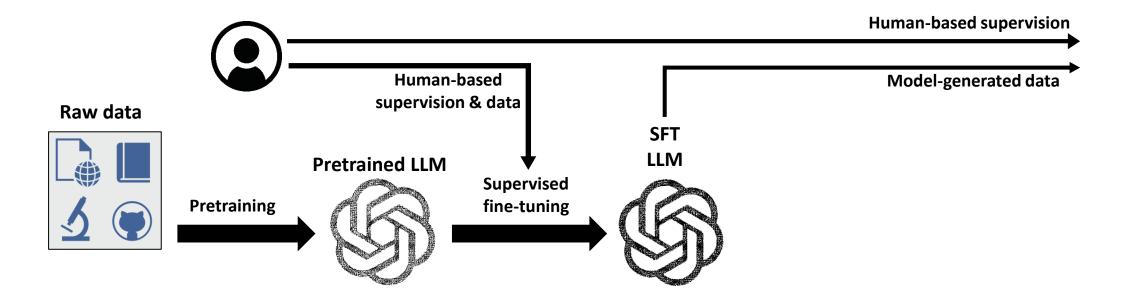








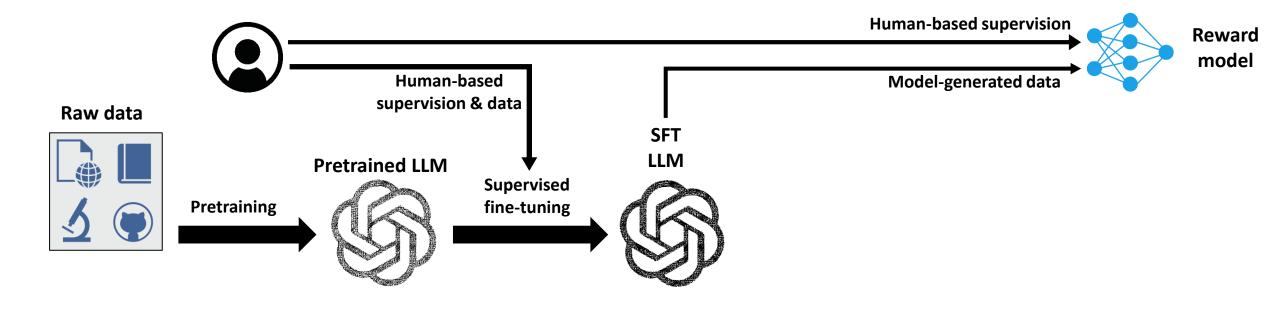








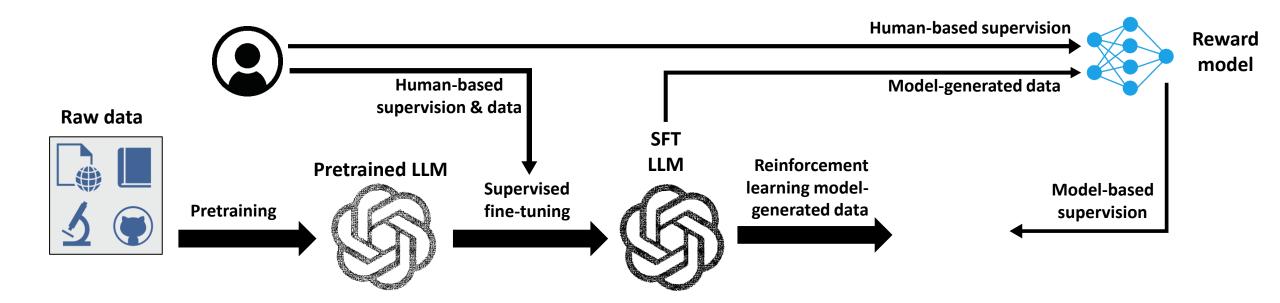








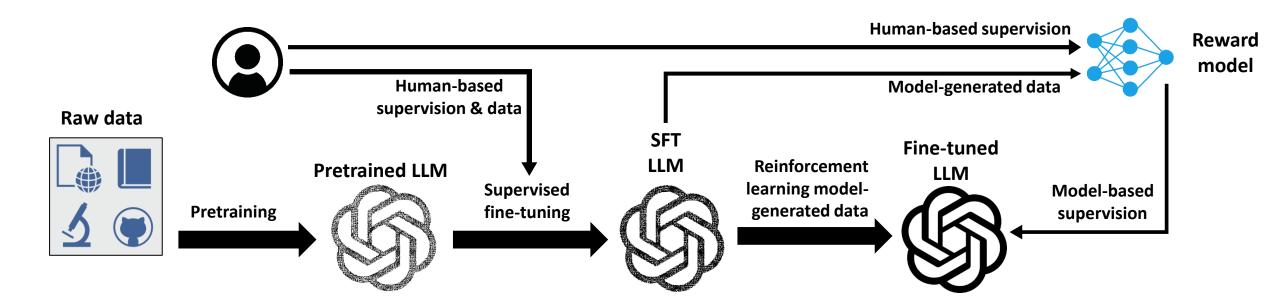








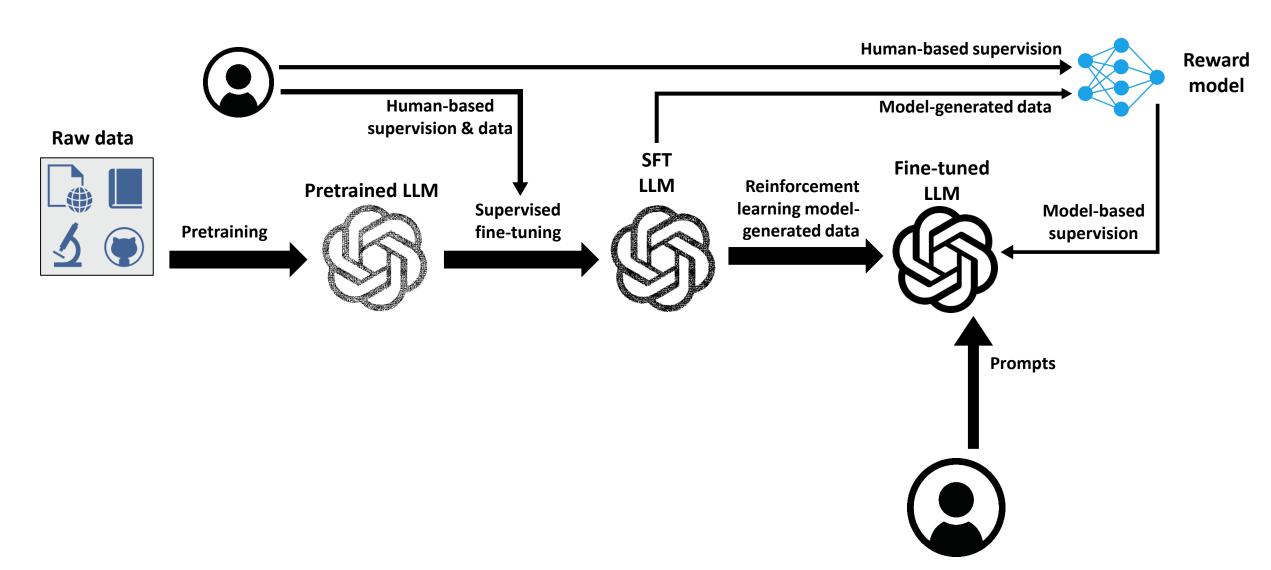








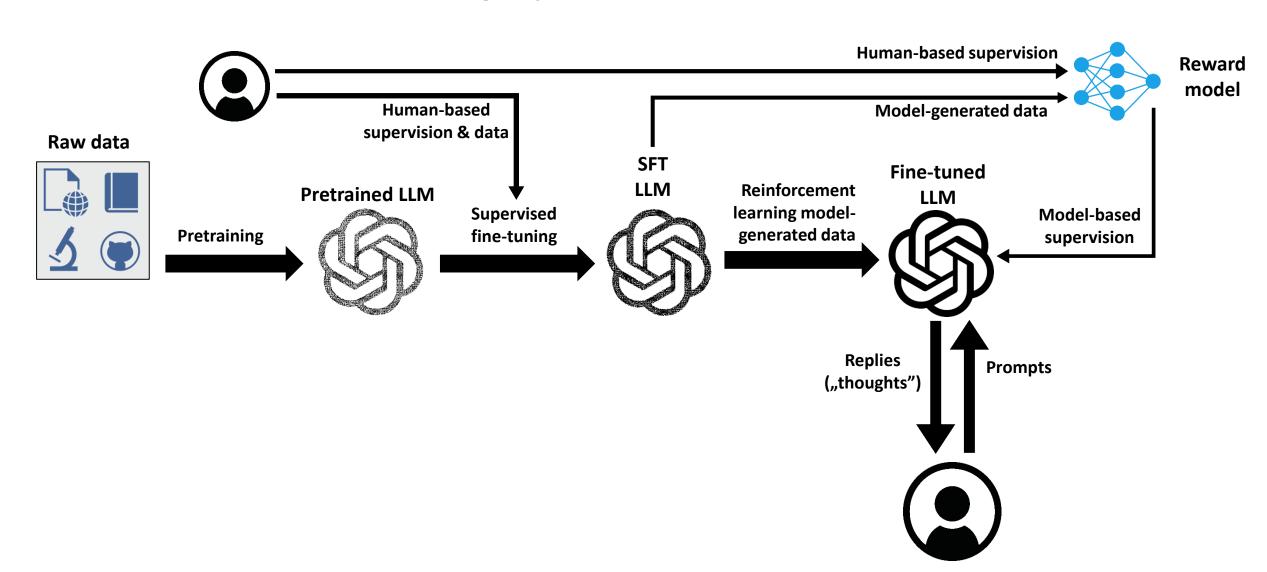








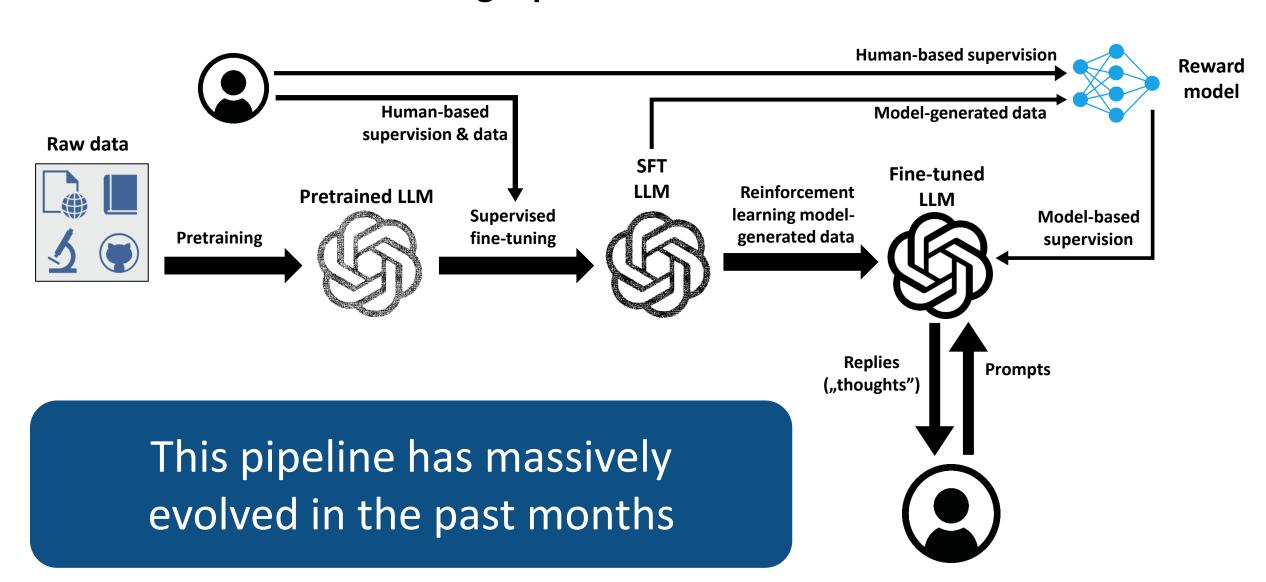








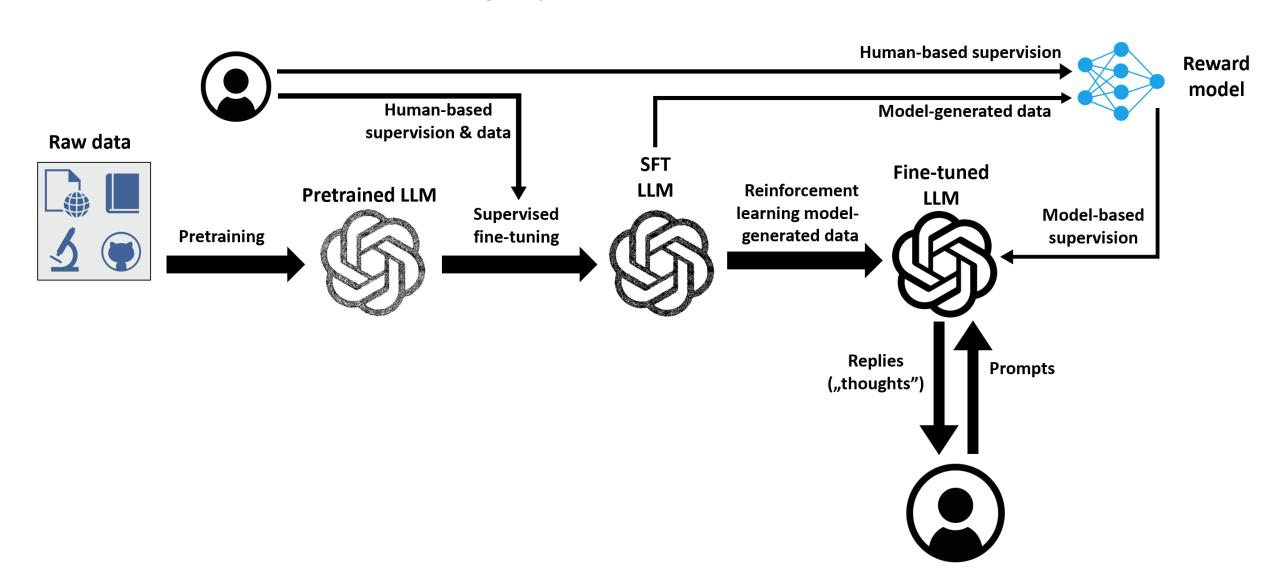








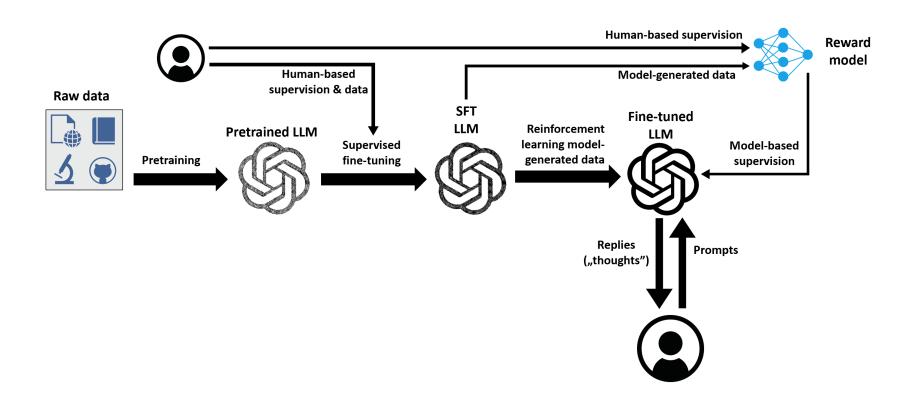








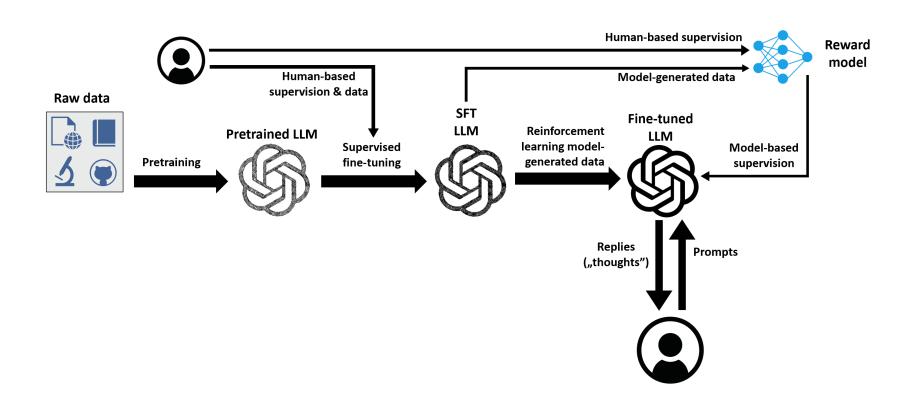








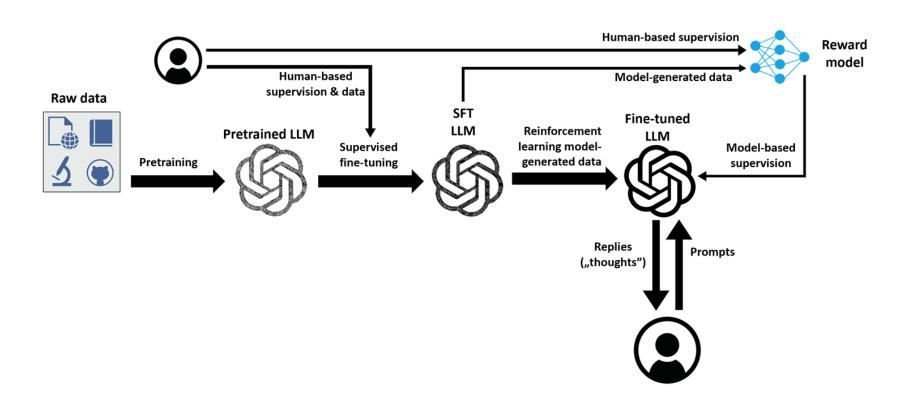








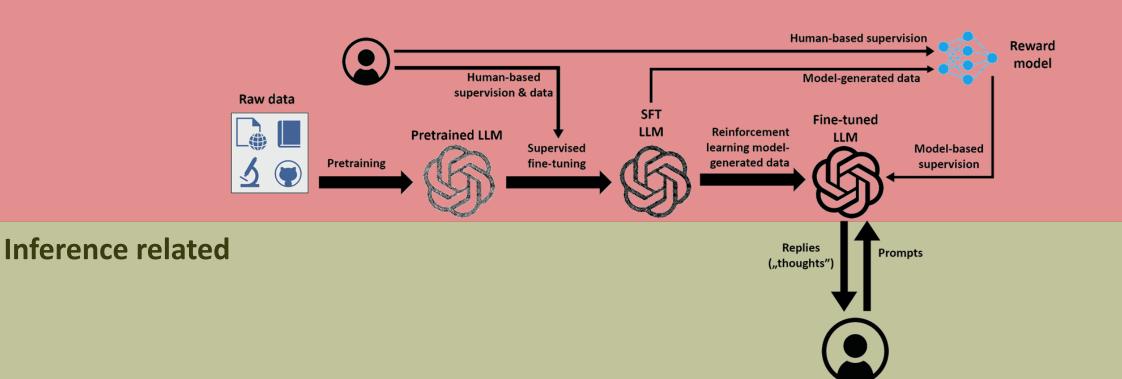








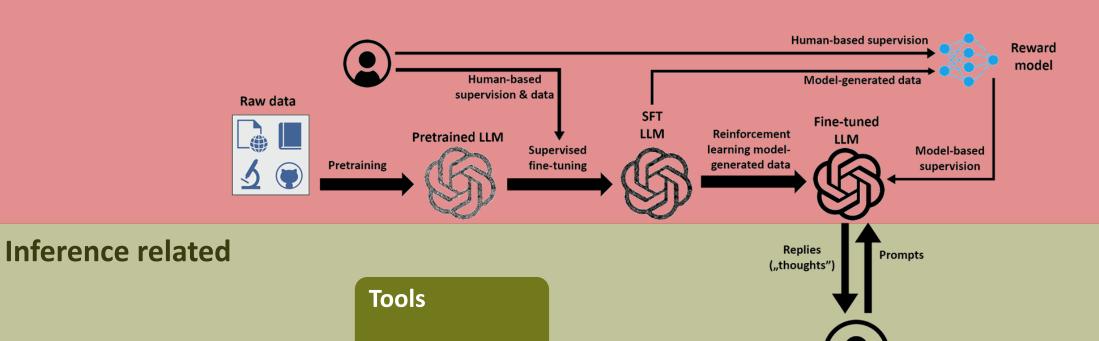








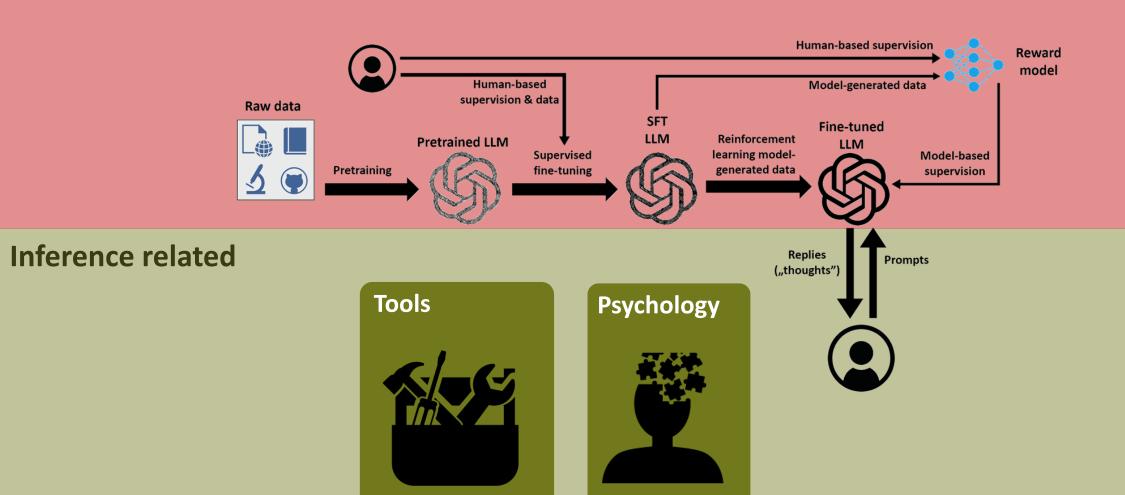








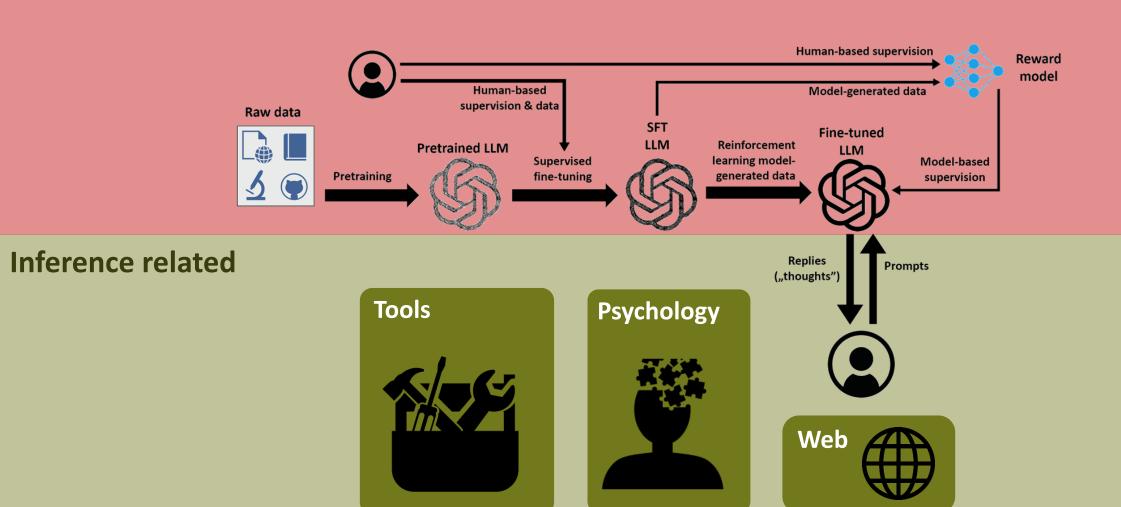








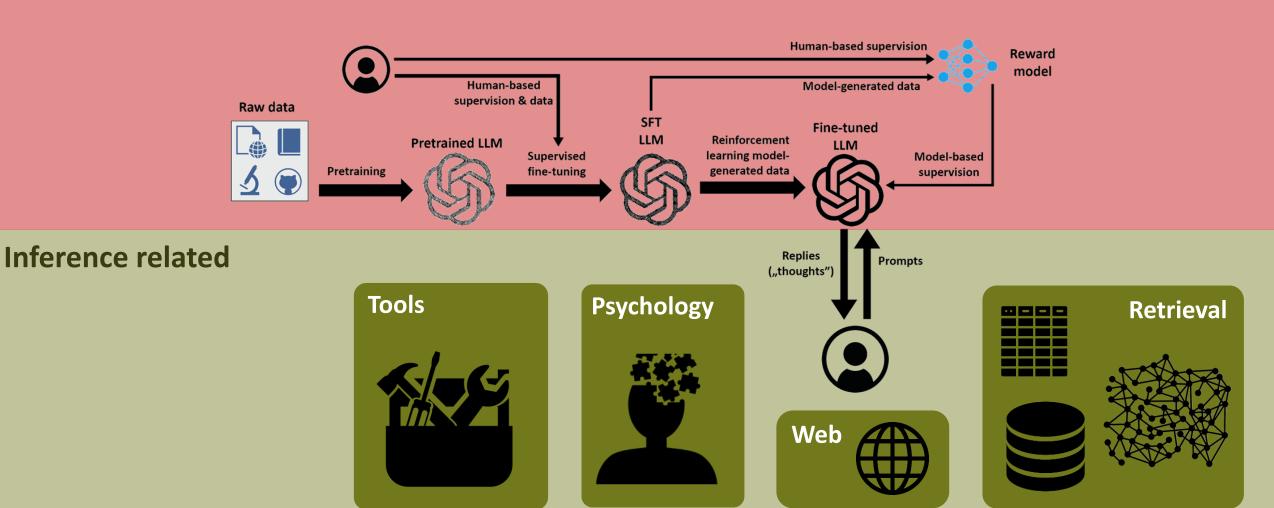








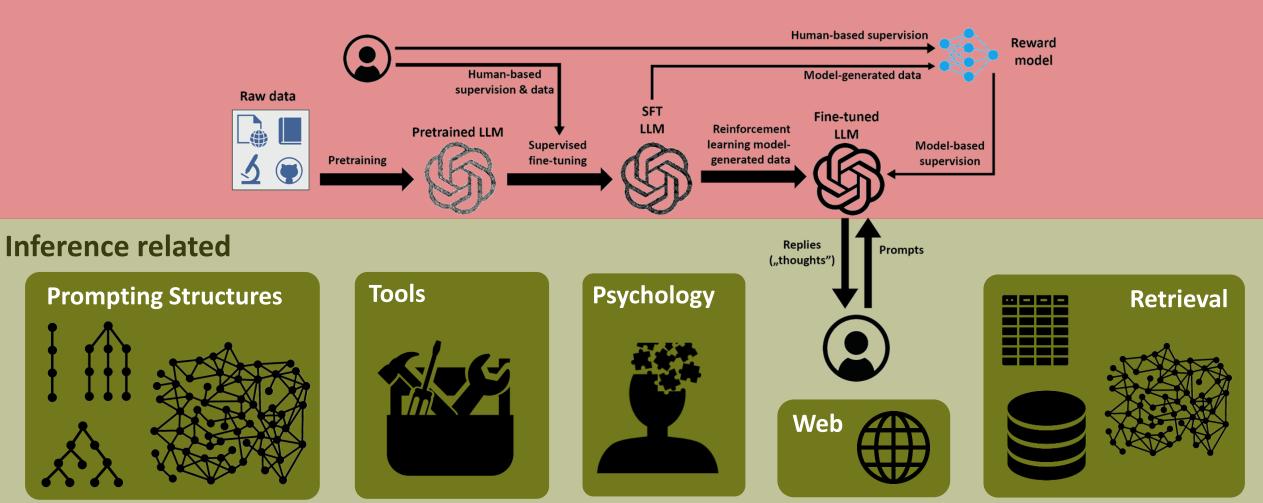








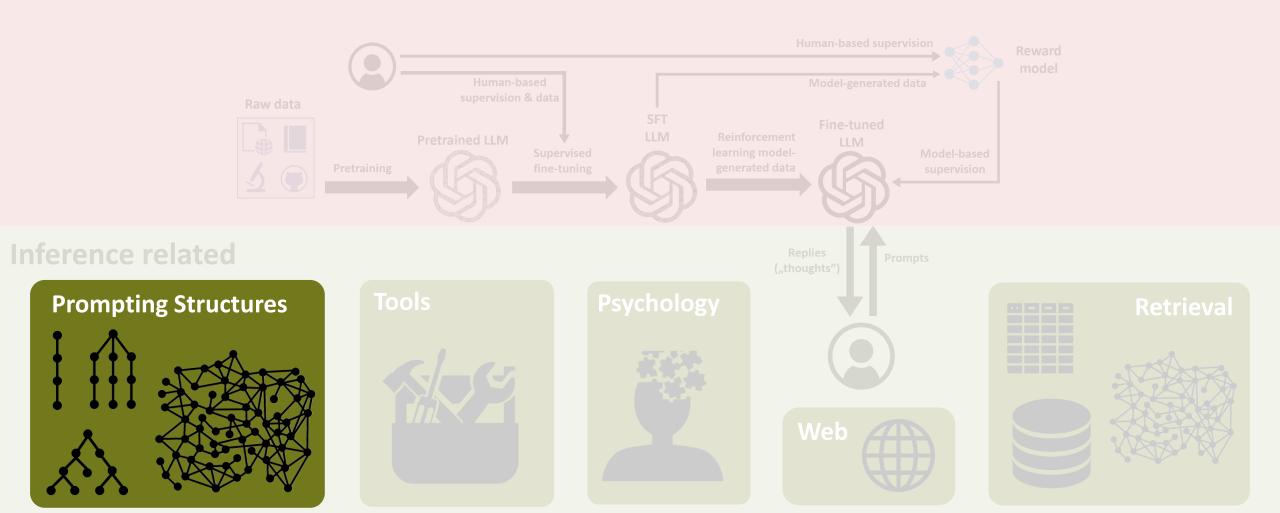














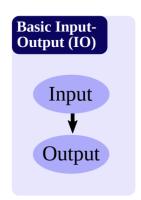


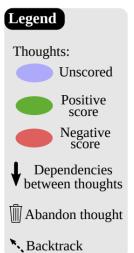








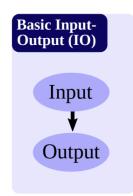


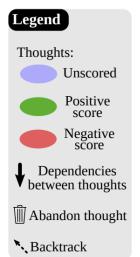












Standard Prompting

Model Input

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how many apples do they have?

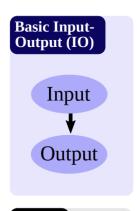
Model Output

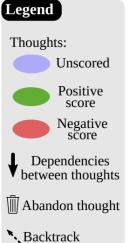












In-context examples

Standard Prompting

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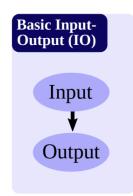
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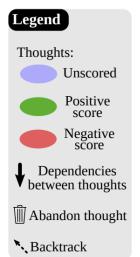












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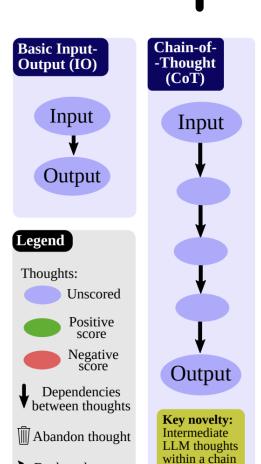








[Wei et al., Jan'22] ■



* Backtrack

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Model Output









[Wei et al., Jan'22]

Key novelty: Intermediate

LLM thoughts within a chain

Chain-of-**Basic Input-**-Thought Output (IO) (CoT) Input Input Output Legend Thoughts: Unscored Positive score Negative score Output Dependencies between thoughts

Abandon thought

* Backtrack

Standard Prompting

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A: The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how many apples do they have?

Model Output

A: The answer is 27.



Chain-of-Thought Prompting

Model Input

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 = 11. The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how many apples do they have?

Model Output

A: The cafeteria had 23 apples originally. They used 20 to make lunch. So they had 23 - 20 = 3. They bought 6 more apples, so they have 3 + 6 = 9. The answer is 9. <





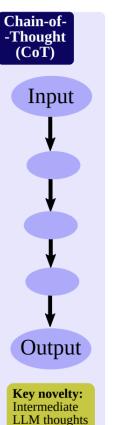


[Wei et al., Jan'22]

Basic Input-Output (IO) Input Output Legend Thoughts: Unscored Positive score Negative score Dependencies between thoughts

Abandon thought

* Backtrack



within a chain

Chain-of-Thought Prompting

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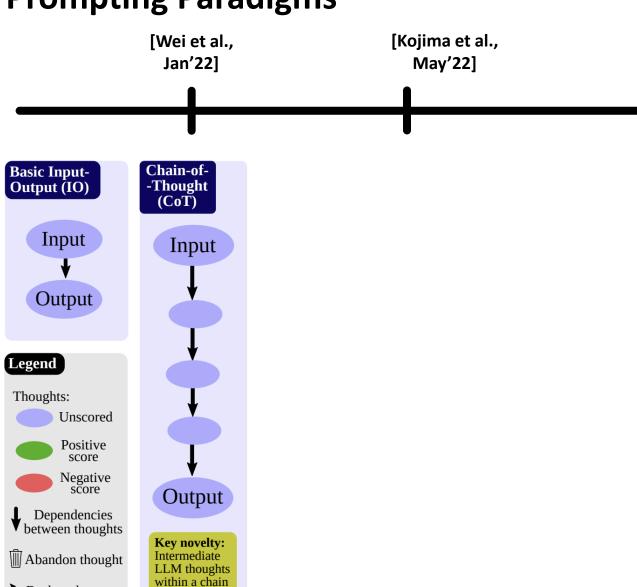


* Backtrack





Prompting Paradigms



Chain-of-Thought Prompting

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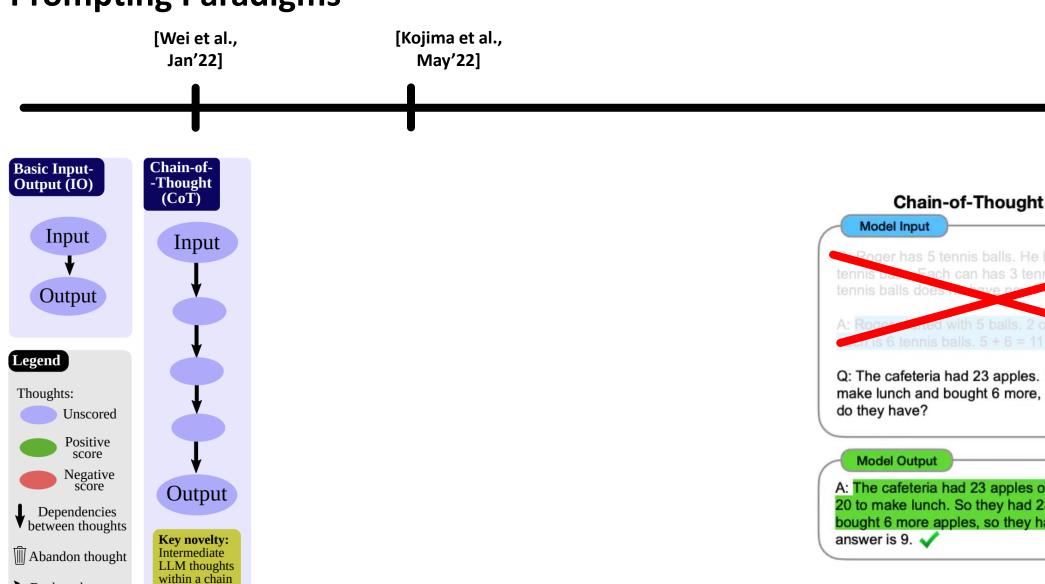


* Backtrack





Prompting Paradigms



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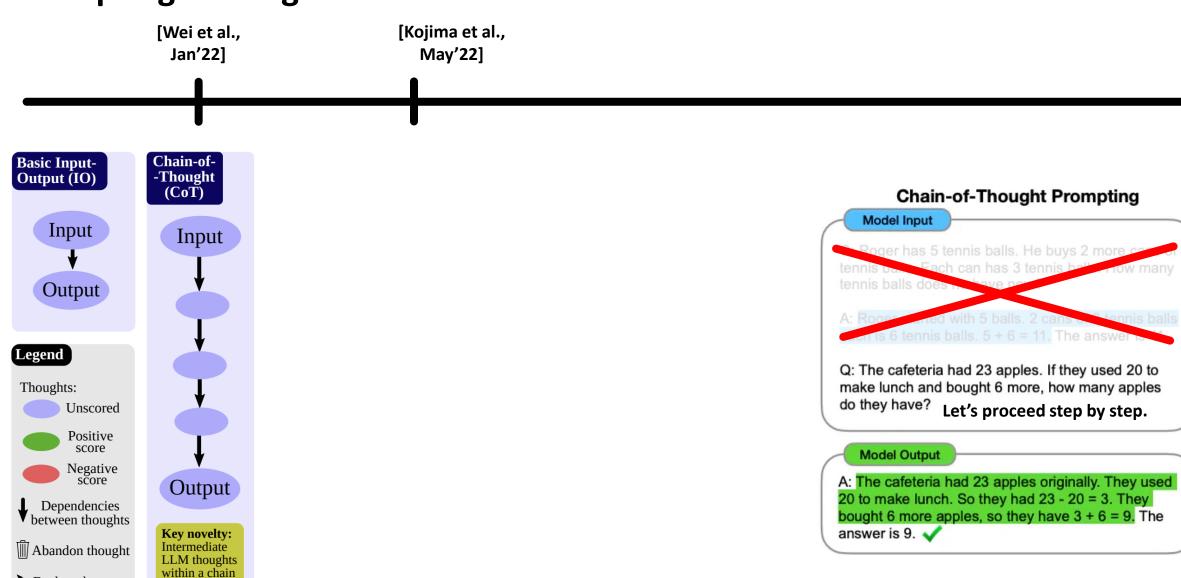


* Backtrack





Prompting Paradigms

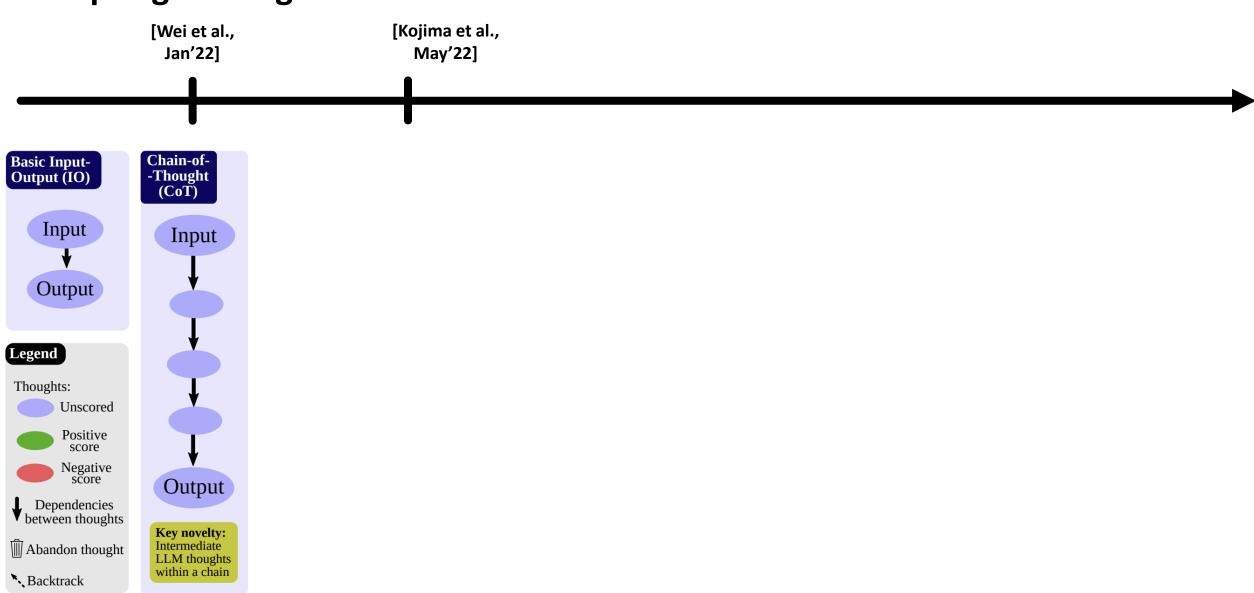








Prompting Paradigms



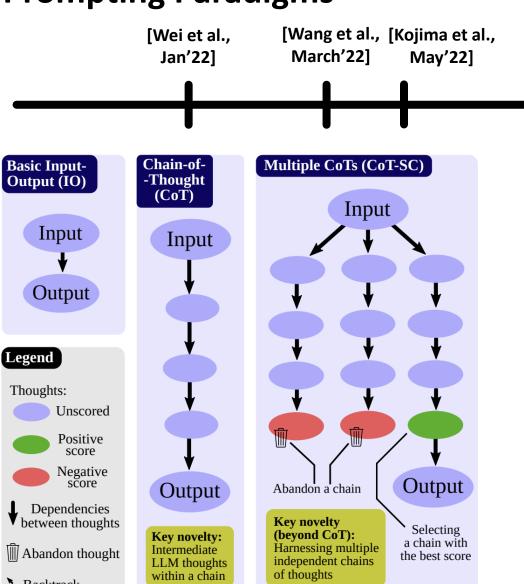


* Backtrack





Prompting Paradigms

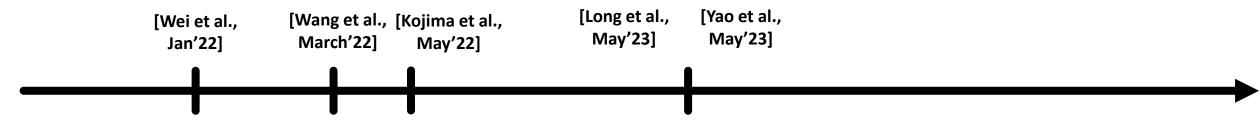


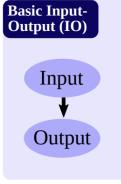






Prompting Paradigms

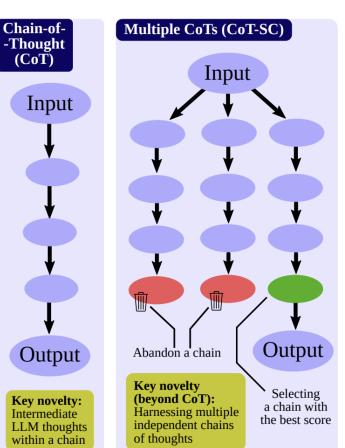


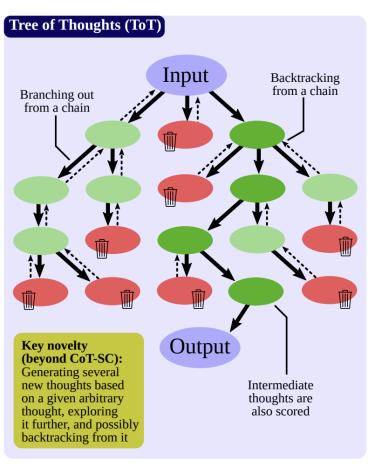




Abandon thought

* Backtrack









The Next Step – Graphical Reasoning





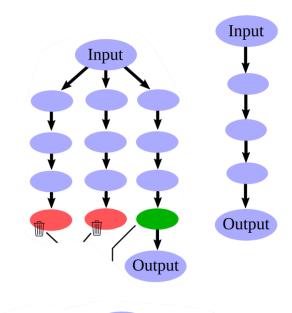
The Next Step – Graphical Reasoning

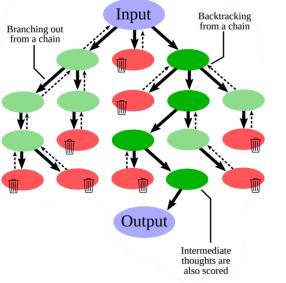
Inspired by human thoughts.









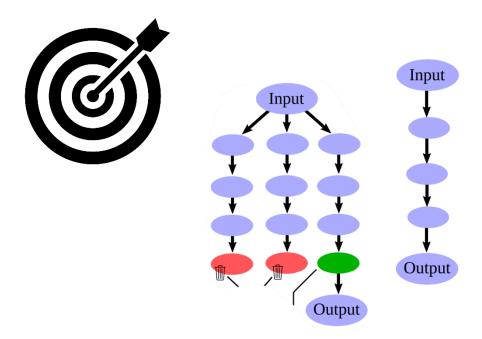


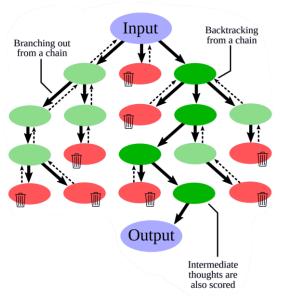








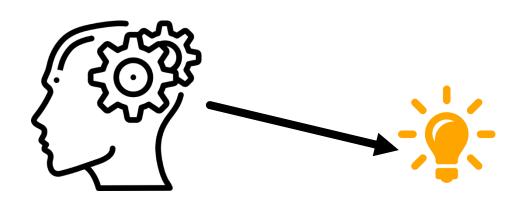


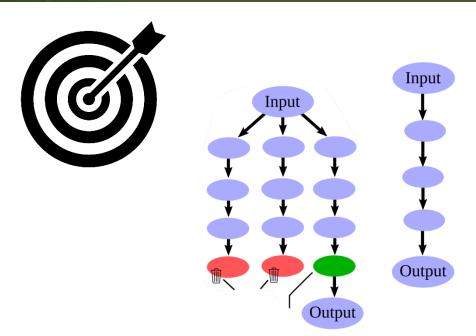


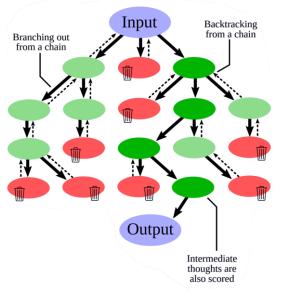








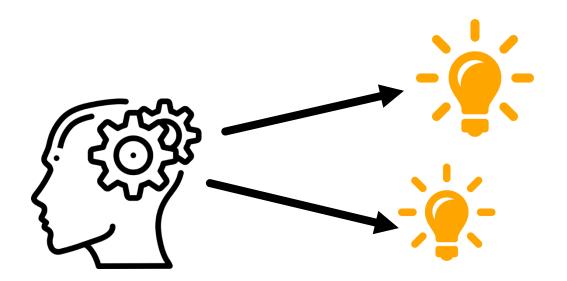


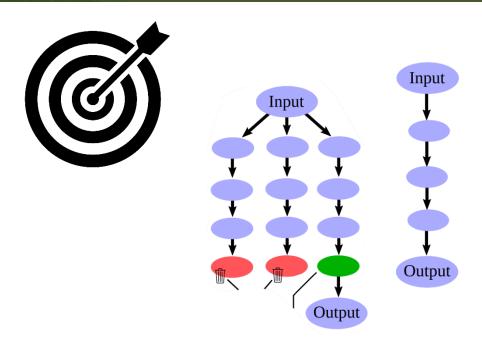


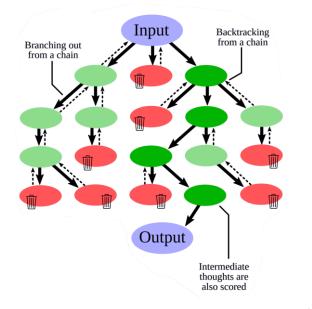










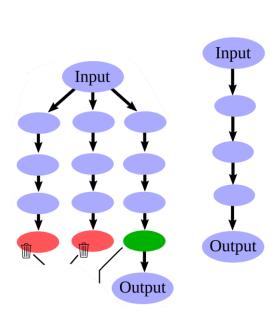


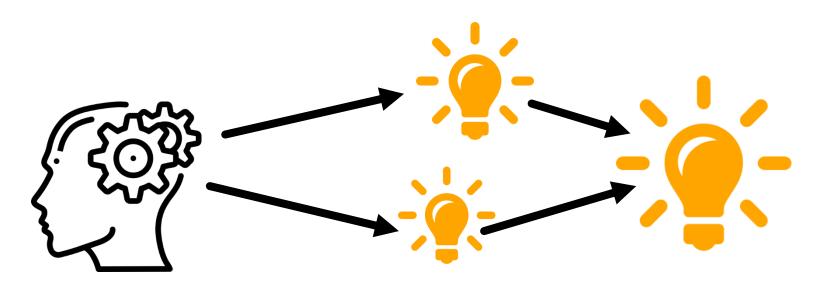


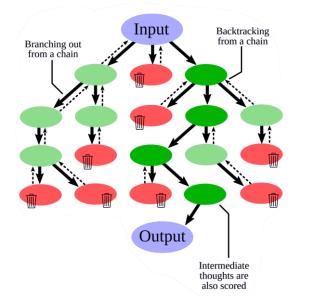










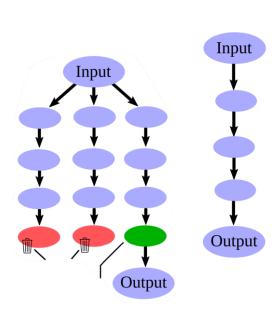


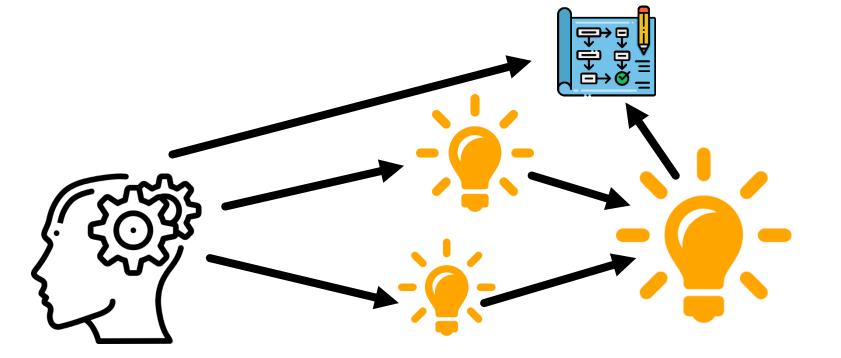


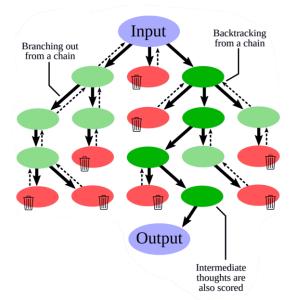








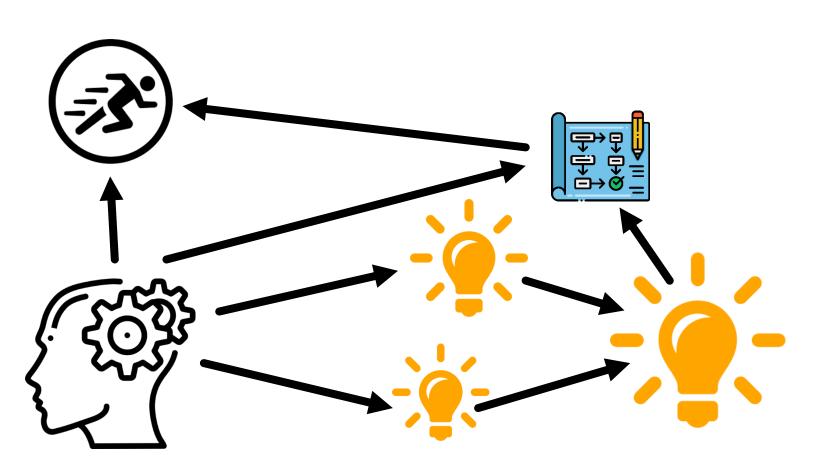


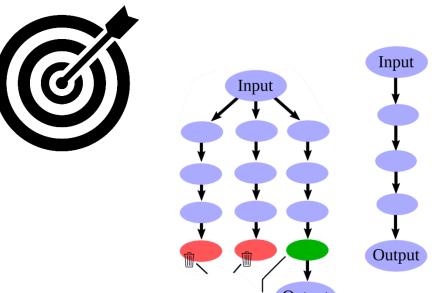


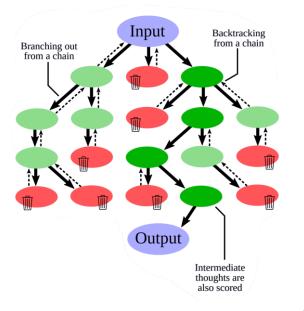








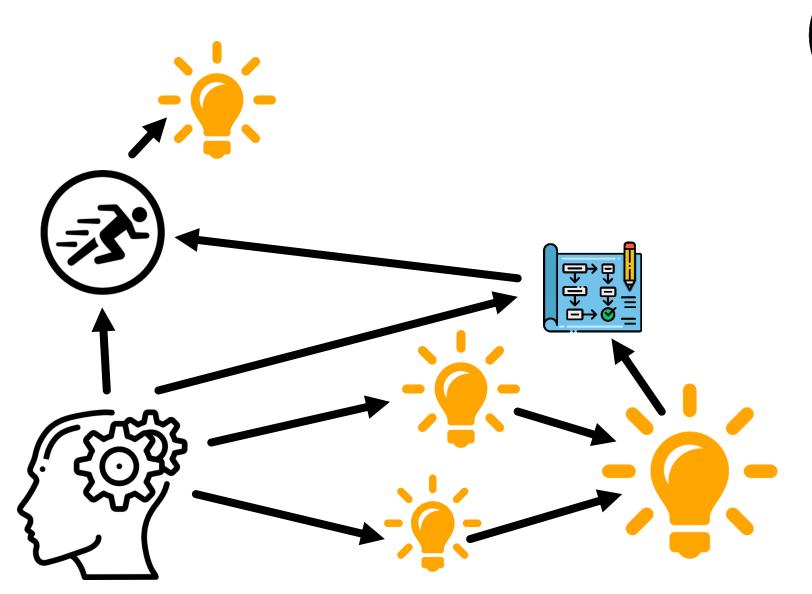


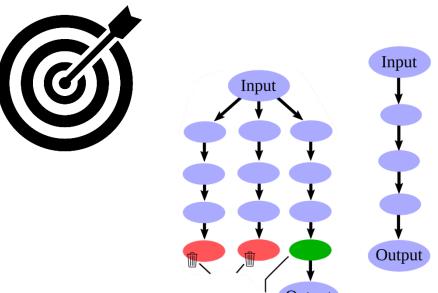


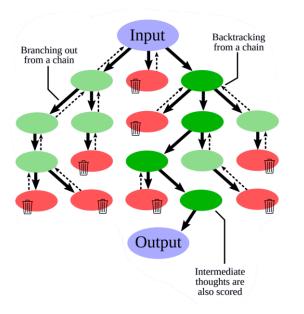








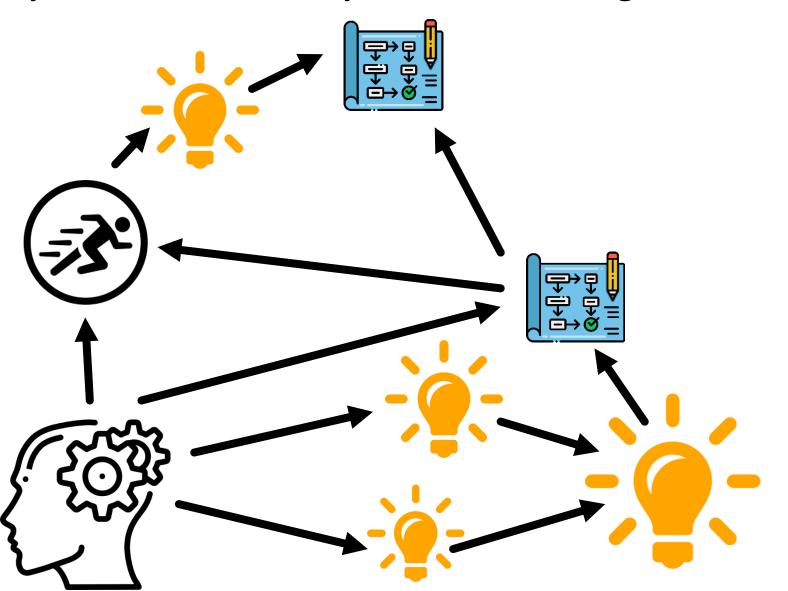


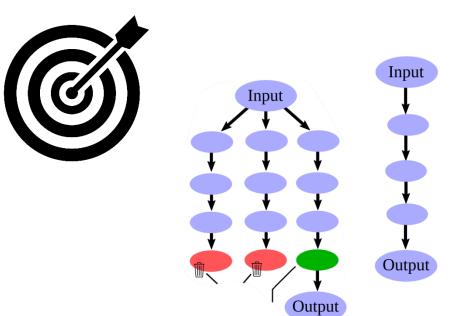


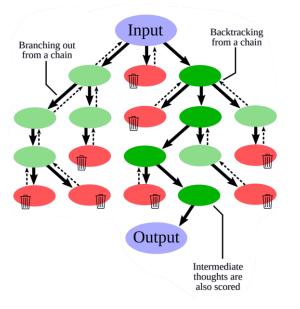








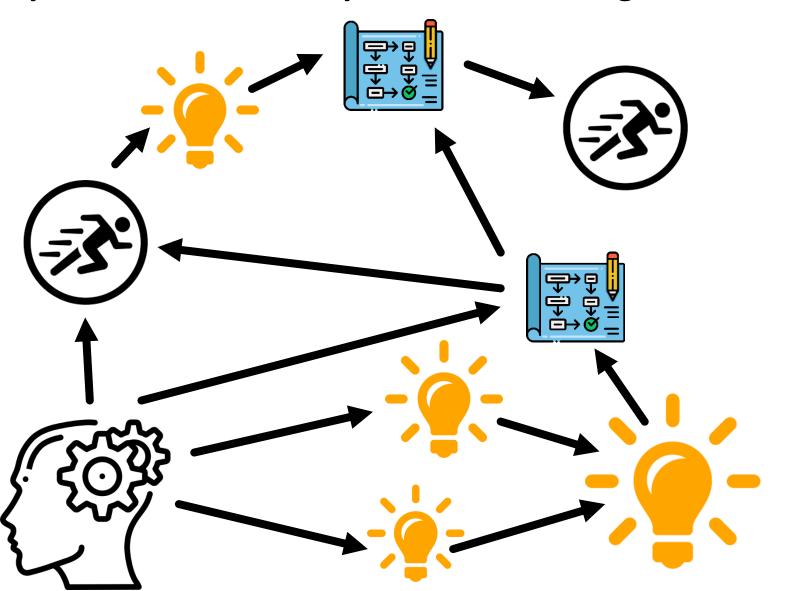


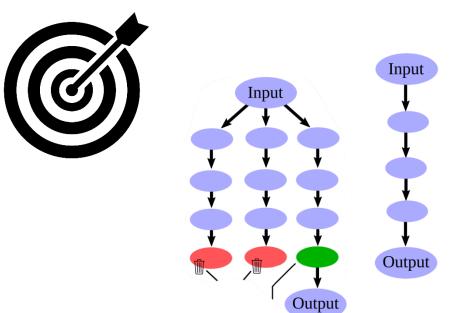


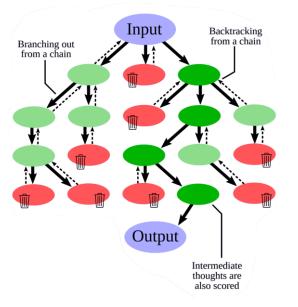








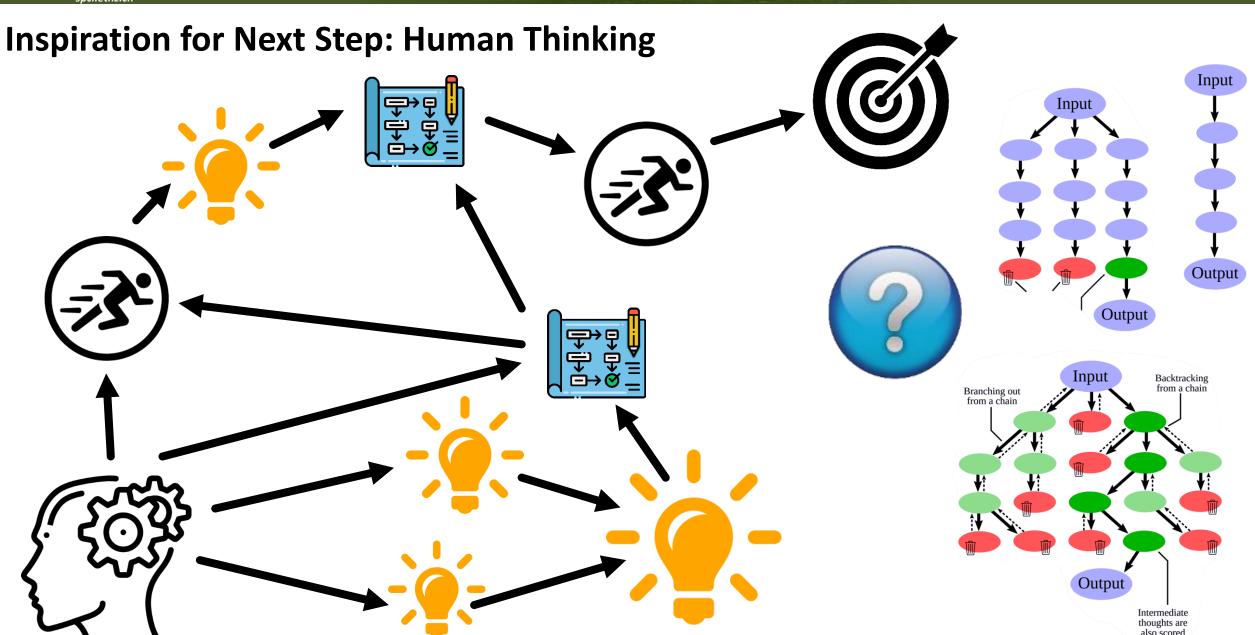








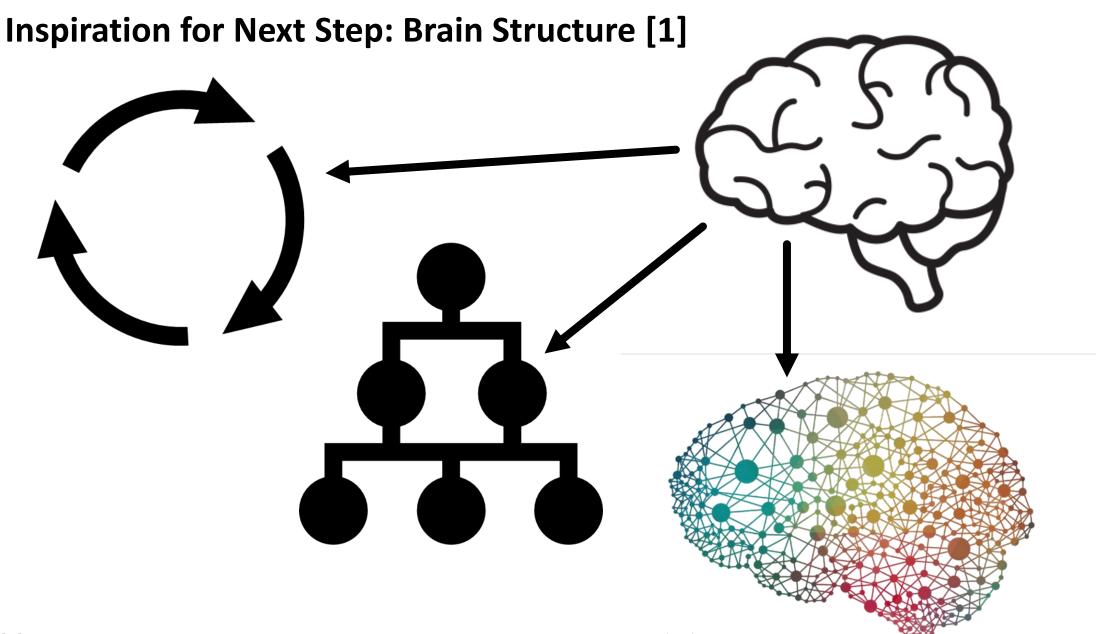










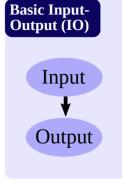










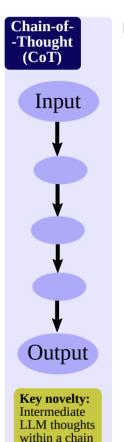


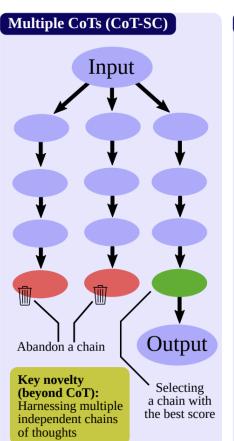


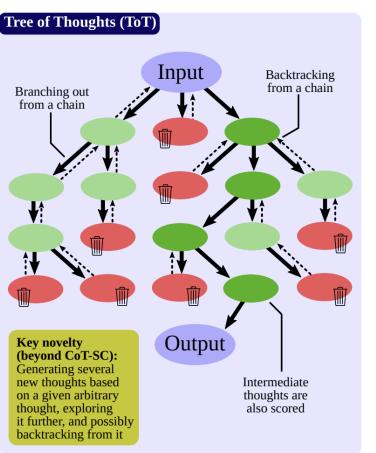
between thoughts

Abandon thought

* Backtrack



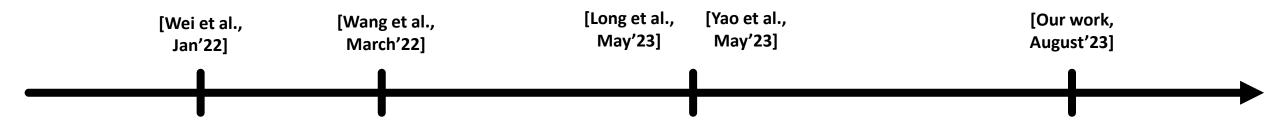


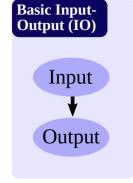




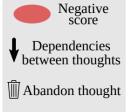












* Backtrack



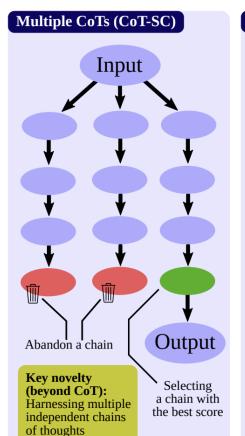
Output

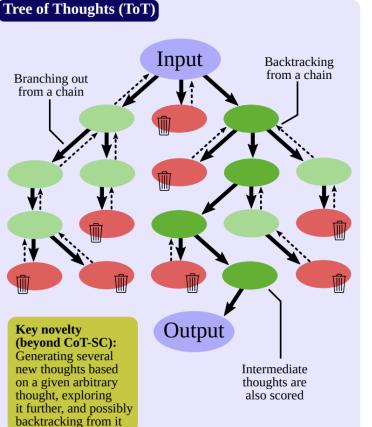
Chain-of-

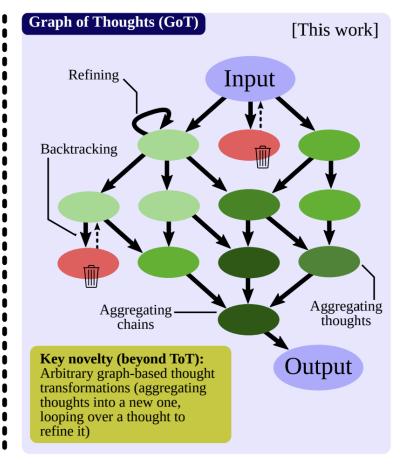
-Thought

(CoT)

Input













[Wei et al., Jan'22] [Wang et al., March'22]

[Long et al., May'23]

[Yao et al., May'23]

"Thought transformations"

Basic Input-Output (IO)



Legend

Thoughts

Unscored

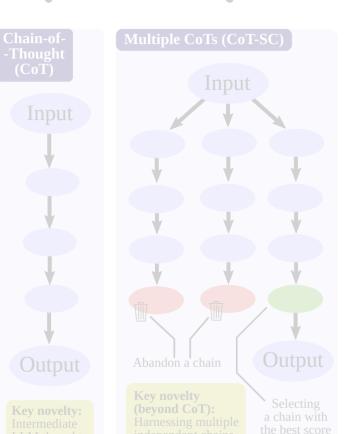


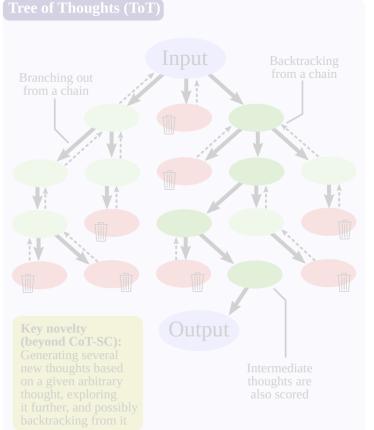


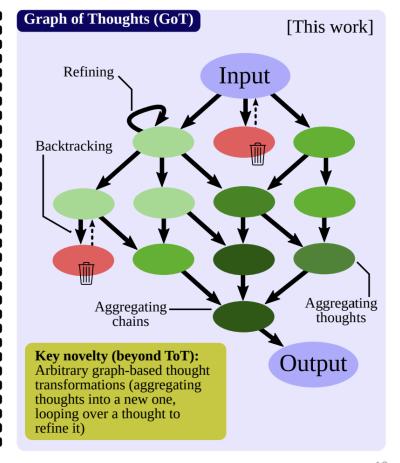


Abandon thought

Backtrack













[Wei et al., Jan'22] [Wang et al., March'22]

[Long et al., May'23]

[Yao et al., May'23] "Thought transformations"

Basic Input-Output (IO)



Legend

Thoughts:

Unscored



Positive score

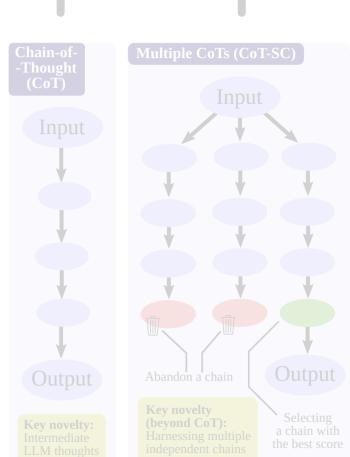


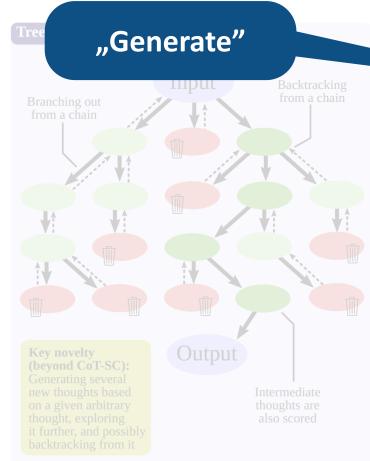
Negative

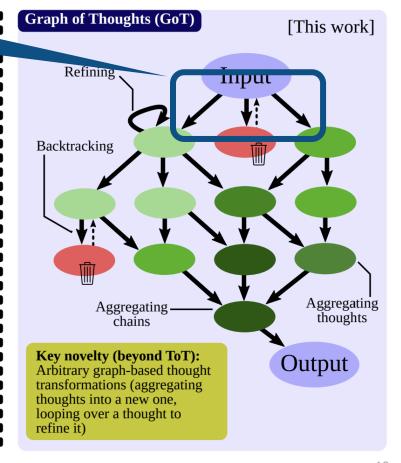


Abandon thought

Backtrack













[Wei et al., Jan'22]

[Wang et al., March'22]

[Long et al., May'23]

[Yao et al., May'23]

"Thought transformations"

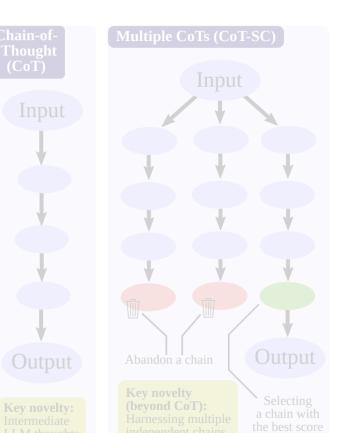


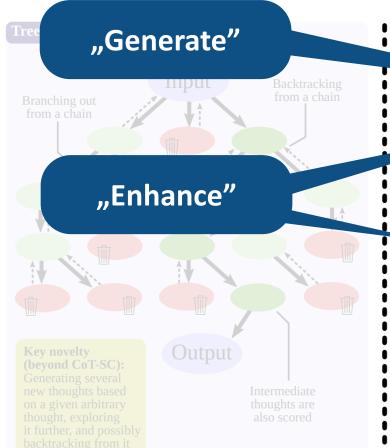


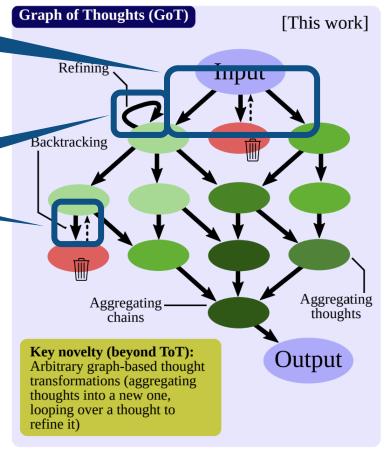




* Backtrack













[Wei et al., Jan'22]

[Wang et al., March'22]

[Long et al., May'23]

[Yao et al., May'23] "Thought transformations"

Basic Input-Output (IO)



Legend

Thoughts:

Unscored



Positive score

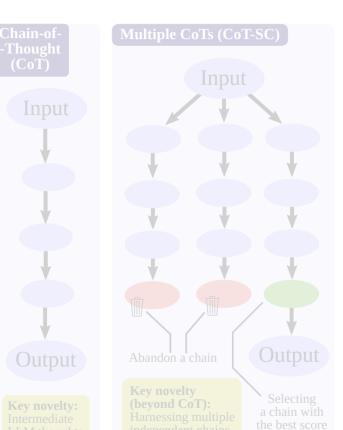


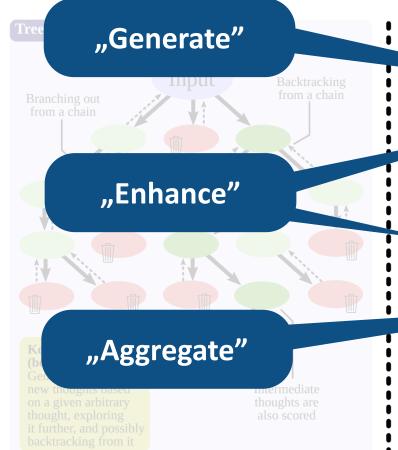
Negative score

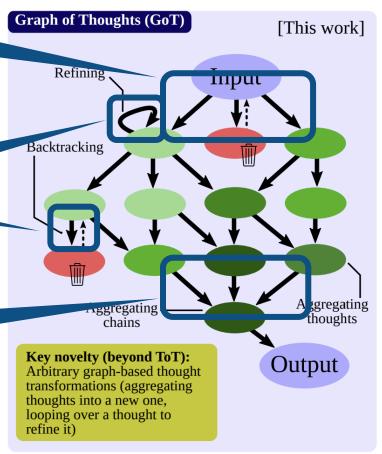


Abandon thought

*, Backtrack

















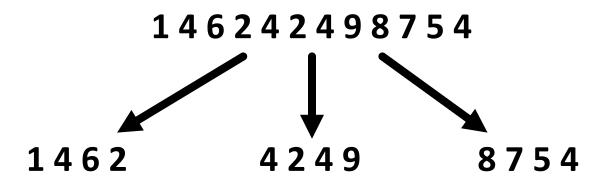


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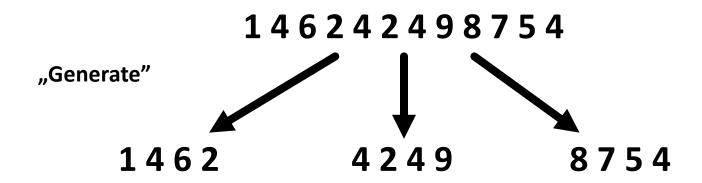








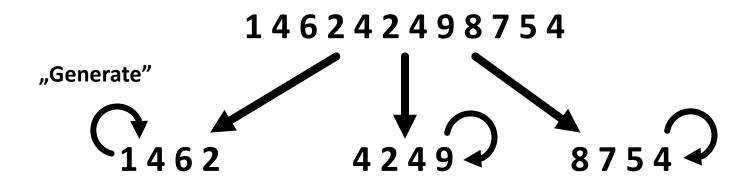








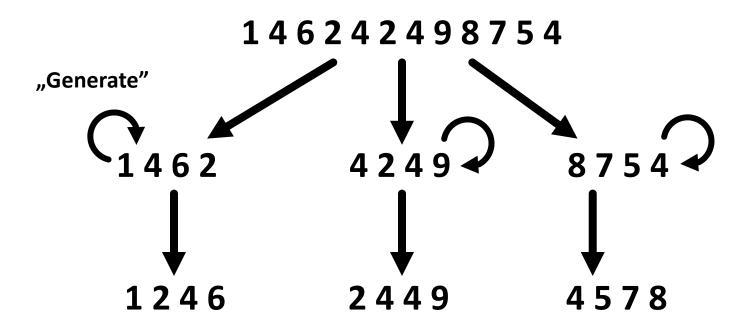








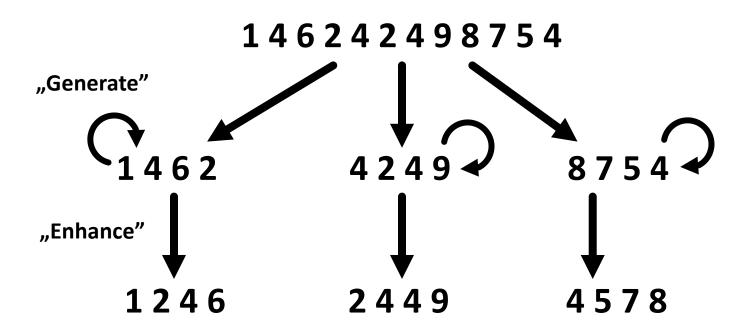








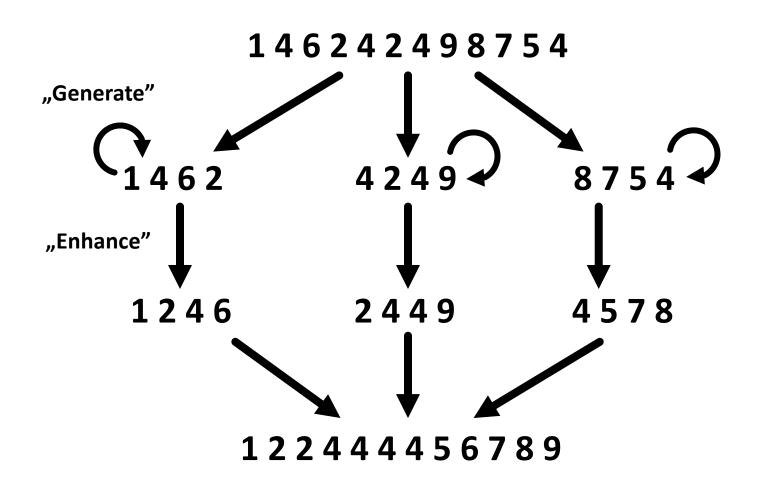








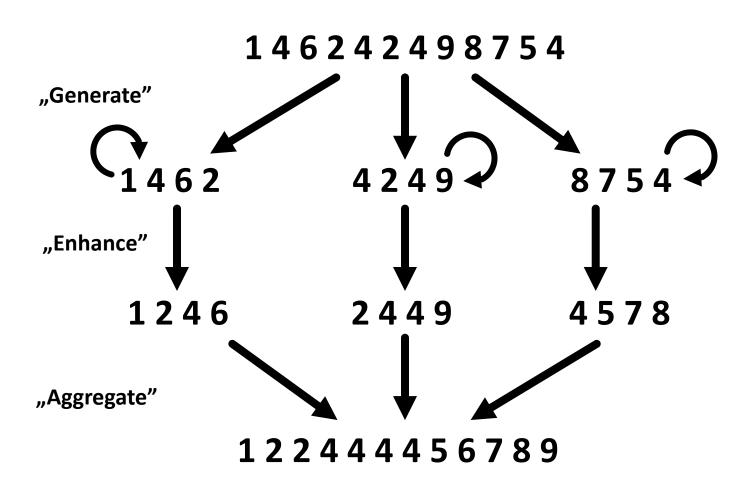










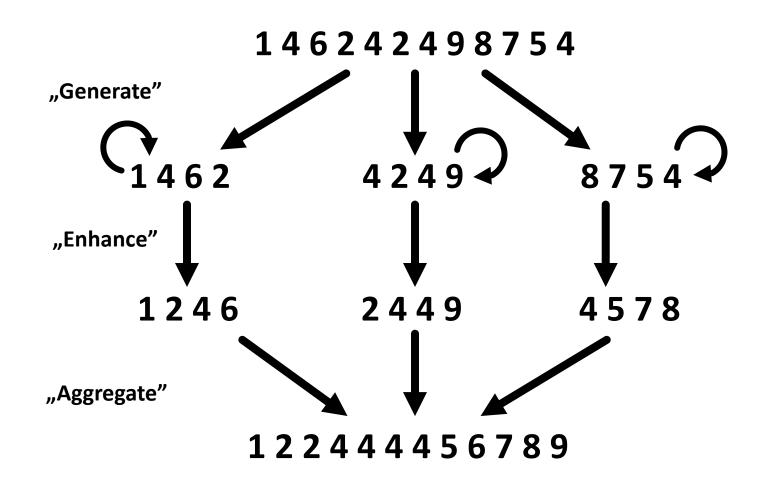








Why sorting? Because it is a fundamental problem in CS, and it still does pose a great challenge for all other baselines



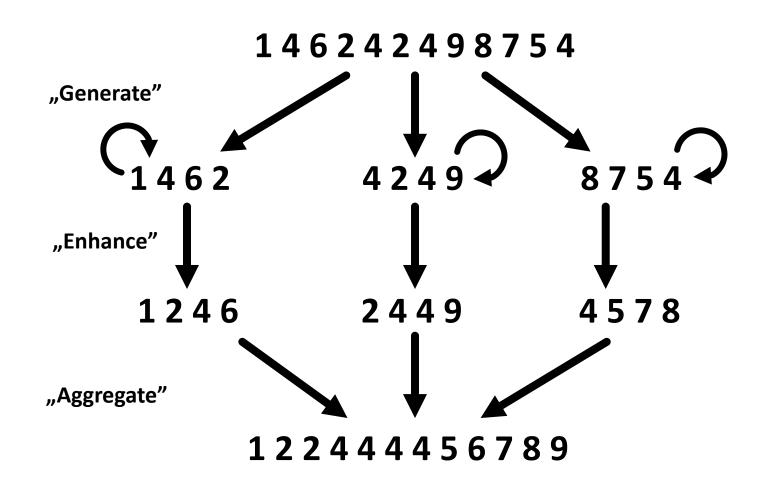






Why sorting? Because it is a fundamental problem in CS, and it still does pose a great challenge for all other baselines

This is a small example; for real use cases, the size is much larger, and the graph gets more complex





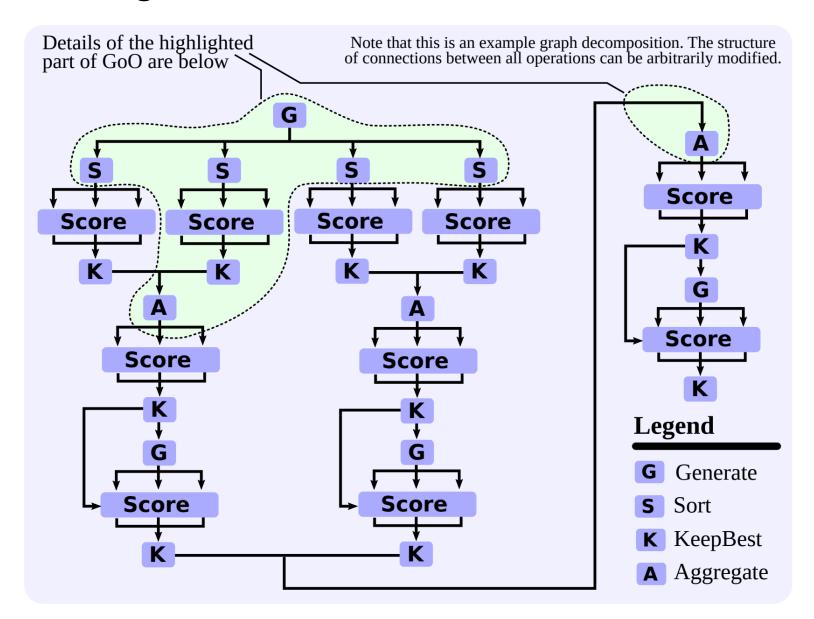




Thought Transformations for Sorting

Why sorting? Because it is a fundamental problem in CS, and it still does pose a great challenge for all other baselines

This is a small example; for real use cases, the size is much larger, and the graph gets more complex

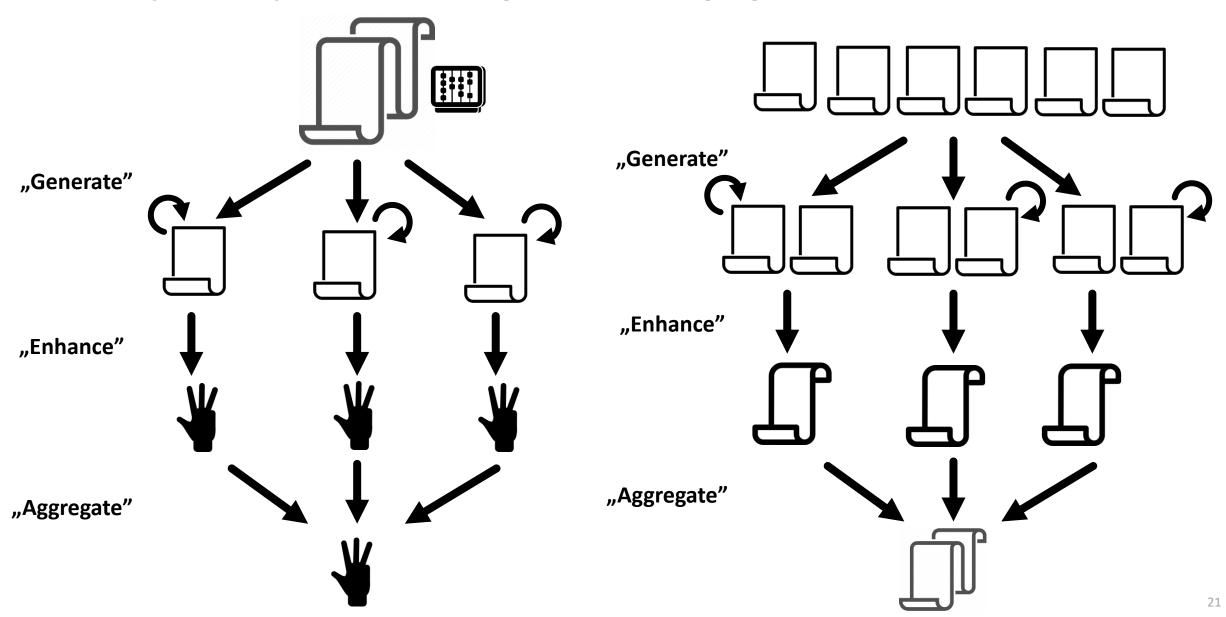








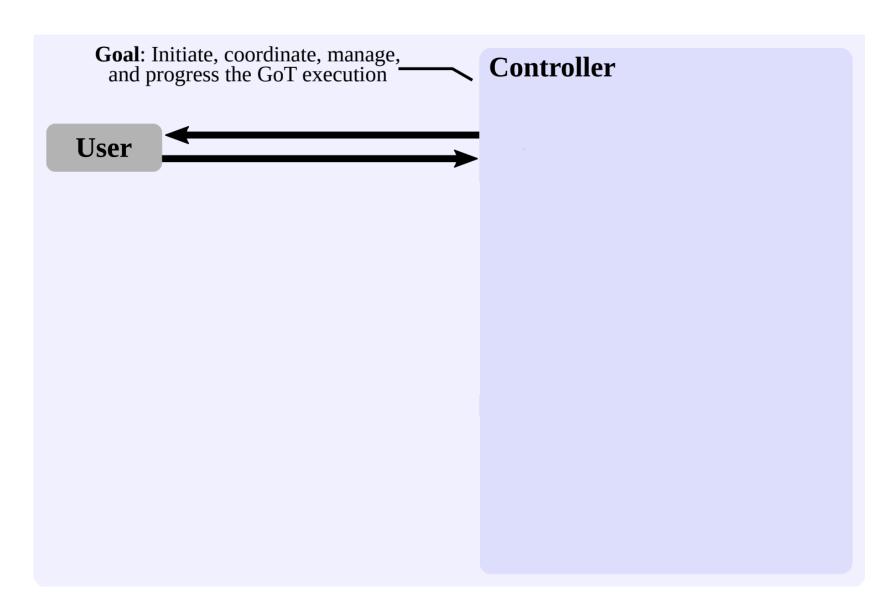
Other Examples: Keyword Counting & NDA merging







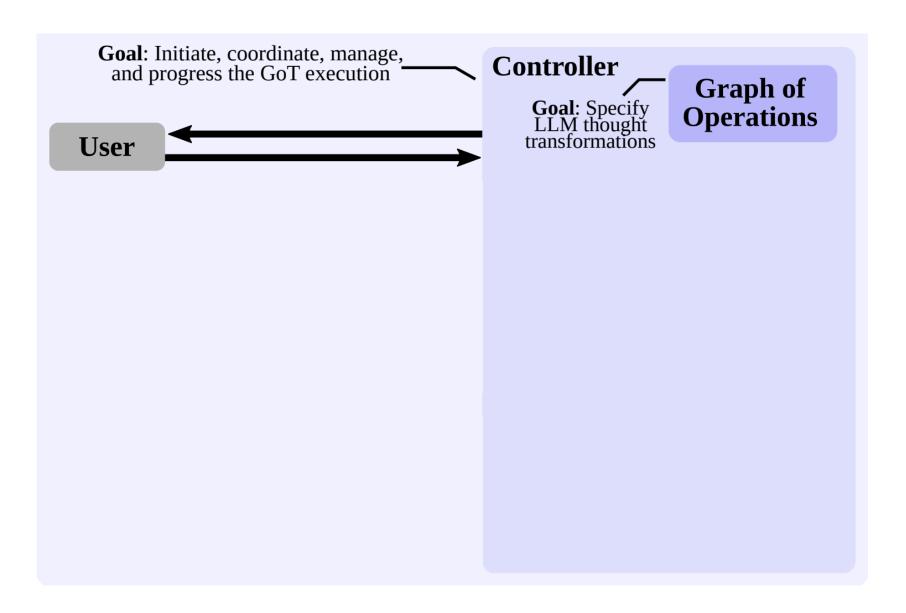








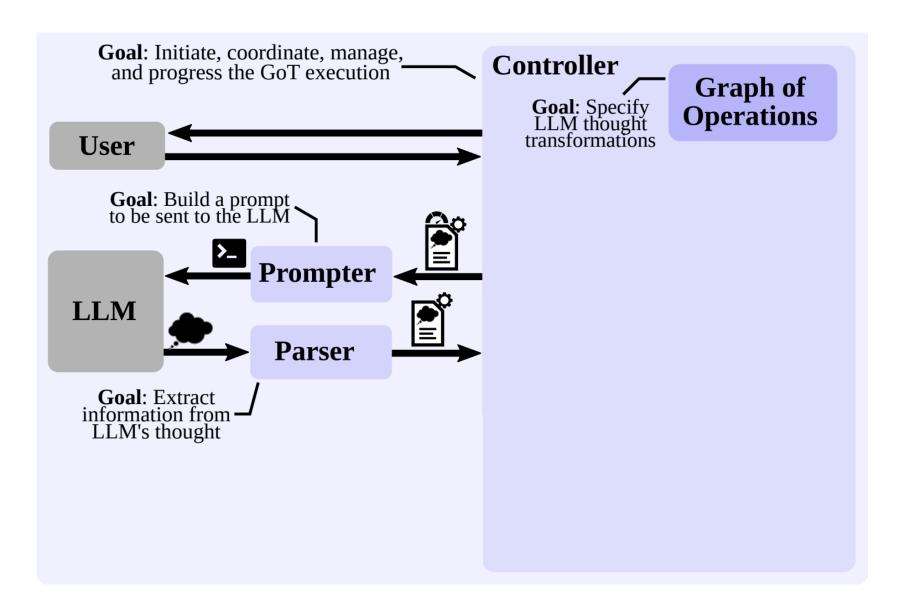








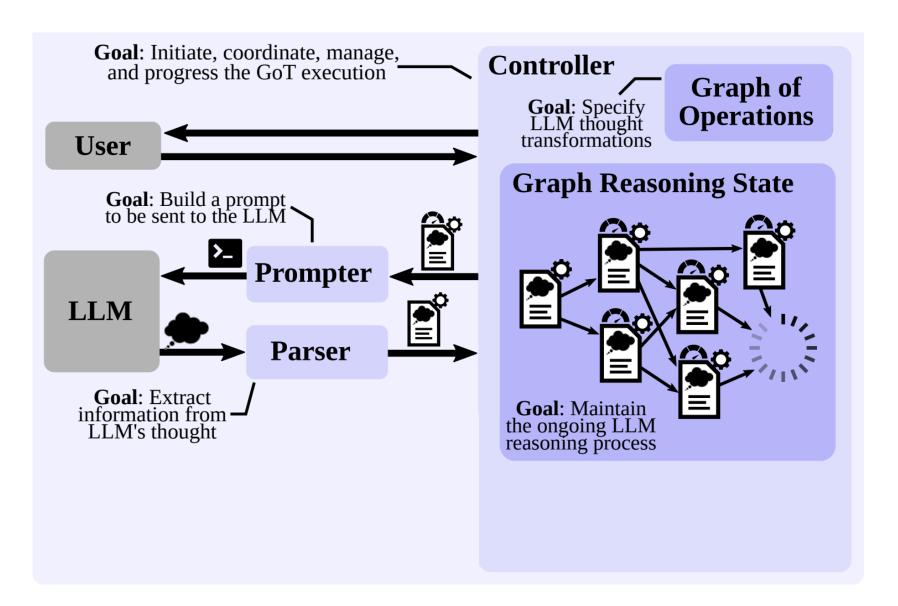








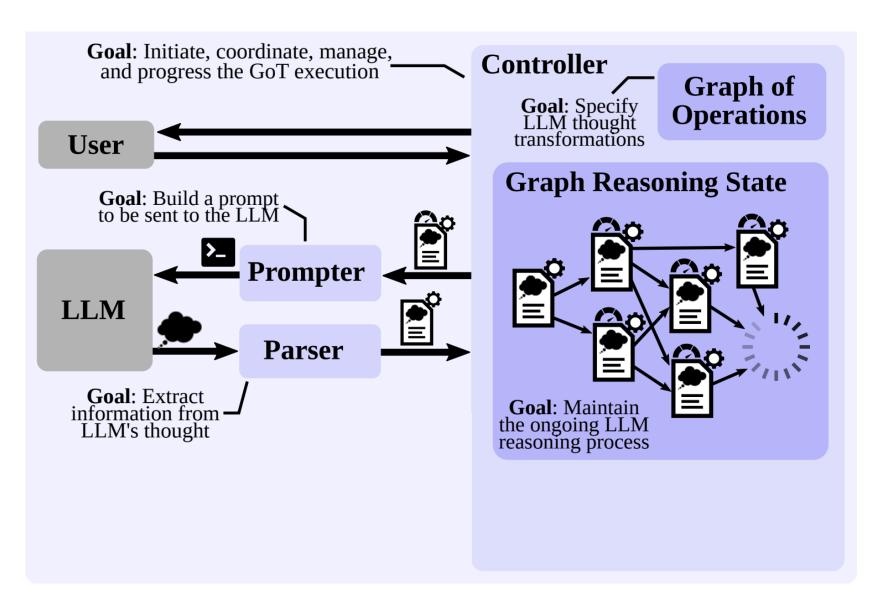










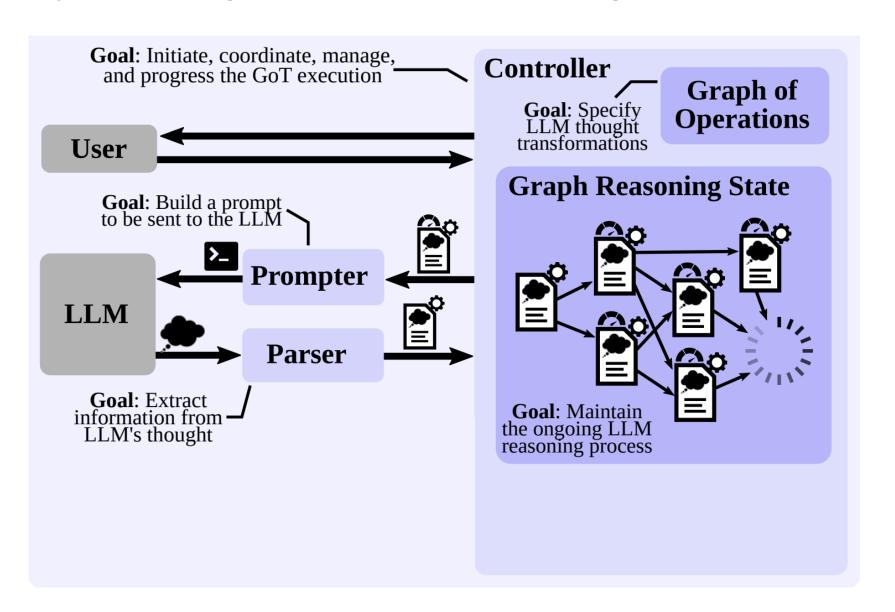


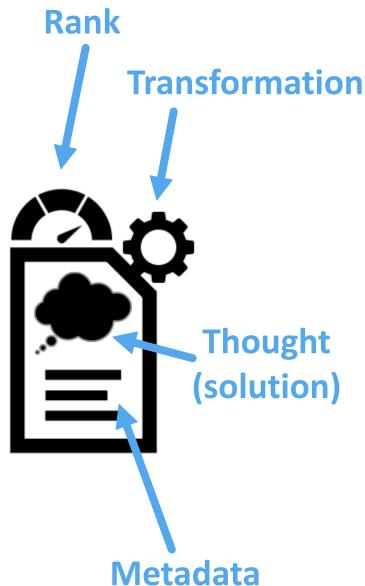








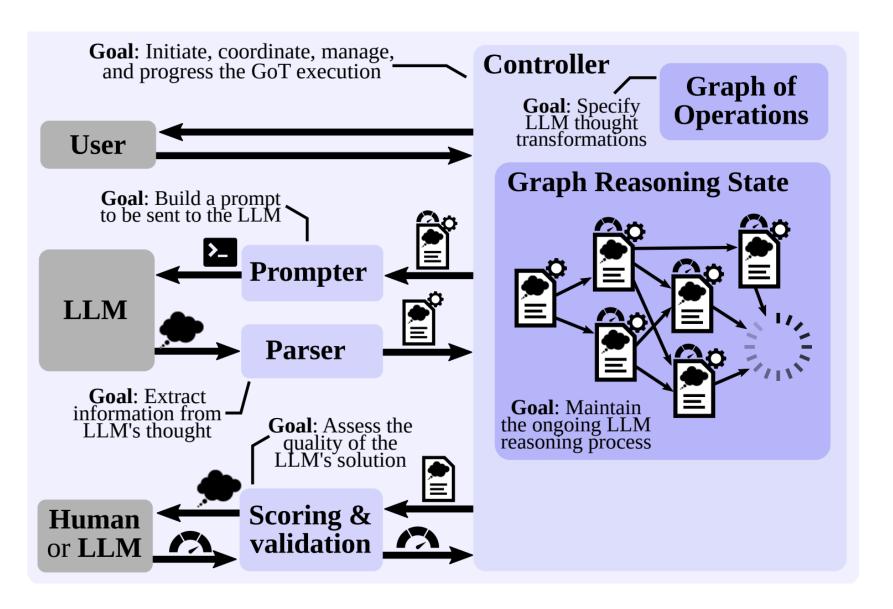


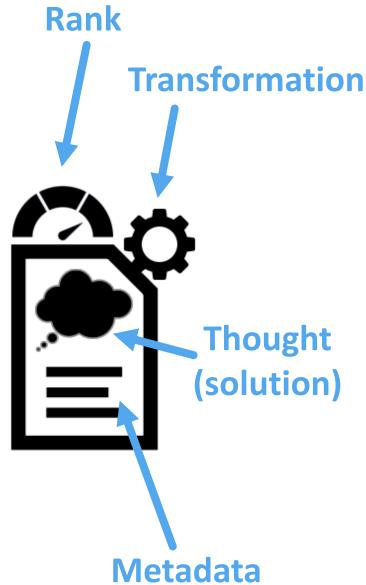








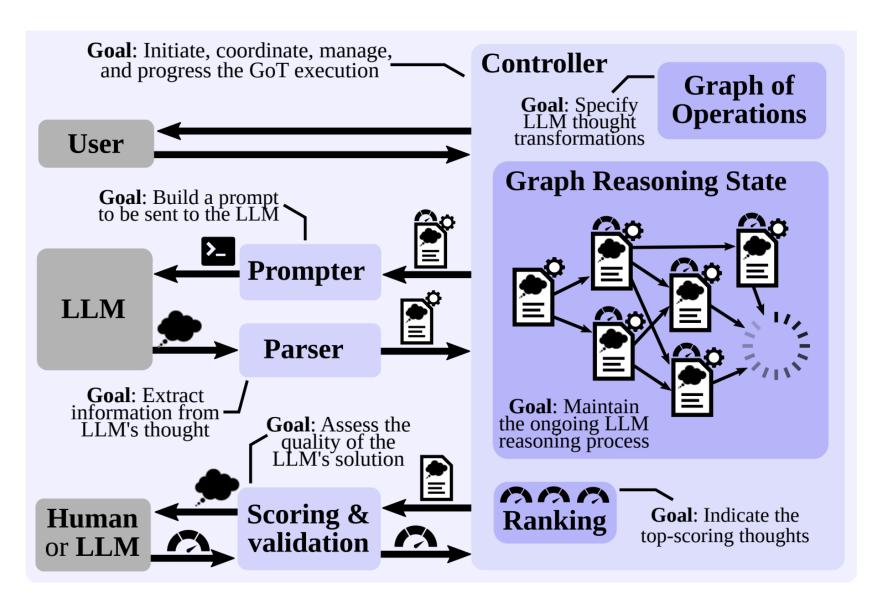


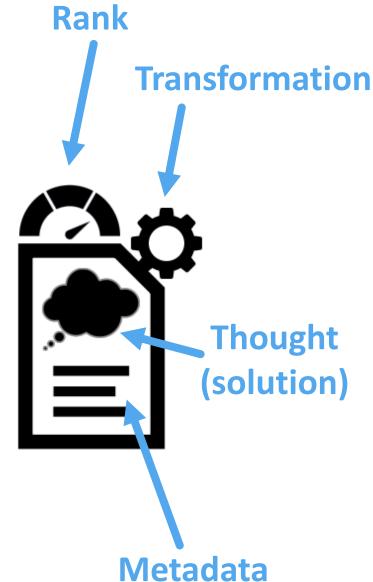








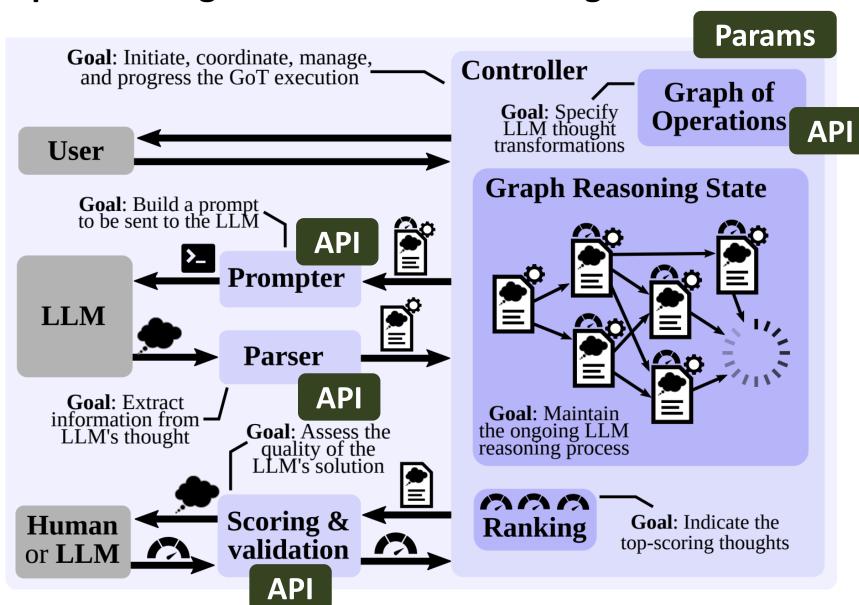


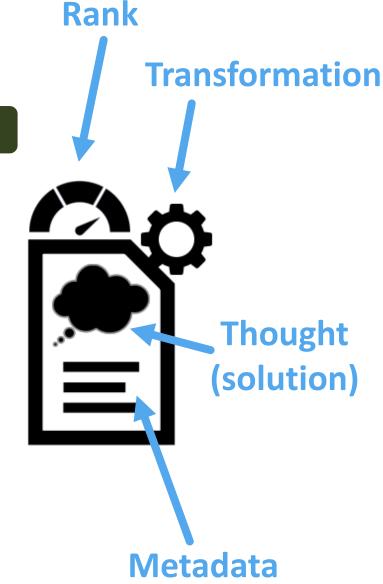








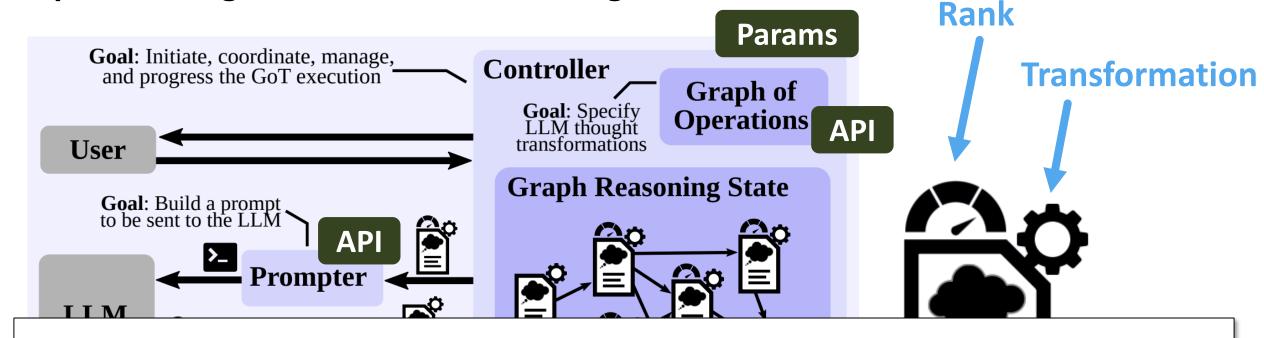












Graph of Thoughts: Solving Elaborate Problems with Large Language Models

Maciej Besta^{1*}, Nils Blach^{1*}, Ales Kubicek¹, Robert Gerstenberger¹, Lukas Gianinazzi¹, Joanna Gajda², Tomasz Lehmann², Michał Podstawski³, Hubert Niewiadomski², Piotr Nyczyk², Torsten Hoefler¹

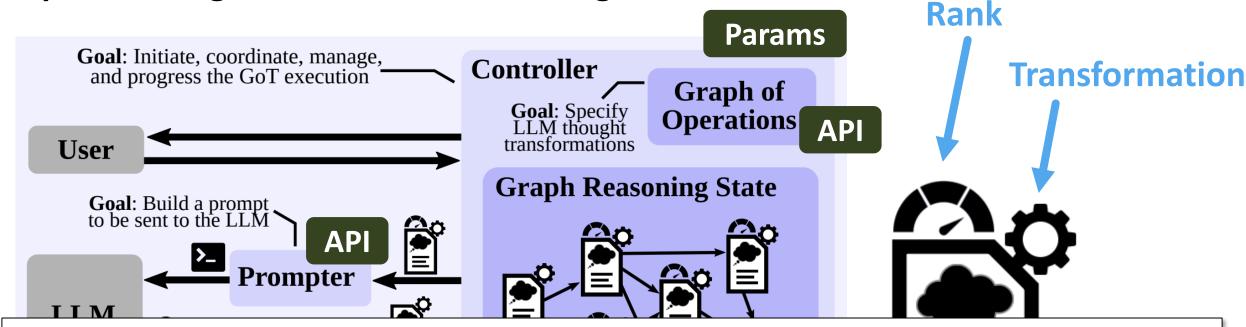
¹ETH Zurich, ²Cledar, ³Warsaw University of Technology bestam@inf.ethz.ch, nils.blach@inf.ethz.ch, htor@inf.ethz.ch

22









https://github.com/spcl/graph-of-thoughts,

@AAAI'24

Graph of Thoughts: Solving Elaborate Problems with Large Language Models

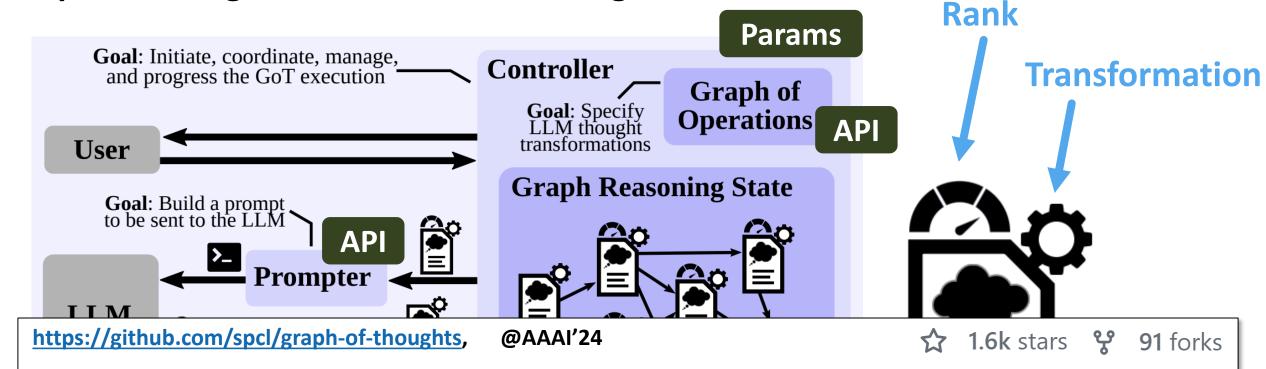
Maciej Besta^{1*}, Nils Blach^{1*}, Ales Kubicek¹, Robert Gerstenberger¹, Lukas Gianinazzi¹, Joanna Gajda², Tomasz Lehmann², Michał Podstawski³, Hubert Niewiadomski², Piotr Nyczyk², Torsten Hoefler¹

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22



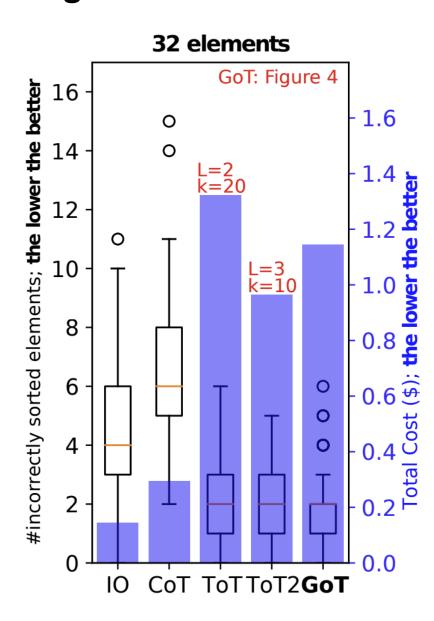








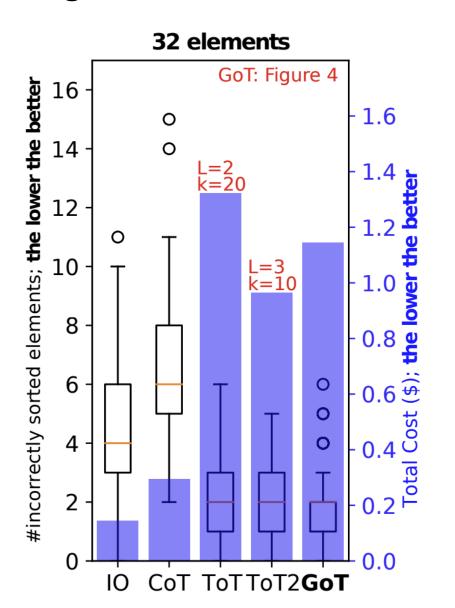


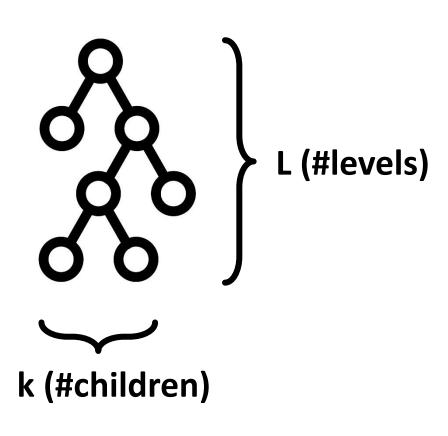








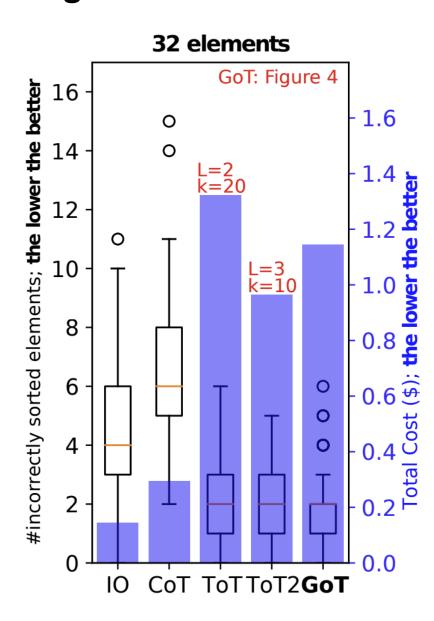








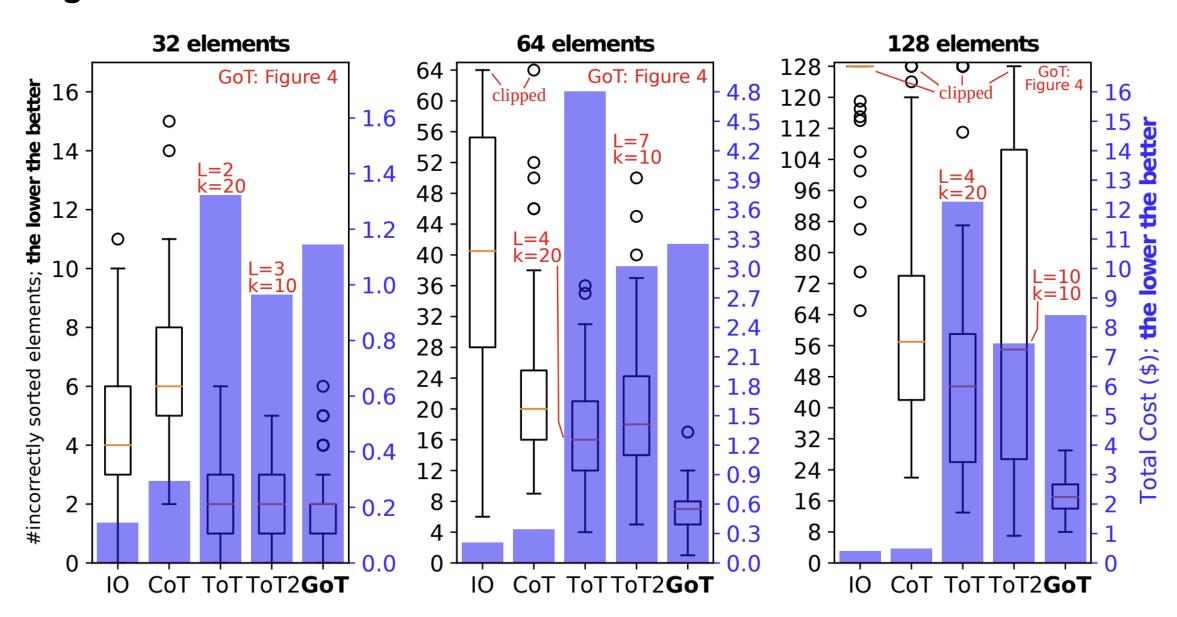










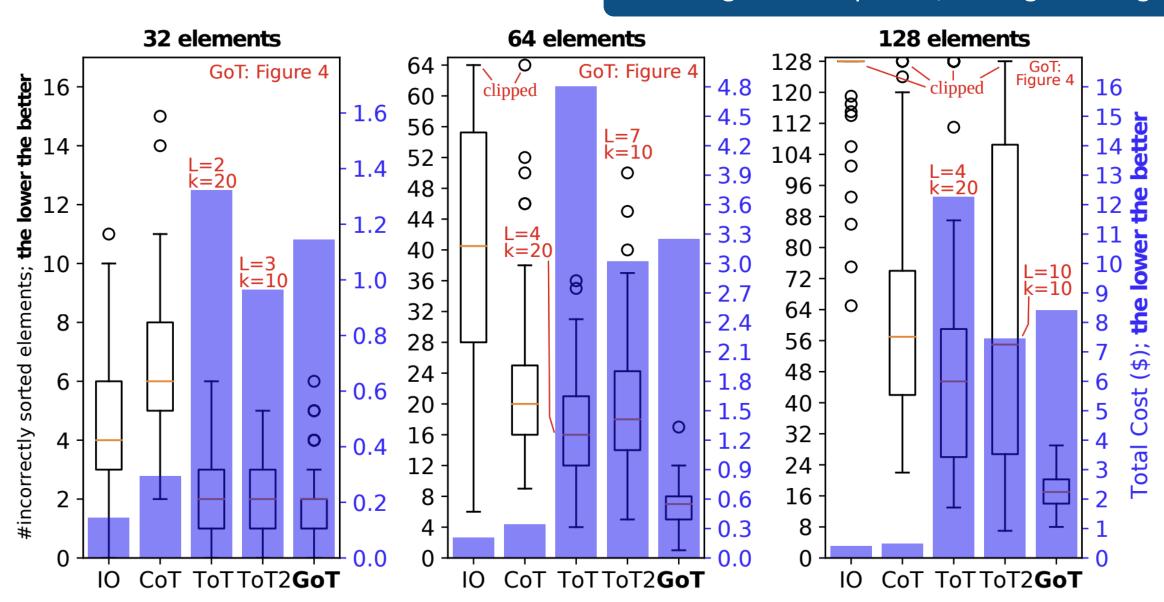








The longer the sequence, the higher the gain

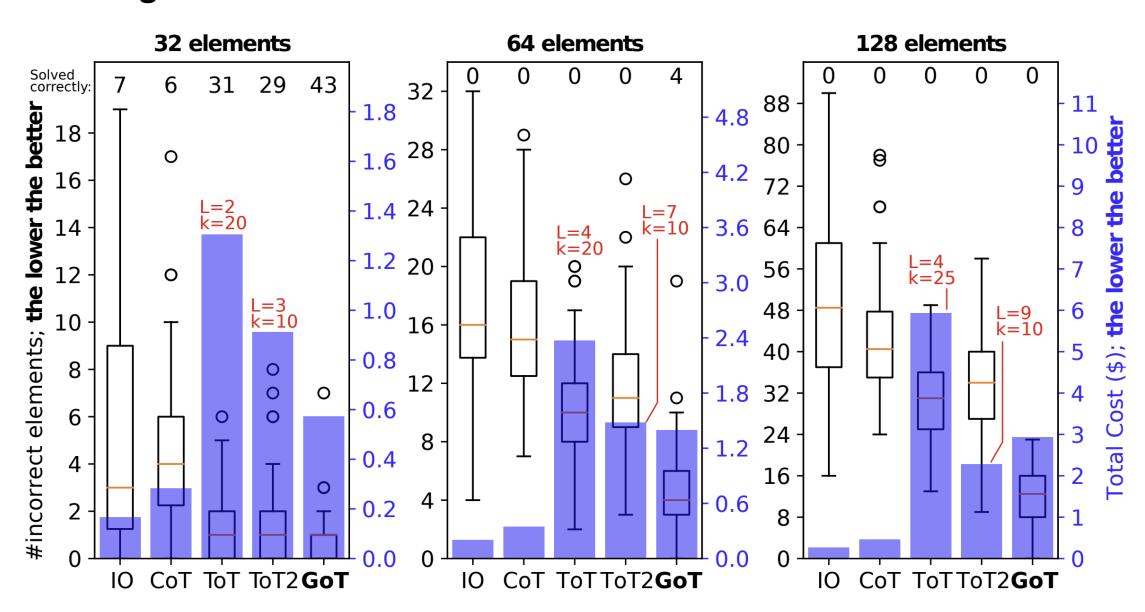








Intersecting Sets of Numbers



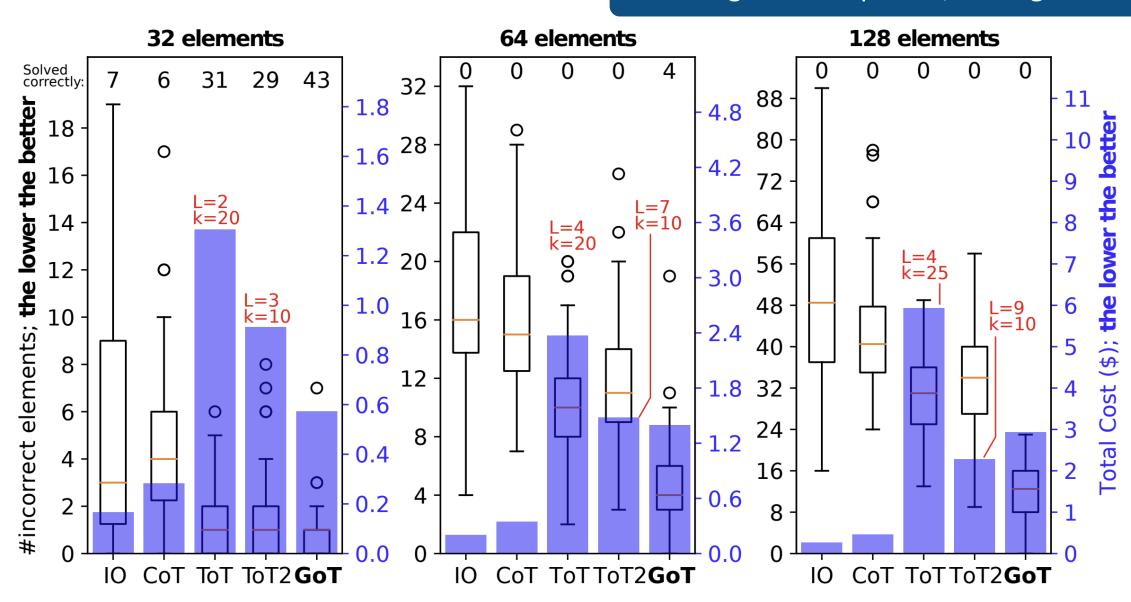






Intersecting Sets of Numbers

The longer the sequence, the higher the gain

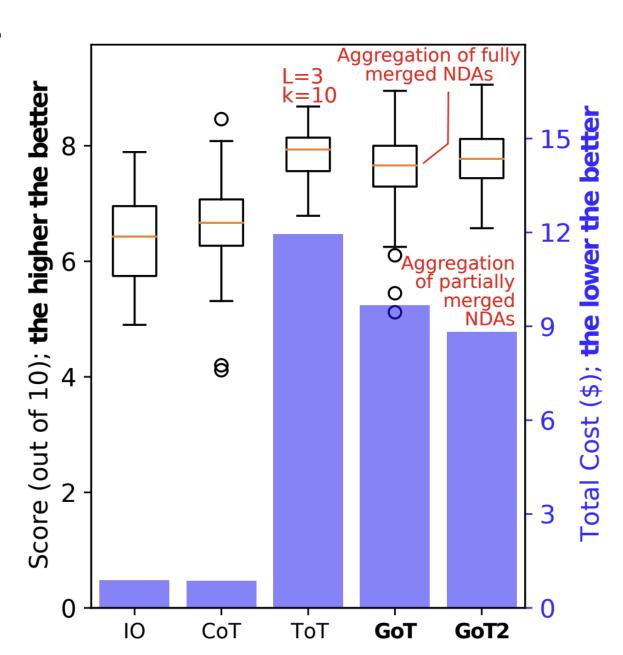








Merging Documents



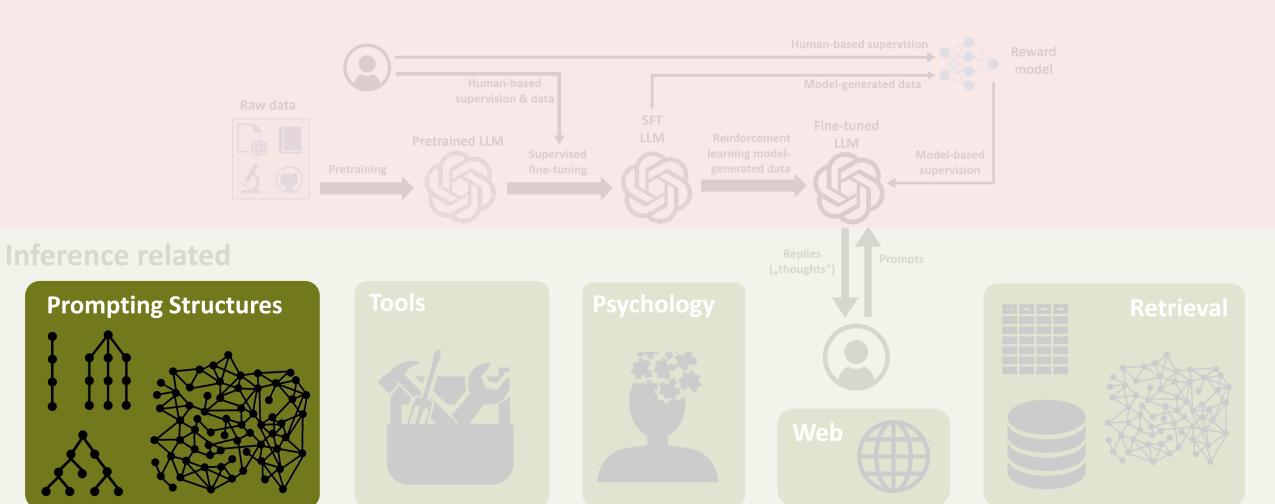






The Emergence of the "Generative AI Ecosystem"

Training related







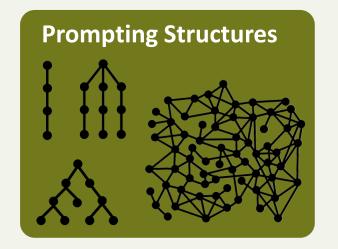


The Emergence of the "Generative AI Ecosystem"

Training related



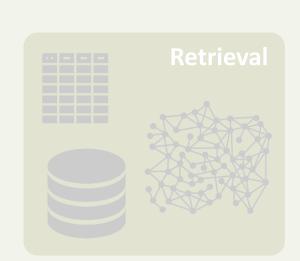
Inference related























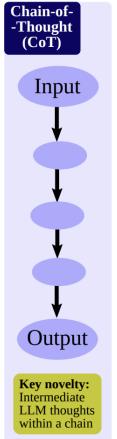
Assume a **fixed thought size** (#tokens) and a **fixed context size** (#thoughts in the LLM context, denoted with N)

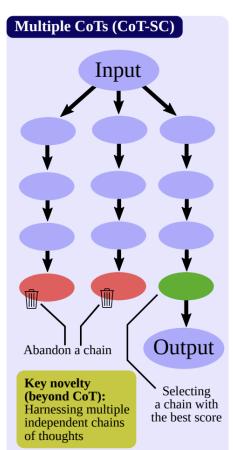
Volume – for a given thought t – is the number of preceding LLM thoughts that could have impacted t

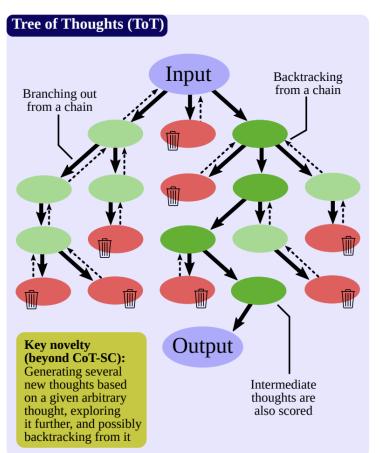


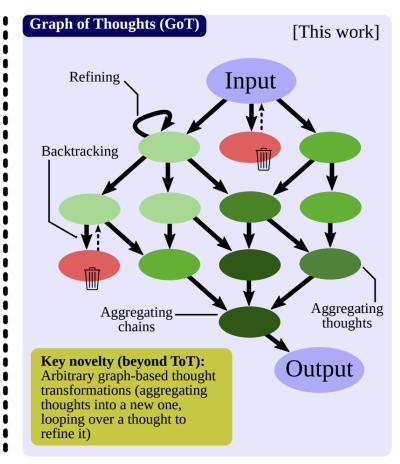








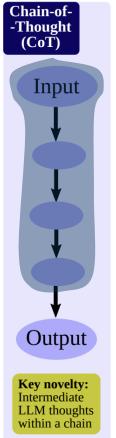


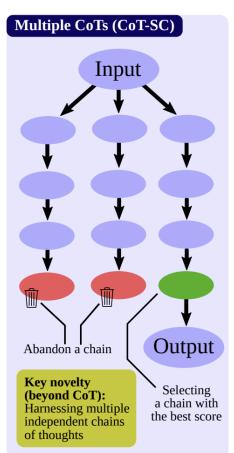


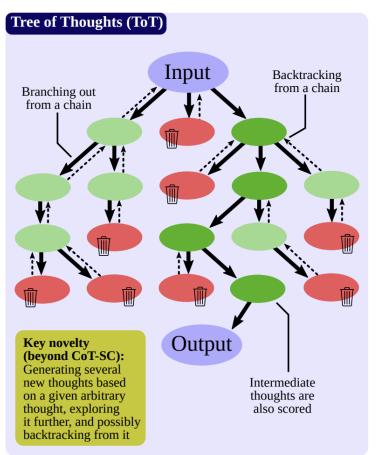


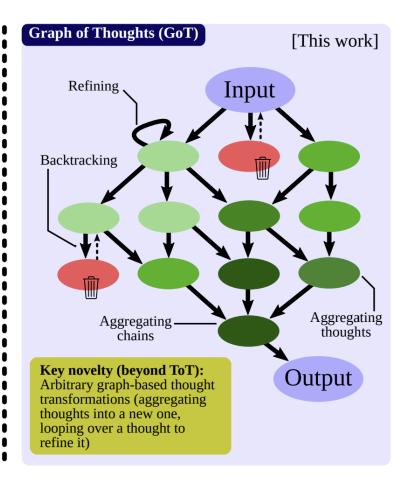








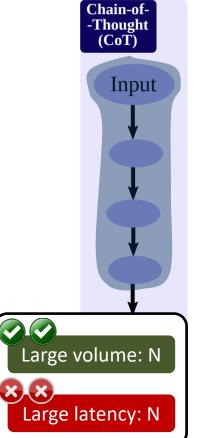


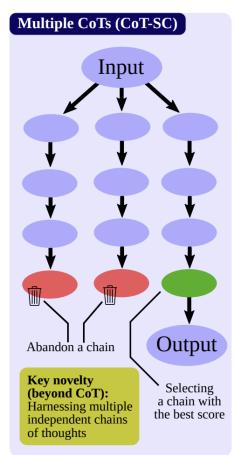


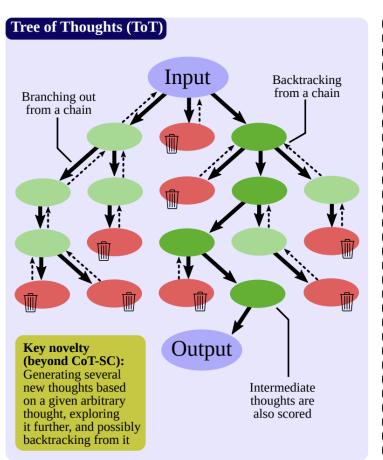


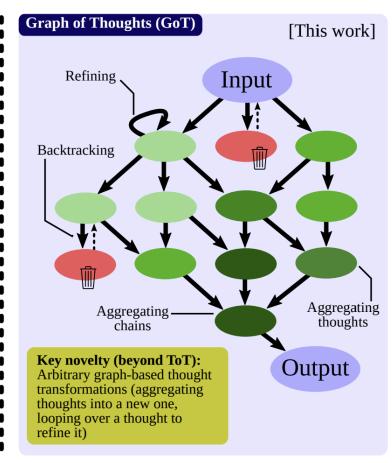








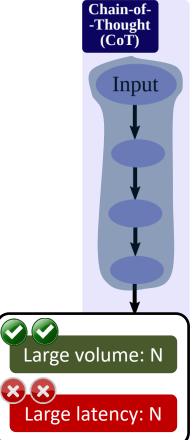


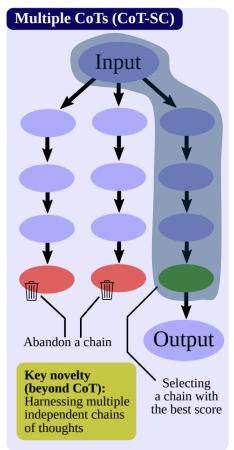


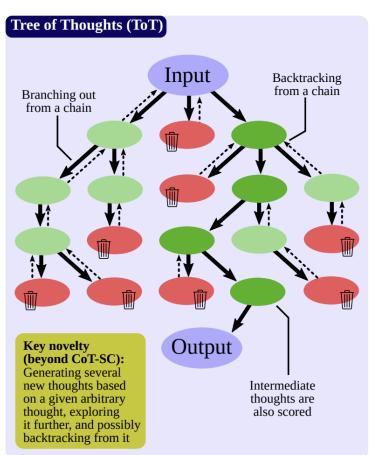


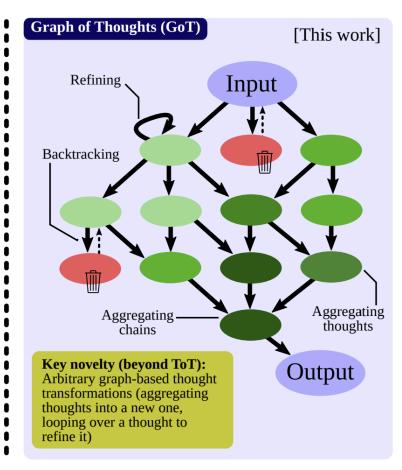








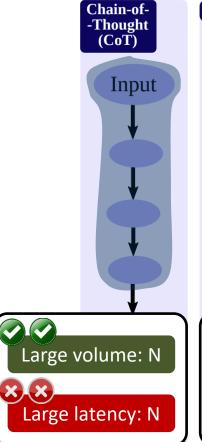


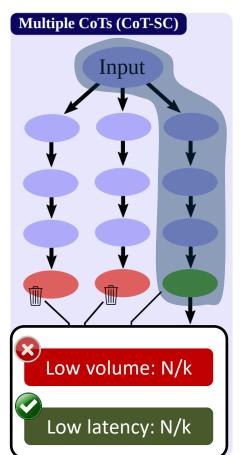


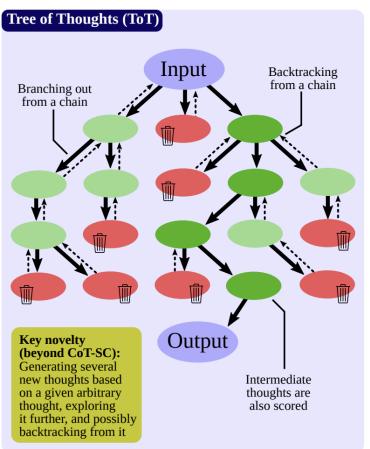


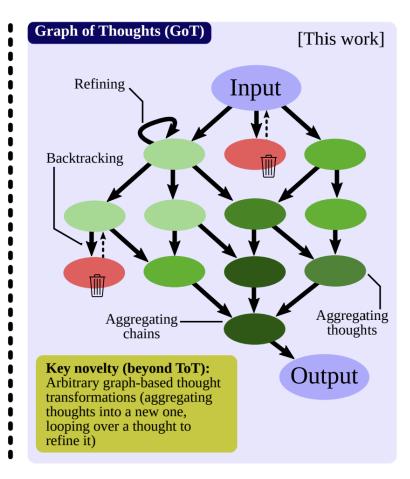








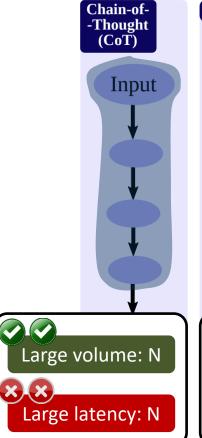


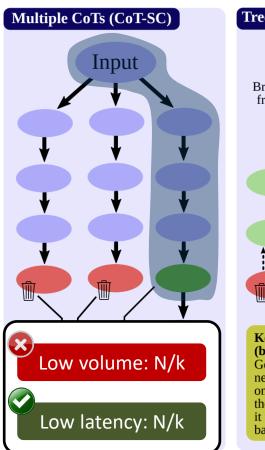


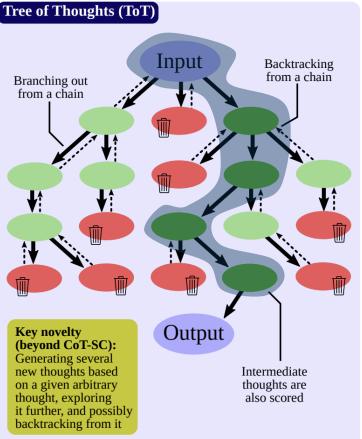


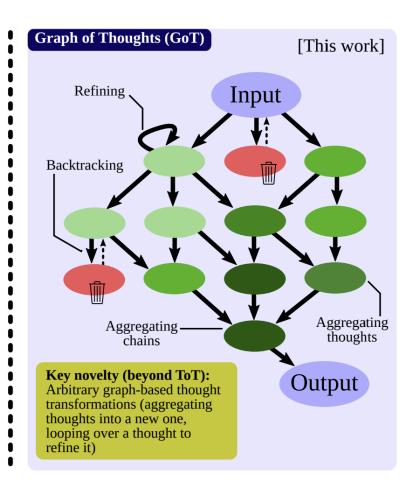










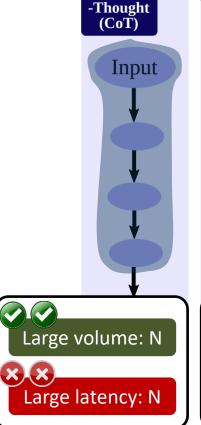




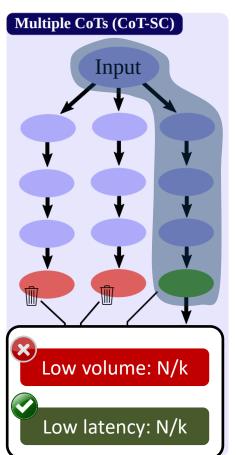


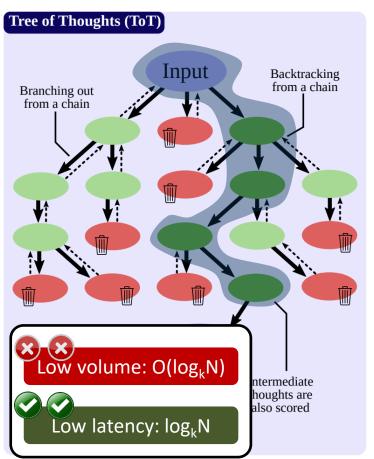


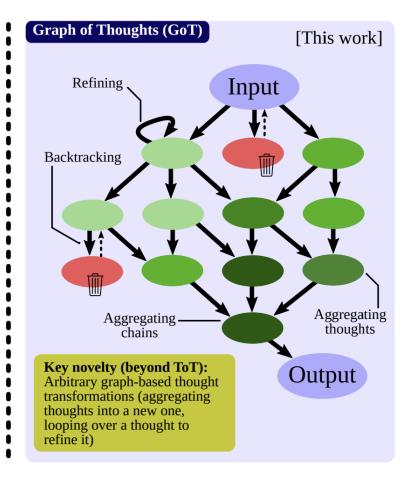
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Chain-of-







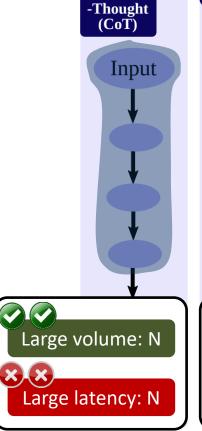




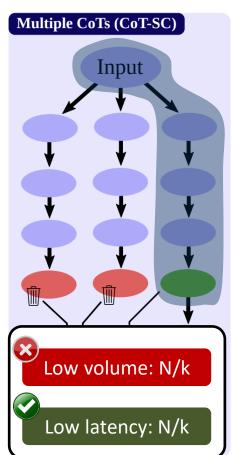


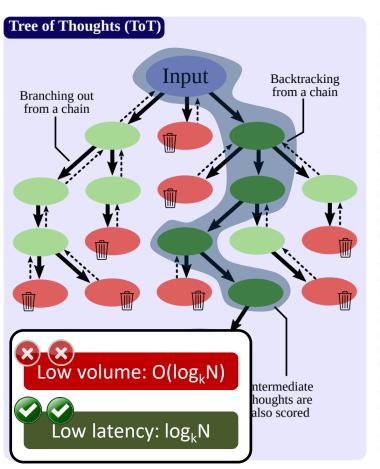
Why Does Structured Prompting Work?

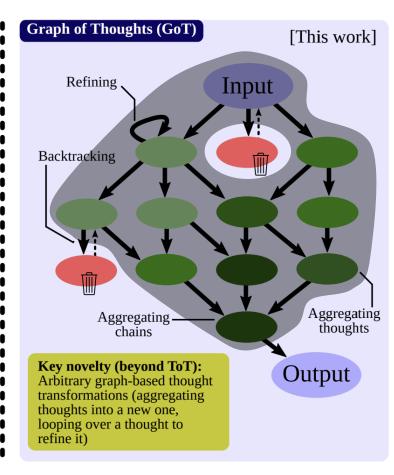
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Chain-of-







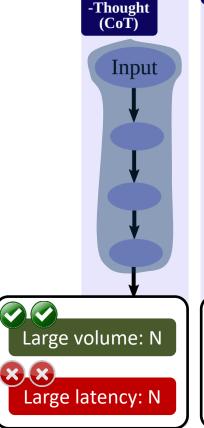




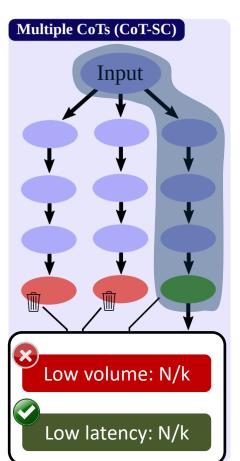


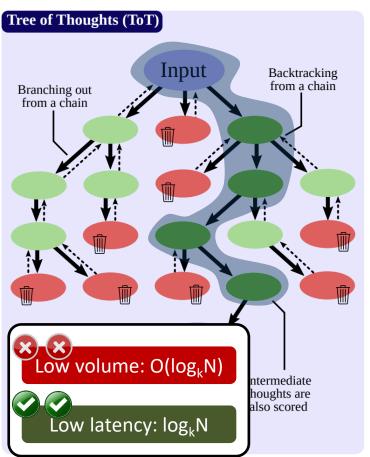
Why Does Structured Prompting Work?

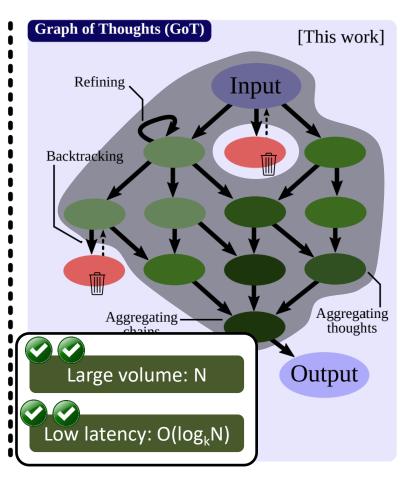
Assume a **fixed thought size** (#tokens) and a **fixed context size** (#thoughts in the LLM context, denoted with N)



Chain-of-



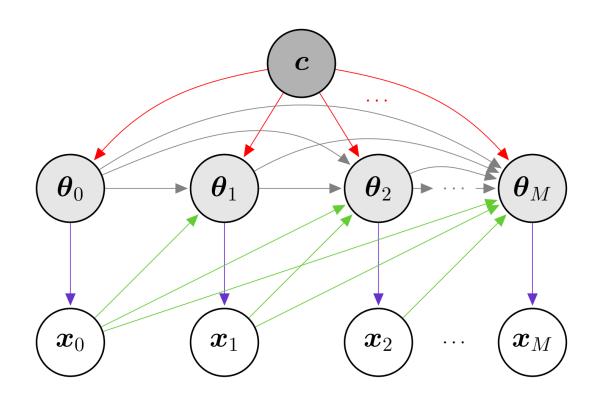








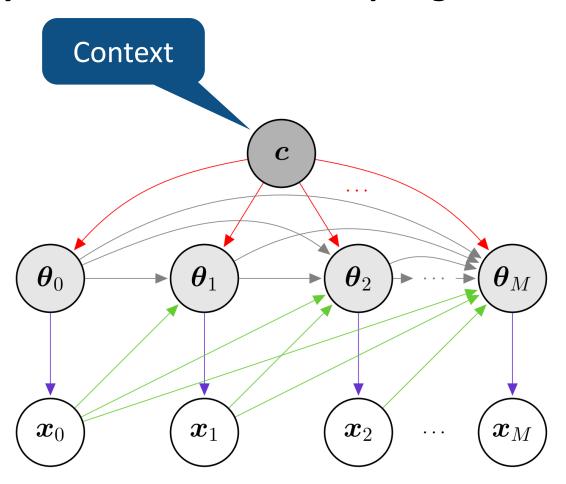








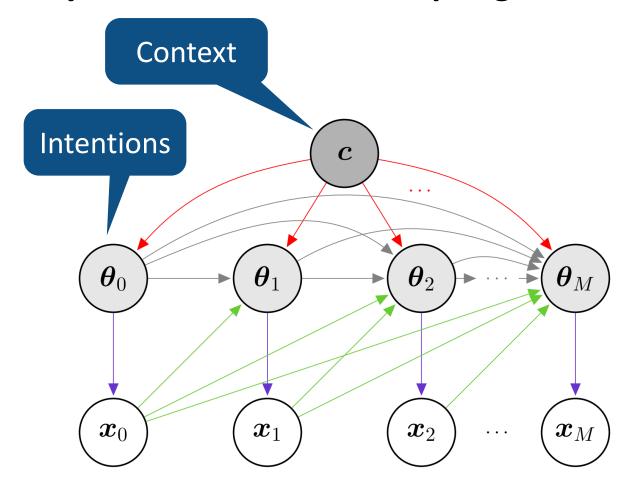








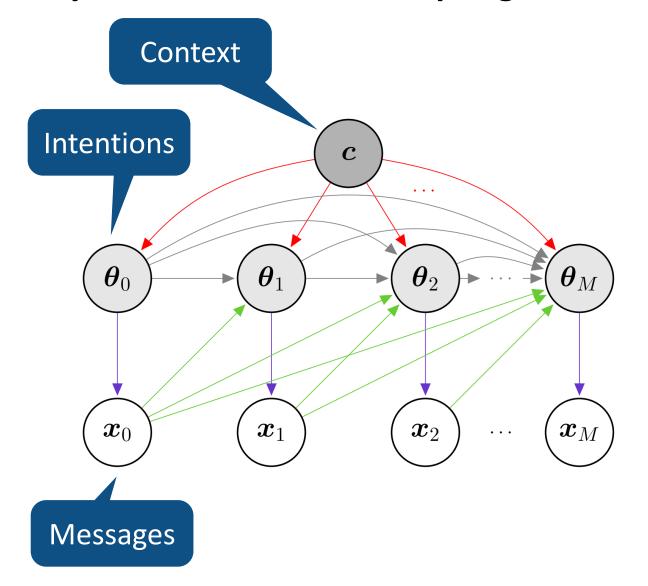








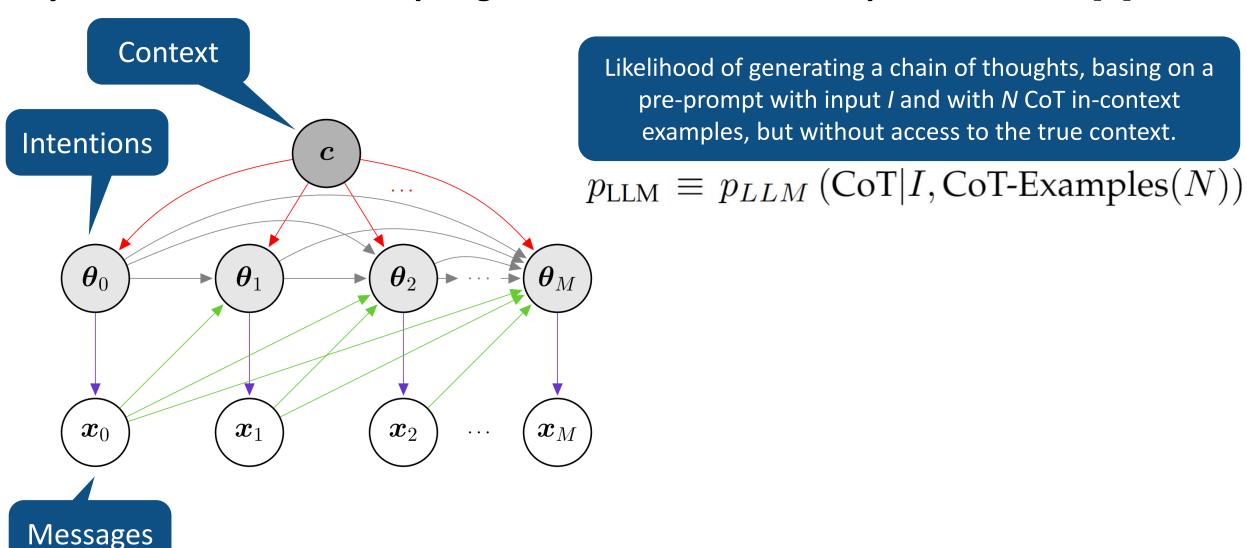








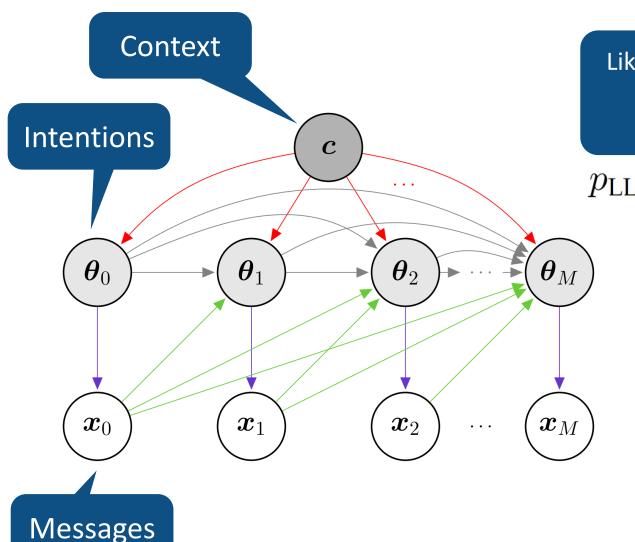












Likelihood of generating a chain of thoughts, basing on a pre-prompt with input *I* and with *N* CoT in-context examples, but without access to the true context.

 $p_{\text{LLM}} \equiv p_{LLM} \left(\text{CoT} | I, \text{CoT-Examples}(N) \right)$

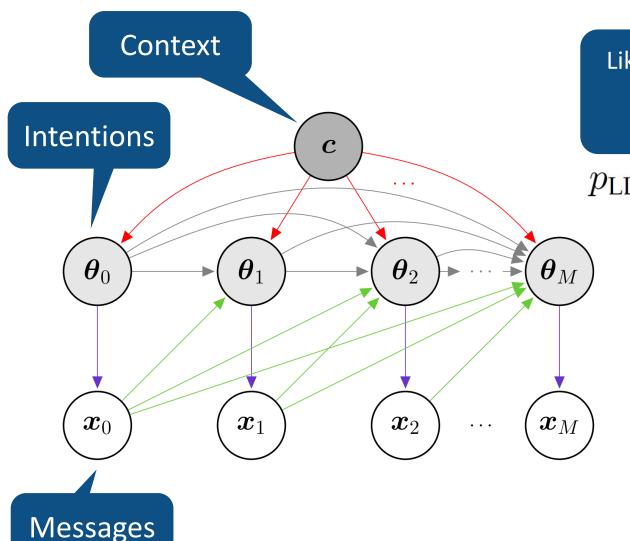
Likelihood of generating the same chain of thoughts as above, but using the true language (and context) conditioned on the same input *I*

 $p_{\text{True}} \equiv p_{\text{True}} \left(CoT | I, \text{True-Context} \right)$









Likelihood of generating a chain of thoughts, basing on a pre-prompt with input I and with N CoT in-context examples, but without access to the true context.

 $p_{\text{LLM}} \equiv p_{LLM} \left(\text{CoT} | I, \text{CoT-Examples}(N) \right)$

Likelihood of generating the same chain of thoughts as above, but using the true language (and context) conditioned on the same input *I*

 $p_{\text{True}} \equiv p_{\text{True}} \left(CoT | I, \text{True-Context} \right)$

$$|p_{\rm LLM} - p_{\rm True}| \le \rho^N$$

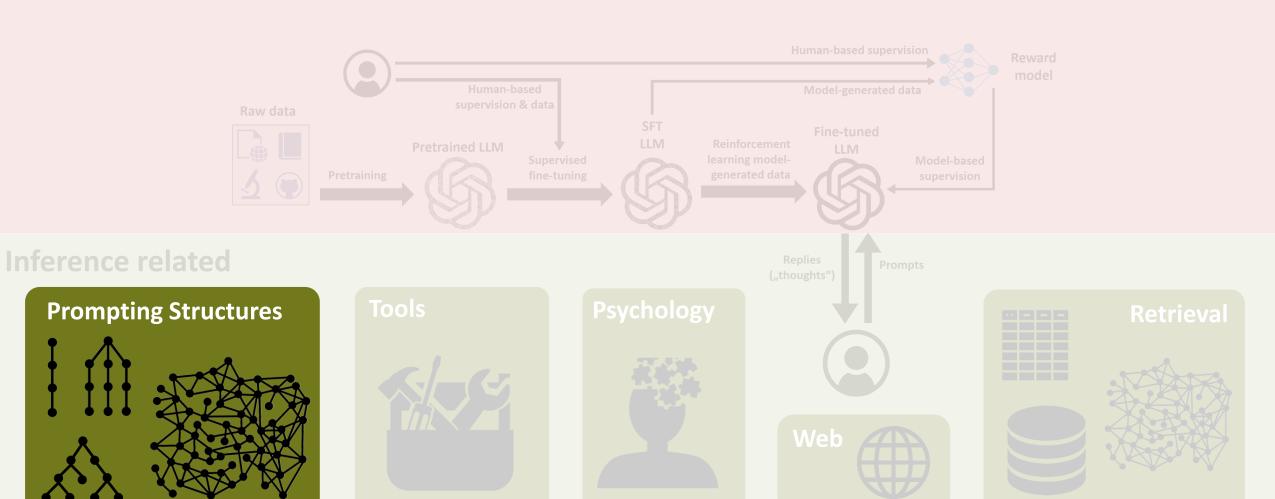
"A function of the language ambiguities", < 1







Training related

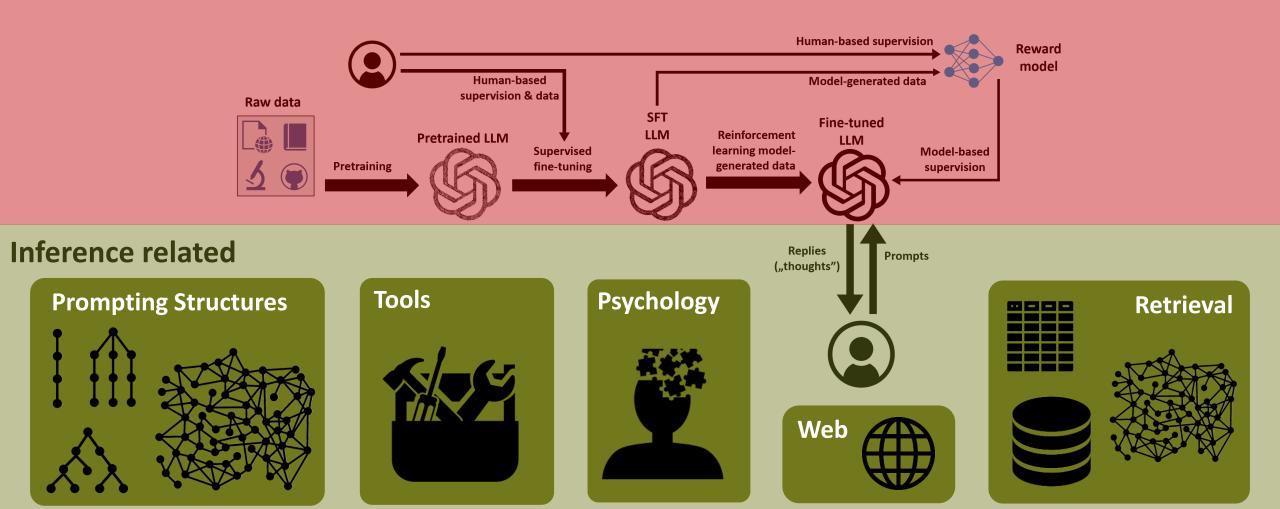








Training related

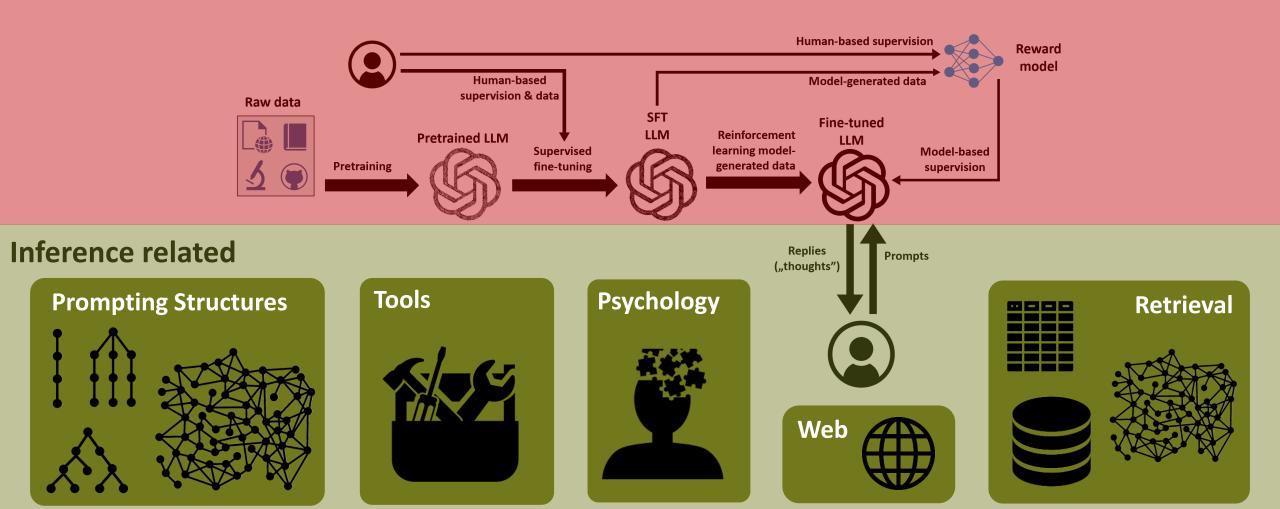








Training related

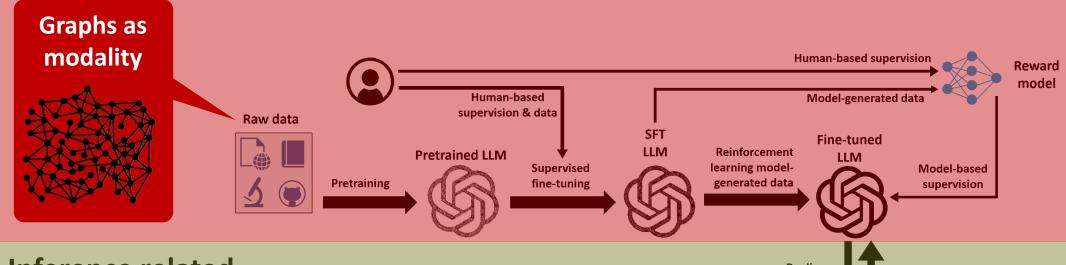




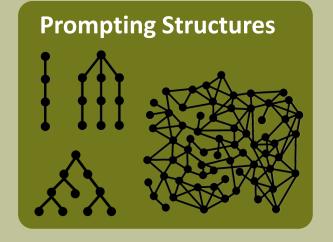




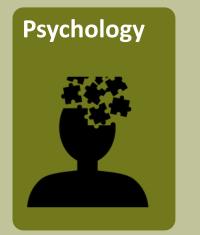
Training related

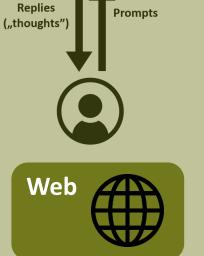


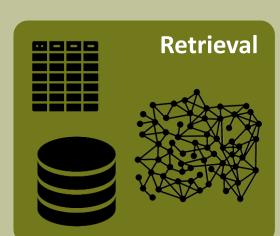
Inference related







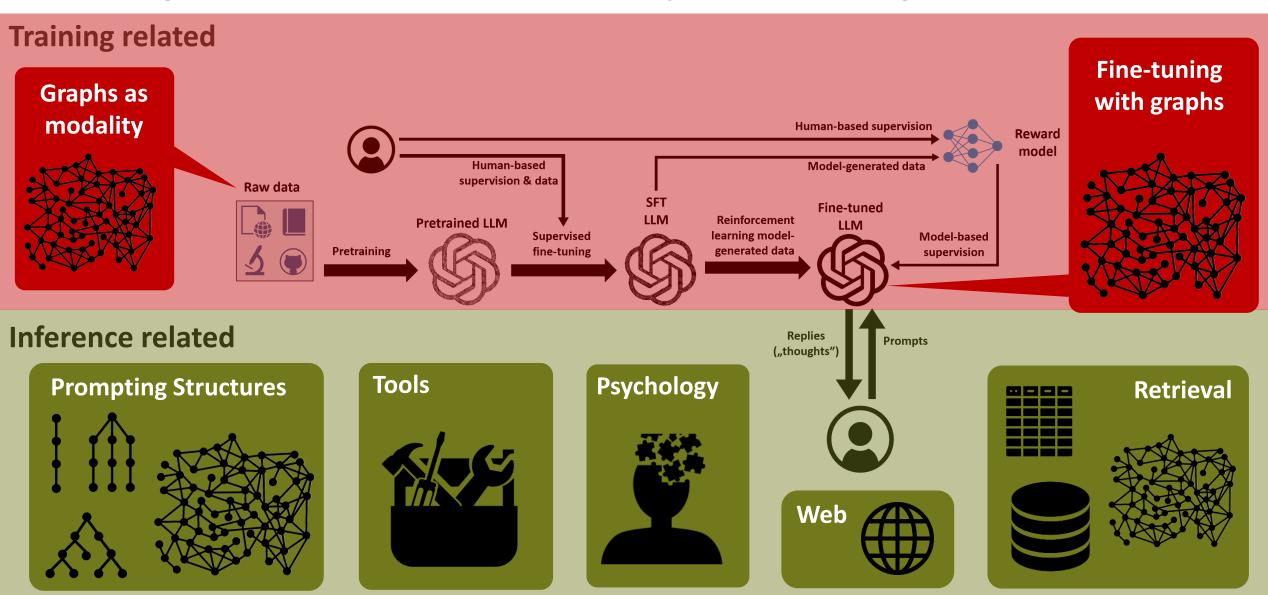












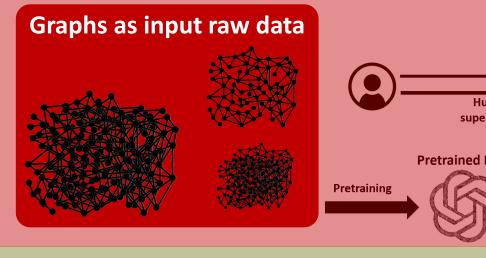


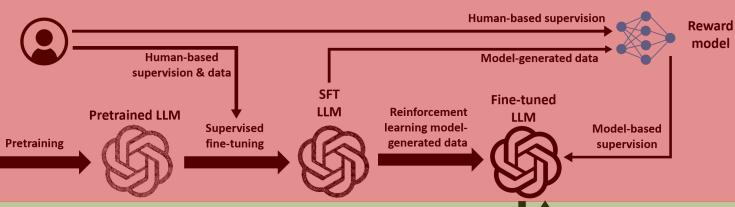




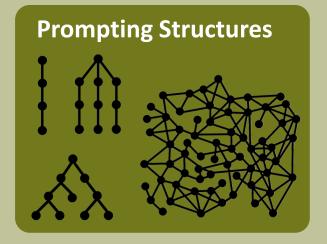
Graph Foundation Models & Graph Prompting

Training related

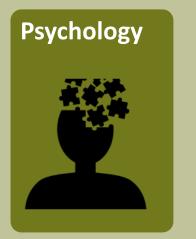


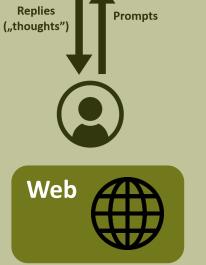


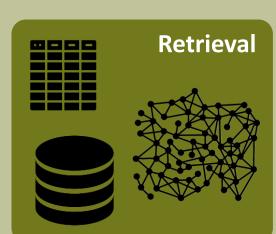
Inference related

















M. Besta, T. Hoefler

WITH N. BLACH, A. KUBICEK, R. GERSTENBERGER, AND MANY OTHERS

Graph of Thoughts: Solving Elaborate Problems with Large Language Models









M. Besta, T. Hoefler

WITH N. BLACH, A. KUBICEK, R. GERSTENBERGER, AND MANY OTHERS

Graph of Thoughts: Solving Elaborate Problems with Large Language Models













Hello. I want to sort the following input sequence of numbers: {input}



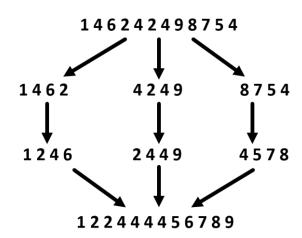




>_

<Instruction> Split the following list of 64 numbers into 4 lists of 16 numbers each, the first list should contain the first 16 numbers, the second list the second 16 numbers, the third list the third 16 numbers and the fourth list the fourth 16 numbers. Only output the final 4 lists in the following format without any additional text or thoughts!

```
{{
    "List 1": [3, 4, 3, 5, 7, 8, 1, ...],
    "List 2": [2, 9, 2, 4, 7, 1, 5, ...],
    "List 3": [6, 9, 8, 1, 9, 2, 4, ...],
    "List 4": [9, 0, 7, 6, 5, 6, 6, ...]
}} </Instruction>
```











<Instruction> Split the following list of 64 numbers into 4 lists of 16 numbers each, the first list should contain the first 16 numbers, the second list the second 16 numbers, the third list the third 16 numbers and the fourth list the fourth 16 numbers. Only output the final 4 lists in the following format without any additional text or thoughts!

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    "List 3": [6, 9, 8, 1, 9, 2, 4, ...],
    "List 4": [9, 0, 7, 6, 5, 6, 6, ...]
}} </Instruction>
```

<Example>

```
Input: [3, 1, 9, 3, 7, 5, 5, 4, 8, 1, 5, 3, 3, 2, 3, 0, 9, 7, 2, 2, 4, 4, 8, 5, 0, 8, 7, 3, 3, 8, 7, 0, 9, 5, 1, 6, 7, 6, 8, 9, 0, 3, 0, 6, 3, 4, 8, 0, 6, 9, 8, 4, 1, 2, 9, 0, 4, 8, 8, 9, 9, 8, 5, 9]

Output:

{{

"List 1": [3, 1, 9, 3, 7, 5, 5, 4, 8, 1, 5, 3, 3, 2, 3, 0],

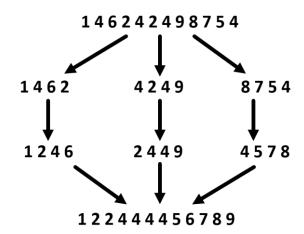
"List 2": [9, 7, 2, 2, 4, 4, 8, 5, 0, 8, 7, 3, 3, 8, 7, 0],

"List 3": [9, 5, 1, 6, 7, 6, 8, 9, 0, 3, 0, 6, 3, 4, 8, 0],

"List 4": [6, 9, 8, 4, 1, 2, 9, 0, 4, 8, 8, 9, 9, 8, 5, 9]

}}

</Example>
```

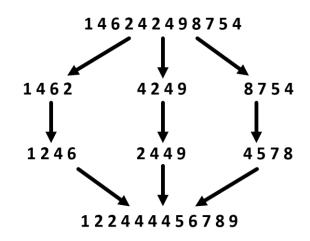








```
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   "List 3": [6, 9, 8, 1, 9, 2, 4, ...],
   "List 4": [9, 0, 7, 6, 5, 6, 6, ...]
}} </Instruction>
<Example>
Input: [3, 1, 9, 3, 7, 5, 5, 4, 8, 1, 5, 3, 3, 2, 3, 0, 9, 7, 2, 2, 4, 4, 8, 5, 0,
8, 7, 3, 3, 8, 7, 0, 9, 5, 1, 6, 7, 6, 8, 9, 0, 3, 0, 6, 3, 4, 8, 0, 6, 9, 8, 4, 1,
2, 9, 0, 4, 8, 8, 9, 9, 8, 5, 97
Output:
   "List 1": [3, 1, 9, 3, 7, 5, 5, 4, 8, 1, 5, 3, 3, 2, 3, 0],
   "List 2": [9, 7, 2, 2, 4, 4, 8, 5, 0, 8, 7, 3, 3, 8, 7, 0],
   "List 3": [9, 5, 1, 6, 7, 6, 8, 9, 0, 3, 0, 6, 3, 4, 8, 0],
   "List 4": [6, 9, 8, 4, 1, 2, 9, 0, 4, 8, 8, 9, 9, 8, 5, 9]
</Example>
                          The input
                          thought t
Input: {input}
```









>_

<Instruction> Split the following list of 64 numbers into 4 lists of 16 numbers each, the first list should contain the first 16 numbers, the second list the second 16 numbers, the third list the third 16 numbers and the fourth list the fourth 16 numbers. Only output the final 4 lists in the following format without any additional text or thoughts!

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"List 2": [2, 9, 2, 4, 7, 1, 5, ...],
"List 3": [6, 9, 8, 1, 9, 2, 4, ...],
"List 4": [9, 0, 7, 6, 5, 6, 6, ...]
}} </Instruction>
```

<Example>

```
Input: [3, 1, 9, 3, 7, 5, 5, 4, 8, 1, 5, 3, 3, 2, 3, 0, 9, 7, 2, 2, 4, 4, 8, 5, 0, 8, 7, 3, 3, 8, 7, 0, 9, 5, 1, 6, 7, 6, 8, 9, 0, 3, 0, 6, 3, 4, 8, 0, 6, 9, 8, 4, 1, 2, 9, 0, 4, 8, 8, 9, 9, 8, 5, 9]

Output:

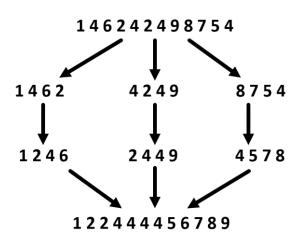
{{
"Hist All's [3, 1, 0, 3, 7, 5, 5, 4, 8, 1, 5, 3, 3, 2, 3, 0]
```

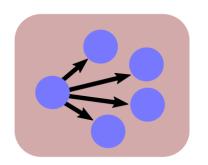
```
"List 1": [3, 1, 9, 3, 7, 5, 5, 4, 8, 1, 5, 3, 3, 2, 3, 0],
"List 2": [9, 7, 2, 2, 4, 4, 8, 5, 0, 8, 7, 3, 3, 8, 7, 0],
"List 3": [9, 5, 1, 6, 7, 6, 8, 9, 0, 3, 0, 6, 3, 4, 8, 0],
"List 4": [6, 9, 8, 4, 1, 2, 9, 0, 4, 8, 8, 9, 9, 8, 5, 9]
```

</Example>

Input: {input}

The input thought t





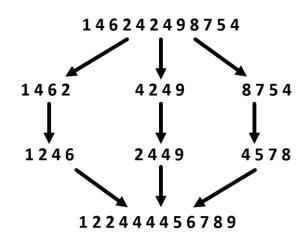


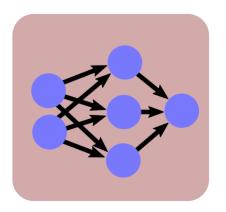




>_

<Instruction> Merge the following 2 sorted lists of length {length1} each, into one sorted list of length {length2} using a merge sort style approach. Only output the final merged list without any additional text or thoughts!
</Instruction>

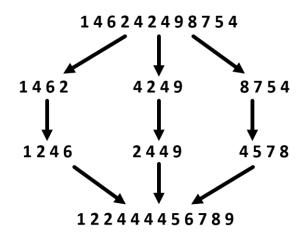












>_

<Instruction> Sort the following list of numbers in ascending order.
Output only the sorted list of numbers, no additional text. </Instruction>

<Example>

Input: [3, 7, 0, 2, 8, 1, 2, 2, 2, 4, 7, 8, 5, 5, 3, 9, 4, 3, 5, 6, 6, 4, 4, 5, 2, 0, 9, 3, 3, 9, 2, 1]

Output: [0, 0, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5,

6, 6, 7, 7, 8, 8, 9, 9, 9]

</Example>

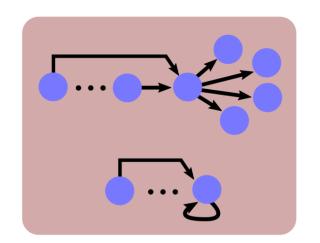
Input: {input}

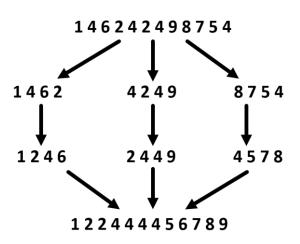














<Instruction> Sort the following list of numbers in ascending order.
Output only the sorted list of numbers, no additional text. </Instruction>

<Example>

Input: [3, 7, 0, 2, 8, 1, 2, 2, 2, 4, 7, 8, 5, 5, 3, 9, 4, 3, 5, 6, 6, 4, 4, 5, 2, 0, 9, 3, 3, 9, 2, 1]

Output: [0, 0, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 7, 7, 8, 8, 0, 0, 0]

6, 6, 7, 7, 8, 8, 9, 9, 9]

</Example>

Input: {input}















