Temporal Logics for Multi-Agent Systems

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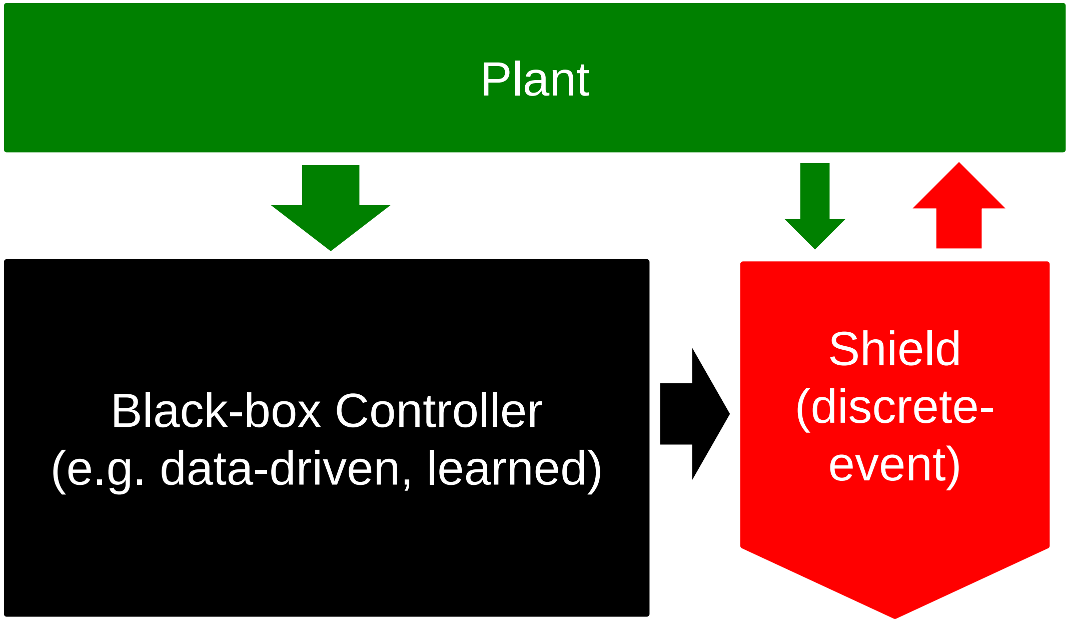
Shielded Control



Shield can ensure safety and fairness (temporal-logic specification), performance (quantitative spec), and/or incremental regimes.

Multiple Agents
(e.g. plant, controller, shield; robotics)

A_1:

```
bool  x := 0
loop
    choice
        | x := 0
        | x := x+1 mod 2
    end choice
end loop
```

Φ_1: □ (x ¸ y)

A_2:

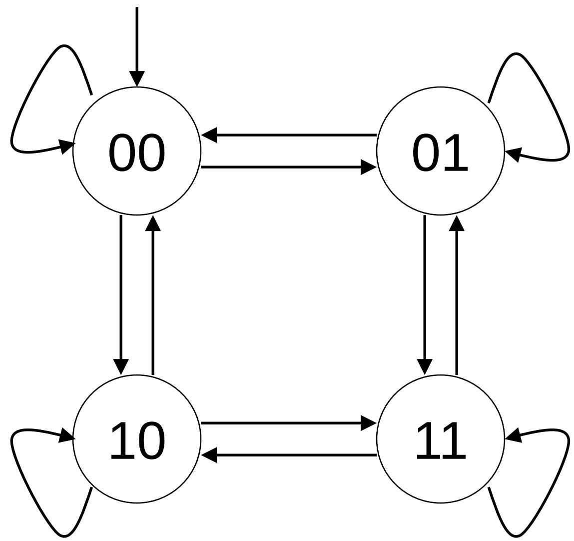
```
bool  y := 0
loop
    choice
        |  y := x
        |  y := x+1 mod 2
    end choice
end loop
```

Φ_2: □ (y = 0)

State Space as Graph

X 8 □ (x ¸ y)

✓ 9 □ (x ¸ y)



State Space as Graph

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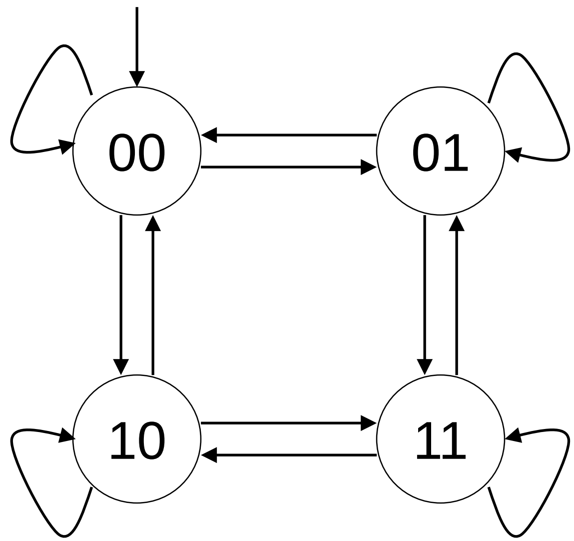


hhA_1ii □ (x ¸ y)

hhA_2ii □ (y = 0)

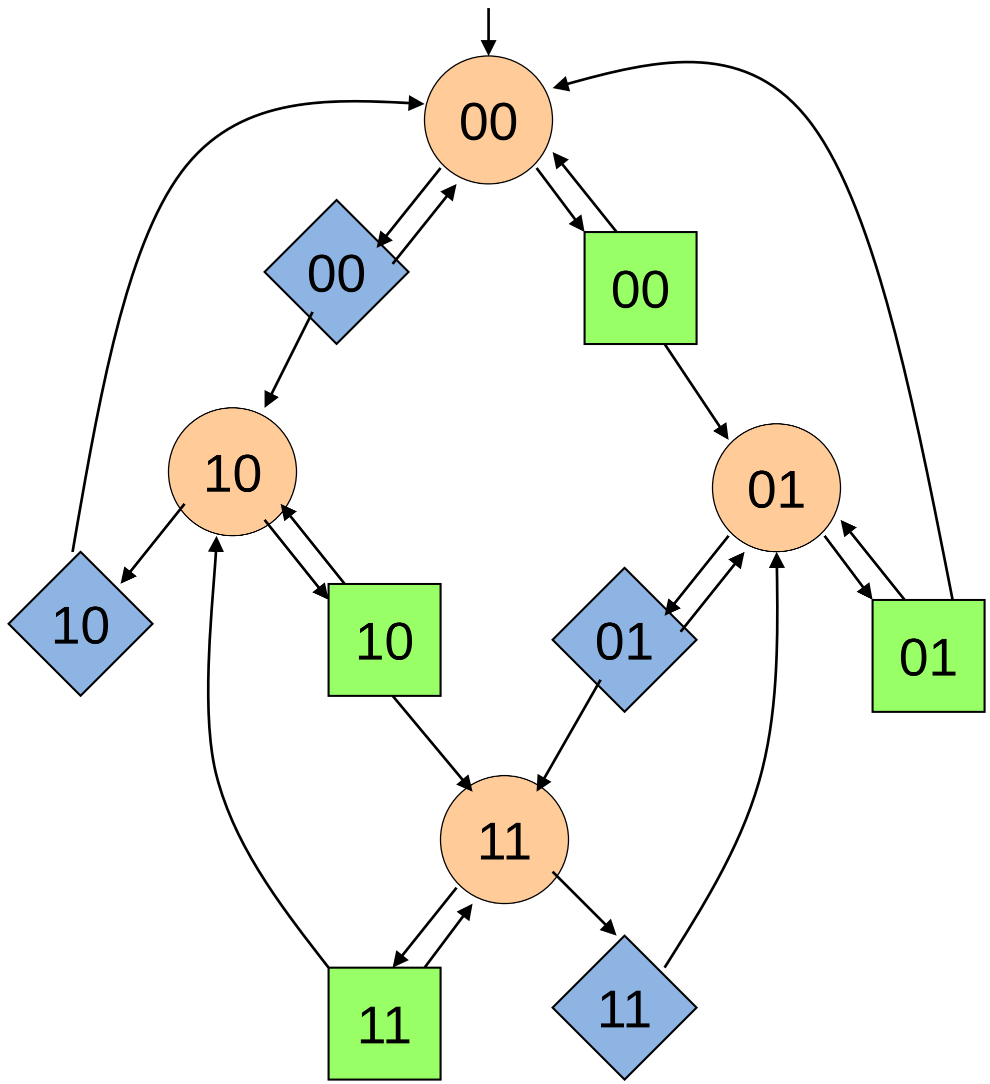
State Space as Game

X 8 □ (x ¸ y)

✓ 9 □ (x ¸ y)

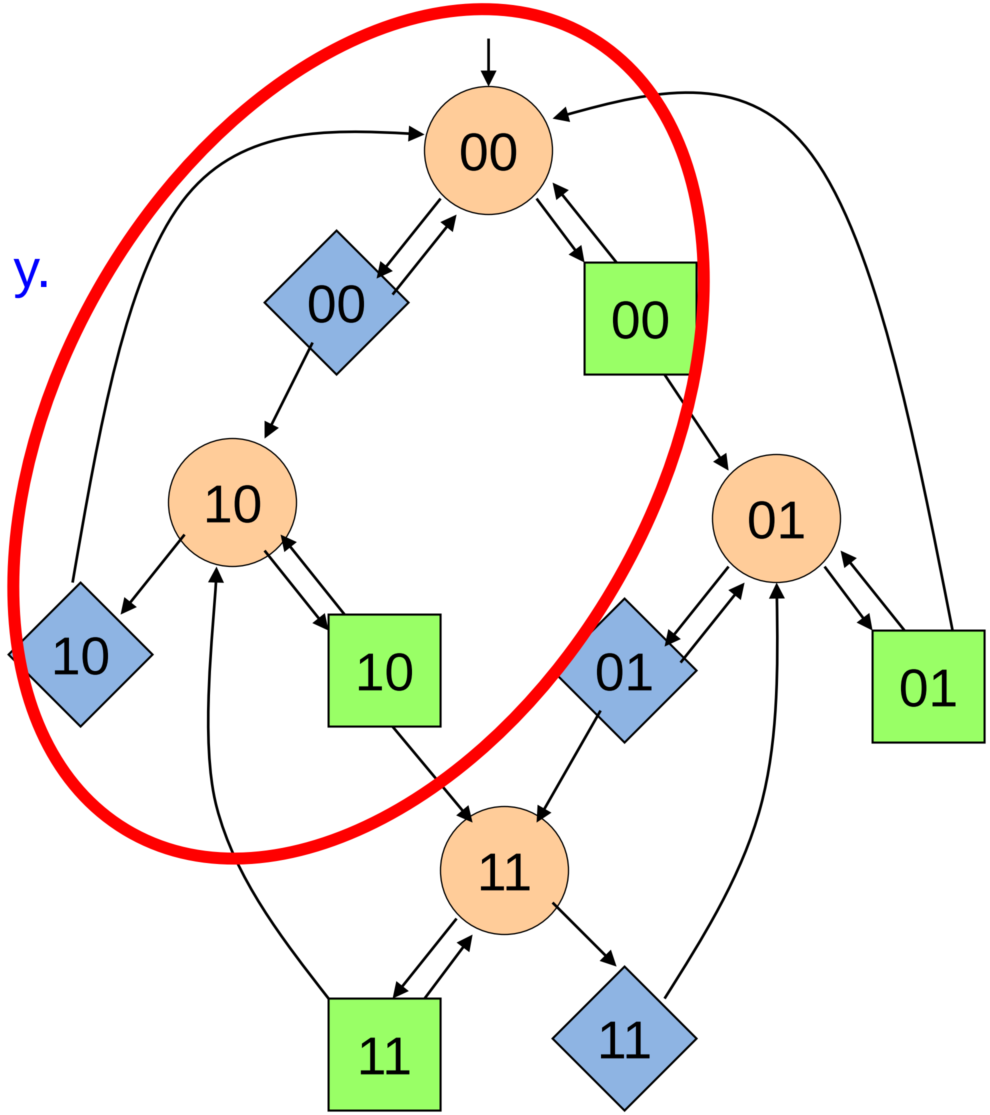
X hhA$_1$ii □ (x ¸ y)

✓ hhA$_2$ii □ (y = 0)



State Space as Game

- If A_2 keeps y = 0,
  then A_1 can keep x ¸ y.



Reactive Synthesis

Agent Synthesis (a.k.a. discrete-event control)

- Given: agent A, specification Φ, and environment E
- Find: refinement A' of A so that A'||E satisfies Φ
- Solution: A' = winning strategy in game A against E for objective Φ

Reactive Synthesis

Agent Synthesis (a.k.a. discrete-event control)

Given: agent A, specification Φ, and environment E

Find: refinement A' of A so that A'||E satisfies Φ

Solution: A' = winning strategy in game A against E for objective Φ

Multi-Agent Synthesis (e.g. shielded or distributed control)

Given:

-two agents A_1 and A_2

-specifications Φ_1 and Φ_2 for A_1 and A_2

Find:

refinements A'_1 and A'_2 of A_1 and A_2 so that

$A'_1||A'_2||S$ satisfies Φ_1 Æ Φ_2 for every fair scheduler S

Mutual Exclusion

```
while( true ) {
  flag[1] := true; turn := 2;

  choice
  | while( flag[1] ) nop;
  | while( flag[2] ) nop;
  | while( turn=1 ) nop;
  | while( turn=2 ) nop;
  | while( flag[1] & turn=2 ) nop;
  | while( flag[1] & turn=1 ) nop;
  | while( flag[2] & turn=1 ) nop;
  | while( flag[2] & turn=2 ) nop;
  end choice;

  CritSec; flag[1] := false;
  nonCritSec;
}
```

```
while( true ) {
  flag[2] := true; turn :=1;

  choice
  | while( flag[1] ) nop;
  | while( flag[2] ) nop;
  | while( turn=1 ) nop;
  | while( turn=2 ) nop;
  | while( flag[1] & turn=2 ) nop;
  | while( flag[1] & turn=1 ) nop;
  | while( flag[2] & turn=1 ) nop;
  | while( flag[2] & turn=2 ) nop;
  end choice;

   CritSec; flag[2] := false;
   nonCritSec;
}
```

Multi-Agent Synthesis Formulation 1

Do there exist refinements A'_1 and A'_2 so that

$[A'_1 \,||\, A'_2 \,||\, S]$ µ (Φ_1 Æ Φ_2)

for every fair scheduler S ?

Solution: game $A_1||A_2$ against S for objective Φ_1 Æ Φ_2

Too weak

(solution has A_1 and A_2 cooperate, e.g. alternate).

Multi-Agent Synthesis Formulation 2

Do there exist refinements A'_1 and A'_2 so that

1. $[A'_1 \mid\mid A_2 \mid\mid S]$ µ Φ_1
2. $[A_1 \mid\mid A'_2 \mid\mid S]$ µ Φ_2

for every fair scheduler S ?

Solution: two games A_1 against $A_2 \mid\mid S$ for objective Φ_1, and A_2 against $A_1 \mid\mid S$ for objective Φ_2

Too strong
(answer is NO, e.g. because agent may stay in CritSec).

Multi-Agent Synthesis Formulation 3

Do there exist refinements A'_1 and A'_2 so that

1. $[A'_1 \,||\, A_2 \,||\, S]$ µ $(\Phi_2\ )\quad \Phi_1)$
2. $[A_1 \,||\, A'_2 \,||\, S]$ µ $(\Phi_1\ )\quad \Phi_2)$
3. $[A'_1 \,||\, A'_2 \,||\, S]$ µ $(\Phi_1$ Æ $\Phi_2)$

for every fair scheduler S ?

Mutual Exclusion

```
while( true ) {
  flag[1] := true; turn := 2;

  while( flag[2] & turn=1 ) nop;

  CritSec; flag[1] := false;
  nonCritSec;
}
```

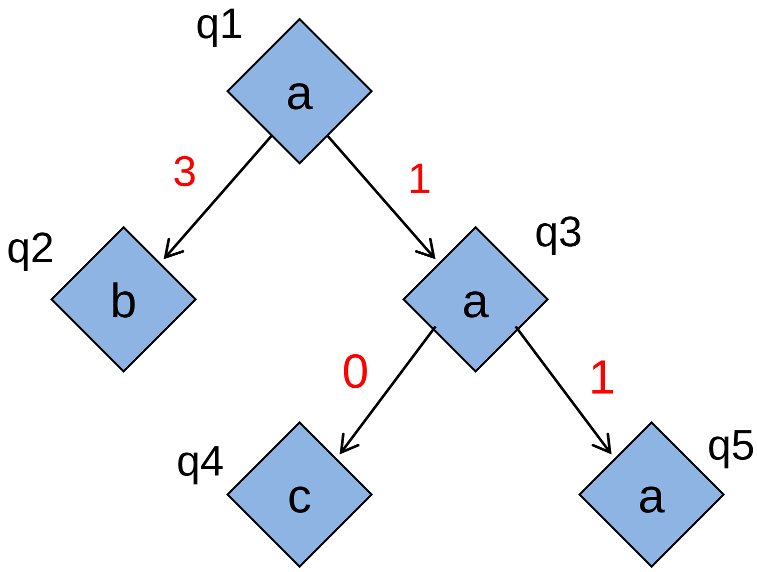
```
while( true ) {
  flag[2] := true; turn := 1;

  while( flag[1] & turn=2 ) nop;

  CritSec; flag[2] := false;
  nonCritSec;
}
```

Solution is exactly Peterson’s mutual-exclusion protocol.

Games on Labeled Graphs

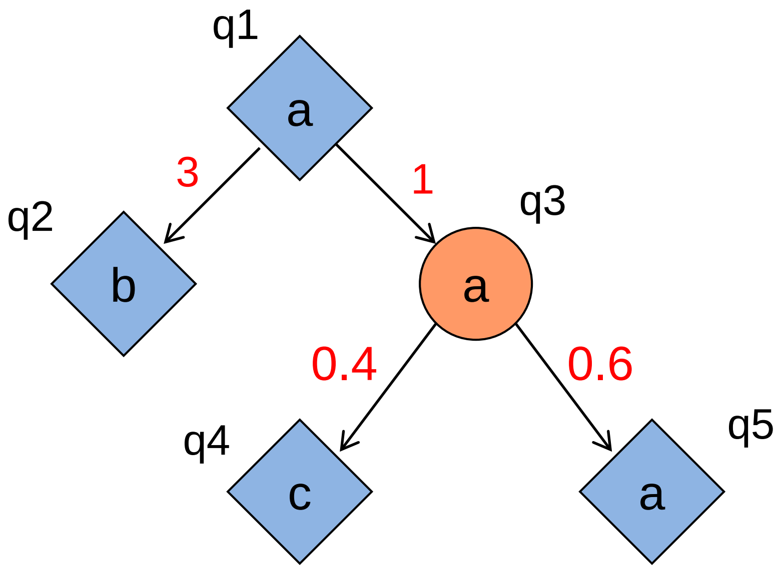
nodes	=	system states
node labels	=	observations
edges	=	state transitions
edge labels	=	transition costs
players	=	agents

Labeled Graph



1-agent system without uncertainty.

Markov Decision Process



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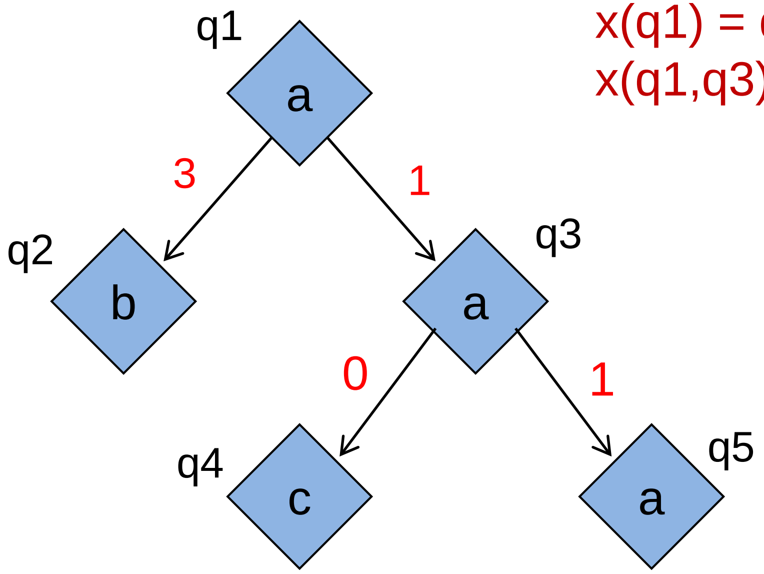
Labeled Graph

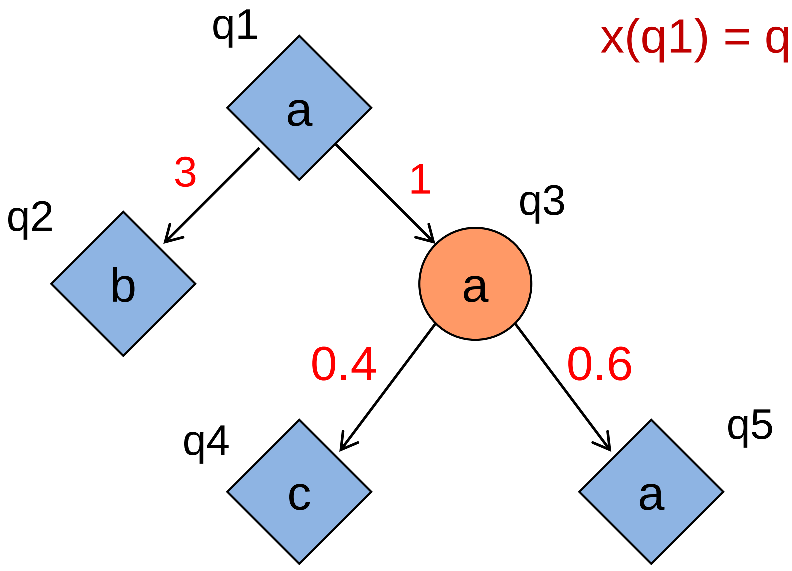


x(q1) = q3
x(q1,q3) = {q4: 0.4; q5: 0.6}

State q 2 Q
Strategy x: Q^* ! D(Q)
x@q: probability space on $Q^!$

} c (x)@q1 = 0.4
avg (x)@q1 = 0.8

Markov Decision Process

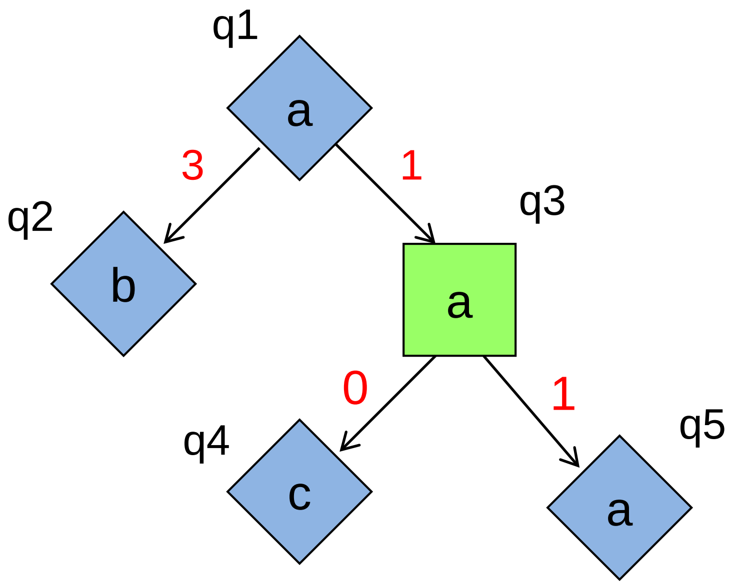


x(q1) = q3

State q 2 Q
Strategy x: Q* ! D(Q)
x@q: probability space on Q!

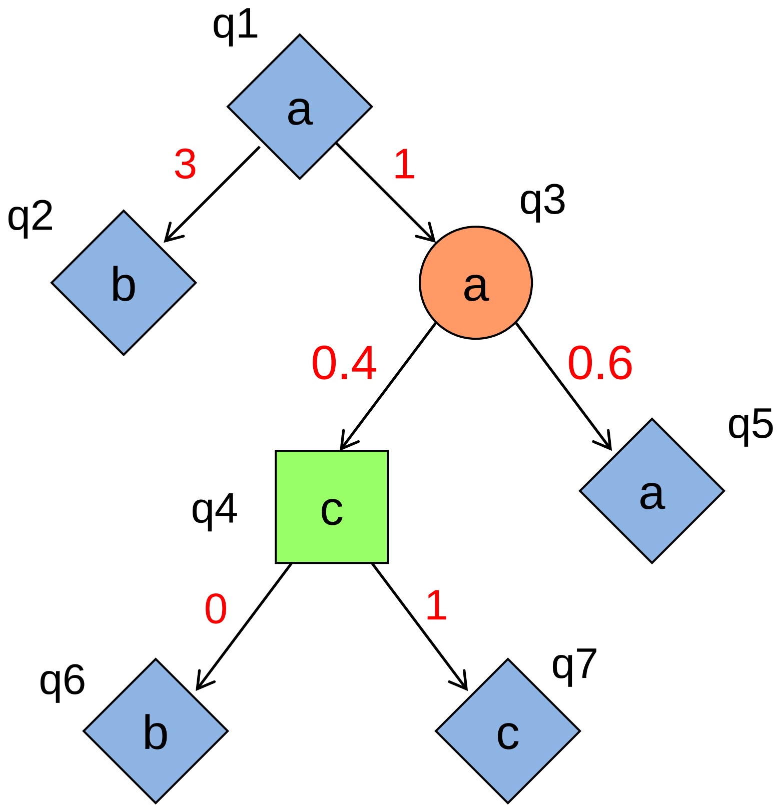
} c (x)@q1 = 0.4
avg (x)@q1 = 1

Turn-based Game



Asynchronous 2-agent system without uncertainty.

Stochastic Game



Asynchronous 2-agent system with uncertainty.

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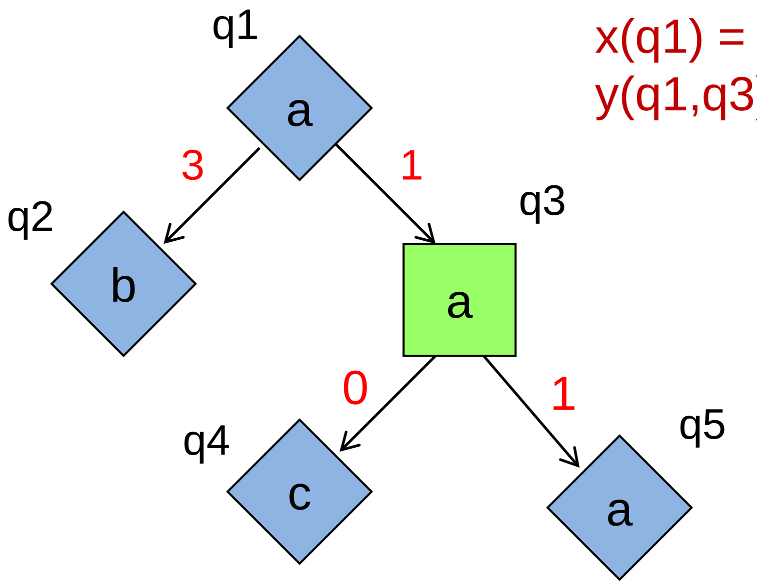
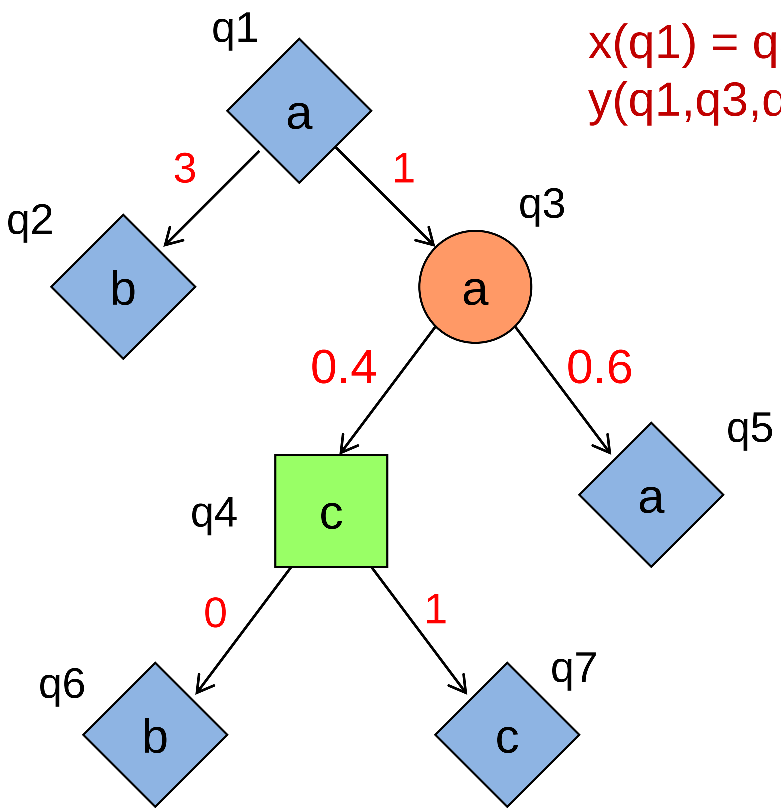


x(q1) = q3
y(q1,q3) = {q4: 0.4; q5: 0.6}

State q 2 Q
Strategies x,y: Q* ! D(Q)
(x,y)@q: probability space on Q!

} c (x,y)@q1 = 0.4
avg (x,y)@q1 = 0.8

Stochastic Game



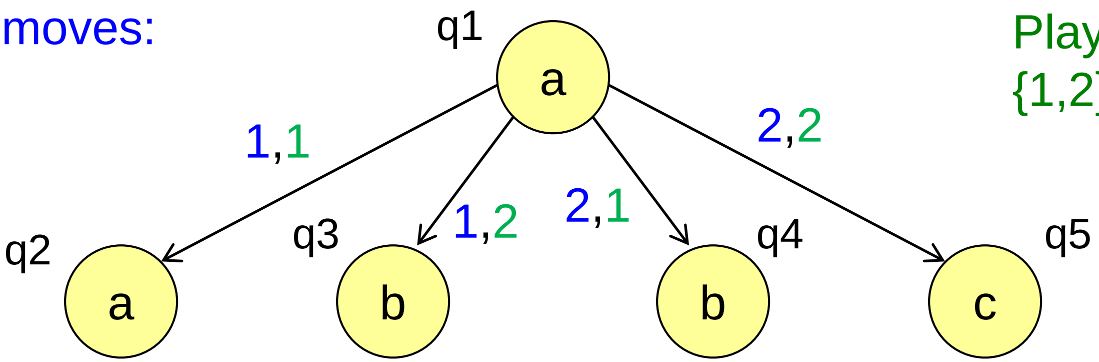
x(q1) = q3
y(q1,q3,q4) = {q6: 0.4; q7: 0.6}

State q 2 Q
Strategies x,y: Q^* ! D(Q)
(x,y)@q: probability space on $Q^!$

} c (x,y)@q1 = 0.4
avg (x,y)@q1 = 0.92

Concurrent Game

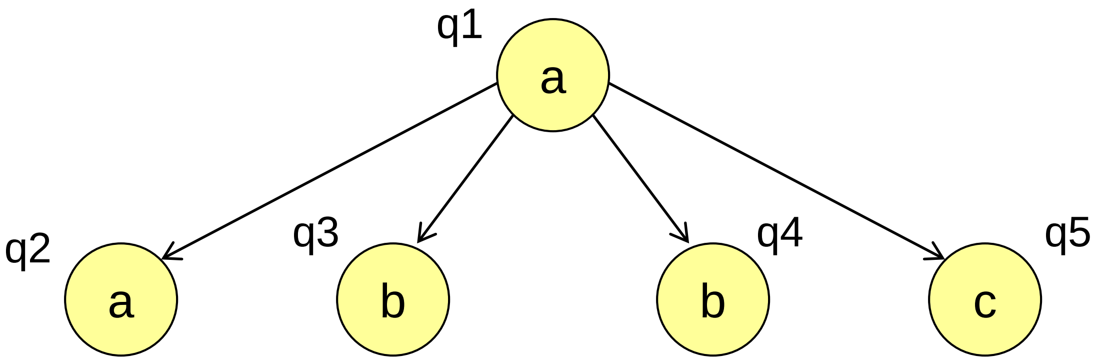
Player Left moves:
{1,2}



Player Right moves:
{1,2}

Synchronous 2-agent system without uncertainty.

Concurrent Stochastic Game



q1:	1	2
1	q2: 0.3 q3: 0.2 q4: 0.5 q5:	q2: 0.1 q3: 0.1 q4: 0.5 q5: 0.3
2	q2: q3: 0.2 q4: 0.1 q5: 0.7	q2: 1.0 q3: q4: q5:

Player Column moves:
{1,2}

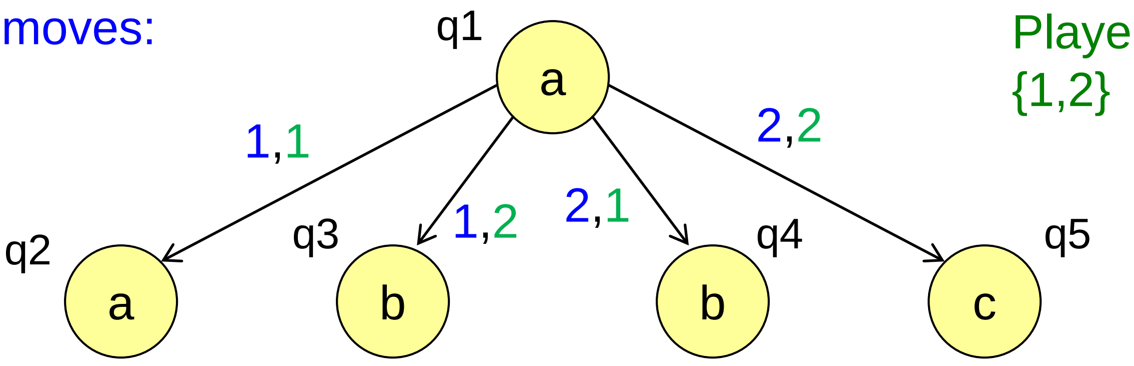
Player Row moves:
{1,2}

Matrix game
at each node.

Synchronous 2-agent system with uncertainty.

Concurrent Game

Player Left moves:
{1,2}



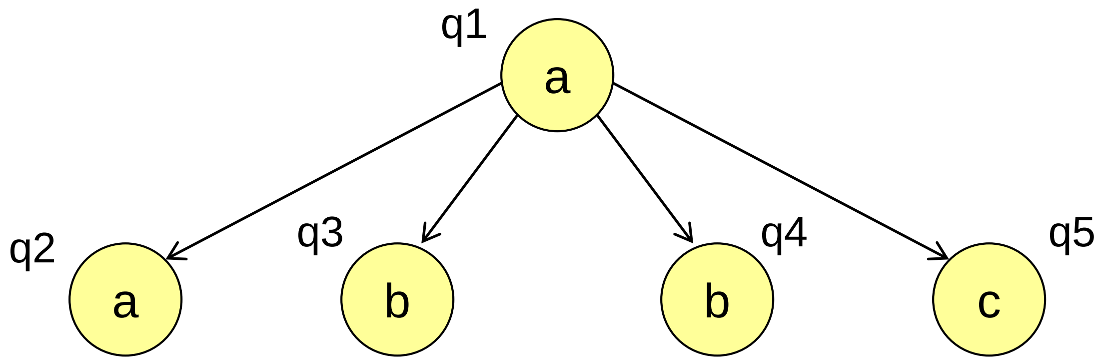
Player Right moves:
{1,2}

State q 2 Q
Strategies x,y: Q* ! D(Moves)
(x,y)@q: probability space on Q!

x(q1) = 2
y(q1) = {1: 0.4; 2: 0.6}

} c (x,y)@q1 = 0.6

Concurrent Stochastic Game



Player Column moves: {1,2}

Player Row moves: {1,2}

q1:

	1	2
1	q2: 0.3 q3: 0.2 q4: 0.5 q5:	q2: 0.1 q3: 0.1 q4: 0.5 q5: 0.3
2	q2: q3: 0.2 q4: 0.1 q5: 0.7	q2: 1.0 q3: q4: q5:

State q 2 Q

Strategies x,y: Q* ! D(Moves)

(x,y)@q: probability space on Q!

x(q1) = 2

y(q1) = {1: 0.4; 2: 0.6}

} c (x,y)@q1 = 0.28

Timed Games, Hybrid Games, etc.

Strategy Logic

1. first-order quantification over sorted strategies
2. linear temporal formulas over observation sequences
3. interpreted over states

q ² (9 x) (8 y) Á iff there exists a player-1 strategy x such that for all player-2 strategies y Á (x,y)@q = 1

Alternating-Time Temporal Logic

1. path quantifiers over sets of players
2. linear temporal formulas over observation sequences
3. interpreted over states

$q \models \langle\langle T \rangle\rangle \varphi$ iff if the game starts from state q
the players in set T can ensure that
the LTL formula φ holds with probability 1

Alternating-Time Temporal Logic

1. path quantifiers over sets of players
2. linear temporal formulas over observation sequences
3. interpreted over states

q ² hhTii Á iff if the game starts from state q
the players in set T can ensure that
the LTL formula Á holds with probability 1

hh;ii Á = 8 Á

hhUii Á = 9 Á where U is the set of all players

[[T]] Á = : hhU\Tii : Á “the players in U\T cannot prevent Á”

ATL* µ SL

$$hhTii\ \acute{A} = (9\ x_1,\ldots,x_m\ 2\ ¦\ _T)\ (8\ y_1,\ldots,y_n\ 2\ ¦\ _{U\backslash T})\ \acute{A}$$

ATL* (SL

Player 1 can ensure $Á_1$ if player 2 ensures $Á_2$:

$$(9\ x)(8\ y)\ (\ ((8\ x')\ Á_2(x',y))\ )\quad Á_1(x,y)\ )$$

ATL* (SL

Player 1 can ensure Á$_1$ if player 2 ensures Á$_2$:

(9 x)(8 y) (((8 x') Á$_2$(x',y)))   Á$_1$(x,y))

The strategy x dominates all strategies w.r.t. objective Á:

(8 x')(8 y) (Á(x',y))   Á(x,y))

ATL* (SL

Player 1 can ensure $Á_1$ if player 2 ensures $Á_2$:

(9 x)(8 y) (((8 x') $Á_2$(x',y))) $Á_1$(x,y))

The strategy x dominates all strategies w.r.t. objective Á:

(8 x')(8 y) (Á(x',y)) Á(x,y))

The strategy profile (x,y) is a secure Nash equilibrium:

(9 x)(9 y) (($Á_1$ Æ$Á_2$) (x,y)
Æ(8 y') ($Á_2$) $Á_1$) (x,y')
Æ(8 x') ($Á_1$) $Á_2$) (x',y))

ATL

ATL is the fragment of ATL* in which every temporal operator is preceded by a path quantifier:

$\langle\langle T \rangle\rangle \bigcirc a$ — single-shot game

$\langle\langle T \rangle\rangle \Diamond b$ — reachability game

$\langle\langle T \rangle\rangle \Box c$ — safety game

ATL

ATL is the fragment of ATL* in which every temporal operator is preceded by a path quantifier:

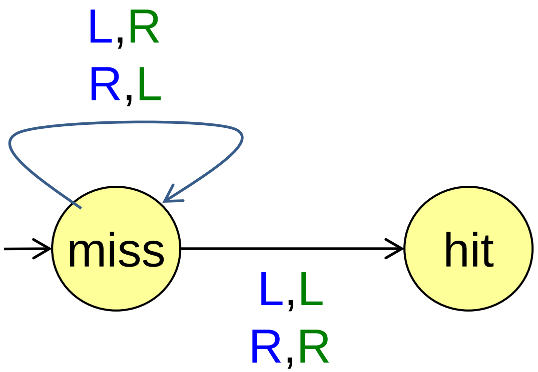
hhTii ° a	single-shot game
hhTii } b	reachability game
hhTii □ c	safety game

Not in ATL:

hhTii □} c	Buchi game
hhTii Á	! -regular (parity) game

Pure Winning

Player 1:
{moveL,moveR}



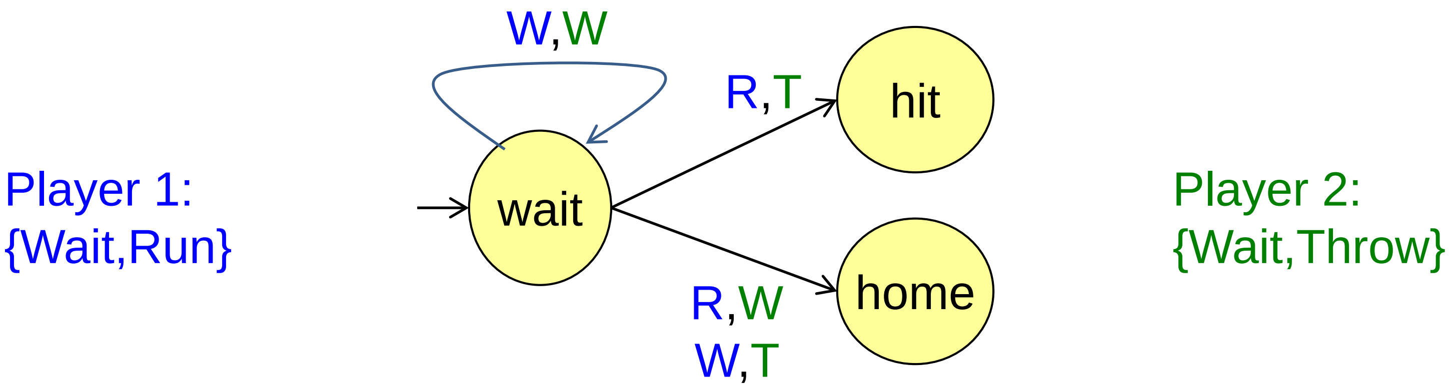
Player 2:
{throwL,throwR}

✗ $\langle\langle P2 \rangle\rangle_{pure} \Diamond\ hit$

✓ $\langle\langle P2 \rangle\rangle \Diamond\ hit$

Player 2 needs randomness to win.

Limit Winning



X hhP1ii } home

✓ hhP1ii $_{\text{limit}}$ } home

Player 1 can win with probability arbitrarily close to 1.

Quantitative ATL

hhP1ii Á = (9 x) (8 y) (Á(x,y) = 1)

hhP1ii $_{limit}$ Á = (sup_x inf_y Á(x,y)) = 1

Quantitative ATL

hhP1ii Á = (9 x) (8 y) (Á(x,y) = 1)

hhP1ii$_{limit}$ Á = (sup$_x$ inf$_y$ Á(x,y)) = 1

hhP1ii$_{val}$ Á = sup$_x$ inf$_y$ Á(x,y)

Complexity of Formula Evaluation (a.k.a. model checking)

CTL: linear in formula, linear/NLOGSPACE in graph
Pure ATL: linear in formula, linear/PTIME in graph
Quantitative ATL: linear in formula, quadratic in graph

CTL^*: PSPACE in formula (convert to word automaton)
ATL^*: 2EXPTIME in formula (convert to tree automaton)

SL: extra exponential for every quantifier elimination

Summary: Classification of Graph Games

1. Number of players: 1 (graph), 1.5 (MDP), 2 , 2.5, k agents
2. Alternation: turn-based or concurrent
3. Formulas: zero-sum (ATL) or equilibria (SL)
4. Strategies: pure or randomized; how much memory needed
5. Values: qualitative (boolean) or quantitative (real)
6. Objectives: Borel 1 (□), 2 (□}), 2.5 (! -regular), 3 (lim avg)

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5. Values: qualitative (boolean) or quantitative (real)
6. Objectives: Borel 1 (□), 2 (□}), 2.5 (! -regular), 3 (lim avg)
7. Full or partial information (can be undecidable!)

Turn-based Games are Pleasant

-optimal strategies always exist [McIver/Morgan]

-in the non-stochastic case, pure finite-memory optimal strategies exist for ω-regular objectives [Gurevich/Harrington]

-for parity objectives, pure memoryless optimal strategies exist [Emerson/Jutla; Condon], hence NP Å coNP

Concurrent Games are Difficult

-determinacy for randomized but not for pure strategies

-optimal strategies may not exist and ε-close strategies may require infinite memory

-sup inf values may be irrational

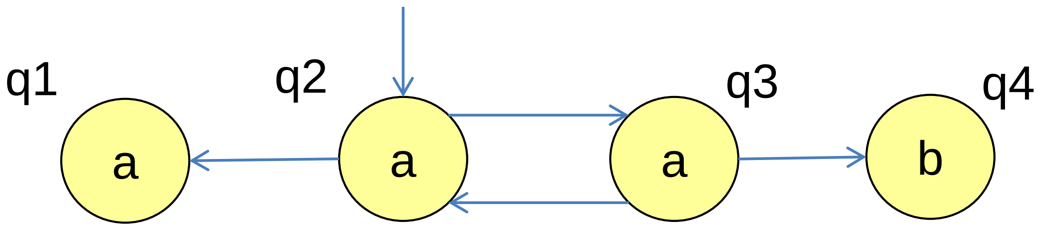
Bidding Game

Each player has a budget.
At each node, each player bids part of their budget.
The winning player chooses the transition.

Richman bidding: the winning bid goes to the losing player.
Poorman bidding: the winning bid goes to the "bank."
Recharging: the budgets are increased by transition weights.

Difficulty: infinitely many possible moves (bids).

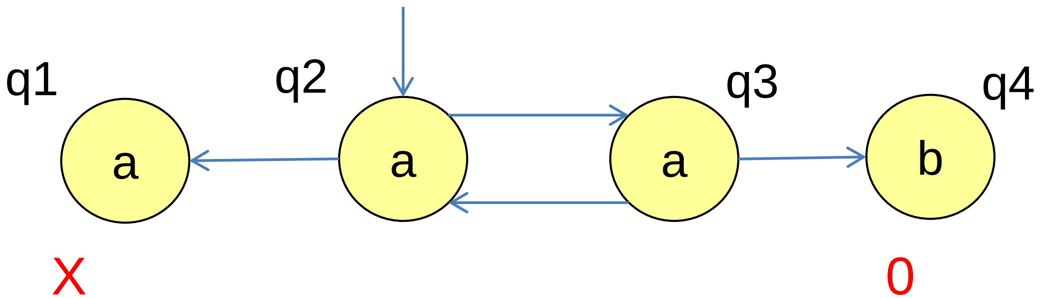
Richman Bidding



The sum of the budgets of players 1 and 2 is 1.

What is the threshold budget for player 1 to win } b ?

Richman Bidding



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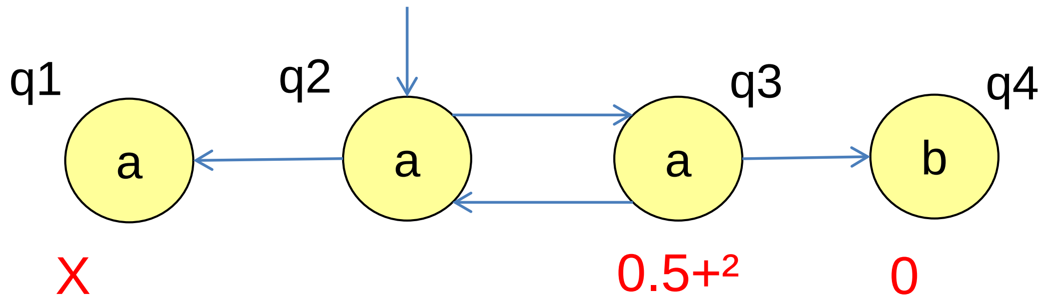

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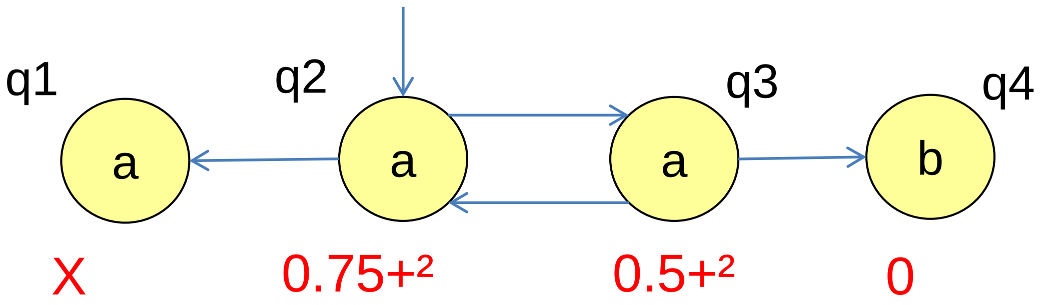

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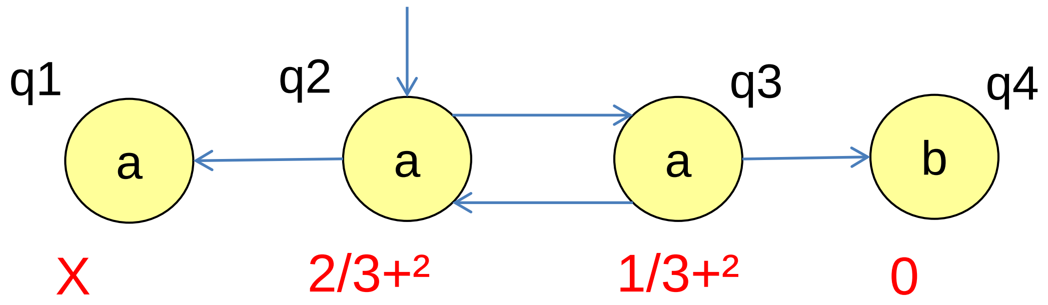

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What is the threshold budget for player 1 to win } b ?

Some References

Alternating-time temporal logic: JACM 2002

Multi-agent (assume-guarantee) synthesis: TACAS 2007

Concurrent reachability games: TCS 2007

Strategy logic: Information & Computation 2010

Infinite-duration bidding games: CONCUR 2017