Rethinking the Micro-Foundation of Opinion Dynamics

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Biography

Wenjun Mei (梅文俊)

- B.S. in Theoretical and Applied Mechanics, Peking University 2007 – 2011
- Ph.D. in Mechanical Engineering, UC Santa Barbara 2011 – 2017

  — Advisor: Prof. Francesco Bullo
- Postdoctoral Researcher, ETH Zurich 2018 –

  — Advisor: Prof. Florian Dörfler
- Editorial board: Journal of Mathematical Sociology

2020 Pre-CDC Workshop on Dynamics in Social and Economic Networks

- Long talk speakers
  - Prof. Tamer Basar (UIUC)
  - Prof. Munther Dahleh (MIT)
  - Prof. Francesco Bullo (UCSB)
  - Prof. Jeff Shamma (KAUST)
  - Prof. Giacomo Como (PoliTo)

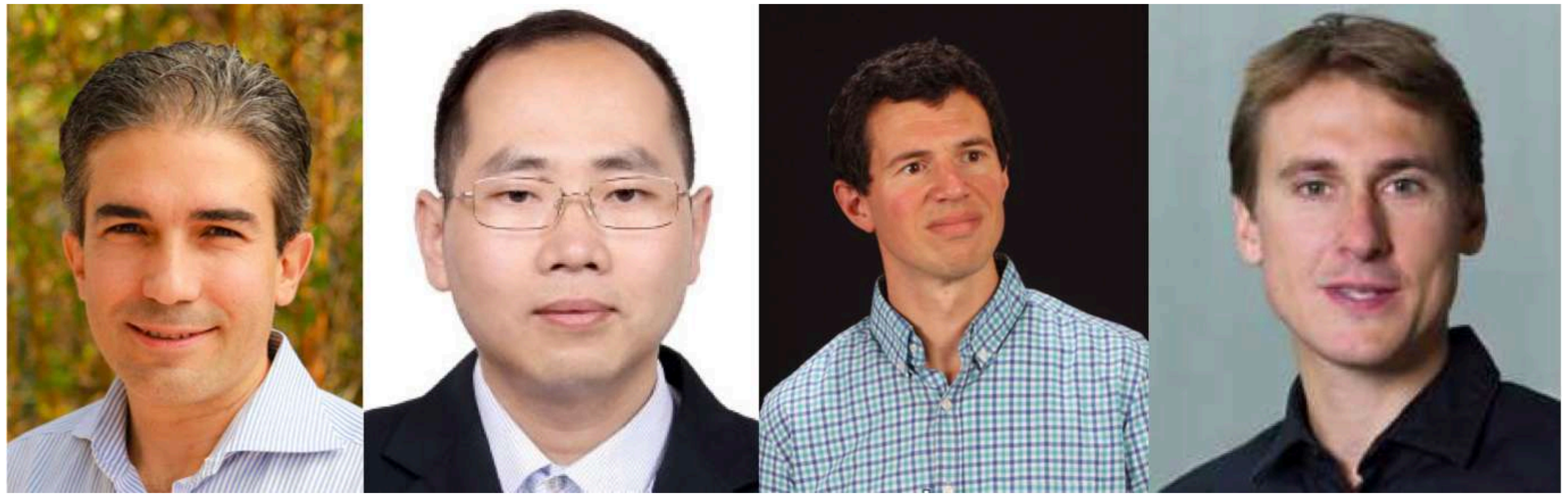
- Subsessions
  - Opinion dynamics
  - Information transmission and network diffusion
  - Network Games and Economic Behavior
  - Network Formation and Network analysis

December 7th, 2020, Jeju Island, Korea

CDC 2020
Jeju Island, Korea

Outline

- Introduction and Motivation
- Derivation of New Model
- Empirical validation
- Comparative Numerical Study
- Theoretical Analysis
- Extensions and Future Directions



Prof. Francesco Bullo   Prof. Ge Chen   Prof. Julien Hendrickx   Prof. Florian Dorfler

Social Science from Control Theorists' Perspectives

Why do control theorists study social science?

— Big data: qualitative → quantitative understanding of social systems

— Engineering problem in social systems, e.g. control of epidemics

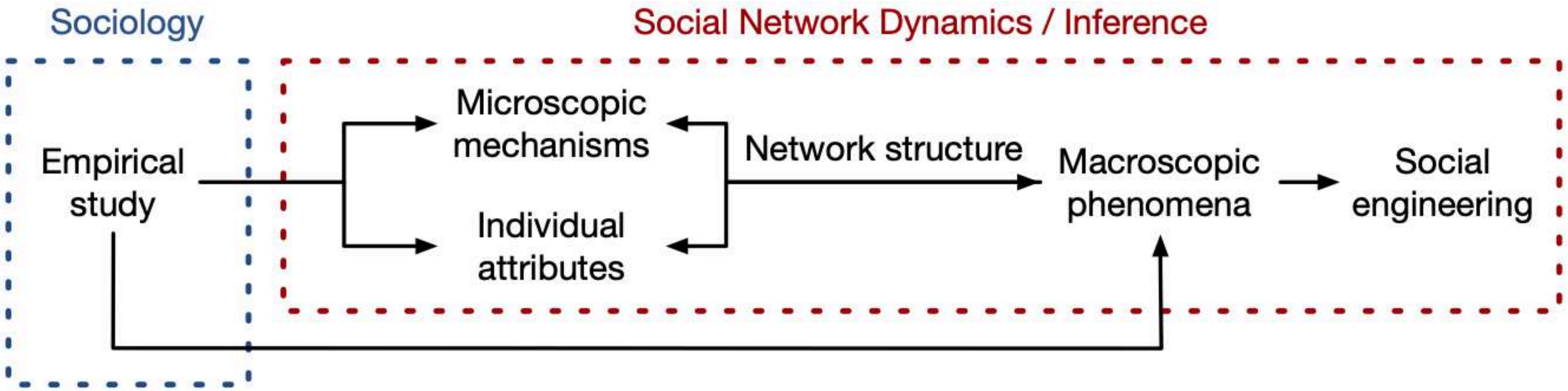
— Social science in engineering systems, e.g. *"human in the loop"*

Why are we able to contribute?

— human groups: networked multi-agent systems

— mathematical tools: graph theory, game theory, stochastic process, linear/nonlinear systems, control theory

How do we contribute to social science?



Mathematical Models of Opinion Dynamics

Motivation: public opinion formation faces unprecedented challenges

— social media/recommendation systems: echo chamber, filter bubble

— opinion radicalization, ideology polarization, misinformation, manipulation

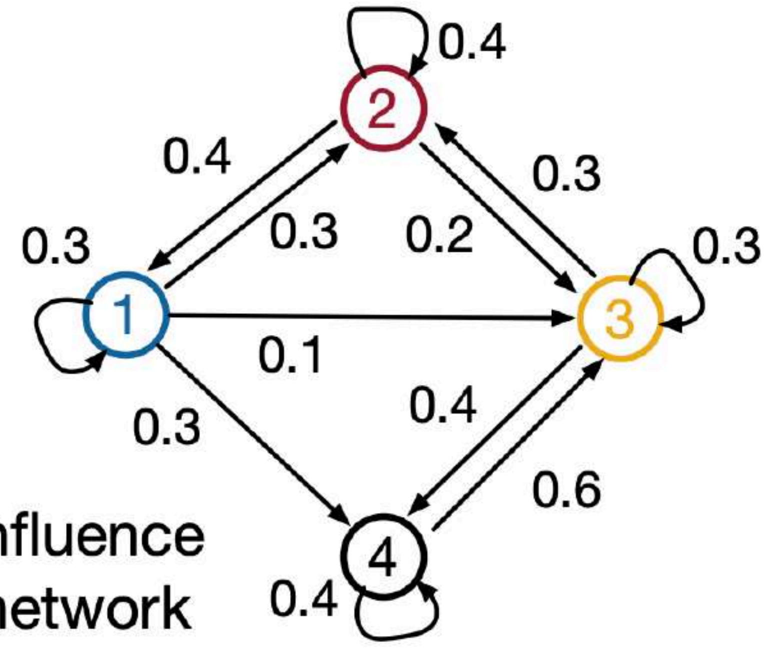
— reliable & quantitative understanding: how social influence shapes opinions

The Classic DeGroot model [JF-56]:

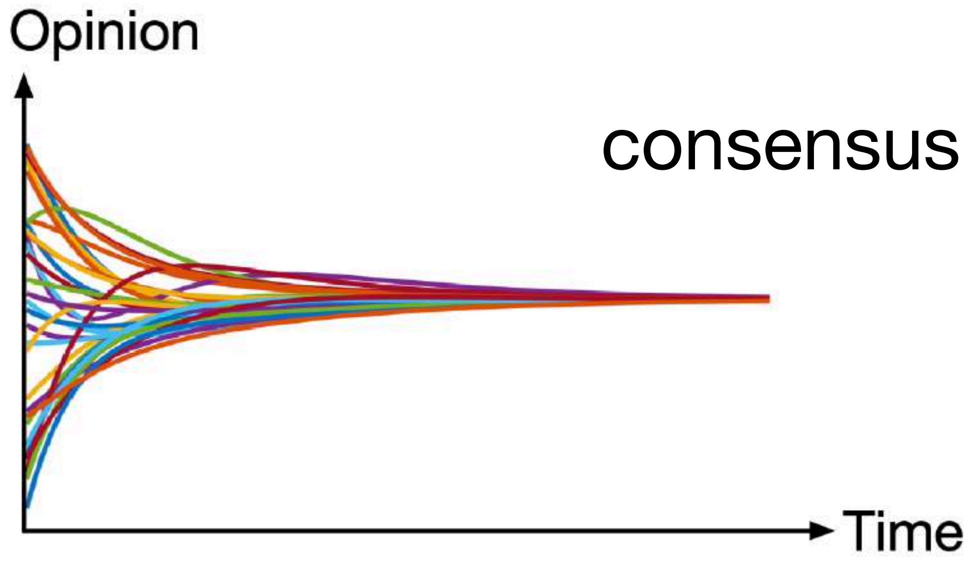
$$x_i(t+1) = \sum_{j=1}^{n} w_{ij}\, x_j(t) \;\Rightarrow\; x(\infty) = \alpha \mathbb{1}_n$$

$$W = \begin{bmatrix} 0.3 & 0.3 & 0.1 & 0.3 \\ 0.4 & 0.4 & 0.2 & 0 \\ 0 & 0.3 & 0.3 & 0.4 \\ 0 & 0 & 0.6 & 0.4 \end{bmatrix} \Leftrightarrow$$

Influence matrix



Influence network



— Assumption 1: opinions as real numbers

— Assumption 2: weighted-averaging update

— Overly simplified prediction: connected network → consensus

[JF-56] J. R. P. French, *Psychological Review*, 63(3):181-194, 1956.

Important extensions

- Absolutely stubborn agents [DA-13]

$$x(t+1) = Wx(t), \;\; \exists\, i \text{ such that } W_{ii} = 1$$

- Bounded-confidence model [RH-02]

$$x_i(t+1) = \sum_{j \in \mathcal{N}_i} x_j \Big/ |\mathcal{N}_i|,$$

where $\mathcal{N}_i = \{j \mid |x_j - x_i| \leq r_i\}$.

- Friedkin-Johnsen model: attachment to initial opinions [NEF-90]

$$x(t+1) = (I - \Lambda)Wx(t) + \Lambda x(0)$$

- Altafini model: antagonistic interactions [CA-13]

$$x(t+1) = Wx(t), \;\; \exists\, i, j \text{ such that } W_{ij} < 0$$

Additional assumptions & parameters → disagreement

[DA-13] D. Acemoglu, G. Como, F. Fagnani, and A. Ozdaglar, Mathematics of Operation Research, 38(1):1–27, 2013.

[NEF-90] N. E. Friedkin and E. C. Johnsen, Journal of Mathematical Sociology, 15(3-4):193–206, 1990.

[RH-02] R. Hegselmann and U. Krause, Journal of Artificial Societies and Social Simulation, 5(3), 2002.

[CA-13] C. Altafini, IEEE Transactions on Automatic Control, 58(4):935–946, 2013.

Further extensions

— Time-varying graph / switching topology

— Gossip-like opinion dynamics

— Quantized opinion dynamics

— Multiple issues with logical constraints

— State-dependent stubbornness

— Unilateral bounded-confidence model

— Private and expressed opinions

……

— Gossip opinion dynamics with negative weights

— Multiple issues with heterogenous logical constraints

— Antagonistic interactions with switching totology

— Gossip-like quantized opinion dynamics

— Convergent rate of gossip-like bounded-confidence model

— Convergent rate of opinion dynamics with negative weights

— Multiple-issue opinion dynamics with negative weights

……

Increasing mathematical sophistication v.s. limited predictive power

— e.g. The more people, the more difficult to reach consensus.

— DeGroot: NO, DeGroot with stubborn agents: NO, Friedkin-Johnsen: NO, Altafini: opposite, Bounded-confidence: opposite

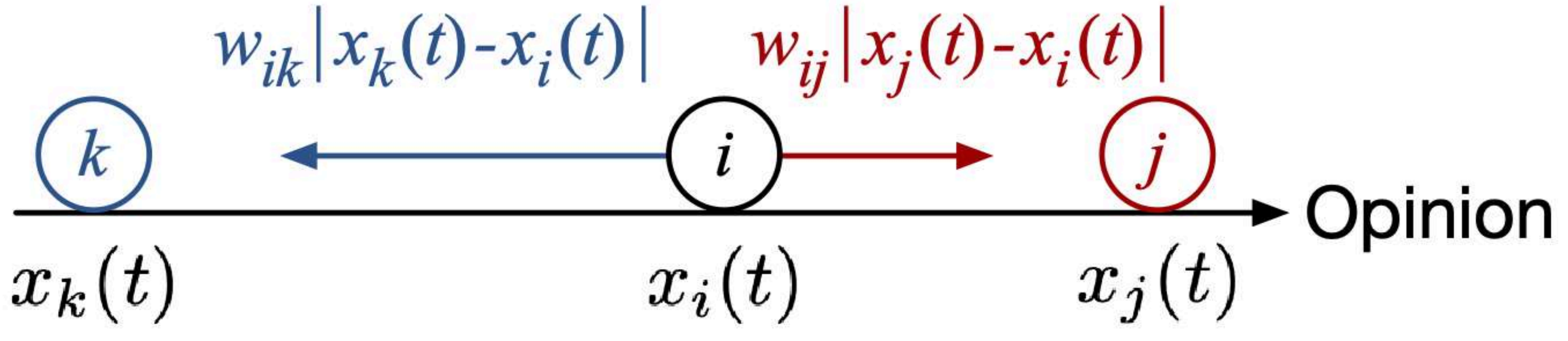
— All the models mentioned above: NO.

What could be wrong?

$$x(t+1) = Wx(t)$$

Weighted-Averaging: Taken for Granted But Unrealistic

- Opinion “attractiveness” ∼ opinion distance
- Intuition behind consensus, inherited by its extensions



$$x_i(t+1)=x_i(t)+w_{ik}(x_k(t)-x_i(t))+w_{ij}(x_j(t)-x_i(t))$$

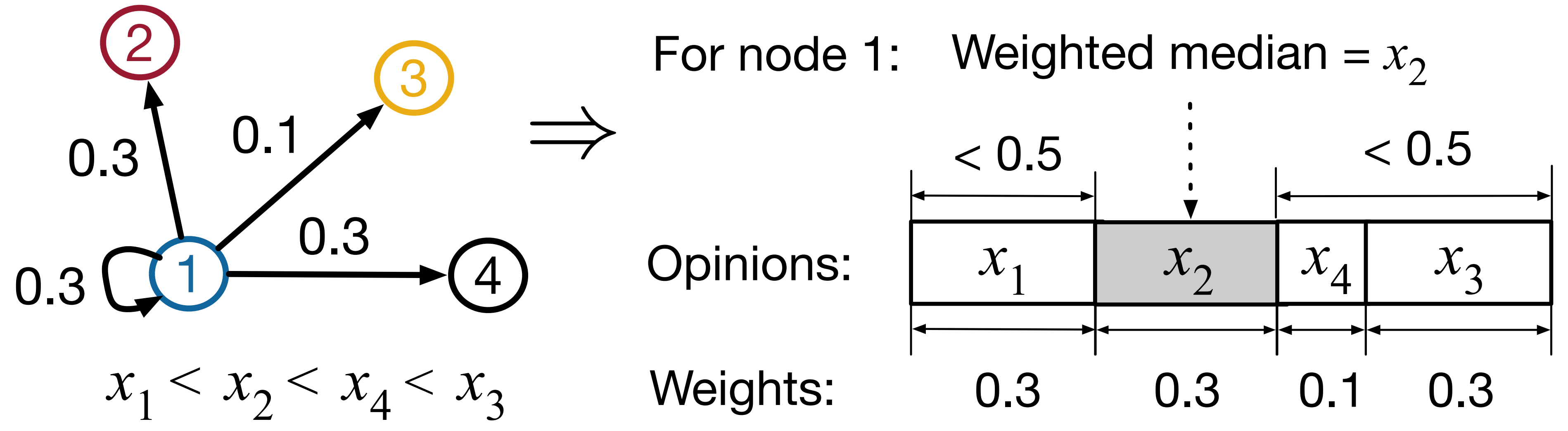
Rethink the micro-foundation of opinion dynamics

- Cognitive dissonance caused by disagreement [LF-62, DM-05]

$$u_i(x) = \sum_j w_{ij}|x_i - x_j|^\alpha, \quad x_i^+ = \text{argmin}_z u_i(z, x_{-i})$$

- $\alpha > 1$: more sensitive to distant opinions (DeGroot: $\alpha = 2$)
- $\alpha < 1$: more sensitive to nearby opinions
- $\alpha = 1$: weighted-median mechanism

$$x_i^+ = \text{argmin}_z \sum_j w_{ij}|z - x_j| = \text{Med}_i(x; W)$$



[LF-62] L. Festinger, “A Theory of Cognitive Dissonance.” Stanford University Press, 1962.

[DM-05] D. C. Matz et al., *Journal of Personality and Social Psychology*, 88(1):22–37, 2005.

Empirical validation

- **Online human-subject experiment** [CK-16]
  - — Each experiment: 6 participants, 30 questions, 3 rounds
  - — Revise answers based on others' answers in previous round

- **Median v.s. Average in predicting opinion shifts**

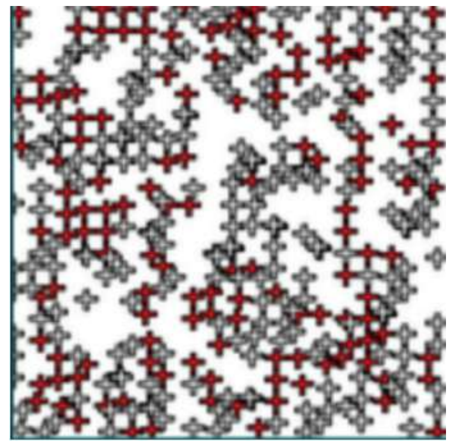
Hypo. 1: $x_i(r+1) = \text{Med}\big(x(r)\big)$

Hypo. 2: $x_i(r+1) = \text{Ave}\big(x(r)\big)$

Hypo. 3: $x_i(r+1) = a_i(r)\, x_i(r) + (1 - a_i(r))\, \text{Med}(x(r))$
Hypo. 4: $x_i(r+1) = b_i(r)\, x_i(r) + (1 - b_i(r))\, \text{Ave}(x(r))$

Hypo. 5: $x_i(r+1) = c_i(r)\, x_i(1) + (1 - c_i(r))\, \text{Med}(x(r))$
Hypo. 6: $x_i(r+1) = d_i(r)\, x_i(1) + (1 - d_i(r))\, \text{Ave}(x(r))$

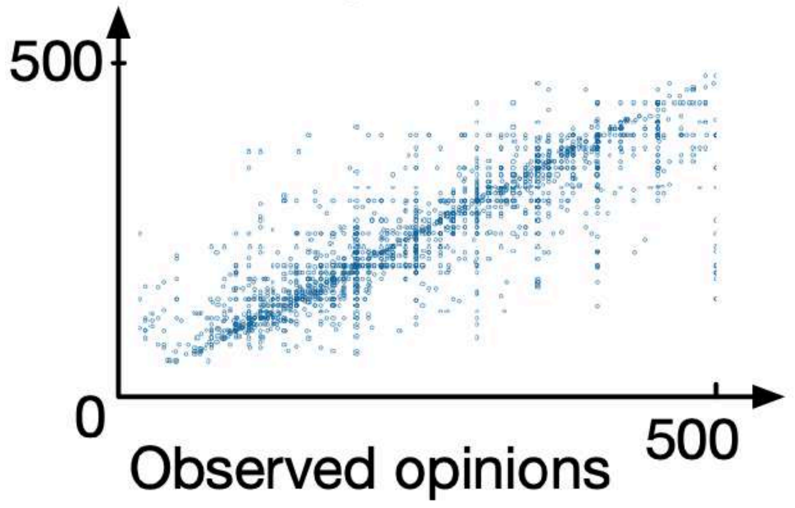
- **From average to median: median error rate reduced by 46.36%**



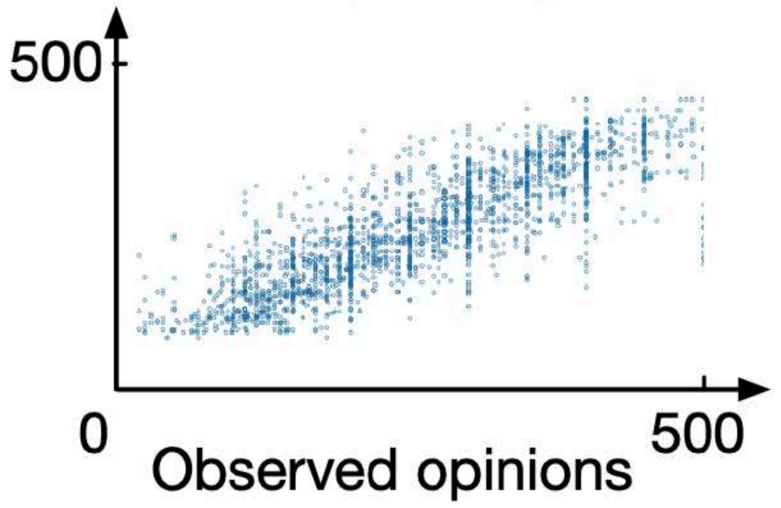
How many red dots do you see?

Answers / Indiv. ID	Question 1		
	Round 1	Round 2	Round 3
55	178	183	194
611	200	200	190
735	250	200	200
780	240	200	200
782	150	160	180
823	120	190	196

Predictions by median



Predictions by average



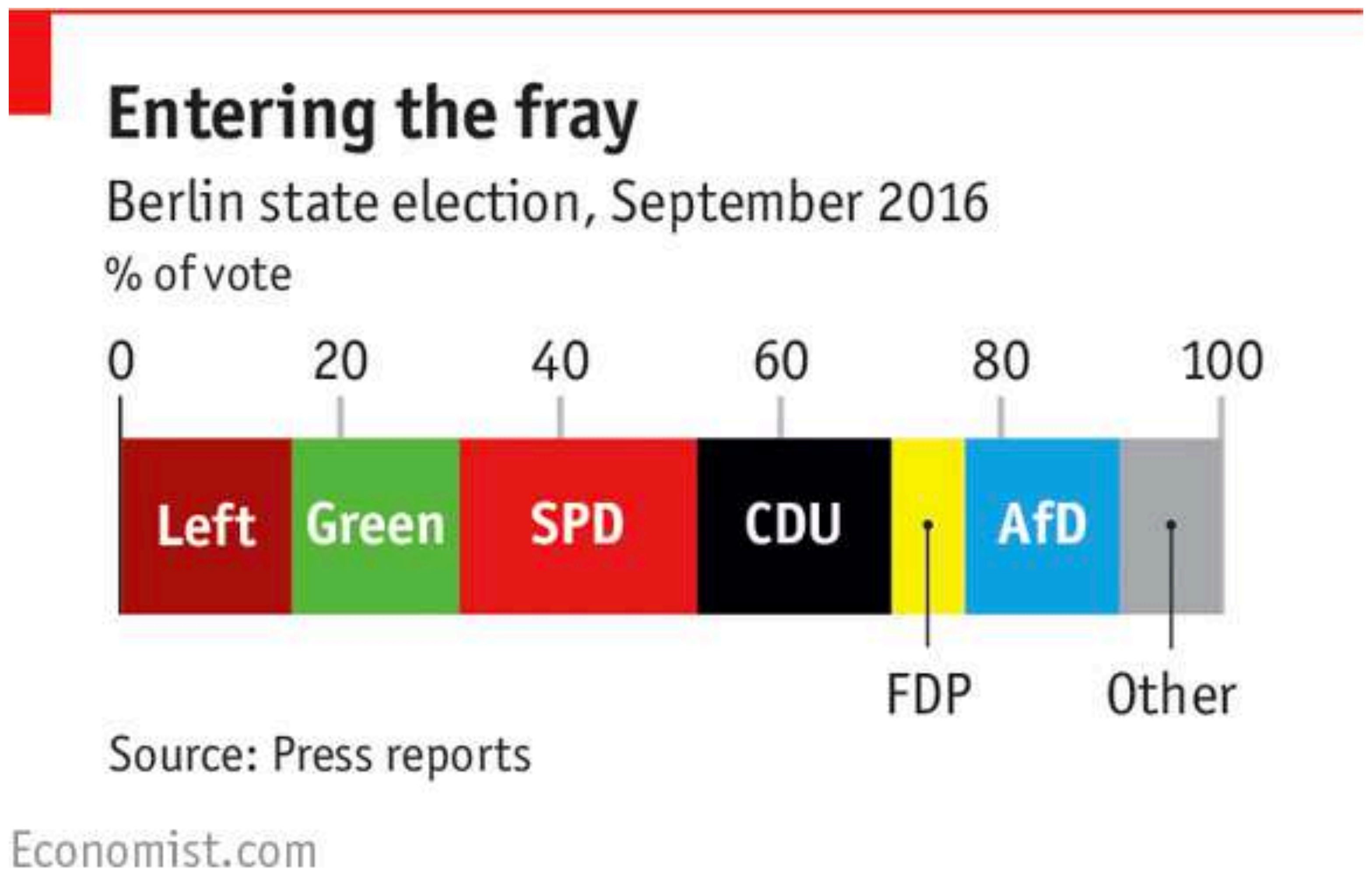
Counting Games, 3rd-round opinions

Predictions by	Median error rate	95% confidence interval	MER
Hypothesis 1	0.0714	[0.0667, 0.0769]	0.1776
Hypothesis 2	0.1331	[0.1230, 0.1408]	0.2332
Hypothesis 3	0.0291	[0.0242, 0.0330]	0.0698
Hypothesis 4	0.0349	[0.0299, 0.0392]	0.0724
Hypothesis 5	0.0507	[0.0435, 0.0592]	0.0939
Hypothesis 6	0.0744	[0.0656, 0.0794]	0.1091

[CK-16] C. Vande Kerckhove et al., *PLOS One*, 11(6):1–25, 06, 2016.

Inconspicuous microscopic change → rich macroscopic consequences

- **Simplest in form**
- **Broader applicability:** ordered multiple-choice issues
- Numerical comparisons: **more realistic predictions**
- Theoretical analysis: **richer & more robust dynamical behavior**
- Open up **new lines of research**



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Models in comparison
(with randomised parameters)

— Weighted-median opinion dynamics
— DeGroot with stubborn agent (DS)
— Friedkin-Johnsen (F-J)
— Networked bounded-confidence* (NBC)

* The ϵ-convergence time for NBC can be arbitrarily large. [RP-19]

[RP-19] R. Parasnis, *IEEE CDC*, 6431–6436, 2019.

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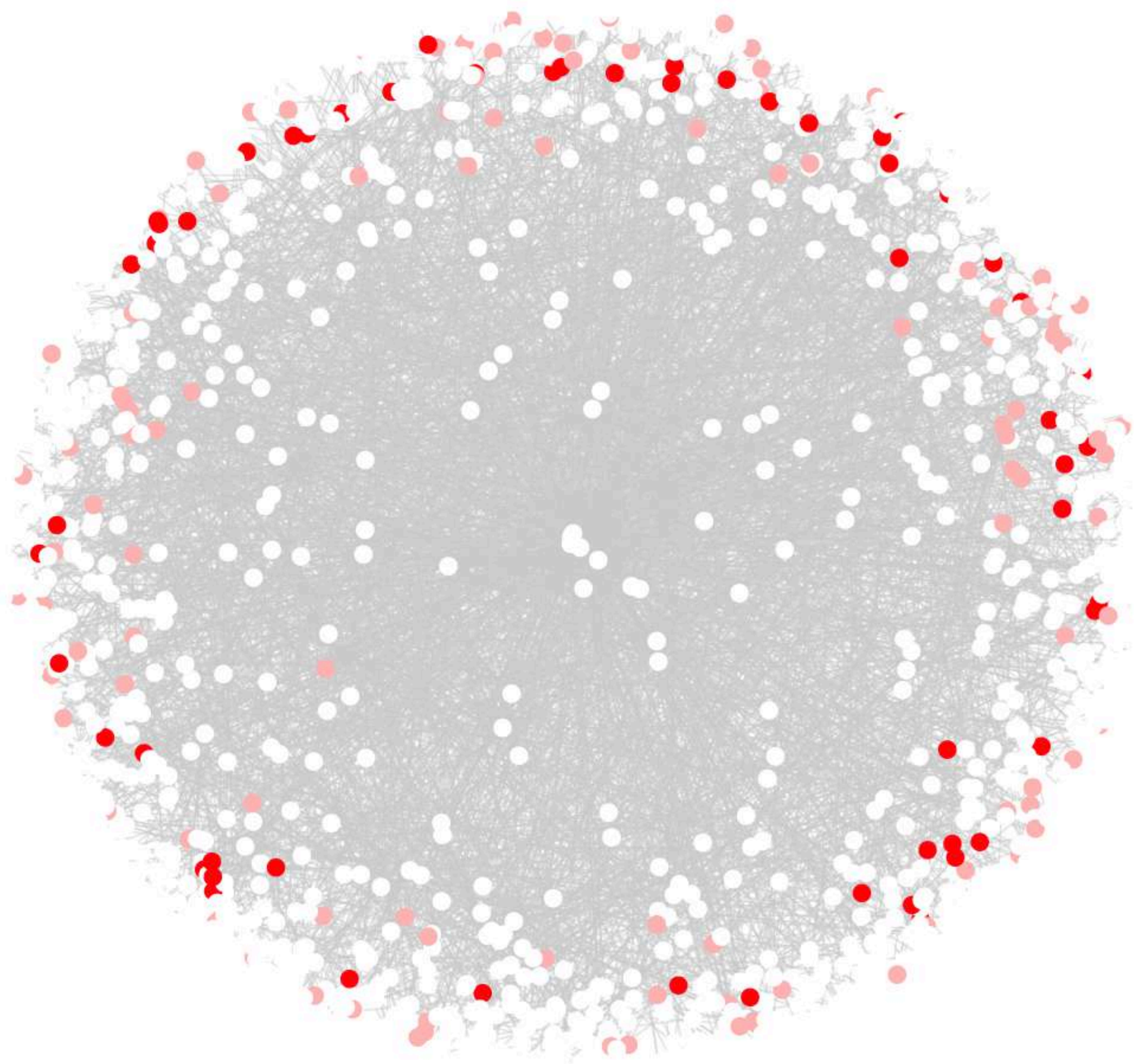
- Scale-free network, 1500 nodes
- 1000 independent simulations
- Initial opinions ~ Unif [0,1]
- Red: prob. of being extreme



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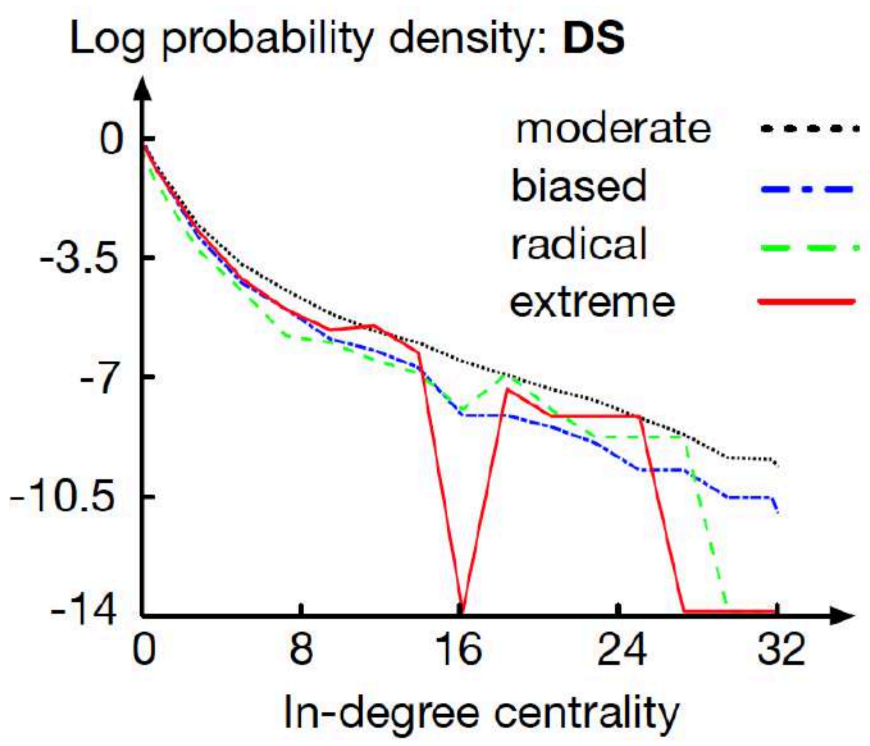
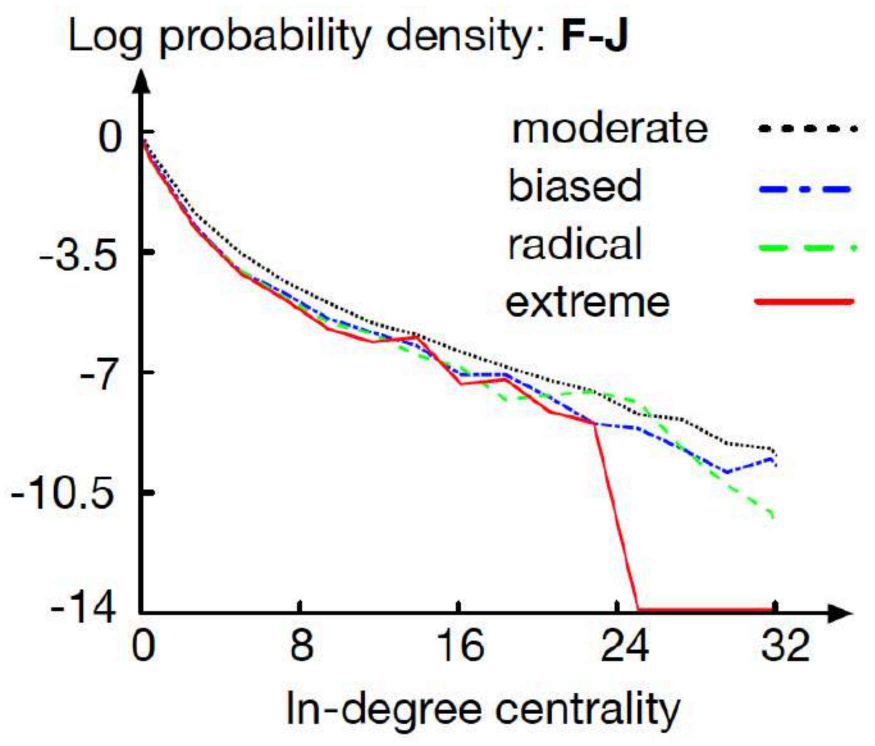
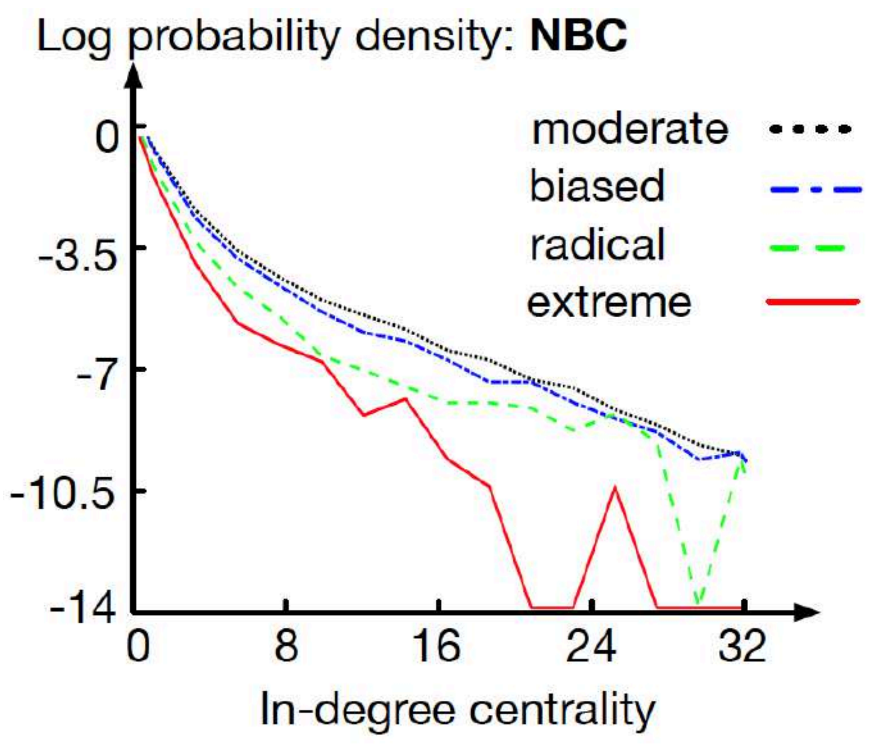
Centrality distributions for different types of opinions.







Moderate: [-0.25,0.25]

Biased: [-0.5,-0.25) ∪ (0.25,0.5]

Radical: [-0.75,-0.5) ∪ (0.5,0.75]

Extreme: [-1,-0.75) ∪ (0.75,1]

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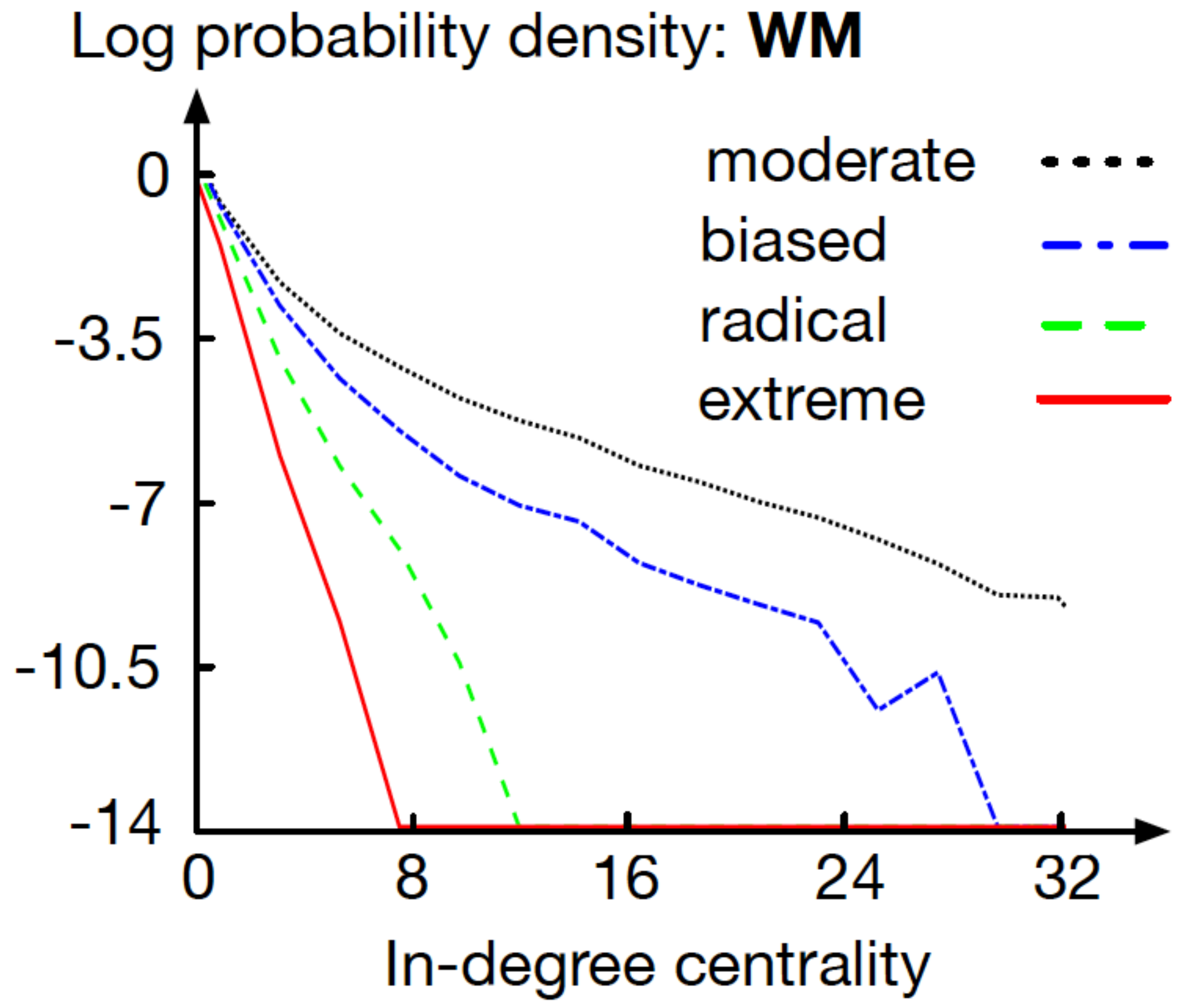

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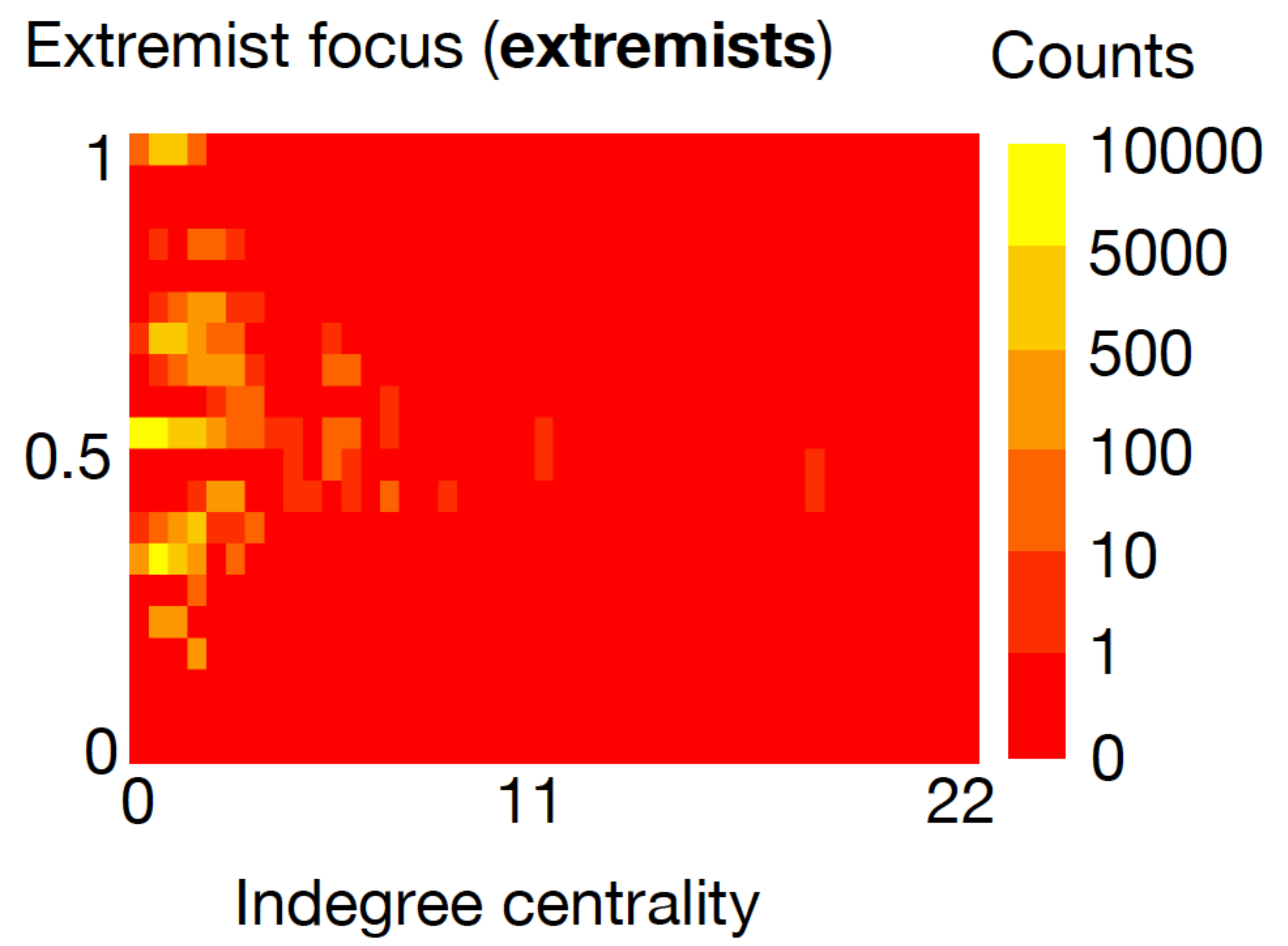
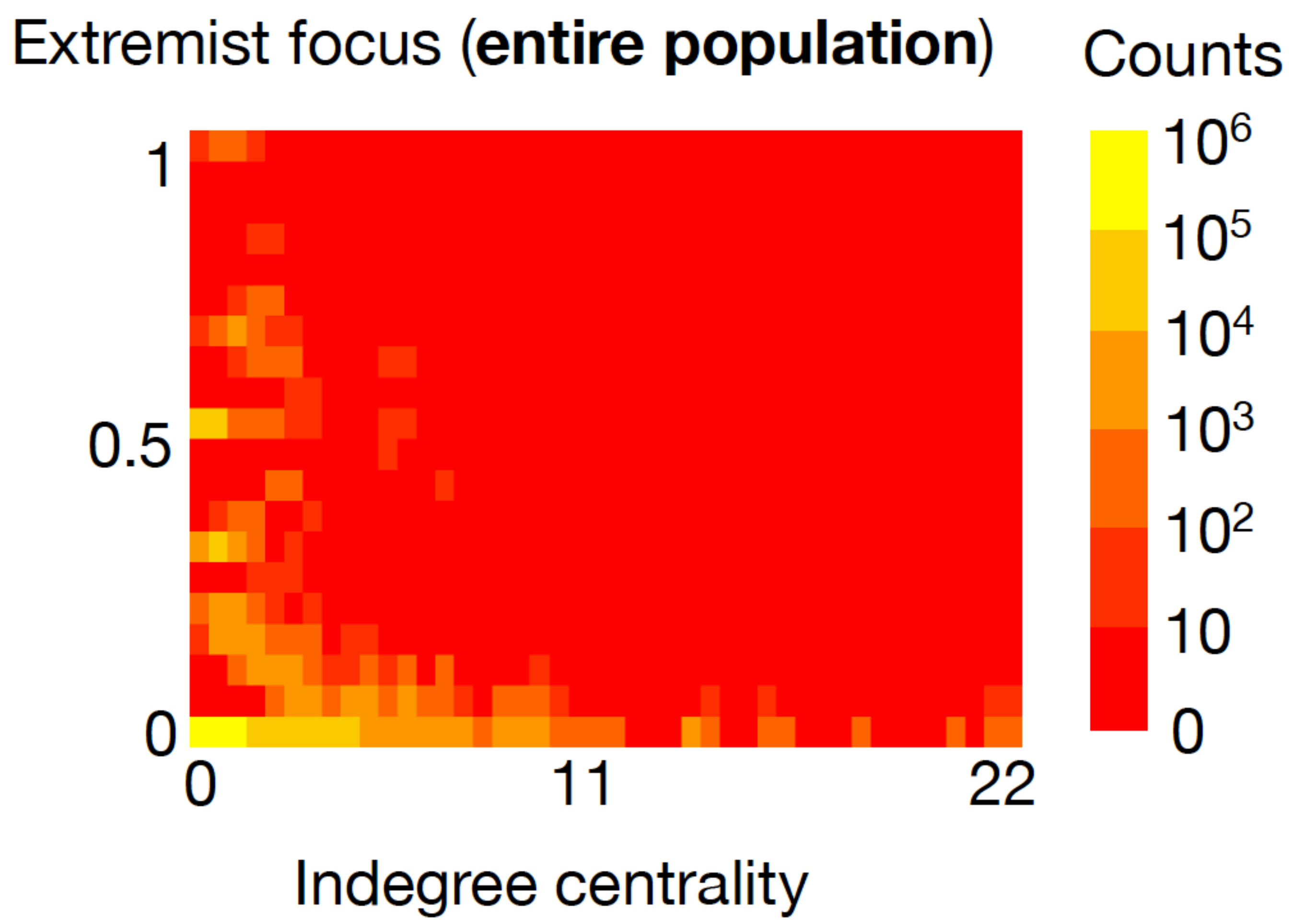
Extremist focus = # extreme neighbors / # out-neighbors





[8] M. C. Benigni et al., *PLOS One*,12 (12): e0181405, 2017.

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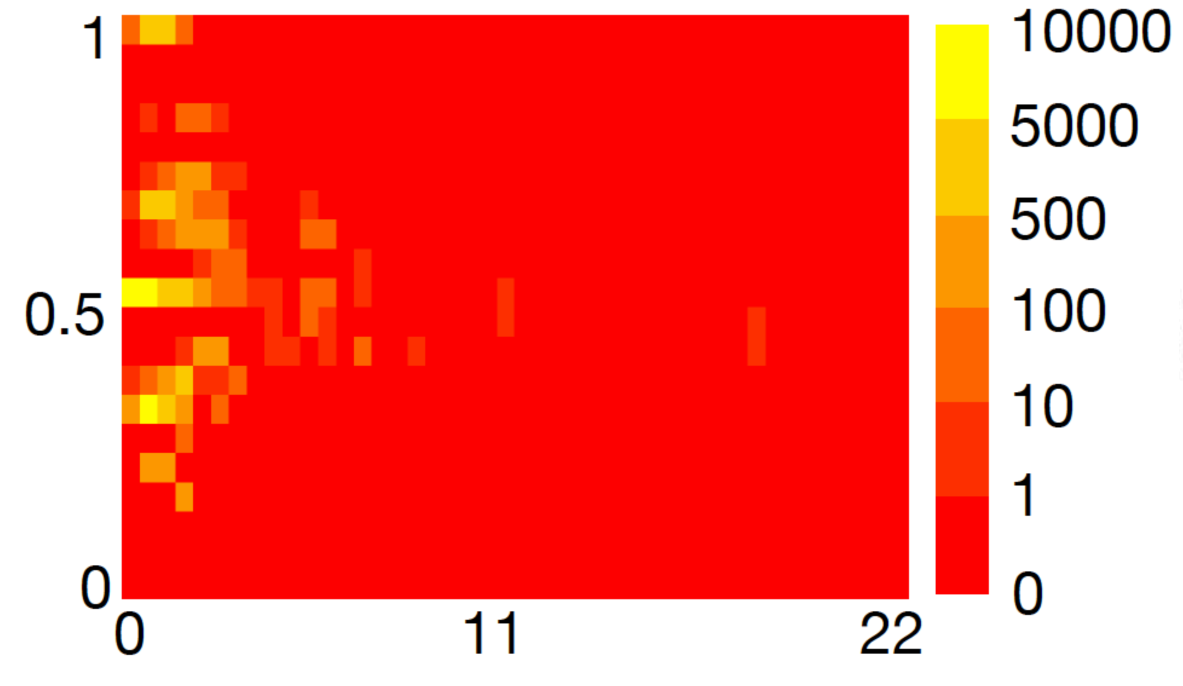
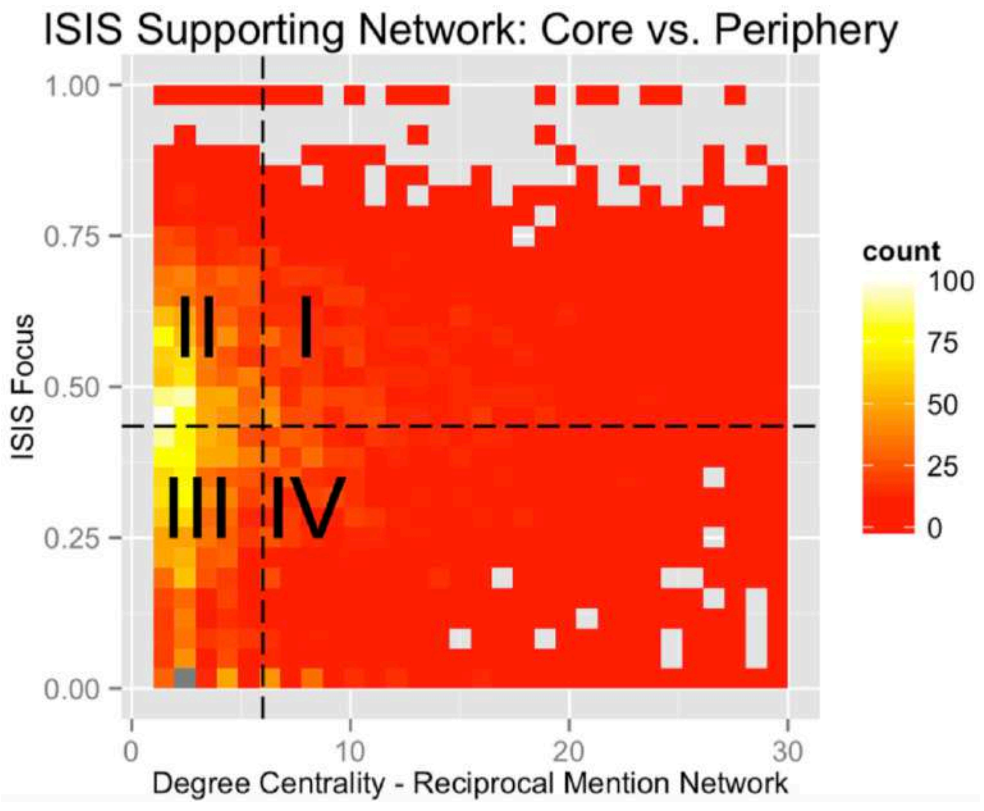
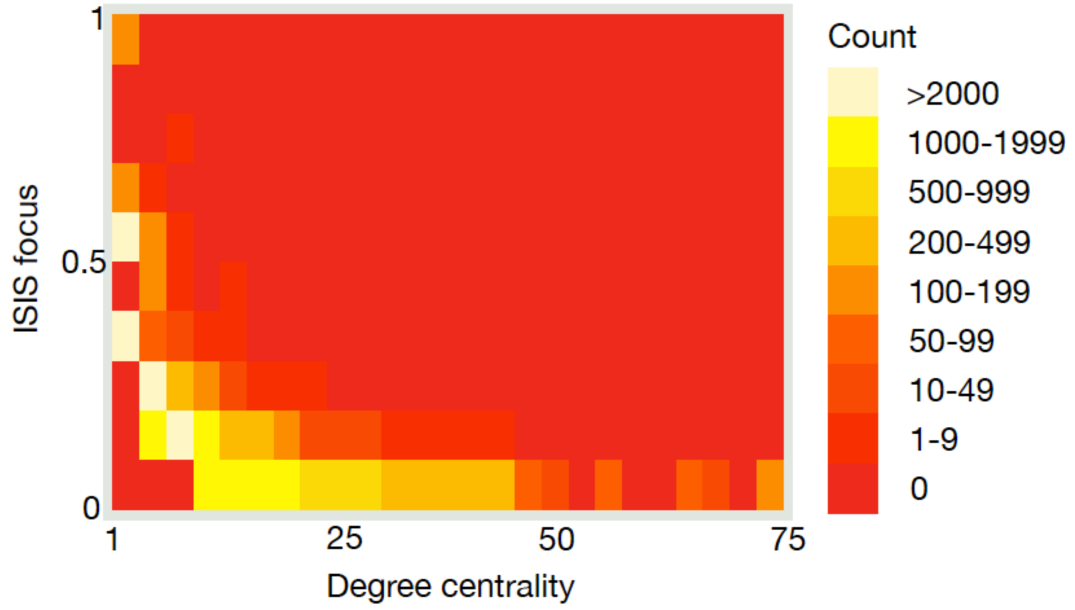
Extremist focus (**extremists**)



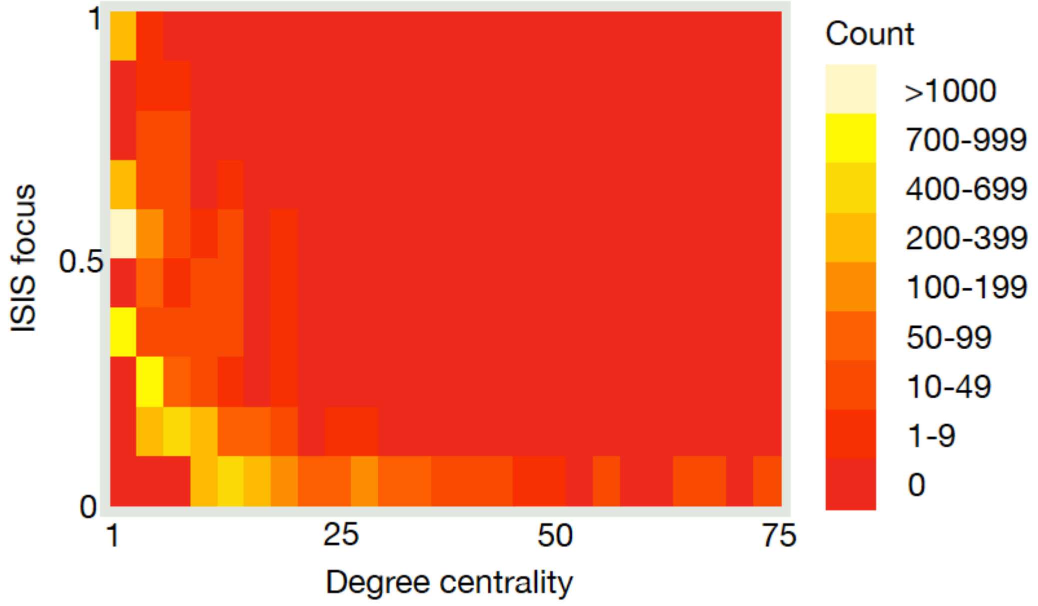
Real data [MB-17]:
ISIS supporters on Twitter



Friedkin-Johnsen model



DeGroot w./ stubborn agents



[MB-17] M. C. Benigni et al., *PLOS One*,12 (12): e0181405, 2017.

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- **Simplest in form**
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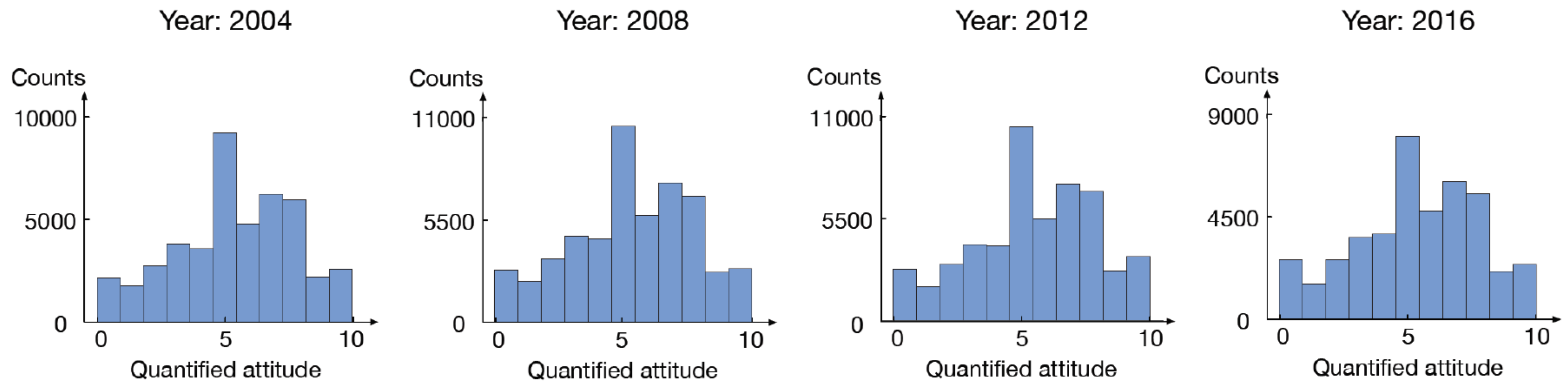
  — Extreme opinions tend to reside in peripheral areas
  — Empirically observed public opinion distributions.
- Theoretical analysis: **richer & more robust dynamical behavior**
- Open up **new lines of research**

- European Social Survey: Do immigrants undermine local culture?



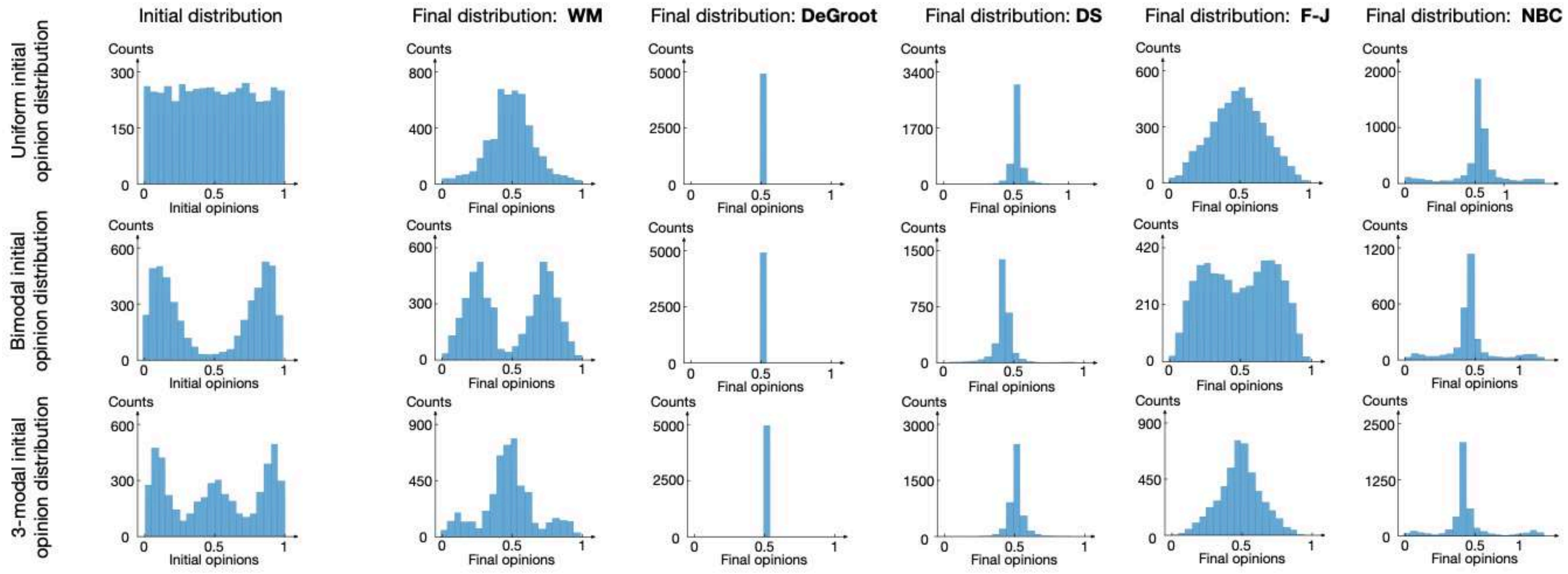
What opinion dynamics models generate various empirically observed opinion distributions [RA-64, NF-15]*?*

Weighted-median model: simplest answer

[RA-64] R. P. Abelson, *Contributions to Mathematical Psychology*, 14:142–160, 1964

[NF-15] N. E. Friedkin, *IEEE Control Systems*, 35(3):40–51, 2015

Setup: scale-free network, 5000 nodes, initial opinions ~ Unif [0,1]



Acronyms: **WM** = the weighted-median model; **DS** = the DeGroot model with absolutely stubborn agents; **F-J** = the Friedkin-Johnsen model; **NBC** = the networked bounded-confidence model.

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- Numerical comparisons: **more realistic predictions**
  — Extreme opinions tend to reside in peripheral areas
  — Empirically observed public opinion distributions.
  — Lower consensus probability in larger networks
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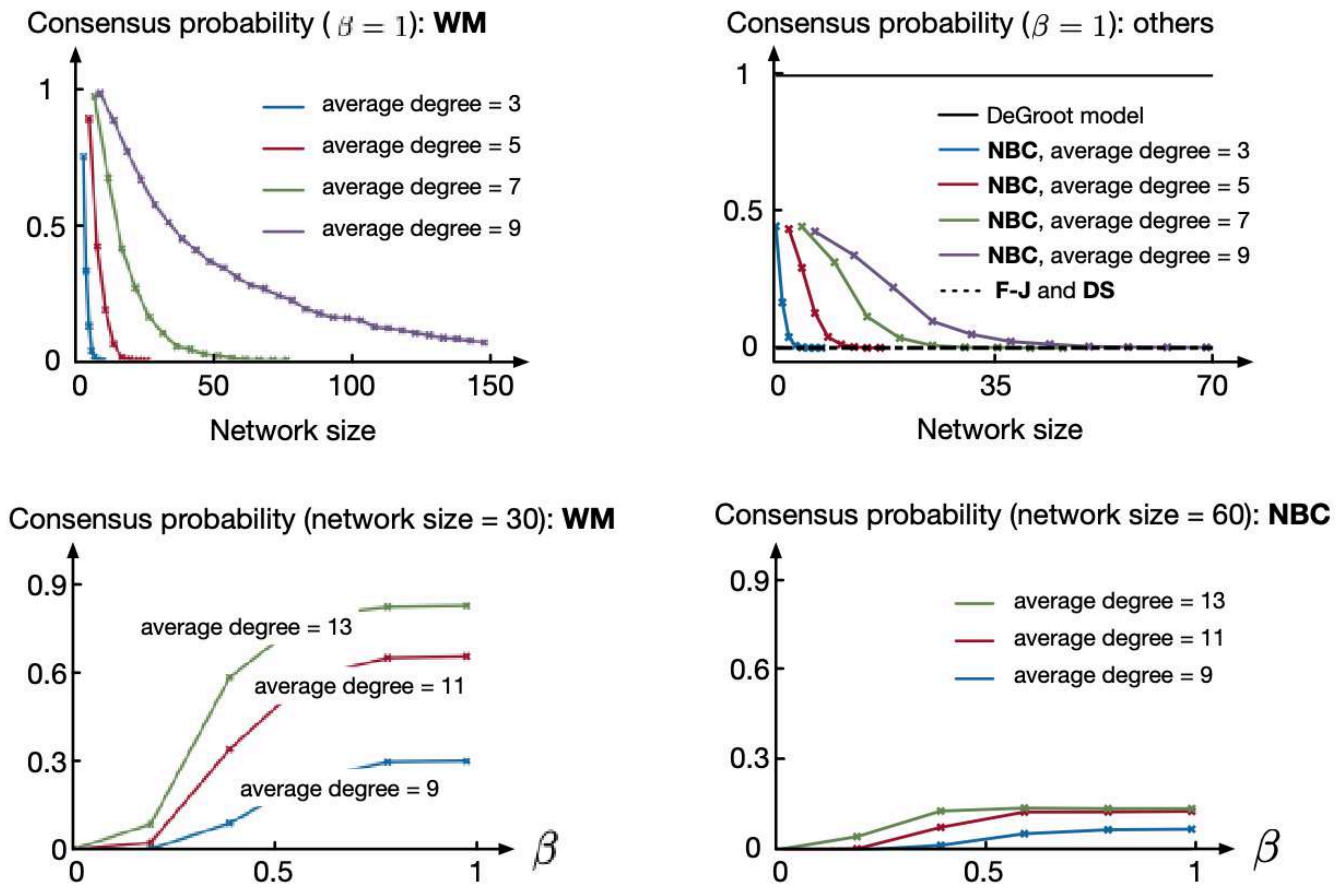
Watts-Strogatz small-world networks:

— n: network size, d: average degree

— β: rewiring prob. (smaller → more clusters)



Physical intuition: why does the weighted-median model behave differently?

DeGroot model and its extensions

$$x_i^+ = \text{argmin}_z \sum_j w_{ij} |z - x_j|^2 + \text{additional terms}$$

Overly large "attractive forces" between distant opinions.
Network structures cannot resist the tendency to consensus.

1. Introduce additional individual dynamics irrelevant to network structure. e.g. stubbornness, attachment to initial opinions.

- Too strong assumptions;
- Artificially generated disagreement;
- Network structures do not play any important role.

2. Artificially truncate the attractive forces between distant opinions. e.g. bounded-confidence.

- Unclear sociological interpretation;
- Role of network does not fully emerge;
- Almost mathematically intractable.

Weighted-median opinion dynamics

$$x_i^+ = \text{argmin}_z \sum_j w_{ij} |z - x_j|$$

Distant opinions are not more attractive. Only the ordering of the opinions and the associated weights matter.

Role of delicate network structures naturally emerge.

— What delicate network structures?
— How do they influence the dynamics?

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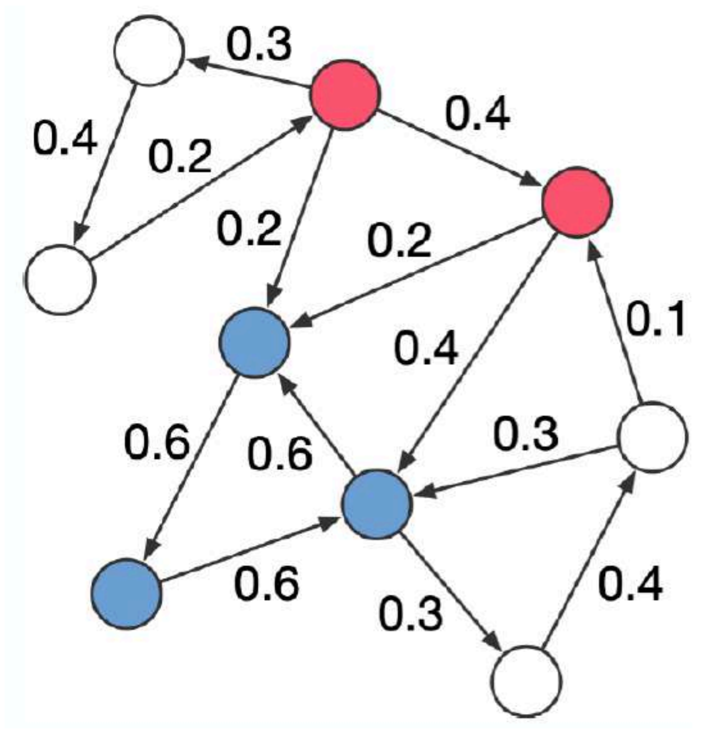
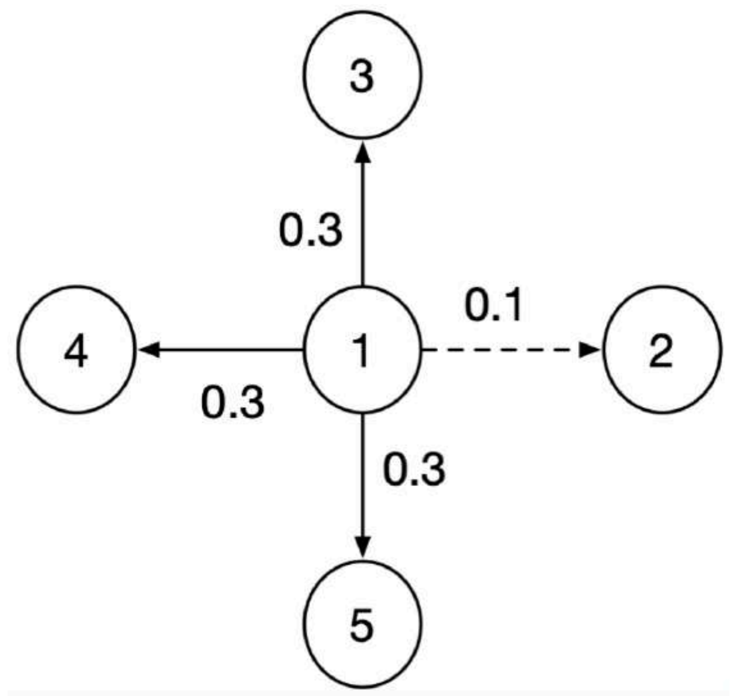


(maximal) cohesive set[9]

Echo chambers



decisive/indecisive links

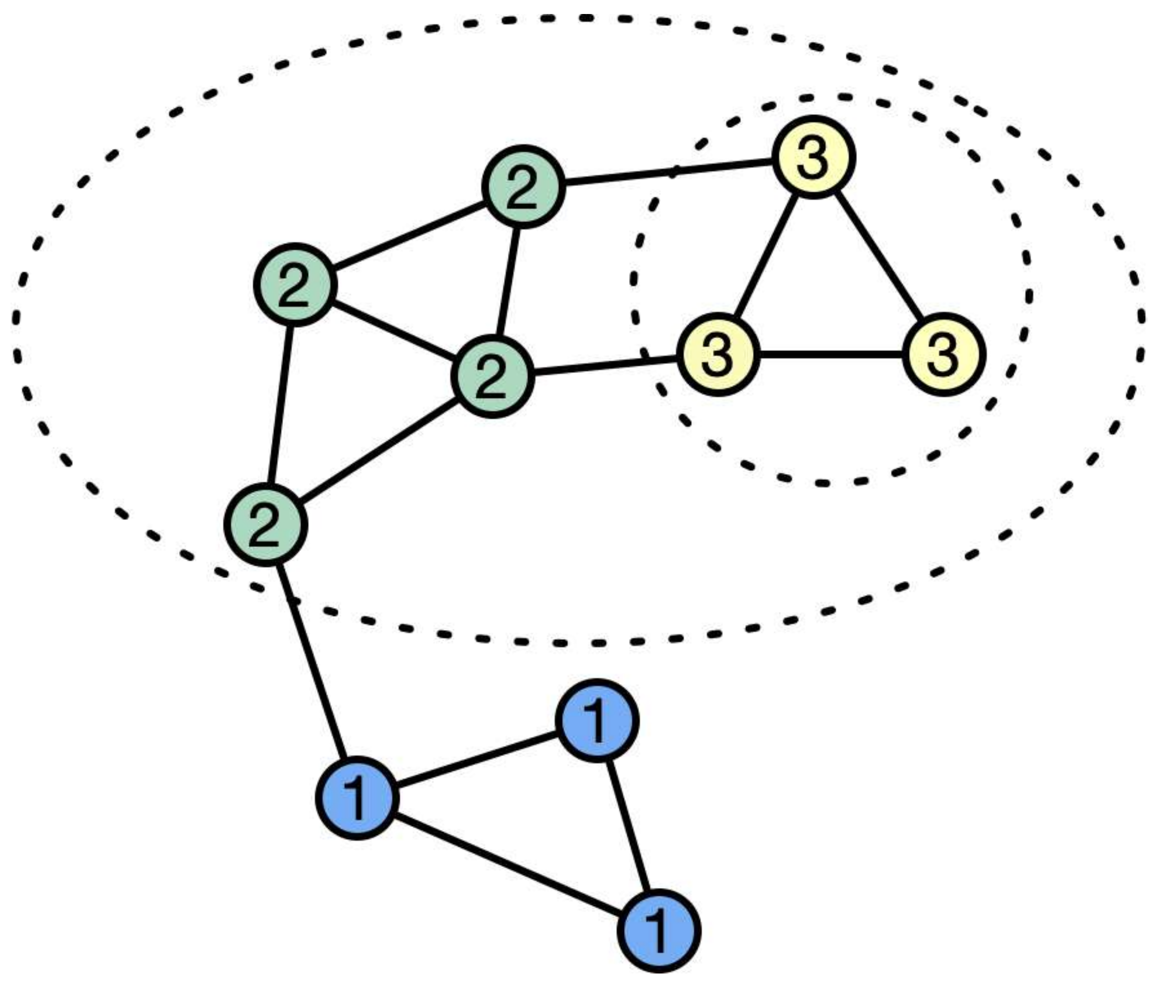
Shapley-Schubik index

1. Cohesive expansion is unique, i.e., independent of the ordering of node addition;
2. M is a cohesive set ⇒
   1). The expansion of M, i.e., $E(M)$, is the smallest maximal cohesive set containing M;
   2). Either $E(M)$ is the entire network;
   3). Or $E(M)$ and the complement of $E(M)$ are both maximally cohesive.

[9] S. Morris. *The Review of Economic Studies*, 67(1):57–78, 2000.

Theoretical Results

Assumption: random & asynchronous updates: to avoid periodic solution.



Set of fixed points

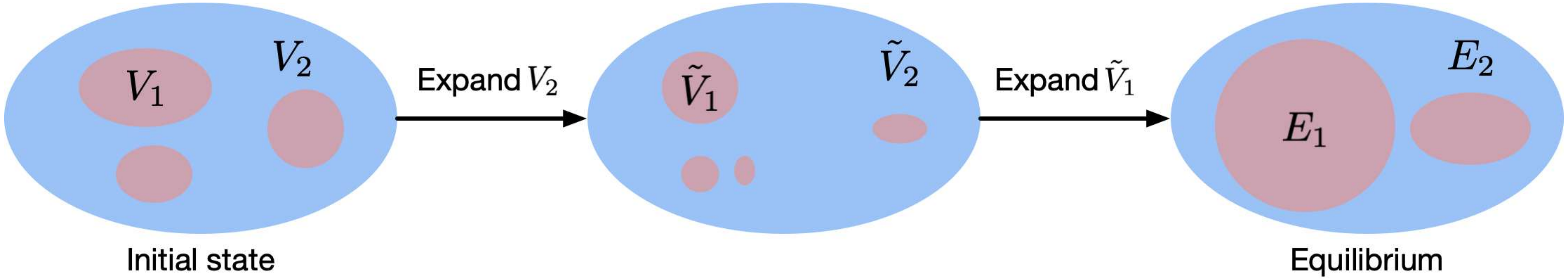
x^* is a fixed point $\Leftrightarrow$ For $\forall\, a$, the node set $\{i \mid x_i^* \geq a\}$ is maximally cohesive.

Convergence and phase transitions

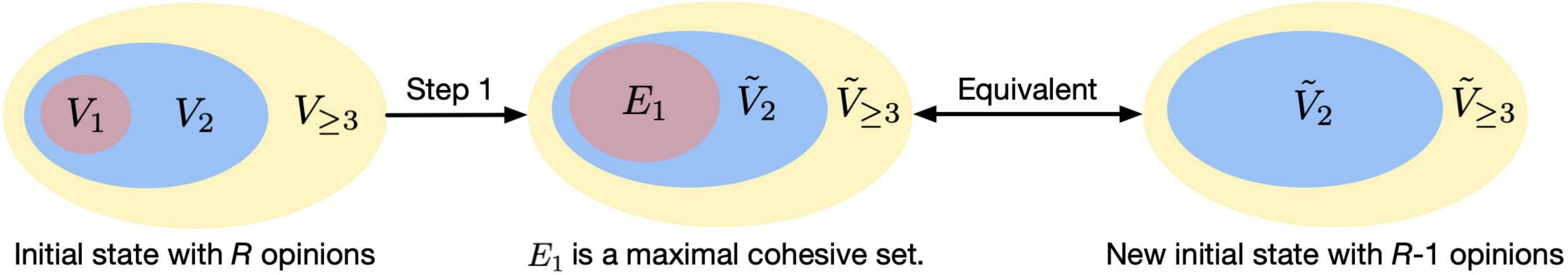
1. Almost-sure finite-time convergence to a fixed point;
2. $\nexists$ non-trivial maximal cohesive set $\Rightarrow$ almost-sure consensus;
3. No globally reachable node in $\mathcal{G}_{\text{decisive}}(W) \Rightarrow$ almost-sure disagreement;
4. $\exists$ non-trivial maximal cohesive set $\Rightarrow$ $\exists\, X_0 \in \mathbb{R}^n$ with non-zero measure, s.t., $\forall\, x_o \in X_0$, $x(t) \rightarrow$ disagreement;

Sketch of proof: “Monkey-typewriter” argument

Step 1. Two-opinion case



Step 2. *R*-opinion case in general



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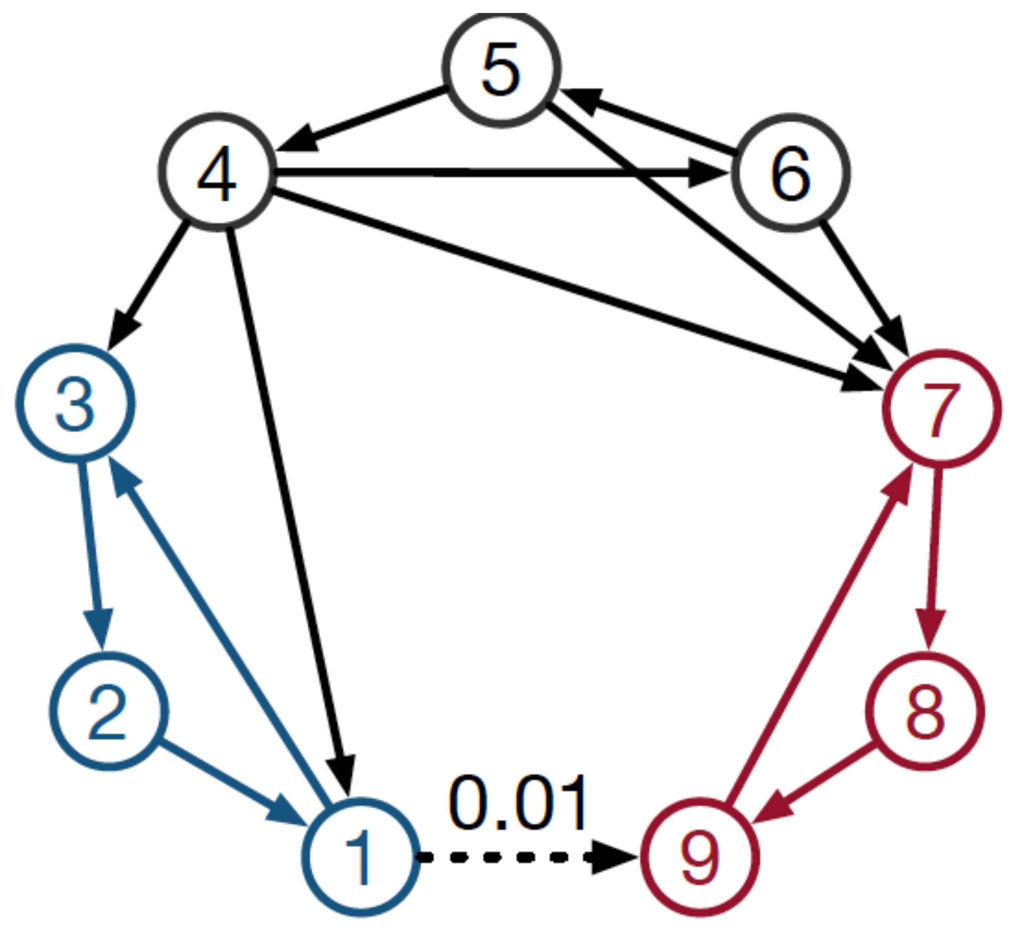

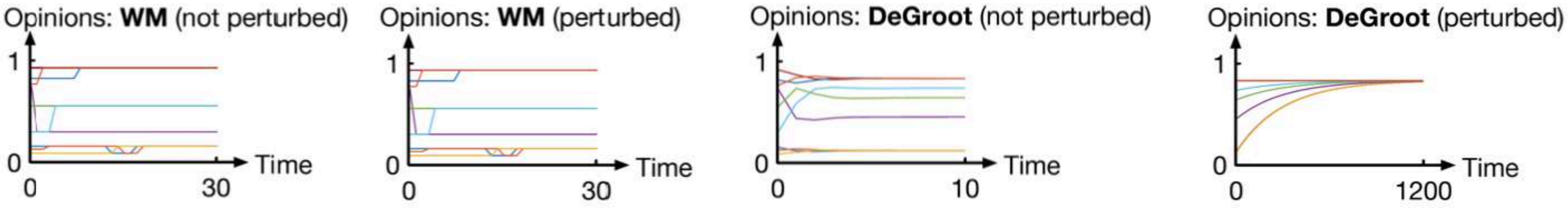

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  - Weighted-median opinion dynamics with compromise behavior

$$\dot{x}_i = \mathrm{Med}_i(x;W) - x_i, \quad \text{or} \quad x_i^+ = \epsilon_i x_i(t) + (1-\epsilon_i)\mathrm{Med}_i(x;W)$$

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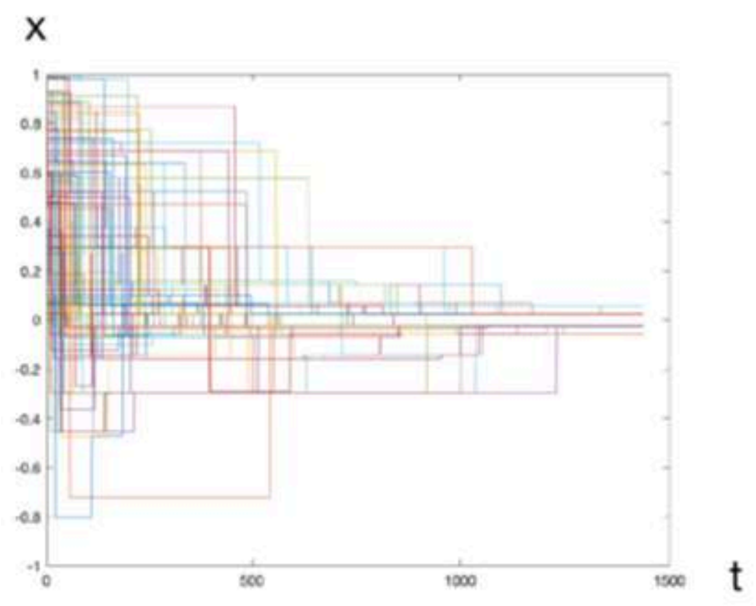
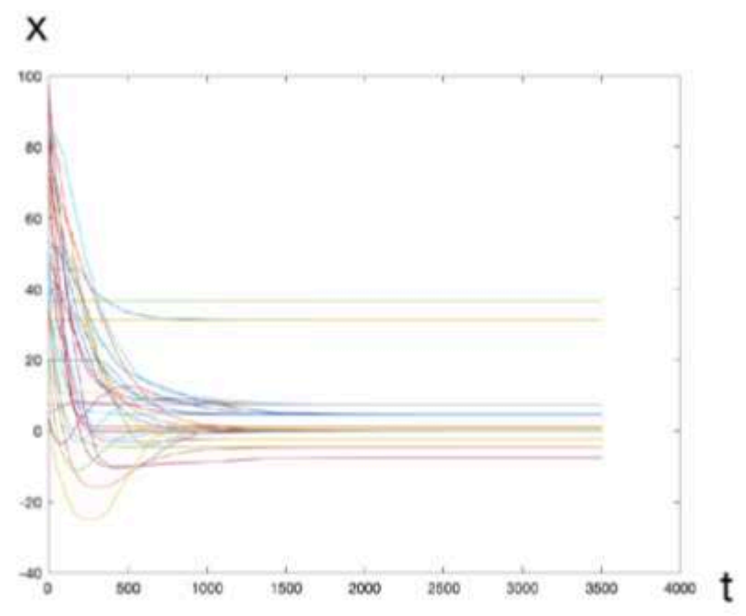
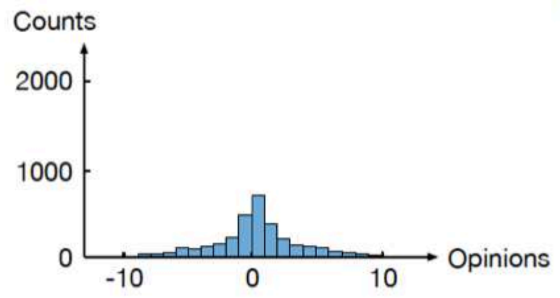


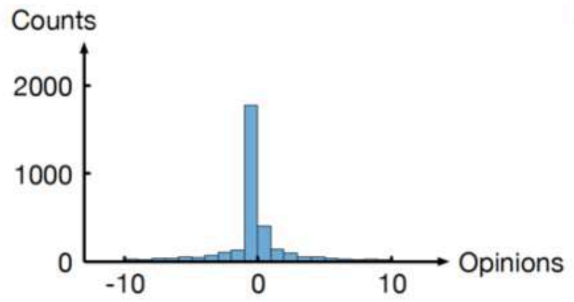
Discrete-time, unbalanced, n=100



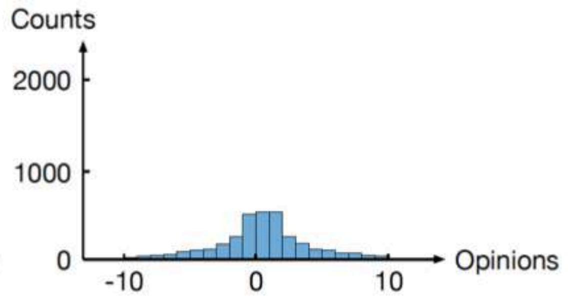
Continuous-time, unbalanced, n=30



$p_{\text{neg}} = 0.1$



$p_{\text{neg}} = 0.5$



$p_{\text{neg}} = 0.9$

— Weighted-Median Altafini Model

1. Motivation: Altafini model implies “structural unbalance $\Rightarrow x(t) \to 0$”
2. Model setup: assigning negative weight to $x_j \Leftrightarrow$ assigning positive weight to $-x_j$
3. Unrealistic feature resolved.
4. New insights

Inconspicuous microscopic change → rich macroscopic consequences

- **Simplest in form**
- **Broader applicability:** ordered multiple-choice issues
- Numerical comparisons: **more realistic predictions**
- Theoretical analysis: **richer & more robust dynamical behavior**
- Open up **new lines of research**
  - Consensus conditions for best-response opinion dynamics

$$x_i^+ = \operatorname{argmin}_z \sum_j w_{ij}|z - x_j|^\alpha$$

1. $\alpha > 1$: $\exists$ aperiodic globally reachable subgraph;
2. $\alpha = 1$: $\nexists$ non-trivial maximally cohesive set
3. $\alpha < 1$: unknown

Inconspicuous microscopic change → rich macroscopic consequences

- **Simplest in form**
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  — Other meaningful extensions to DeGroot model

Thank you!