Economic impact of residential photovoltaics with battery storage

Master thesis
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Abstract

Using a battery on a household level has become feasible after the launch of Tesla’s Powerwall, a battery developed to be installed in residential houses. Storing electricity during daytime’s PV overproduction or charging the battery during night-time with an attractive tariff are the most prominent applications described.

This master thesis explores the economic impact of the usage of residential battery storage combined with solar photovoltaics (PV) based on real load data from California.

First, the need for storage in a changing landscape of power production with rising stochastic electric power infeed is elaborated and different policies to foster battery or PV usage are reviewed.

Second, different models to minimize electricity costs for single households are presented. The results of a deterministic model are compared to those of a stochastic one and it is shown that for a hourly resolution, the deterministic model is a good benchmark, due to its short computation time.

Third, the battery size for single and multiple households is derived. It is shown that a high proportion of the electricity savings of a PV/battery system are needed for investment payback. Also, only for a few cases, the cost minimizing battery size corresponds to the one of Powerwall.

Fourth, the impacts on the distribution grid level, if PV/battery systems are installed on a household level, are investigated. Overvoltages due to over-sized PV installations and consequent in feed to the grid, are expected to be the major concern.

Fifth, the impacts of residential PV/battery systems on the power markets with a utility that adapts retail to wholesale prices are estimated. The evaluation is done in terms of production and consumption costs and pollutant emissions as a function of battery/PV penetration. It is shown that market prices are most sensitive to battery installations during night time hours and that photovoltaic systems seem to have a higher potential to reduce emissions than battery systems.

Finally, a simple algorithm, determining the optimal PV/battery size combination for a given maximum allowed peak load as well as for a given self-sufficiency rate is presented.
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List of Abbreviations

PV  photovoltaics
TS  tariff structure
RES renewable energy sources
BES battery electricity storages
IRR internal rate of revenue
PGE Pacific Gas and Electric
PDF probability density function
TOU time of use pricing
NPV net present value
FIT feed-in-tariff
SOC state of charge
(R)MPC (randomized) model predictive control
ARIMA autoregressive integrated moving average
(P)ACF (partial) autocorrelation function
EAC equivalent annual cost
OPF optimal power flow
eGRID Emissions and Generation Resource Integrated Database
EIA Energy Information Agency
CAISO California Independent System Operator
PP power plant
LMP locational marginal price
Chapter 1

Introduction

1.1 Motivation

1.1.1 Rising use of Photovoltaics in the future

The rising atmospheric CO$_2$ level over the past 150 years is said to be the one of the main contributors to climate change, whose reduction is one of United Nations’ declared goals. In addition to this, high uncertainty in fossil fuel prices and the political will to reduce the dependency on these commodities (including uranium for safety reasons) have had an important impact on the implementation of renewable energy sources (RES) (cf. Reference [24]). For a long time, RES had not been cost competitive to conventional electricity production (cf. Reference [18]), but recent policies, such as tax reliefs and feed-in-tariff (FIT)s, have fostered the use of RES especially in US and Europe.$^1$

The different growth rates of RES and electricity demand illustrate the rising importance of wind and PV. According to BP (cf. [16]), the annual growth of electricity demand in 2014 was 0.4 % and -1.6% for North America and Europe respectively. However the growth rates of PV/Wind installed capacity were at the same year in Europe 8.9%/10.8% and North America 50.5% /10.2% respectively.

Figure 1.1 depicts the installed capacity of PV and wind for North America and Europe. Wind is the predominant source of both RES, whereas the actual installed capacity of PV in North America of 78 GW is minim compared

$^1$As the focus of this work is on California and the author’s home University is in Europe, the focus is on Europe and the US. Surely also other countries have promoted the implementation of photovoltaics (PV). However, China for example has fostered its PV production massively in the past years, nevertheless, according to [39] this has mostly be done to increase export and strengthen the PV industry and less to increase domestic usage.
1. Introduction

to the potential this RES might have: Porro et al. [33] determine the pure potential for PV in the US. A total potential capacity of about 155 TW\(^2\) is striking, even though their estimates exclude some important features\(^3\) necessary for the implementation of PV. Consequently, the impact of PV-usage on households, the grid and the electricity market is likely to rise, especially, when considering laws that foster the deployment of RES. So requires the ‘Californian Renewables Portfolio Standard’ investor-owned utilities, electric service providers, and community choice aggregators to ‘increase procurement from eligible renewable energy resources to 33 % of total procurement by 2020’ (see [19]). Also the European Union (Directive 2009/28/EC) has set its goal of 20 % renewables for energy production, and thus is likely to increase the use of renewables for electricity production as well.\(^4\)

The market already shows an increased penetration of RES, which is likely to grow further due to the above argumentation. The shape of the spot

\(^{2}\)Urban utility-scale-, rural utility-scale- and rooftop PV-potential is given with 1’200 GW, 153’000 GW and 664 GW respectively.

\(^{3}\)The authors determine ‘technology-specific estimates of energy generation potential based on renewable resource availability and quality, technical system performance, topographic limitations, environmental, and land-use constraints only [...]’ and ‘do not consider economic or market constraints’.

\(^{4}\)2013, the share of renewables in the electricity production was already at 25 %, which lead to an overall share of RES at the energy production of 15 %, see [27].
prices in Germany is said to have changed due to increasing PV integration. As an example, the averaged epex spot prices for the month of August are shown in Figure 1.2. The price peak at noon and the overall price fluctuation seems to be reduced over the past years. In addition it is likely that part of the overall electricity price reduction is due to the increasing RES deployment (cf. Reference [42]).\footnote{It is also argued that a reason for the reduced wholesale price is the low coal price that had been observed in the last years, which was at about 130 USD/tonne in 2008 and at about 40 USD/tonne in 2015.}

![Figure 1.2: Averaged spot prices for August in an hourly resolution for electricity in Germany (source: epex).](image)

### 1.1.2 Increasing need for storage

**Balancing overproduction and high demand over time on a macro-level**

Due to their intermittent nature, the increasing RES penetration will likely increase the need for measures to mitigate the effects of RES on a macro level.

Peaking power plants are dispatched in order to guarantee reliable power supply for these hours in the year, when there is more demand than supply. For example, when RES cannot meet the actual demand, these peak power plants can be used. With rising RES however, electricity prices are
likely to stay low and therefore peak power plants will be less profitable than they were before. For example, the operator of the newly constructed gas fire plant ‘Irsching’ plans its decommission already, without any electricity production having taken place. Capacity markets guarantee utilities income from owning flexible units, so that peak power plants are not shut down. Opponents of this approach argue however that this would not be economically efficient. Also, it is argued that capacities markets cannot be introduced on a national level, since electricity markets in Europe are connected and tax payers of a single country would subsidize a potential price ceiling.

An increase of electricity storage could alleviate the need for flexible plants, as overproduction can be stored and used during peak hours. Thus it is argued that storage is an essential requirement for an increasing RES penetration (see e.g. Reference [23]). The load shifting process for an increased RES penetration is illustrated in Figure 1.3. It can be seen that – if storage is widely implemented – the baseload can be increased and peaks in demand are satisfied by storage. Overproduction will consequently be captured by storages and the need for peak power plants is reduced.

For these scales, battery technology has been considered to be less suited until now and pump-storage power plants still dominate (see Figure 1.4).

Batteries can quickly charge and discharge in the event of low wind speed or solar radiation. Their comparably high storage capacity costs make them still cost inefficient to bridge supply for longer time periods than minutes. To provide ancillary services however, battery storage is discussed (cf. Reference [28]).

**Balancing overproduction and demand on the distribution-grid level**

Houses with PV-panels can implement storage in order to balance out overproduction during noon to use it e.g. during evening hours. There are several benefits since the battery solves different problems that occur in combination of distributed generation and the actual grid topology (cf. Reference [26]).

The actual transmission/distribution system is designed to transmit electricity that is produced at a different location than it is consumed: (extra-) high voltage transmission grids transmit electricity through many transformation steps and regional and local distribution grids to the end consumer and are designed to be regulated from the generation side. Distribution grids may not be designed to withstand overproduction occurring through infeed of distributed generation. Furthermore, electricity injection into a higher distributional (e.g. regional)
1.1. Motivation

Figure 1.3: Example of storage use during the day with high RES penetration and reduced use of flexible units, in accordance with Reference [25].

Figure 1.4: Different storage types and their application, according to [41].

level can arise for which the grids are not designed and congestions can appear. Even if this problem were solved, transformation to a higher voltage level and at some place to a lower one, induces unnecessary losses. The benefits of combining (battery-) storages with distributed PV are apparent: the grid stability on the distribution level is preserved and batteries with high round-trip efficiencies potentially reduce losses.
Price arbitrage, which has been discussed to be another benefit of battery storages, seems to be uneconomical for residential customers for most battery types and tariffs, given the actual investment costs for battery storage (cf. Reference [17]). Especially, the higher the amount of storage implemented is, the lower the price arbitrage possibilities become as prices converge.

1.1.3 Different storage technologies

Depending on the application, different storage technologies are chosen.

Of all commercialized technologies, hydropower can store electricity longest and is therefore used for bulk power management. Reservoirs are capable to store water from one season to the other (in Switzerland from September, when most glacial ice is melt until February, when electricity prices are high). Pump storage has a lower capacity and is able to store water in the time scale of days to weeks (it could be stored longer, but the plant has to be profitable). These storage times are linked to the traditional timescale of price (and demand/supply) fluctuations, which were at the scale of days to weeks. This, since fluctuations in demand were linked to large temperature and day-light changes.

Ancillary services are provided, in the case when either a power plant fails or demand or the production of RES are forecasted wrongly. This response has to come within seconds to minutes (depending on the service provided). Traditionally, spinning reserves in turbines have been used for this, but also other applications such as supercapacitors are implemented on smaller scales.

Chemical storage technologies, such as batteries have a short history of application on a level that could impact markets. They are located in terms of storage time in between of the two above mentioned technologies. Traditionally, they have been used to supply autonomous systems. However, prices for battery storage on a large scale (in terms of storage capacity) have dropped significantly over the past years and price differences during short periods of time (minutes – hours) in electricity markets are likely to increase. This is why already the implementation of this technology for the use as an energy storage has begun. California for example committed itself to implement 1.3 GW of storage until 2020 (see Table 1. in ). The unit GW seems unclear in this context and so is the legislation (no storage capacity is indicated, only the discharge rate). However, it can be concluded that ambitions are high to increase storage based on battery technology.
1.1.4 Policies subsidizing PV and storages

Finnsson (cf. [29]) evaluates the instruments in California which foster the usage of RES combined with battery electricity storages (BES) focusing on PV and batteries. It is shown that over 11 policies are already implemented in California that mostly subsidize – either directly by tax credit or by levies – the implementation and investment into a RES/BES system. The author concludes that PV/battery systems are already profitable, with an internal rate of revenue (IRR) of about 4%.

Most part of the IRR however is contributed (in this study a value of 52% is determined) to the different support schemes in place, such as investment support and FIT. These results suggest that the implementation of distributed RES/BES are increasingly gaining importance for the future if subsidizing policies are not changed.

1.1.5 Potential effects of storage on the market

Different effects and benefits, such as bridging low distributed renewable generation, omitting transfer of electricity from the local to the regional distribution grid and increasing self-sufficiency for customers have been discussed above.

Effects of storages on the market are generally summarized by their smoothing effect on prices. As the principle is to charge electricity when supply exceeds demand and vice versa, this principle has been used by many power producers.

For centralized storages, the operation is mostly undertaken by a utility. Since the utility can participate in the wholesale market, it will flatten out price differences with its operation. Using distributed electricity storage however is different, as the type of stakeholder changes.

Decentralized storage is operated by a diversity of stakeholders that respond differently to prices from those in the wholesale market. Prices they face are handed down by the utility from the wholesale market to the consumer, including markups for transmission, provision of ancillary services and other costs the utility encounters. This lets the consumer with its storage operation react to this tariff, to production of distributed generation (as PV) and to incentives from the regulator. As a result, the reaction of the sum of battery storage owners is hard to predict and makes the consumer – the former price taker – to an uncoordinated price maker.
1. Introduction

1.2 Goal and overview of the thesis

In Section 1.1, the need for an evaluation of the impact of storage on different levels was motivated. Therefore, the goal of this thesis is to scrutinize the impact of combined residential PV/battery storages.

The potential savings for single households are quantified in monetary terms in Section 2.1 (Methods) and 3.1 (Results), where they are contrasted to the investment costs for PV/battery systems. Also the impact in terms of load, time of consumption and infeed to the grid is considered. Therefore different modeling approaches are used. A deterministic model with full knowledge of consumption and PV production is run, a model predictive control (MPC) approach as well as a stochastic scenario based model are presented.

In Section 2.2 (Methods) and 3.2 (Results) the optimal cost minimizing battery size for single and multiple, aggregated households is determined using a simulation-based approach that incorporates the methods of Section 2.1. For the single households, for different battery lifetimes and discount rates assumed, the optimal size is derived and related to the potential savings. For multiple households, clusters of houses with PV are contrasted to clusters without PV to define differences in total costs (composed of battery and electricity costs) for both cluster types.

Section 2.3 (Methods) and 3.3 (Results) evaluate the impact of houses equipped with PV/battery storages on the distribution grid. Thereby different tariff structures and PV/battery systems combinations are used and it is shown that a high PV penetration leads to for example to overvoltages that can – to a small extent – be damped by choosing the right tariff structure and battery combination.

In Section 2.4 (Methods) and 3.4 (Results), the potential impact of the implementation of PV/battery systems on the residential level on powermarkets is assessed. Therefore, a stochastic and time-dependent marginal cost curve of generation with real data of all power plants in California and a time-dependent demand curve, derived from real load data, are used. Therefrom, the hourly wholesale market price and emissions in California are obtained and the impact of PV/battery systems at the residential level is estimated.

In order to mitigate the effects of PV and high demand, batteries can also be designed for load shaving. An algorithm is presented in Section 2.5 (Methods) that first determines the needed battery storage capacity for a given PV capacity to meet the requirement of a defined, maximum load and then finds the optimal combination of both by making use of the isocost curve for investment. Also a load-dependent electricity tariff is developed, that is be
able to let people reduce their peak load, while being so simple that private customers will be able to comply with. The results are presented in Section 3.5.
2.1 Model: cost minimization for a single household

2.1.1 Database and input models

Load profiles

Modeled load profiles  For a first modeling approach, data from the OpenEI [5] were used. This open-source platform provides generic load profiles which are modeled for specific locations for residential buildings which are based on the Building America House Simulation Protocols [1] and the Residential Energy Consumption Survey [6]. In consistency with the radiation and temperature data, in the first modeling approach, data for Bakersfield, California were taken.

Load profiles are given in three different scenarios: Low, Base, High for the electricity consumption of households with different building structure, design and fuel types. Every simulation was run scenario-wise with these consumption profiles.

Load profiles from Pacific Gas and Electric residential customers  Hundred-thousand realized load profiles of residential customers for one year from 486 Zip Codes in Northern California could be retrieved from PGE. 1923 ($\approx 2'000$) of these load profiles were used in the modeling approach to estimate the monetary savings potential for residential battery users in California.

Solar radiation and photovoltaic generation

Modeling of solar radiation  According to the literature (see [35] or [32]) the solar radiation is modeled with a $\beta$-distribution. Data were taken from the National Solar Radiation Database and the documentation can be found in [51].
For the modeled load profiles, data were taken from the station location Bakersfield Meadows, California, where data are of highest possible quality (Class I from I-III) and are given as mean and standard deviation of the radiation \([W/m^2]\) for every hour of the month (a 12x24 matrix \(\cong 288\) elements). The most recent data available stem from 2010.

For the realized PGE load profiles, data from the same source and with the same methodology were taken and every customer’s ZIP-Code was matched with the closest Class I meteorological station.

The \(\beta\)-distribution is given

\[
PDF(r) = \begin{cases} 
\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} r^{\alpha-1}(1-r)^{\beta-1} & \forall r \in [0,1], \alpha \geq 0, \beta \geq 0 \\
0 & otherwise
\end{cases}
\] (2.1)

where

- \(PDF(r)\): the \(\beta\)-distribution probability density function
- \(r\): radiation \([W/m^2]\)
- \(\Gamma\): the Gamma function
- \(\alpha\): parameter of the PDF [-]
- \(\beta\): parameter of the PDF [-]

The parameters \((\alpha&\beta)\) of the PDF are estimated from the data as follows (for derivation refer to Section A.1.1):

\[
\alpha = \left(\frac{(\mu_r - a) * (b - \mu_r)}{\sigma_r^2} - 1\right) * \frac{\mu_r - a}{b - a}
\] (2.2)

\[
\beta = \left(\frac{(\mu_r - a) * (b - \mu_r)}{\sigma_r^2} - 1\right) * \frac{b - \mu_r}{b - a}
\] (2.3)

where

- \(\mu_r\): mean of radiation \([W/m^2]\) for a given hour and month
- \(\sigma_r\): standard deviation of radiation \([W/m^2]\) for a given hour and month
- \(a\): the lower bound of measured data, in this case 0 \(W/m^2\)
- \(b\): the upper bound of measured data, in this case 1200 \(W/m^2\)

The 288 derived probability density functions (PDF) of the \(\beta\)-distribution have an upper bound of 1 and a lower bound of zero. The documentation of the meteorological data states that the upper bound of the radiation corresponds to 1200 \(W/m^2\). Thus, the upper bound of 1 of the distribution corresponds to a radiative energy of 1200 \(W/m^2\) and the lower bound to 0.
2.1. Model: cost minimization for a single household

\[ W/m^2 \] respectively.

To get time series of the radiation \([ kW/m^2 ]\) for the time series of the data, a random sample of the radiation distribution of the correspondent hours of the month was taken and multiplied with 1.2 \(^1\). As the radiation is likely to change relatively from one hour to the next due to clouds, no filtering was applied as done in the above mentioned literature. \(^2\)

**Modeling of photovoltaic system**  
A Si-PV Cell System was taken as a basis, as according to \([4]\) ‘Solar cells based on silicon semiconductors account for nearly 90 percent of 2011 sales of PV products’. Si (multicrystalline) modules of Trina Solar exist with an efficiency of 20.8 % (tested by \([30]\)), but are not widely used yet.

In order to model typically installed panels, a very common panel was taken as basis: *Trina Solar TSM-245PA05A, 245W Watt Solar Panel*. The efficiency of the panel is 15 %, and the system size is 64.95 \( \times \) 39.05 inches (about 1.6 \( m^2 \)). As a result for the sizing of the PV cells, a possible installation of 150 W/m\(^2\) is taken as basis. \(^3\) Here, the calculation is done on an area basis in order to keep in mind the surface requirement for an installation. Therewith the power output \((\vec{G}_{\text{en},p_v})\) is given in Equation 2.5 (first the efficiency is derived before explaining the equation).

Capital costs (w.o. installation costs) are given in June 2015 170 USD/Panel which is equal to approx. 106 USD/m\(^2\) installation. Two different inverter types were assumed: the research case was that houses that possess PV had it installed before the battery was installed. Since PV installations only require unidirectional inverters, an additional inverter for the battery had to be installed (thus, the older PV inverter might not have an efficiency as high as the battery inverter). The PV inverter efficiency was adopted from \([38]\) with 85 %, and the typical costs for an inverter (or the sum of all microinverters), are at about 2000 USD (e.g. *Magnum Energy MS4448PAE*), for a power of up to 17.6 kW.

The efficiency was modeled according to \([36]\) as a linear function of the radia-

\(^1\) \( \hat{\beta} = \times \frac{1200}{1000W/kW} \)

\(^2\) The distribution of radiation in California during the day and the year is much different to the one in Switzerland. The probability of a constantly strong radiation without breaks during the day is relatively high and smooth radiation profiles are prevalent.

\(^3\) 150W/m\(^2\) \( \leq 245W/1.6m^2 \). The efficiency is included in the power rating, therefore the 150 W/m\(^2\) correspond to an efficiency of 15%, as the standard radiation is at 1000 W/m\(^2\). A 2kW PV-installation would equate to about 13 m\(^2\) of installation.
2. Model Description

The model description was for simplification reasons, the air temperature was assumed to be similar to the cell temperature:

\[ \eta_{pv} = \eta_{ref} \times [1 - 0.0042\left(\frac{E_{rad}}{18} + \theta_a - 20\right)] \times \eta_{inv} \]  

(2.4)

with

- \( \eta_{pv} \): the PV cell efficiency as a function of temperature and radiation [-]
- \( \eta_{ref} \): the PV cell’s reference efficiency (15 %)
- \( \eta_{inv} \): the inverter’s efficiency (85 %)
- \( E_{rad} \): radiation [kW/m²]
- \( \theta_a \): temperature [°C]

To model the air temperature, the National Solar Radiation Data Base [51] was used and the closest meteorological station to the customer’s ZIP-code was taken as basis. Also an analogue procedure for modeling was used: for every hour of a month, a mean and a standard deviation was calculated (28-31 data points), from which a random time series sample of a normal distribution could be drawn. The normal distribution was chosen, as the data points for a specific hour of a month align well on qqplots (see Fig. A.2 in the Appendix as an example) and the Chi-square goodness-of-fit test, does not reject the null-hypothesis of normal distribution on p=5 %-level for randomly chosen temperature distributions of different hour-month combinations. As normal temperature profiles do not exhibit big jumps, a filtering with a window of [0.05 0.2 0.5 0.2 0.05] was applied.

**Modeling of photovoltaic generation**  The PV-production time series for a given area of PV-panels deployed was calculated by multiplying element-wise a randomly drawn time series sample of radiation and efficiency and the area of the module which was varied during the simulation.

\[ \vec{Gen}_{pv} = \text{diag}(\vec{r}_{pv}) \times \vec{\eta}_{pv} \times A_{pv} \]  

(2.5)

with

- \( \vec{Gen}_{pv} \): the PV cell generation [kW]
- \( \vec{r}_{pv} \): the radiation on the cells [kW/m²]
- \( \vec{\eta}_{pv} \): the PV cell’s efficiency [-]
- \( A \): the area of PV deployed [m²]
In Figure 2.1, the distribution of residential PV systems in California is shown. It can be seen that most installation sizes vary between 2 kW and 6 kW. Therefore, in the following models, when the impact of different PV sizes are determined, capacities of 2 kW, 4 kW and 6 kW are chosen.

Figure 2.1: Distribution of installed PV capacity in California according to [7].

Modeling of the battery

According to different sources, such as the Washington Post [9], the announcement of Tesla’s new Powerwall was taken as ‘The coming revolution in energy storage’. Tesla’s Powerwall is a battery storage that can be installed on the household level, is designed to bridge high price hours and to store excess electricity supply from solar PV. Therefore, the specifications of the two Powerwall-models (weekly- and daily cycle) were taken from the press release of Tesla [8] as simulation basis (state of data correspond to time of the preparation of the thesis), as no other data for this specific battery were available at the preparation of this model. A linear charge/discharge rate and efficiency as given in the Powerwall’s specifications and listed in Table 2.1 was assumed. As most houses equipped with PV posses a unidirectional inverter already, but for a battery, a bidirectional inverter is needed, it is assumed that a state-of-the-art bidirectional inverter for the battery with an efficiency of 95 % is installed.

Tariff profile

The PGE time of use pricing (TOU) price profile for residential customers was used as basis.
PGE is the major electricity supplier in northern California and for a long
Table 2.1: Tesla’s Powerwall Specification based on its press release.

<table>
<thead>
<tr>
<th>Type</th>
<th>Daily Cycle</th>
<th>Weekly-Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage Capacity [kWh]</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Continuous power [kW]</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Peak power [kW]</td>
<td>3.3</td>
<td>3.3</td>
</tr>
<tr>
<td>Round trip efficiency [%]</td>
<td>92</td>
<td>92</td>
</tr>
<tr>
<td>Inverter efficiency [%]</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>Charge/discharge efficiency $^a$ [%]</td>
<td>96</td>
<td>96</td>
</tr>
<tr>
<td>Warranty [years]</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Number of cycles warranted</td>
<td>25000</td>
<td>5200</td>
</tr>
<tr>
<td>Price of 1 kWh $^b$ [$]</td>
<td>0.12</td>
<td>0.57</td>
</tr>
</tbody>
</table>

$^a$Estimated by square root of Round Trip Efficiency
$^b$Based on the guaranteed cycle number and capital costs only, w.o. depreciation

Table 2.2: Rate scheme of PGE for electric vehicles.

<table>
<thead>
<tr>
<th>Price [cents/kWh]</th>
<th>Off-Peak</th>
<th>Half-Peak</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter $^a$</td>
<td>10</td>
<td>17</td>
<td>29</td>
</tr>
<tr>
<td>Summer $^b$</td>
<td>10</td>
<td>22</td>
<td>42</td>
</tr>
</tbody>
</table>

$^a$Oct – Apr, Off-Peak: 22-11, Half-Peak: 11-14 and 20-22, Peak: 14-20 o’clock
$^b$May – Sept, Off-Peak: 22-11, Half-Peak: 11-14, Peak: 14-22 o’clock

time, customers were only provided with flat price schemes which varied between (climatic) regions and season but not during the days. Since recently, PGE offers TOU-rates which account for the typical wholesale price profile over a day (see introduction). Many different rate schemes for different user type exist and until now, no rate exists for clients possessing a battery installation at home. Consequently, the rate plan for clients having electric vehicles was chosen as basis and is listed in Table 2.2. One can see that the rate plan is high during summer due to high cooling loads in California. Also the price is high during high consumption hours, such as in the evening, when people cook or watch TV. The low price hours are during night, when general demand is low and EV can be charged.

In some of the following analyses, different tariff structure (TS) are used. Some of them (in this case TS 3 and 4) are derived later. For the sake of completeness, the terminology used later is listed here.

- Tariff structure 1: PGE’s Time of use price and a FIT of 8.92 cents, the
2.1. Model: cost minimization for a single household

current average FIT in California, according to [20].

• Tariff structure 2: PGE’s Time of use price and a FIT of 0 cents.

• Tariff structure 3: PGE’s Time of use price, a FIT of 8.92 cents and max peak price (see formulation in Equation 2.47a). Peak price is chosen to be 3 USD/kW for weekly evaluations and 50 USD/kW for yearly evaluations.

• Tariff structure 4: PGE’s Load dependent pricing following the reasoning of Section 2.5.4 and with the same numbers as there.

Installation costs for battery/PV systems

In Table 2.1, the capital costs for the Powerwall are listed, in the following estimates for the total costs, including installation, are made.

Costs of PV systems Data were obtained from [7] and are shown in Figure 2.2. The costs plotted are empirical costs in USD/W for a given system size, including installation and administrative costs. Due to economies of scale, the per Watt-price sinks with an increasing system size strongly between 1 and 2 kW and keeps sinking up to a size of about 10 kW. For installations between 2-6 kW, the average costs of 8250 USD/kW were assumed.

![Figure 2.2: Empirical per Watt costs for PV systems in the US, source [7].](image)

Costs of the battery installation The 7-kWh capacity battery is given with capital costs of 3’000 USD (10-kWh capacity battery with 3’500). Including taxes and shipping capital costs of 3’500 USD for the 7-kWh Powerwall
type are assumed. For other overhead costs, data sources are limited as the Powerwall is not on the market yet. This makes the net cost of the 7-kWh capacity version to be about high as the gross costs for the 10-kWh version. To incorporate installation costs, SolarCity, a common provider for solar PV in California, offers the 10-kWh capacity battery for about 7’000 USD, including installation and inverter but excluding taxes. Since this offer is not existent at the moment of the thesis preparation for the 7-kWh capacity battery, it is assumed that the total net costs for the 7-kWh version are the same.

Net present value and payback time for PV/battery systems The net present value (NPV) is calculated based on the following formula

\[
\text{NPV} = \sum_{t=1}^{t=\text{end}} \frac{S_t}{(1+r)^t} - C_0
\]

where \(S_t\) stands for the savings incurred in time period \(t\) in USD, \(r\) the discount rate in % and \(C_0\) the investment costs in USD.

The payback time (or break even) is defined as the time period \(t = \text{end}\), for which the NPV becomes zero for the first time. In this analysis only years, thus discrete time steps were taken into account for the payback time. Therefore, the year for which the NPV turns zero or positive for the first time is chosen.

Thereby efficiency losses, replacement after a lifetime and maintenance costs were neglected. Also, the estimation was done based on the results of one year and it has to be kept in mind that electricity prices and consumption underly uncertainty.
2.1. Model: cost minimization for a single household

2.1.2 Problem formulation

The house which was assumed to have the objective of cost minimization is depicted in Figure 2.3. The house (load) is able to choose whether to (dis-)charge the battery or use the grid to buy or sell electricity.

Figure 2.3: Overview on house with PV, battery and grid connection.

The overall goal of the model was to minimize the costs of electricity consumption for a household with PV, battery storage and a given load profile, resulting in a net-load profile with grid-import and -export and the minimized costs for a specific time period. CVX, a Matlab software to solve convex optimization problems was used. As cost minimization is the only goal.

\[
\begin{align*}
\text{minimize} & \quad \sum_{t=1}^{t=\text{end}} (r_t \times g_t^i - \text{FIT}_t \times g_t^e) \\
\text{subject to} & \quad l_{\text{net}}^t + \delta_c^i_t - \delta_d^i_t + g_t^e - g_t^i = 0 \\
& \quad 0 \leq g_t^i \leq \text{cap}_{\text{max},\text{in}} \\
& \quad 0 \leq g_t^e \leq \text{cap}_{\text{max},\text{out}} \\
& \quad 0 \leq \delta_c^i_t \leq \delta_{\text{max},c} \\
& \quad 0 \leq \delta_d^i_t \leq \delta_{\text{max},d} \\
& \quad 0 \leq \text{soc}_t \leq \text{c}_{\text{max}} \\
& \quad \text{soc}_t(t = 1) = \text{soc}_t(t = 0) \\
& \quad \text{soc}_t(t + 1) = \text{soc}_t(t) \times \text{soc}^c + \delta_c^i(t) \times \eta_c - \delta_d^i(t) \times 1 / \eta_c
\end{align*}
\]
2. Model Description

\[ g_i^i \]: electricity import from the grid [kW]
\[ g_i^e \]: electricity export to the grid [kW]
\[ soc_t \]: the battery’s state of charge [kWh]
\[ soc_t^l \]: the battery’s self-discharge rate [-]
\[ \text{net}^i_t \]: net load profile [kW]
\[ \text{cap}^{\text{max/in/out}}_t \]: the maximal line capacity to/from the grid [kW]
\[ r_t \]: profile of the tariff [$/kWh]
\[ \text{FIT}_t \]: feed-in-tariff [$/kWh]
\[ \delta^{\text{max/c/d}}_t \]: max charge and discharge rate of the battery [kW]
\[ \delta^c_t \]: the battery’s rate of charge [kW]
\[ \delta^d_t \]: the battery’s rate of discharge [kW]
\[ \eta^c_t \]: the battery’s charging efficiency, as derived from round-trip efficiency, discharging efficiency \( \eta^d_t = 1/\eta^c_t \) [-]
\[ c^{\text{max}} \]: max battery capacity [kWh]

\( a \) Import from and export to the grid were defined as separate variables, so that setting the FIT to zero, export is eliminated without changing constraints.
\( b \) The net load profile is the difference of the household’s load profile and PV-production.
\( c \) Explanation why charge and discharge are separated can be found below.

Explanation of the formulation

**Equation 2.7a** is the cost-function of the model household which is minimized subject to the constraints (2.7b to 2.7i).

The first part of the objective function represents the positive costs that a household purchasing power from the grid faces: for every hour, there is a price per kWh. This net load can be balanced (see balancing equation 2.7b) by dis/(charge) and ex/import, but every import and charge operation is costly.

The second part of the objection function is a possibility for the household to make a profit (negative costs) and is only relevant, if a) the PV-production exceeds the load after the battery is charged b) the FIT is higher than the electricity price\(^4\) (either for the same hour, when import would directly be sold, or for an other hour during the optimization horizon, when stored energy is sold)\(^5\). Thus the household can directly sell electricity from PV production or via battery discharge to the grid.

\(^4\)This does not hold in this simulation, here the FIT was always lower than the lowest electricity price
\(^5\)Incorporating efficiency and losses.
Equation 2.7b is the power balance equality. For every point in time, the load (plus eventual export) can either be balanced by buying power from the grid or by discharging the battery. Vice versa, every charging of the battery can only be balanced, if the grid import minus export does not exceed the net load.

Equation 2.7c incorporates two inequality constraints for the grid import: the import from the grid is constrained by the capacity of the line which represents the upper bound and was set to the maximum occurring load of the year. As negative import (‘export’ for this line and therefore selling electricity for the normal rate) is not allowed, the lower bound is zero.

Equation 2.7d analogously incorporates two inequality constraints for the grid export: the export to the grid is constrained by the capacity of the line which represents the upper bound. As negative export (‘import’ for this line and therefore buying electricity for the FIT) is not allowed, the lower bound is zero.

Equation 2.7e and 2.7f describe the lower and upper (dis-) charge rate of the battery.

Equation 2.7g the inequality constraints state that the state of charge (SOC) of the battery cannot turn negative (one cannot withdraw more energy from the battery than it has stored) and that the SOC cannot exceed the battery’s capacity.

Equation 2.7h defines that at the beginning of the optimization cycle the battery has a certain SOC. This is either given as exogenous input (for a time series of a year it can be chosen arbitrarily), or for the MPC and stochastic approach (see Section 2.1.2 and 2.1.3), it is the SOC of the last realized time step which is the starting time for the new optimization of the MPC.

Equation 2.7i defines the SOC for the battery: the SOC of the battery is equal to the SOC of one time step before times the self-discharge rate plus charging operation times the charging efficiency and discharging operation times the discharging efficiency.

Most importantly, in order to secure a convex formulation of the problem in Equation 2.7i, the charging and the discharging operation are separated.

Second, the electricity output is equal to energy $\times$ efficiency.

$$E_{\text{battery} \rightarrow \text{cons.}} = E_{\text{battery}} \times \eta_d$$
2. Model Description

\[ E_{\text{cons.}\rightarrow\text{battery}} = E_{\text{grid}} \times \eta_c \]

From the viewpoint of the battery, however \( E_{\text{battery}\rightarrow\text{cons.}} \) and \( E_{\text{battery}\leftarrow\text{cons.}} \), the battery needs more electricity to fulfill the requirement of the consumer.

\[ \frac{1}{n_c} \times E_{\text{battery}} \hat{=} \eta_d \times E_{\text{battery}} \]

[31] proposes a further constraint is the product of the charging and discharging operation must be zero for all times, so that only either one of both takes place. This is done, to prevent the battery to charge and discharge at the same time. In order to be able to express it in YALMIP, the problem is reformulated (see Eq. 5.15 in [31]), into a mixed integer problem. This is not needed here: as the buying price is always greater than zero and efficiency is lower than one, each simultaneous charge and discharge operation would waste electricity and thus rise the electricity costs. As the objective is to minimize them, these actions never take place simultaneously (which was tested and verified).

Deterministic model-predictive control vs. the multi-period deterministic approach

The multi-period deterministic approach minimizes the electricity costs of the household in an optimal way, given the (net) load profile of a year (including a random realization of the radiation and the temperature and the load profile) and the electricity cost profile. The main benefit is the short computational time for the calculation, as only one realization is taken as basis. This is however unrealistic in many ways: neither the demand of a household is known for an entire year, nor the PV-production. On the other hand, this approach can be used as an estimation for the saving potential, given a battery could be scheduled in an optimal way.

The deterministic model-predictive control approach takes some of the uncertainty into account: a time horizon \( t_h \) over which the whole simulation is carried out (e.g. 365 days) is defined. The optimization is now not carried out over the whole simulation horizon, but over a defined optimization horizon \( t_{\text{pred}} \) (e.g. 3 days), for which it is more realistic to have complete knowledge. After a realization time \( t_{\text{real}} \) (e.g. 1 day), a new optimization from \( t_{\text{real}} \) to \( t_{\text{real}}+t_{\text{pred}} \) is carried out and the last SOC of the battery at \( t=t_{\text{real}} \) is taken as input for \( \text{soc}_t(t=1) \). Then a new random sample from the PV-production is taken and the load profile is updated. The output of the optimization is one realization and therefore seems to be deterministic. However, as the uncertainty of the varying load profile is taken into account, suboptimal solutions for the charging of the battery are calculated.
and therefore the savings’ estimation is more realistic.

One drawback of the approach is the computational time which is roughly about $t_h/t_{\text{real}}$ times longer than the deterministic approach. As a result, to carry out the simulation in a reasonable computational time, $t_{\text{real}}$ cannot infinitely be reduced which would be necessary to incorporate the uncertainty in a time step. Therefore, different $t_{\text{real}}$ are used to represent the increased realisticity of the simulation that can be gained by increasing computational time. Also $t_{\text{pred}}$ is altered to define the needed prediction horizon to make right decisions.

### 2.1.3 Scenario based stochastic optimization

Both models derived in Section 2.1.2 are of deterministic nature, as for a given time horizon, perfect knowledge is assumed. In order to better account for the uncertainty, a scenario based stochastic optimization is presented here.

Different work has been carried out in this area. An example for applied work is the Randomized MPC (RMPC) proposed in [52]. In this paper, the uncertainty of the weather prediction and of internal gains and house occupants is taken into account. In the deterministic MPC, constraints are tightened in order to account for the uncertainty in the prediction. The stochastic MPC uses constraints which do not have to be fulfilled at every time step, as ‘chance constraints’ and secures that those are fulfilled with a defined probability. The RMPC is the result of both techniques, taking the uncertainty of the disturbances into account without tightening constraints. Reference [49] presents a MPC scheme to control buildings with a variety of appliances that can exhibit demand response features, applying electricity prices that vary over time and by using perfect predictions of future disturbances of weather conditions or internal gains.

The model used in this thesis is built up according to [14] which gives an insight on how to account for uncertainty in MPC-systems and [44] which is basic tutorial on stochastic programming.

A number of different consumption scenarios are taken into account for every household. These scenarios (in the following called net load($i,n,t$)), in analogy to a Monte Carlo simulation, are the basic load profiles of the households minus a random sample of electricity PV-production (according to the parameters of the household’s PV installation and geographic location), plus a random term accounting for the consumption uncertainty.
2. Model Description

net load\((i, n, t)\) \(= \) load\((i, t)\) \(-\) PV\((i, t)\) \(+\) \(\delta_{cons}(i, t)\) \hspace{1cm} (2.8)

where \(i\) indicates the household and \(n\) the scenario. The consumption uncertainty is given

\[ \delta_{cons}(i, t) \sim N(0, \sigma(i, t)^2) \] \hspace{1cm} (2.9)

where the variance is the historic variance at a specific hour of the day. For very few hours during night, negative demand can occur in the scenarios, as a normal distribution with zero mean is added. These cases are set to zero.

The objective of this model and its constraints are the same as for the deterministic formulation. However only the current net load is known. Therefore, the optimization is carried out from the current state to the optimization’s horizon \(T\) for all scenarios \(n \in N\).

\[
\begin{align*}
\text{minimize} & \quad r_1 \times g^i_1 - FIT_1 \times g^e_1 + \sum_{t=2}^{T} \sum_{n=1}^{N} E[r_{t,n} \times g^i_{t,n} - FIT_t \times g^e_{t,n}] \\
\text{subject to} & \\
& \quad l_{net}^{n} + \delta_{c}^{t,n} - \delta_{d}^{t,n} + g^{e}_{t,n} - g^{i}_{t,n} = 0 \hspace{1cm} (2.10b) \\
& \quad 0 \leq g^{i}_{t,n} \leq \text{cap}^{\text{max}_{\text{in}}} \hspace{1cm} (2.10c) \\
& \quad 0 \leq g^{e}_{t,n} \leq \text{cap}^{\text{max}_{\text{out}}} \hspace{1cm} (2.10d) \\
& \quad 0 \leq \delta_{d}^{t,n} \leq \delta^{\text{max}_{d}} \hspace{1cm} (2.10e) \\
& \quad 0 \leq \delta_{c}^{t,n} \leq \delta^{\text{max}_{c}} \hspace{1cm} (2.10f) \\
& \quad 0 \leq soc_{t,n} \leq c^{\text{max}} \hspace{1cm} (2.10g) \\
& \quad soc_{t,n}(t = 1) = soc_{t,n}(t = 0) \hspace{1cm} (2.10h) \\
& \quad soc_{t,n}(t + 1) = soc_{t,n}(t) + \delta_{c}^{t,n}(t) \times \eta_{c} - \delta_{d}^{t,n}(t) \times 1/\eta_{c} \hspace{1cm} (2.10i)
\end{align*}
\]

where \(E[\ldots]\) represents the expected value for the future.

After the optimization is carried out for a single time step (in this case an hour) and its optimization horizon, time is moved forward, the ‘real’ value for the next time step is taken and a new expectation for the coming time steps is derived. The SOC of the battery is the only memory which is carried from one hour to the next.

The expected value can be derived in many ways. In this context an autoregressive integrated moving average ARIMA\((24, 1, 1)\) time series model was used. The differencing \((D=1)\), was applied to remove the trend, as looking
2.1. Model: cost minimization for a single household

at the plots of the autocorrelation function (ACF) and partial autocorrelation function (PACF) suggested that an autoregressive process with a lag=24 and a moving average process with lag=1 was prevalent.

![Sample autocorrelation function](image1)
![Sample partial autocorrelation function](image2)

Figure 2.4: Autocorrelation and partial autocorrelation of a differenced net load.

Typical household consumption is subject to daily seasonality which is represented with the AR(24) part. The dependence on the last hour is expressed with the MA(1) part. The t-statistics for the tested time series were highly significant for the MA and AR part and support the choice of the model.

For the actual simulation, the net load (including added consumption uncertainty) from the previous week was taken as a training set and the last 24 hours of the Monte-Carlo scenario were taken as predictors. The prediction horizon has the same length as the optimization horizon, for which 24 hours seemed to be reasonable. An advantage of using the ARIMA-model in comparison to taking other, simpler techniques to derive the expected value, is that the standard error of the estimation is provided which can be used to e.g. run a simulation for the 95% percentiles of net load.
2.1.4 Cost of anarchy

This short subsection gives an overview on the costs of anarchy for the sizing of batteries. The concept of the cost of anarchy applied to battery storage is that a single large storage, with a capacity lower than the sum of several distributed, small storages, could fulfill the same requirements as the distributed, small storages. This excludes all services provided by distributed storages, so e.g. directly balancing out demand on a household level not to influence the regional distribution grid. Therefore, this is a simple estimation, on how big is the cost of anarchy for a sample of ≈ 2'000 customers.

To do so, first, the total electricity costs for these ≈ 2'000 customers were calculated if everybody had a single storage at home (once with, once without PV). Second, the aggregated load of all customers was exposed to a single, big storage. The size of the big storage was incrementally increased. At a certain point, the summed costs for using the centralized, big storage and the decentralized storages is the same. However, the total capacity needed (big storage, vs. sum of small storages) is smaller for the centralized battery. The difference of the total capacities is called the cost of anarchy, expressed in terms of storage capacity. This could also be expressed in terms of costs, where in addition to the material costs, coordination costs would need to be added to the costs of the distribution storages.
2.2 Model: cost minimizing battery size

To find the optimal size of a battery for single and multiple households, a cost minimization can be conducted that includes investment costs.

2.2.1 Costs incurred from installing a battery

In order to account for the costs incurred by the battery for a given time period, the investment costs were broken down into equivalent annual cost (EAC) and are calculated according to

\[
\text{EAC} = \frac{\text{Investment} \times r}{1 - (1 + r)^{-n}}
\]

(2.11)

where \( r \) is the applied discount rate, \( n \) is the number of time periods in years and \( \text{Investment} \) is recorded in USD.

According to Tesla, the pure capital costs of the Powerwall are at 3'500 USD. Sunrun is willing to sell the Powerwall for \( \approx 7'000 \) USD, including inverter and installation. The guaranteed lifetime for the Powerwall is 10 years which was altered in order to see the effect on the EAC and the sizing. Even though, the Powerwall is not sizable per kWh storage capacity, it is assumed to be possible to derive an optimal size. Therefore, the costs were calculated on the basis of one kWh storage capacity. Table 2.3 lists the costs for different lifetimes, discount rates and including installation costs.

2.2.2 Cost minimization including battery sizing for a single household

Most solvers that were explored were not able to solve the following optimization due to the high number of decision variables and the long time series used.

The CVX solver \textit{SeDuMi} however was able to solve it in a reasonable time if the formulation of Section 2.1.2 was reduced in its complexity. Therefore, the whole adapted formulation that was used is presented here.

The charging and discharging rate was used as a function of the battery storage capacity following the specifics of the Powerwall, for which storage capacity is equal to 7 kWh and the peak (dis-)charging rate 3.3 kW.

The (dis-)chaging rate was adapted linearly to the used storage capacity (i.e. a storage capacity of 1 kWh corresponds to 1/7 of the original capacity, therefore the peaking (dis-)charging rate corresponds to \( 3.3 \times 1/7 \approx 0.47 \) kW per kWh storage capacity).
Table 2.3: Equivalent annualized costs of the Powerwall on a unit and on a kWh-storage capacity basis.

<table>
<thead>
<tr>
<th>Expected lifetime [a]</th>
<th>discount rate</th>
<th>1 %</th>
<th>2 %</th>
<th>5 %</th>
<th>1 %</th>
<th>2 %</th>
<th>5 %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Incl. install.</td>
<td>915</td>
<td>956</td>
<td>1083</td>
<td>131</td>
<td>137</td>
<td>155</td>
</tr>
<tr>
<td></td>
<td>Cap costs</td>
<td>457</td>
<td>478</td>
<td>542</td>
<td>65</td>
<td>68</td>
<td>77</td>
</tr>
<tr>
<td>10</td>
<td>Incl. install.</td>
<td>739</td>
<td>779</td>
<td>907</td>
<td>106</td>
<td>111</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>Cap costs</td>
<td>370</td>
<td>390</td>
<td>453</td>
<td>53</td>
<td>56</td>
<td>65</td>
</tr>
<tr>
<td>15</td>
<td>Incl. install.</td>
<td>505</td>
<td>545</td>
<td>674</td>
<td>72</td>
<td>78</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>Cap costs</td>
<td>252</td>
<td>272</td>
<td>337</td>
<td>36</td>
<td>39</td>
<td>48</td>
</tr>
</tbody>
</table>

1. Assumed to be 7'000 USD/battery.
2. Assumed to be 3'500 USD/battery.

For a part of the simulations, indicated in the Results part, no remuneration was assumed. This was done since this could drastically reduce computational time. In this case, grid export is excluded in the objective function 2.12a but is still formulated to balance Equation 2.12b. Also, it is noted that the inequality constraint limiting the battery size to positive numbers is not necessary, as it is a consequence of Equation 2.12g.

\[
\begin{align*}
\text{minimize} & \quad c_{\text{max}} \times C_{\text{Bat}} + \sum_{t=1}^{t=\text{end}} (r_t \times g_t^i - \text{FIT} \times g_t^e) \\
\text{subject to} & \quad l_t^{\text{net}} + \delta_t^i - \delta_t^d + g_t^e - g_t^i = 0 \quad (2.12b) \\
& \quad 0 \leq g_t^i \quad (2.12c) \\
& \quad 0 \leq g_t^e \quad (2.12d) \\
& \quad 0 \leq \delta_t^d \leq \delta_{\text{max}} \quad (2.12e) \\
& \quad 0 \leq \delta_t^i \leq \delta_{\text{max}} \quad (2.12f) \\
& \quad 0 \leq \text{soc}_t \leq c_{\text{max}} \quad (2.12g) \\
& \quad \text{soc}_t(t = 1) = \text{soc}_t(t = 0) \quad (2.12h) \\
& \quad \text{soc}_t(t + 1) = \text{soc}_t(t) + \delta_t^c(t) \times \eta_c - \delta_t^d(t) \times 1/\eta_c \quad (2.12i)
\end{align*}
\]
2.2. Model: cost minimizing battery size

$g^i_t$ : electricity import from the grid [kW]
$g^e_t$ : electricity exported to grid, to balance 2.12b and/or to reduce costs, when FIT was assumed [kW]
$c^{max}$ : max. battery storage capacity, a decision variable in this case [kWh]
$C^{\text{bat}}$ : EAC/kWh (see Table 2.3) [USD/kWh*a]
$soc_t$ : the battery’s state of charge [kWh]
$l^{\text{net}}_t$ : net load profile [kW]
$r_t$ : profile of the tariff [USD/kWh]
$\delta^{\text{max}}_{c/d}$ : max charge and discharge rate of the battery [kW], a linear function of the decision variable $c^{max}$
$\delta^c_t$ : the battery’s rate of charge [kW]
$\delta^d_t$ : the battery’s rate of discharge [kW]
$\eta^c_t$ : the battery’s charging efficiency, as derived from round-trip efficiency, discharging efficiency $\eta^d_t=1/\eta^c_t$ [-]

2.2.3 Cost minimization including battery sizing for multiple households

The general formulation is the same as for the single household presented in Section 2.2.2, whereby here the total costs of the community were minimized. Two different arrangements of the battery-house system were considered that are depicted schematically in Figure 2.5.

**Simple aggregation** The only adaption in this case is that the net load is considered the sum of all load profiles of houses in an aggregation. The formulation is similar to the one of cost of anarchy in Section 2.1.4.

$$\text{Net load}_{\text{agg}} = \sum_{\text{house}=1}^{n} \text{Net load}_{\text{house}}$$ (2.13)

The hypothesis was that with an increasing size of aggregated houses, synergy effects reduce the average total costs (battery + electricity costs) per house. However, especially when considering remuneration for electricity infeed, some houses may have a higher electricity bill than before (as surplus electricity is not sold to the grid, but transfered to the neighbors house). Therefore, a Pareto constraint is introduced.

**Aggregation with Pareto constraint** The battery can be charged and discharged by all houses and can itself charge from the grid and is only used in order to minimize the overall costs of the community. The battery usage
is still constrained subject to maximal charging and discharging rate, so that all houses have to share the charging and discharging rate for a given point in time.

The Pareto constraint is expressed as follows. Equation 2.14 represents the formulation when considering remuneration for grid export, Equation 2.15 when considering no remuneration.

for every house

$$r_t \cdot l_{i}^{net+} - FIT \cdot l_{i}^{net-} \geq r_t \cdot (g_{i}^e + cr_{grid} / n_{houses}) - FIT \cdot g_{i}^e$$ (2.14)

$$r_t \cdot l_{i}^{net+} \geq r_t \cdot (g_{i}^e + cr_{grid} / n_{houses})$$ (2.15)

whereby $l_{i}^{net+}$ and $l_{i}^{net-}$ stand for a positive and negative net load of the single house respectively, FIT stands for remuneration with FIT, $r_t$ for the TOU-tariff and $g_{i}^e$ and $g_{i}^i$ stand for the grid export and import of the house when actually being in the community. $cr_{grid}$ is the charge rate of the battery from the grid and $n_{houses}$ are the number of houses in the cluster.

The left part of the equation consists of constants and is known before running the simulation.

Thus the resulting costs of a house in the community with battery are not allowed to be greater than those of the single household without battery, while the additional costs for direct battery charging are shared.

The costs of the battery do not enter the Pareto constraint due to the following reason: a cost minimizing battery size would not be chosen for the community, if no house would profit from a battery, thus would have lower total costs than without the battery. As the research question is, whether community costs can be reduced while sharing a battery, introducing the constraint for the battery costs as well would a priori make this solution equal to the one of the single houses.

2.2.4 Optimal charging rate and storage capacity combination for single houses

One shortcoming of the approach outlined in Section 2.2.2 is that the charging rate is always coupled to the storage capacity proportion of charging rate to storage capacity of the Powerwall. The formulation is analogous to the one in Section 2.2.2, however the rate and the capacity can take values separately, why only the difference to this formulation is outlined here.
2.2. Model: cost minimizing battery size

\[
\begin{align*}
\text{minimize} & \quad c_{\text{rate}}^{\max} \times C_{\text{rate}} + c_{\text{max}}^{\max} \times C_{\text{Bat}} + \sum_{t=1}^{t=\text{end}} (r_t \times g^t_t) \\
\text{subject to} & \quad \text{constraints}
\end{align*}
\]

\begin{itemize}
\item $g^t_t$: electricity import from the grid [kW]
\item $r_t$: tariff profile [USD/kWh]
\item $c_{\text{max}}^{\max}$: max. battery storage capacity, a decision variable in this case [kWh]
\item $c_{\text{rate}}^{\max}$: max. battery charging rate, a decision variable in this case [kW]
\item $C_{\text{bat}}$: The costs (EAC) per unit storage capacity [USD/kWh*a]
\item $C_{\text{rate}}$: The costs (EAC) per unit charging rate [USD/kW*a]
\end{itemize}

The total annual costs of the battery (the specifics are still related to those of the Powerwall) consist in this case of a part of allocated to the charging rate and a part allocated to the storage capacity.

\[
\text{EAC}_{\text{bat}} = C_{\text{rate}} \times c_{\text{rate}}^{\max}_{\text{Powerwall}} + C_{\text{bat}} \times c_{\text{max}}^{\max}_{\text{Powerwall}}
\]

Where $\text{EAC}_{\text{bat}}$ [USD/a] refers to the total EACs of the battery. As this is one equation with two unknown, different weightings ($\alpha$) for the
2. Model Description

capacity and rate were chosen in order to find for which weighting, the cost minimizing combination is the one of the Powerwall.

\[ C^{bat} = \left( \frac{EAC_{bat}}{e^{max}_{Powerwall}} \right) \times \alpha \]  \hspace{1cm} (2.18)

and

\[ C^{rate} = \left( \frac{EAC_{bat}}{c r^{max}_{Powerwall}} \right) \times (1 - \alpha) \]  \hspace{1cm} (2.19)
2.3 Impact on the grid level

The usage of PV/battery systems results in a changing pattern of electricity exchange from and to the grid. In order to evaluate the potential impacts and to find the TS that mitigates these effects best, an analysis with Matpower was conducted.

Matpower is a package of Matlab which is used to solve optimal power flow (OPF) problems. The following introduction into the modeling approach is based on [54], [53] and the Matpower manual.

2.3.1 Introduction into Matpower's OPF

Matpower offers a simple and standardized way of analyzing the potential effects on the grid as a function of network topology, demand and supply. In this case, the formulation of an OPF is used and costs and the impacts (constraint violations) are recorded for each node and point in time.

In general, two different ways to model the OPF exist. The DC power flow approximation is mainly used in the application of transmission grids, as the resistance/reactance ratio is small. For distribution grids, where the $R/X$ ratio is about 1, the linearization of the power flow equations would not be justified that is why in this evaluation the AC-OPF is used.

In general, an OPF problem can be described as follows:

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad g(x) = 0 \\
& \quad h(x) \leq 0 \\
& \quad x_{\text{min}} \leq x \leq x_{\text{max}}
\end{align*}
\]

where the objective function $2.20a$ is the summation of the generation cost functions of active and reactive power injections, that are minimized over the voltage angle $\Phi$, the voltage magnitude $V$ and the active and reactive power $P$ and $Q$.

The equality constraints in equation $2.20b$ represent the power balance equations for real and reactive power.
The inequality constraints expressed in equation 2.20c, are based on branch flow limits for both ends of a line and are non-linear functions of the voltage angles and magnitudes.

Finally, 2.20d summarizes all upper and lower limits on bus voltage magnitudes, real and reactive power generator injections and the reference bus angle.

### 2.3.2 IEEE 123 node test grid

The basis of this analysis is the IEEE 123 node test grid for medium voltage distribution grids. The documentation can be found on the IEEE webpage [40]. It consists of the feeder, the connection to a higher level of the distribution grid (‘the generator’ on which the cost function of the electricity price is applied) and 85 loads which for each of them can make up to 1000 houses. The feeder is chosen to be the slack bus with a voltage of 1kV, $U_{\text{slack}} = 1\text{pu}$ and $\Phi_{\text{slack}} = 0$.

In this case a smaller number of houses was chosen not to stress the grid by the normal operation, as the objective was to evaluate the impact of PV/battery combinations. Therefore, the number of houses for each load was chosen to be a random number between 1 and 200, as the peak load of the summed load profiles of 200 homes corresponds to the average load in the original file (throughout the analysis the distribution stayed the same).
2.4 Modeling the impact on the power market

In order to estimate the impact of PV/battery systems on the power markets, an evaluation on the Californian power plant (PP) landscape was conducted in order to model the supply side behavior of the wholesale market. Then a default load demand was modeled on which the load profiles of residential houses with solar and PV can be added, in order to evaluate their respective impact on the market. Default load is not to be mixed up with base load: this model handles the existing load through customers not equipped with a PV/battery system as default load, in order to be able to understand the impact of houses with battery/PV on the market.

To model the power market, different approaches are possible (see also [48]): the most realistic one is to run an OPF model on the transmission level with known grid and generator parameters for every time step as a function of the net demand of the households with PV/battery systems and the default load. Also different markets would need to be included. As this would go beyond the scope of this thesis, a simple equilibrium model that is able to provide sufficient insights to estimate the impact of PV/battery on the market was implemented. Reference [43] is an example for an analysis with a similar goal but different approach. The authors model the Californian power market to estimate feedback on a potential charging of electric vehicles and derive an optimal charge control for these.

In this work, a stochastic marginal cost curve for California based on data of e-grid [2] is generated. Thereby transmission losses and dispatches due to different locations and congestions are neglected. The generators marginal costs are calculated based on their expected marginal costs (i.e. the generator’s heat rate or hydro opportunity costs) and are taken as bids into the market. The uncertainty of production due to maintenance or availability (renewables) is estimated based on past data on a generation-type basis.

The default load demand is modeled based on average monthly consumption data for residential, commercial and industrial customers and their respective hourly load shapes. Depending on the depth of battery/PV penetration, the residential demand is partially and incrementally replaced by the shape of the aggregated demand curves of houses with battery/PV systems.

In order to model the prime time responsiveness of residential households with PV/battery, a feedback mechanism on the retail and wholesale market price is introduced. The feedback of the market on the pricing is modeled by correlating the
median wholesale price with the residential TOU-pricing. Reference [47] has conducted a similar approach: in their work, the retail price consists of the wholesale market price plus a mark-up. However, as this could not explain the large variation of actual TOU-prices during the day, here a linear regression is used. Figure 2.6 gives an overview of the modeling approach.

![Image of the power market feedback loop](image.png)

Figure 2.6: Overview on the power market feedback loop.

**Data**

**Supply curve** Data for Californian PPs are given by the U.S. Environmental Protection Agency via the Emissions and Generation Resource Integrated Database (eGRID) [2]. Data included are inter alia PP fuel type (hydro, nuclear, gas,...), the nameplate capacity [MW], the production [MWh] of a given year (latest update from 2010), the capacity factor [-] and the heat rate [Btu/kWh]. Since the sum of gas, coal, solid waste, wind, geothermal, solar, nuclear and water PPs amounted 2010 up to 99 % of the total generation, other fuel types were neglected. The composition of California’s generational mix in terms of capacity and generated electricity for 2010 is given in Fig. 2.7a and Fig. 2.7b respectively.

**Emissions for generation** Figure 2.8 gives an overview on the regional distribution of pollutants. The thickness of the circles correspond to the marginal amount of pollutant (by weight) emitted by MWh generation. However, the scaling differs for each of the plots (the same circle size of N\textsubscript{2}O does not correspond to the same marginal emission in grams of CO\textsubscript{2}). Emission-rates of pollutants that harm on a local scale, such as SO\textsubscript{2} or ozone...
2.4. Modeling the impact on the power market

(a) Capacity mix split, 100% corresponds 71.9 GW.

(b) Generation mix split, 100% corresponds 198 TWh.

Figure 2.7: California’s capacity and generation mix for electricity production based on data from e-grid for 2010.

and NO$_x$\textsuperscript{6} are mostly released in very few regions. These are mainly north of Los Angeles and San Francisco – regions where most consumption is located due to the high population. Comparing it to Fig. A.7 in the Appendix, where the power plant’s capacity and its marginal costs based on location are depicted, it can be seen that these regions also domicile the biggest power plants in California, among which there are plants with higher marginal costs. This naturally coincides with plants based on fossil fuels, having high direct pollutant’s emissions.

**Demand curve** To find a typical default load demand curve, yearly data on the total demand of residential, industrial and commercial customers for 2013 from the U.S. Energy Information Agency (EIA) \cite{3} were used.

In order to find the typical yearly and daily demand patterns, data from PGE for 50'000 industrial and commercial customers and $\approx$ 2'000 residential customer were taken. For each of the customer segments, the aggregated load was taken on an hourly basis and set as 100% (normalized by the yearly sum). The yearly total electricity demand from the EIA for each of the segments was then distributed over the yearly demand shape and taken as typical default load. In other words, for the load curve of the PGE data, the hourly demand-curve was divided by the sum of total yearly demand for all customers. This resulting shape indicates the share of the hourly demand on the total yearly sum of the demand. With summed yearly demand obtained from the EIA, the total demand for each segment was determined.

\textsuperscript{6}Both together by nature, as most of the tropospheric ozone formation occurs when NO$_x$, CO and volatile organic compounds react with the help of sunlight.
2. Model Description

2.4.1 Calculation of the marginal costs for power plants

The general assumption of this evaluation is that power plant owners bid in the wholesale market based on their marginal costs. Strategic bidding, as reviewed in [22] for competitive electricity markets is not taken into account. Only a one-stage market without day-ahead/ intraday /ancillary services is modeled. The intersection of supply and demand is expected to be the clearing price for a given hour which the utility and the producers fulfill in terms of payments and generation. In the long term, the TOU price for retail customers adapts following the change of the wholesale market price. The feedback on the consumption and respectively on the costs is evaluated in Section 2.4.4.

The marginal prices were calculated differently for different fuel types. In the following the methodology for deriving the marginal costs for all types of power plants is evaluated.

Power plants using fossil fuels

Power plants, such as coal-, oil- or gas fire plants need commodities traded on the market. Depending on their specific efficiencies they differ by their heat rates. The heat rate of a generator defines the amount of fuel needed to produce 1 kWh of electricity. They are given on a power plant level in e-grid’s database and the averages per fuel type are depicted for illustra-
2.4. Modeling the impact on the power market

tive purposes in Figure 2.9. To derive the (average) marginal costs for every
generator, the heat rate is multiplied with the price and the fuel’s cost [USD/ Btu] according to eq. 2.21.

\[ mc_f = f_c f \times r_\text{heat}_{pp} \times 10^3 \]  
(2.21)

Where \( mc_f \) are the marginal costs [USD/MWh] for a given type of power plant, \( f_c f \) are the fuel costs [USD/Btu] of the respective fuel, \( r_\text{heat}_{pp} \) is the heat rate in [Btu/kWh] for a given fuel and power plant and the factor \( 10^3 \) (kWh/MWh) is used for unit conversion.

In this study, the simulation is done on an hourly basis. Thus the average marginal production costs [USD/MW], excluding plant specific efficiency curves, can directly be drawn from the marginal costs in [USD/MWh]. Table 2.4 gives an overview of different fuel costs. The fuel price for solid waste was estimated to be zero.

**Power plants without fuel costs**

In this category renewables such as hydro, solar, geothermal and wind power and nuclear power plants are taken into consideration. Especially for nuclear and hydro, different bidding strategies exist. Strategies for hydro are reviewed in detail in [46]. It is also known that hydro power producers face opportunity costs for their water. Storage power plants only
Table 2.4: Prices used to evaluate the California’s marginal electricity generation cost curve

<table>
<thead>
<tr>
<th>Fuel type</th>
<th>Costs</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil</td>
<td>13.67 USD/10^6 Btu</td>
<td>OECD average 2010 (^a)</td>
</tr>
<tr>
<td>Coal</td>
<td>71.63 USD/tonne (^b)</td>
<td>US Central Appalachian coal spot price 2010</td>
</tr>
<tr>
<td>Gas</td>
<td>4.39 USD/10^6 Btu</td>
<td>Henry Hub 2010 average</td>
</tr>
</tbody>
</table>

\(^a\)Prices from 2010 were taken, as the most actual generator’s data stem from this year.
\(^b\)Later conversion calculated with 27'778'243 Btu/tonne.

have a determined amount of water to run through their turbines and – in contrast to run-of-river power plants – the consumption can be scheduled. Thus, power plant owners will only bid in the market, when they expect the market clearing price to be high enough to meet their opportunity costs.

**Hydro** Therefore, for hydro the following approach depicted in Figure 2.10a is chosen.

Given the capacity factor \(c_f\), the yearly number of full-load-hours of the generators can be derived (for simplicity, it is assumed, the power plant only runs with full or zero load).

The definition of the capacity factor is

\[
c_f = \frac{\text{Yearly electricity production}}{\text{nameplate-capacity} \times \text{hours of the year}}
\]

In [34], monthly hydropower generation in the Sacramento Municipal Utility District is given in percent of the total yearly production. Therefrom the number of hours of production for every month can be obtained.

Historic average electricity prices for California Independent System Operator (CAISO) can be downloaded for 2012-2014 at [21]. Sorting the historic electricity prices for a month and having information about the possible number of full-load hours \((h_{f_l})\), the operator chooses its bidding price, such that the market price will be higher than the bidding price in \(h_{f_l}\) of the month.

As an example, the knowledge of full load hours equivalents of production in a given month (past data) and the sorted correspondent realized prices lead a distribution from which it can be read off how many hours with prices that were higher than a given price were realized (e.g. 1 hour realized a price higher than 200 USD/MWh, 50 hours of a month realized a price
2.4. Modeling the impact on the power market

higher than 60 USD/MWh, 150 hours price higher than 35 USD/MWh etc.). The hydro producer can estimate the full load hours of production for a month (e.g. 150h), so he will always bid in the market with a price that was in 150 hours of the month higher than the bidding price (35 USD/MWh).

This approach is not dynamic, so that in this model for each month each generator bids into the market with its marginal costs, independent of whether it has been dispatched or not. Also, ramping-up or -down times and depreciation of turbines due to dispatch are not considered. The resulting marginal prices are shown in Figure 2.10b.

**PV, wind, geothermal and nuclear** In this analysis, only the short run marginal costs are taken into account. These can be greater than zero, when overhead and maintenance are functions of turbine usage. However, empirically, especially for PV, wind and nuclear, power producers aim for getting dispatched under any condition, as their ramping times are too long, or just because their electricity would get wasted otherwise. Thus, for simplifications, marginal costs are set to zero for wind, solar, geothermal and nuclear.
2. Model Description

(a) Approach to derive hydro-power-plants marginal bidding price.

(b) Distribution of bidding prices for Californian hydro based on historic data.

Figure 2.10: Approach and results of hydro’s bidding price.
2.4.2 Accounting for the uncertainty in production

Not every producer is able to bid into the market for every hour of the year, as for large power plants, such as coal or nuclear, maintenance is undertaken. For hydro power, water might not be available and the primary energy source is intermittent. In order to account for the uncertainty for every hour, different approaches are used for different power plant types.

**Coal, Wind and Hydro**

For these types of power plants, the capacity factor from the e-grid is chosen as availability indicator. Every generator was considered to bid \( f(bid)_{gen,h} \) into the market for every hour with the probability of its capacity factor \( c_f \).

More detailed approaches for wind, coal and hydro might lead to more precise results. This one seems viable, as the plant’s respective marginal costs are so low that it gets mostly dispatched if it is available.

The capacity factor \( c_f \) can be seen to consist of two parts: first, the power plants availability and second the probability of being dispatched, when the power plant bids into the market. For PP with low marginal costs, one can consider that they get dispatched most of the time when they bid into the market, so their \( c_f \) reflect their availability. For PP with high marginal costs, such as gas, the chance of being dispatched while bidding in the market is much lower and consequently the capacity factor is not an indicator of their availability.

\[
\begin{align*}
f(bid)_{gen,h} &= \begin{cases} 
  bid = 1 & \text{if } \text{unif}(0, 1) < c_f \\
  bid = 0 & \text{if } \text{unif}(0, 1) > c_f 
\end{cases} 
\end{align*}
\]  

One limitation of this approach is, that it does not capture the fact that if a PP is offline at hour \( t \), it will most likely be offline at hour \( t+1 \) as well.

**Nuclear power plants and geothermal**

In [10], the annual availability of nuclear power plants in Europe is evaluated. It is shown that most maintenance is done from June-August and thus for this generation type, an expected availability of 50 % can be expected for ca. 2000 h during summer.

In California, demand patterns over the year differ to those in Europe, as loads are – due to cooling – higher in summer as in winter (while in central Europe the distribution is vice versa). Therefore, the capacity factor was taken as an average availability for nuclear and geothermal. The reasoning is the same as for wind, hydro and coal: nuclear and geothermal power plants
are dispatched whenever they are available, due to the low marginal costs. Therefore, the capacity factor is a good approximation for the availability in the year. The average capacity factor was used, as in contrast to wind, coal and hydro, only two nuclear power plants are installed in California. Thus, if nuclears are considered to be dispatched if a random number is smaller than their capacity factor, this could influence the model outcome in a strong way. Therefore, nuclears were considered to be available at every point in time, but their output was reduced by the multiplication with their capacity factor. The same approach was chosen for geothermals as well, since their availability is in general not of intermittent nature either.

**Gas**

Gas power plants were expected to bid into the market at every hour of the year. Gas fired power plants are among those with the highest marginal costs, thus they are only dispatched when the market price exceeds or is equal to their biding price (which is directly linked to the respective marginal costs). Therefore, it is assumed that they are available for every hour of the year and maintenance is done during the season, when the electricity demand is low.

**Solar**

For solar, the mean of a random sample from beta-distributions from a variety of Californian stations of solar radiation (see methodology in Section 2.1.1) was independently taken as basis for every electricity producer of solar energy.

### 2.4.3 Modeling the demand in the power market

The demand was expected to consist of a default load as explained above based on the sum of the yearly consumption spread over the year based on empirically aggregated consumption profiles. In addition to that, the penetration level of residential houses with a PV/ battery system was gradually increased, in order to scrutinize the effect of this system.

Equation 2.23 describes the derivation of the total demand:

\[
D(t) = D(t)_{i,c} \ast s(t)_{i,c} + (1 - \alpha) \ast D(t)_{r} \ast s(t)_{r} + \alpha \ast D(t)_{PV/bat} \tag{2.23}
\]

Where the total demand \( D(t) \) for a given hour is the sum of the following parts:
2.4. Modeling the impact on the power market

- The default demand for industrial and commercial customers \( D(t)_{ic} s(t) \), with \( D(t)_{ic} \) the total demand for industrial and commercial customers, and \( s(t) \) the shape of their consumption

- The default residential load \( D(t)_{r} s(t)_{r} \), with \( D(t)_{r} \) the total demand for residential customers, and \( s(t)_{r} \) the shape of their consumption, weighted with \( (1-\alpha) \), where \( \alpha \) represents the penetration depth of PV/battery systems

- The demand \( D(t)_{PV/bat} \) of houses with PV/battery systems, weighted with \( \alpha \), their penetration depth.

2.4.4 Feedback on the residential electricity price

The actual PGE TOU-price for summer and the average wholesale electricity price for California in August for 2013 and 2014 from [21] are depicted in Figure 2.11a. Figure 2.11b shows the cross correlation of the median wholesale price and the TOU-price. It can be seen that the highest correlation is at lag=0 hours giving significant results (out of 95 % intervall). Therefore, the regression parameter of a linear regression of the median realized historic electricity price for a month and the TOU pricing for this time period were taken as basis, as an axis intercept is similar to the grid-usage fee and the slope to the electricity price. For every feedback, these regression parameters were taken to derive the residential TOU-price \( (r_t) \) from the wholesale price \( w_t \). The minimum residential price was set to 0.1 cents (negative wholesale price are not handled down to the customer).

\[
r_t = -0.007 + 0.0061 \times w_t \quad (2.24)
\]

\[
\text{if } r_t < 0, r_t = 0.001 \text{ USD/kWh} \quad (2.25)
\]
2. Model Description

(a) Comparison of the median electricity price for Aug 2013 and 2014 and the actual TOU for residential customers.

(b) Crosscorrelation of median electricity price for Aug 13/14 and the actual TOU for residential customers.

Figure 2.11: Comparison of the electricity wholesale and the TOU-price for residential customers.
2.5 Sizing of battery for peak shaving and self-sufficiency

2.5.1 Peak shaving

High peaks in residential loads can lead to many drawbacks for the grid and the power markets.

On the one hand, grids are designed for a certain load. If this load is exceeded, security of supply cannot be insured. Therefore, batteries cannot only be used to reduce the electricity bill of a single consumer, but also to flatten its consumption profile in order to provide security of supply.

On the other hand, as explained in Section 2.4, power plants are aggregated in the market sorted in increasing order of their bids – which are highly dependent on their marginal costs. As for a high load, plants with high marginal costs such as gas fire plants are dispatched, a reduction in the peak demand can reduce the electricity costs super-linearly.

Thus as motivated in Section 2.5.4, for a single customer it makes sense to reduce the peak load, if tariff schemes and incentives are set correctly.

In [37] a peak shaving algorithm using batteries is developed, that reduces the electricity bill of industrial customers. It is stated that power demand (in contrast to traditional electricity billing) is a main driver of the industrial’s plant electricity bill and makes up to 50% of it. Therefore, the optimal peak shave is determined by finding the intersection of reduced electricity (‘power demand’) costs and investment into the battery. This pricing is not implemented for residential customers yet. However the changing landscape of power-supply indicates that such a pricing might be available for residential customers soon as well and therefore the need of an evaluation for residential customers with actual PV/battery cost combinations is carried out.

2.5.2 Self-sufficiency requirement

Beside typical peak shaving, for many households far off from cities, it is attractive to have a self-sufficient (zero peak) PV/battery system.

Reference [45] evaluates different combination of existing PV-panels and batteries to draw conclusions on a cost-efficient way to reduce loss-of-load events. Similarly, in this thesis, also the needed capacities are derived from smooth iso-cost/quant curves for every percentage of self-sufficiency. Therefrom – just as for the typical peak shaving approach – optimal PV/battery
2. Model Description

combinations are chosen.

Reference [15] presents a method to determine the economically optimal PV/battery size combination for a given maximum load. Thereby the derivation is of the iso-quant curves is different here, but the intersection of iso-quant and iso-cost curves are also used. Also, the sizing methodology for the battery is kept simple, the battery/PV size combinations are given as functions of reliability and max. peak and data used are up to date. In the following reliability will stand for

\[ \text{reliability} = \frac{\text{Expected yearly hours of self-sufficiency}}{\text{hours of the year}} \]  \hspace{1cm} (2.26)

The distribution of needed battery capacity as a function of reliability is recorded on a yearly basis on real data. Therefore, for each given percentage of reliability, an optimal PV/battery combination can be determined.

2.5.3 Sizing requirements for different peak loads

In this Section, an algorithm for sizing of a battery (in terms of storage capacity (i.e. energy capacity) and rate (i.e. power capacity)) as a function of the maximal allowable peak load is derived. First, the method to calculate the battery size to keep the grid demand below a given threshold for a single household is presented. Then, it is discussed, how an optimal battery size can be derived from these results. The algorithm presented is similar to the sizing approach of [50] in which RESS-BESS systems are designed for small, isolated grids.

Storage capacity

An overview on this approach is given in Figure 2.12.

Let \( l_{\text{max}} \) denote the household’s maximal allowable load on the grid. For all time steps, for which the difference \( l_{\text{net}}^t - l_{\text{max}}^t \) is positive, the battery has to provide this difference by discharging. For all time steps with \( b_{\text{rest}}^t \) being negative, the battery can charge.

\[ b_{\text{rest}}^t = l_{\text{net}}^t - l_{\text{max}}^t, \quad \begin{cases} b_{\text{rest}}^t < 0 & \text{charging possible} \\ b_{\text{rest}}^t > 0 & \text{discharging required} \end{cases} \]  \hspace{1cm} (2.27)

In order to differentiate \( b_{\text{rest}}^t \), is introduced.

The storage capacity requirements for all time periods \( k \) when a consecutive discharging operation \( Dc(k) \) is required is given with
2.5. Sizing of battery for peak shaving and self-sufficiency

\[ Dc(k) = \int_{t=start(k)}^{t=end(k)} b_t^{rest} + \eta_d^{-1}dt \]  

(2.28)

and a possible charge operation \((Cc(j))\) is given for all time periods \(j\) with

\[ Cc(j) = \int_{t=start(j)}^{t=end(j)} b_j^{rest} - \eta_c^{-1}dt \]  

(2.29)

where \(\eta_{c/d}\) stands for the charge and discharge efficiencies and \(j\) stands for all time periods, when a charging operation is possible.

In this notation, time period \(j\) is always precessing time period \(k\), and for each time period \(j\) and \(k\), \(t\) can take all values of points in time during these periods \((t \in j\) and \(t \in k))\).

Therefore, the possible SOC of the battery at the beginning of time period \(k\) is given by \(^7\)

\[ SOC(k)_{t=0} = SOC(k-1)_{t=end} + Cc(j) \]  

(2.30)

The SOC at the beginning of the time period \(k\) is the SOC at the end of one discharge period before, plus what has been charged in the period \(j\) in between.

For time periods, when a discharge operation is required, a possible charging operation must have taken place beforehand. The amount of electricity withdrawn from the grid or PV \(b^w\) in this time period is equal to

\[ b^w(j) = \eta_c^{-1} * Dc(k) - SOC(k-1)_{t=end} \]  

(2.31)

If, for all time periods \(SOC(k-1)_{t=end}\) is zero (i.e. \(SOC(k) \leq Cc(j)\)), the required storage capacity of the battery \(Cb\) is given by the maximum of all storage requirements that occur (the charging operation of the preceding period can always satisfy the electricity need for \(Dc(k)\), thus \(Dc(k)\) determines the battery size).

\[ Cb = max[Dc(k)] \text{ for all } k \]  

(2.32)

For all households, for which \(SOC(k-1)_{t=end} \neq 0\), the required storage capacity can be derived by moving back from the last point of the time series

\(^7\)Here the self-discharge of the battery is neglected, as a typical self-discharge rate is at about 2% per month and it is worked with hourly data.
2. Model Description

Figure 2.12: Overview on the sizing approach for storage capacity.

All electricity withdrawn from the battery at period \( k \) must be satisfied with electricity stored at period \( j \) (possible charging period before discharge period \( k \)) plus \( SOC(k-1)_{t=\text{end}} \), and thereby the total storage capacity needed at this time period is equal to \( SOC(k) \).

By moving backwards from the last to the first time period \( k \), the additionally needed storage capacity (which cannot be satisfied through withdraw during a previous time period) is added recursively on the required SOC.

\[
SOC(k-1)_{t=\text{end}} = SOC(k-1)_{t=\text{end}} + [Dc(k) - Cc(j)] + SOC(k)_{t=0} \quad (2.33)
\]

Intuitively, in this case, the storage capacity needed is

\[
Cb = \max\{SOC(k)\} \quad \text{for all } k \quad (2.34)
\]
2.5. Sizing of battery for peak shaving and self-sufficiency

**Charging rate**

For dimensioning a battery, the charging and discharging rates are crucial, too. In the approach used here, it is assumed that the charging and discharging rate are equal in absolute terms in order to reduce the number of parameters to be determined. Also, it is assumed that the charging rate is independent of the storage capacity. An overview is given in Figure 2.13.

The required discharge rate relates to the difference of the load and the maximal allowed peak load to:

\[ D_r(t, k) = \eta_d^{-1} |b_t^{rest+}| \quad \text{for all } t \text{ and } k \]  

(2.35)

and the possible charging rate is analogically

\[ C_r(t, j) = \eta_c^{-1} |b_t^{rest-}| \quad \text{for all } t \text{ and } j \]  

(2.36)

For all periods \( k \), when \( \text{SOC}(k)_{end} > 0 \), the required discharge rate is given by

\[ C_{req} = \max[C_r(t, j), D_r(t, k)] \quad \text{for all } k, j \text{ and } t \]  

(2.37)

as the charging rate will always be used up to its maximum to fulfill all discharging requirements.

For some periods, when \( \text{SOC}(k)_{end} = 0 \), the required charge rate might be smaller than its maximum.

Therefore, for all time periods \( j \), for which \( \text{SOC}(k)_{end} = 0 \), all possible charging rates. \( |b_t^{rest-}| \), with \( t \in j \), will be sorted in ascending order

\[ bs_{t_{ij}}^{rest-} = \text{sort}(|b_t^{rest-}|) \]  

(2.38)

The number of points in time \( t \) needed to fulfill the charging requirement is given by solving Equation 2.39 for \( x \)

\[ \frac{\int_{t=1}^{t=x} (bs_{t_{ij}}^{rest-})dt}{Dc(k) * \eta_d} \]  

(2.39)

\( x \) might be \( \in \mathbb{Q} \).

Thus the charge rate needed for this time period \( k \) is either given by
2. Model Description

Generally, the (dis-)charge rate $C_{\text{req}}$ is determined by $\max(D_r(k,t), C_r(j,t))$ as long as $SO\ C(k_{\text{end}})$ has to be $\geq 0$ for all periods $k$, as the $C_r(j,t)$ never satisfies $D_r(k,t)$.

For all $j$, where $SO\ C(k_{\text{end}})$ is equal to zero, the minimal needed $C_r(j,t)$ that to satisfy $D_r(k,t)$ will be determined to find $C_{\text{req}}$.

Figure 2.13: Overview on the sizing approach for the required (dis-)charging rate.

$$C_{r,\text{floor}}(j) = b_s z^\text{rest} - z = \lfloor x \rfloor$$

or

$$C_{r,\text{ceil}}(j) = (x - z) * b_s z^\text{rest} - z + 1$$

so that

$$C_{\text{req}} = \max[C_r(t,j), D_r(t,k), C_{r,\text{ceil}}(j), C_{r,\text{floor}}(j)]$$

2.5.4 Optimal peak shaving capacity

If the battery sizing approach is performed for many different PV system sizes, an iso-quant curve for all peak-load requirements can be derived. This iso-quant gives the optimal PV/ battery size combination for a given maximum peak load, $m_l$, (see A in Figure 2.14). With the iso-cost curve to implement a PV/ battery system (B in Fig. 2.14), the optimal PV/ battery and its costs can be derived for each maximal allowed load (C in Fig. 2.14) by setting the slope of both curves equal.

Setting the max. peak load to be zero, the requirements for sizing the battery/PV system for a self-sufficient system can be derived. This can be done...
2.5. Sizing of battery for peak shaving and self-sufficiency

Table 2.5: PV/ battery cost specifications used.

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Capacity</th>
<th>Capital costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tesla’s Powerwall</td>
<td>3’500 USD</td>
<td>7 kWh</td>
<td>500 USD/kWh</td>
</tr>
<tr>
<td>Trina Solar</td>
<td>170 USD</td>
<td>245 W</td>
<td>695 USD/kW</td>
</tr>
</tbody>
</table>

for all probabilities of loss-of-load events. Therefrom conclusions on the self-sufficiency case can be drawn.

For different reasons, a load dependent electricity pricing (D in Fig. 2.14, and overview of derivation in Fig. 2.16) can be a solution to hand over increasing marginal costs of production for electricity to the residential end consumer. If this is done, the consumer is highly incentivized to reduce its peak load. Therefrom a monetary benefit of reducing to a given peak load can be determined. Together with the costs incurred from the implementation, total costs for the customer can be derived. If for a given time horizon these total costs are lower than the original electricity costs, the investment into this PV/ battery system will be realized and this maximum peak load requirement will be fulfilled (D and E in Fig. 2.14).

Optimal PV/ battery size combination

In order to derive the optimal PV/ battery size combination for a given maximal peak load \((m_t)\), the following equation for the system cost was taken:

\[
C_{tot} = C_{PV} \times Size_{PV} + C_{Bat} \times Size_{Bat}
\]  

so that the iso-costcurve is equal to

\[
Size_{Bat} = \frac{C_{tot} - C_{PV} \times Size_{PV}}{C_{Bat}}
\]  

where \(C_{PV}/Bat\) represent the costs in USD/kW for a PV or battery system and \(C_{tot}\) the total costs, \(\frac{-C_{PV}}{C_{Bat}}\) is the slope and \(\frac{C_{tot}}{C_{Bat}}\) the axis intercept.

As described above, the values for the battery/PV cost and size implemented in the model are based on Tesla’s Powerwall and the PV panel used above, Trina Solar. To be able to get smooth cost functions, it was assumed that the system was not built modularly, but sizable per kW capacity. Therefore, the costs listed in Table 2.5 are assumed.

At the optimal size combination, the slope of the iso-costs curve has to be equal to the slope of the PV/ battery iso-quant.
Load dependent electricity pricing

<table>
<thead>
<tr>
<th>Load [kW]</th>
<th>Costs [cent/kWh]</th>
</tr>
</thead>
</table>

Sizing requirements

- Battery storage capacity [kWh]
- PV-system capacity [kW]
- Decreasing $m_i$

Relative battery/PV costs

$C_{rel}(m_i) = \frac{C_b(m_i) - C(PV)}{C_b(0) - C(PV)}$

Total Cost for each max load

$C_{tot}(m_i) = C(m_i) - B(m_i)$

Benefit of each max load

$B(m_i) = C_b(0) - C_b(m_i)$

Net present value maximization

$C_{NPV}(m_i) = C(m_i) - B(m_i)$

Figure 2.14: The approach used to evaluate the sizing of a PV/battery system, and the optimal maximum peak load $m_i$. 
2.5. Sizing of battery for peak shaving and self-sufficiency

**PV/battery system sizing as a function of reliability for households with self-sufficient electricity supply**

The results are derived for the case that the household at no point in time exceeds the maximum peak load. For a self-sufficient system, this means that for the time period of the time series, it would be 100% reliable. However, also in real electricity supply, no hundred-percent reliability can be reached. Reference [11] states that the actual reliability in the US electricity grid is at about 99.9% (which equals to about 8 hours of outage per year).

In the following, the PV/battery system size is determined as a function of the reliability. Therefore, the self-sufficient case for an household is taken for two reasons.

The approach described above and fitting methods work best for low-peak allowances, due to the resulting L-shape of the iso-quants. Self-sufficiency is an obvious benchmark in the context of electricity supply. Many papers discuss the reliability and the loss of load events for houses equipped with RES/storage facilities such as [13], in which the design of PV/BESS/diesel stand alone systems is discussed.

In addition, determining the costs of self-sufficiency for a household as a function of the reliability can give an indication of the value of the grid for the user. It is discussed whether the grid of the future will only be the insurance for the time when the distributed (renewable) generation is not working. The costs of individual households for the additional percentages of reliability towards the 99.9% of the grid can be seen as an upper limit of the user’s valuation of the grid. Given that in the future, residential households are charged based on other tariffs structures such as locational marginal prices or peak load pricing, power and not electric energy might be the cost driver for residential houses.

Therefore, PV/battery systems will be designed, that maximize the consumers benefit by reducing total costs and thus may reduce the (peak) load.

If a PV/battery combination is chosen that does not ensure complete self-sufficiency, the grid is needed to secure reliable consumption at any time (as long as the costs of the grid (connection fee plus electricity) are lower than the additional costs for the household to be self-sufficient).

The approach used to evaluate the sizing is two-fold. First, the optimal battery/PV system size is determined for all households as a function of the grid supply security. As a result, investment costs for the system (as a function of the reliability) are contrasted to the savings the customer would, given a certain pricing scheme, have. Therefrom, the user’s preferred PV/battery size combination and costs and the consumer’s aspired self-sufficiency rate are derived.
Load dependent electricity pricing

Motivation As explained in [37], for many industrial and commercial customers power demand can make up to the same costs as the electricity itself, as they are charged dependent on their load (e.g. through peak load pricing). For these customers for a given month, a consumption of 100 MWh consumed during 100 hours with a load of 1 MW would therefore cost less than the same electricity consumption during 10 hours with a load of 10 MW.

The problem of a flat pricing system is that people’s behavior is not influenced by the actual production cost, as – independent of the consumption time and the load – they are charged independently of the consumption time and the load. For private customers this TS might seem convenient. However, as production costs vary throughout the day, the flat electricity price is a mix calculation of all production costs occurring and therefore people with low peak demand are ‘subsidizing’ those with higher peak demand. Another drawback is that peaking power plants which have high marginal costs and emissions are not shut down.

Consequently TOU pricing was introduced at many places. At times with generally high demand, electricity is more expensive than at times with low demand. This tariff is due to changes throughout the year and location. The hypothesis is that people adapt and change their consumption patterns according to the tariff profile from high cost to low hours (demand response, controlled with a variety of techniques). The obvious drawback is that new peaks might develop if the aggregated change, due to the reaction of PV/battery systems is large enough. The consequence is a feedback cycle, where the TOU will adapt to the wholesale market price, the aggregated demand will change accordingly and so on.

It has also been proposed to use locational marginal prices (LMPs) for residential customers. These include the wholesale market price which is highly dependent on marginal costs of production and the aggregated demand and also possess parts depend on congestion. The hope is that passing over these prices to the customer will flatten out the aggregated demand and reduce losses, as prices rise in regions with congestions. The problem is that reliable responses to LMPs can only be achieved by automated control, as most people cannot pay attention to quickly changing LMPs. Therefrom new problems of coordinated behavior can arise, whose answer is in the scope of this thesis.

Another pricing scheme, known as ‘Peak Demand Charges’ is often applied to commercial and industrial customers. Electricity is charged based on a
2.5. Sizing of battery for peak shaving and self-sufficiency

typical flat, TOU or day/night tariff. However, in addition to this, charges for the maximal occurring peak load during a time period apply. This is used to reduce the consumer’s peak load, but does not relate to its energy consumption for a given peak. The cost minimization that a household performs, given this TS, is shortly explained in Section 2.5.4.

The following Section derives a different pricing scheme, a load dependent electricity pricing derived from the marginal costs of production is developed. Therefrom, for every customer, the savings based on his/her maximum peak load can be derived. This will then be an input for the optimal sizing of the PV/battery system in Section 2.5.

Derivation of the load dependent price profile An overview of the approach chosen here is given in Fig. 2.16.

In order to derive a load dependent price profile, it would also be possible to look at the wholesale market, subtract the demand for commercial and industrial customers and take the marginal costs incurred in the market to produce an additional kWh electricity for residential customer. Mark-ups for the producer, trader and for the grid-transmission might apply, that may change through the day, depending on the aggregated demand of industrial and commercial customers.

The drawback of this method is that the residential electricity price can – widely – be chosen arbitrarily from the utility. Thus, this method would suggest a precision which is not given, since a large part of the derivation is based on assumptions on the mark-ups for traders, utility and transmission.

Consequently a simpler approach is chosen. The distribution of the occurring loads of the PGE dataset is mapped to the realized wholesale market price distribution. Thus, for example the 5 % percentile of the residential loads is mapped to the 5 % percentile of the realized prices. Therefrom the correspondent curve is estimated. The actual numbers in USD are then defined, so that the average residential electricity bill is the same with this pricing, just as with the TOU-pricing. Also, to make it easier, a step function is applied that defines a price for each load interval other than a smooth curve that would incrementally increase costs based on the load (i.e. different electricity costs for each kWh between 0 – 1 kW, 1 – 2 kW and so on).

This does not let the resulting load dependent electricity price curve to have the same shape as the marginal production curve. However, the marginal cost curve cannot be seen independent of the distribution of consumption. A mapping of e.g. the highest occurring load to the highest marginal production cost and a sub-sequential interpolation to zero demand and cost
2. Model Description

would overweigh the high cost area in the marginal cost curve.

Figure 2.15 gives an overview on the distribution of the realized wholesale market prices for electricity in California in 2013/14 and the distribution of the load in the dataset. The distribution of the prices is centered around the average price of ca. 40 USD. The occurrence of high load hours is much smaller, the higher the load is. This is another indicator that a pure interpolation would not be consistent with the realized demand and cost distributions.

Figure 2.15: Distribution of load in the PGE dataset and wholesale market prices in California 2013/2014.
Figure 2.16: Overview on the derivation of load-dependent electricity pricing and the savings estimation.
2. Model Description

Derivation of electricity savings from peak reduction using the load dependent price profile. The optimal PV/ battery size combinations, as derived in Section 2.5.3, are used and the electricity cost savings for all peak load reductions are calculated.

This is done by running a minimization model in analogy to Section 2.1.2.

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} \sum_{t=1}^{t=\text{end}} (p(kW_k) \times g_{tk}^i - FIT_t \times g_{tk}^e) \\
\text{subject to} & \quad 0 \leq g_{tk}^i \leq \text{cap}_{\text{max}^i}, k \in 1,..,K \\
& \quad 0 \leq g_{tk}^e \\
& \quad \text{l}_{\text{net}} + \delta_c^t - \delta_d^t - \sum_{k=1}^{n} g_{tk}^i + g_{tk}^e = 0 \\
& \quad \text{other constraints as in 2.1.2}
\end{align*}
\]

\(g_{tk}^i\): electricity import from the grid for all intervals of the load dependent tariff \(^a\) [kW], \(k \in 1,..,K\)

\(l_{\text{net}}\): net load profile [kW]

\(\text{cap}_{\text{max}^i}\): the size of the interval for the load dependent tariff [kW]

\(p(kW_k)\): profile of the tariff \(^a\) \(k \in 1,..,K\) [USD/kWh]

Each \(k\) indicates a load dependent tariff interval. \(k=1\) can for example be the interval 0-1 kW, for which a different per kWh price is used than for the interval 1-2 kW.

For every interval of the load dependent tariff, a correspondent interval of the load determines the costs incurred. Therefore the sum of independently chosen decision variables is minimized. If the load dependent electricity pricing was a smooth curve, one decision variable would be sufficient (the step function was chosen and motivated due to the intuition for customers).

To determine the electricity costs of a household under this TS, \(g_{tk}^i\) does not have to have an upper interval limit, as the price per kWh will not increase after a certain threshold.

However, for the load-shaving case, where no load higher than the maximum peak load \((p_{\text{max}})\) is allowed, the number of intervals and an upper limit for \(g_{tk}^i\) can be chosen, so that
2.5. Sizing of battery for peak shaving and self-sufficiency

\[ \sum_{k=1}^{n} g_{t}^{ik} = p_{t}^{max} \] (2.46)

By allowing \( g_{t}^{ik} \) to have an unlimited upper bound, backtesting of the peak shaving model is possible. Given a new sample of solar radiation and the respective optimal PV/battery size for a given \( p_{t}^{max} \) as an input, it can be tested if a cost-minimizing household would fulfill the maximum peak load requirement.

**Peak load charging**  When a household faces a peak load charging in addition to the energy charging, and it minimizes its costs, the following optimization is conducted:

\[
\text{minimize} \quad \sum_{t=1}^{t=\text{end}} (r_{t} \times g_{t}^{i} - FIT_{t} \times g_{t}^{e} - \max(g_{t}^{i}) \times m_{p}(\max(g_{t}^{i})))
\]

subject to

constraints given in Section 2.1.2

where \( r_{t} \) is the tariff profile for energy (here the TOU is used), \( g_{t}^{i} \) is the grid import, \( FIT_{t} \) the FIT, \( g_{t}^{e} \) the grid export and \( m_{p}(\max(g_{t}^{i})) \) the peak load price which itself can be a function of the peak load (or is just a fixed price).
3.1 Single household optimization

The following chapter presents the outcomes derived from deterministic and stochastic modeling approaches with modeled and real load data. Firstly, the optimization output of the modeled load data is presented to show how the single household optimization changes the household’s grid exchange. The modeled load data are used, as load profiles from PGE are under a non-disclosure agreement and thus cannot be disclosed in this thesis.

Secondly, the outcome of the single household optimization for the PGE data is shown. Thereby, the focus is put on potential savings and different methods to derive these, such as a purely deterministic model, a MPC and a stochastic approach.

In the following, different expressions are used:

- **Cost load** stands for incurred costs of the original load profile (without PV and battery and TOU).
- **Cost net load** is the hypothetical cost of the net load (including PV production).
- **Bill** represents the realized costs, including the usage of the battery (without profit generated from export to the grid).
- **Export remuneration** is the profit from exporting electricity to the grid.
- **Total costs** stand for Bill minus Export remuneration.
- **Consumption costs** are Cost load minus Consumption costs.
- **Savings [%]** are (Cost load minus Consumption costs)/ Cost load.
3. Results

A boxplot is a common and simple way to display results. In Figure 3.1, a typical boxplot is shown. The annotation was prepared according to Matlab’s documentation.

![Boxplot Diagram]

Figure 3.1: Interpretation of boxplots depicted in this thesis, based on the Matlab documentation.

### 3.1.1 Single household with modeled load data

The optimization was run for the three model households with a random sample from the PV production distributions, the Tesla battery with 7 kWh capacity (daily cycle) and nearly unconstrained line import/export capacity (max. occurring load in profile). The peak power charging rate (3.3 kW) was taken as basis. The tariff profile corresponds to the PGE TOU pricing. In this simulation, the self-discharge rate was neglected and for the cases with PV, a 2 kW PV installation was assumed. The two profiles are depicted in Figure 3.2.

Figure 3.2a shows the consumption profile for the model household with a high load. It can be seen that electricity is not exported to the grid and the maximal storage capacity of the battery is only reached rarely. For this household, the sizing of the PV/ battery system seems to be appropriate. The battery is discharged and charged nearly to the full extent during a daily cycle so that cost savings can be maximized. Increasing the PV capacity from this point on would result in grid export, since the battery is not capable to store more.
3.1. Single household optimization

Figure 3.2b shows the consumption profile for the model household with a low load. Here, electricity is exported to the grid. As the simulation is on a discrete timestep basis, it looks as if grid import and export occurred simultaneously which is not the case (when grid import is zero, grid export starts). For this household, the sizing of the PV/battery system seems to be inappropriate. The battery is never discharged or charged to the full extent. Also the PV capacity seems to be oversized, as the average net load of this household is negative.

Shortcomings of the results presented here are that the model loads do not change their consumption for the week-ends and cycles are limited to daily patterns. They also do not posses any peaks/jumps which makes the analysis less realistic, even though the stochastics of the solar production infer jumps to the net profile during the day. The PGE data (mean load of 0.89 kW) are in the range of the model load data for the base case (mean load for the low, base and high case are 0.45 kW, 0.86 kW and 1.14 kW respectively).
3. Results

Figure 3.2: Modeled load profile high and low with a 2 kW PV installation and the respective battery action for the Powerwall.

(a) Modeled load profile “High” and battery action for first week of August.

(b) Modeled load profile “Low” and battery action for first week of August.
3.1. Single household optimization

3.1.2 PGE load data for single households

Two-thousand households were analyzed to determine the savings potential with battery. The formulation of the optimization for the model including PV is equivalent to Equation 2.7a. The formulation of the model without PV is analogous, only excluding the possibility to sell electricity back to the grid. First, the results of the full deterministic models are presented, then the implications of the MPC approach are evaluated. The different model settings are listed in the appendix in Section A.2.

Savings through the installation of a PV/ battery system without considering investment costs

Figure 3.3a depicts the distribution of savings in percentage compared to houses without any PV or battery.
For houses only using the battery, median savings are roughly 14 % of the annual electricity costs. These savings are induced through the TOU pricing: the battery can charge during the night with a low tariff, and discharge during the day, when electricity prices are high.
For the PV/ battery system, savings rise to a median of 70 %. In California, 5-6 peak sun hours/day are normal, which can highly influence the net load of small consumers. 17 percent of all load profiles examined are making profit from the installation (i.e. savings > 100 %) through export remuneration. As most of these households have low electricity costs a priori, on a percentage basis the PV/ battery system can have such a big impact.
Average savings are 1’070 USD per household and year, with a standard deviation of 265 USD/a, 95 % of all households save between 540 and 1’600 USD per year.
Figure 3.3b depicts the costs for different scenarios i.e. for the deployment of PV alone and or the PV/ battery combination.

Cost load is the cost for the actual load profile without using PV or battery. For Bill no PV (only battery), the deviation of the costs as well as the median decrease. Considering Cost net load (PV only, grid-export at the FIT), some households make a profit and the whole distribution is noticeable shifted towards lower costs, even though the deviation is still very high. For Bill PV (incl. PV/bat, without remuneration for export), the deviation of the distribution shrinks and for the Consumption costs, the median and the upper whisker are reduced compared to Costs net load.
It is striking that the cost reduction from using the battery only (compare Bill no PV to Cost load) is much lower than the reduction of a 2 kW PV installation (compare Cost net load and Cost load). This is the case, since batteries are not providing electricity ‘for free’ and remunerated electricity infeed does not make sense if no PV is deployed.
3. Results

(a) Distribution of savings for different PV/battery combinations.

(b) Distributions of costs for different scenarios.

Figure 3.3: Savings and costs for different deployments of battery and PV/battery systems.
Self-sufficiency  No household could in practice be self sufficient (electricity demand would need to meet solar production/battery discharge every single hour of the year). Self sufficiency is defined as follows.

\[
\text{Self-sufficiency}_{\text{house } i} = \frac{\sum \text{hours without grid import}_{\text{house } i}}{\sum \text{hours of the year}} \tag{3.1}
\]

The distribution of self sufficiency for houses with PV only is depicted in Figure 3.4b and the distribution for houses with combined PV/battery systems is shown in Figure 3.4b (for the original load profiles is on average at around 0.5 %). It can be seen that the implementation of PV highly increases the self sufficiency rate of houses. Adding a battery additionally increases the self sufficiency rate.

The objective of the simulation was to minimize costs and not maximize the self sufficiency rate which is only a result of the cost optimization.
3. Results

(a) Distribution of self sufficient electricity consumption for households with 2 kW PV only.

(b) Distribution of self sufficient electricity consumption for households with PV and battery.

Net present value and payback time for PV/ battery installations  The average NPV was negative when including installation costs for all cases (for an assumed battery lifetime of up to 20 years, minimum discount rate of 1 % and TS 1-4, see Section 2.1.1 for detailed reference on TS).
Therefore, only the payback time for the PV/battery system is depicted in Figure 3.5. Overall, the payback time takes values much higher than the guaranteed lifetime of the Powerwall (10 years). Considering tariff structure (TS) 1 and 2, the only difference is that in TS 2 no FIT is paid (both have TOU-pricing). The distribution of payback time however, is not much shorter for TS 1 than for TS 2. This is due to two reasons. First, the combination of PV and battery leads to a lower infeed and thus the renumeration for the infeed plays a minor role for the profitability of the system. Second, the costs of the PV/battery system are very high in comparison to the potential savings, of which parts occur in the far future and are discounted. A relative small change in additional savings through the FIT therefore has a minor impact on the payback time of the PV/battery system.

TS 3 (peak load pricing with a peak load price for the highest load during a year of 50 USD/kW) has the lowest median for the resulting payback times. Increasing annual electricity cost savings is the best way to reduce the payback time. As one feature of a PV/battery installation is the possibility to reduce peak load, this leads to high savings for this tariff structure.

In the derivation of TS 4 (load based pricing), one requirement was that
the average electricity bill of TS 2 and TS 4 without using a battery should be the same. For payback times derived for TS 4, the distribution is broader than for TS 2 but has a higher median. Therefore, batteries in the cases examined were better in saving costs, when a normal TOU was applied (load shifting) than for a load based pricing (reducing overall load). The distribution of loads is very positively skewed. The load dependent electricity pricing has steps that are 1 kW wide, so that only loads higher than 4 kW are heavily penalized, but reducing these does not significantly reduce total costs, since there are only a few hours. Between loads of 1 kW and 2 kW, it is however hard to further reduce and to fall below a given threshold, which is then only slightly rewarded in terms of cost reduction. In this thesis, a step function for the load dependent pricing was introduced to make the pricing more transparent for the consumer. A smoothly increasing function however would better account for the quickly decreasing density of loads. Thus is can be concluded that a load dependent electricity pricing that smoothly increases instead of a step function would be more beneficial (even though this would be harder to communicate).

Implications for the grid by the installation of a PV/ battery system

Households normally face flat or TOU prices which are decoupled from the actual demand. Time of use pricing is linked to the general peak demand, in order to let consumers participate in higher prices induced through higher wholesale costs and to flatten peaks in the residential consumption pattern. As some loads can be shifted, such as using washing machines, others are essential. One possibility to further flatten the load profile and reduce peaks is to install a PV/ battery system.

Figure 3.6 depicts the distribution of hourly loads for different PV/ battery combinations for single houses. An increase in the mean consumption can be observed when installing a battery without having PV. This happens as a (dis-)charging efficiency lower than 1 leads to losses that have to be balanced by withdrawing more electricity form the grid. Overall, the balancing effect of the battery deployment can be observed when considering the 95- and 99-% percentiles which are lower for battery and PV/ battery systems of the single houses.

When aggregating houses, problems that stress the grid can occur, since charging and discharging operations happen at the same time. Figure 3.7 depicts the average load for all households in the data set for a day in August and January. The peak is shifted from the time with the highest price to hours with lower prices and can exceed the original peak load. For houses with PV only, infeed occurs when electricity production exceed the houses’ demand which can lead to additional stresses.
3.1. Single household optimization

In other words, through aggregation, the normal load is smoothed, as random behavior cancels out. Load occurring through a cost minimizing battery control, however can lead to additional peaks, as the control mechanism (cost minimization) and the price signals are the same for all households.

Therefore, in Section 3.3, the impact on the distribution grid is evaluated in more detail. Also, different TS are applied to determine the most suitable one to mitigate the effects on the grid.

---

**Figure 3.6: Distributions of hourly loads for houses with combination of PV and battery.**
Figure 3.7: Average daily load profile in August and January for different PV/battery combinations (PV of 2 kW), and TOU pricing for summer and winter.
3.1. Single household optimization

Battery usage

In order to evaluate if the installation of PV has a considerable impact on the usage of the battery, the parameter battery usage is defined as

\[
\text{usage}(i) = \frac{\sum_{t=1}^{t_p} \text{charge}(i, t)}{t_p \times B_C}
\]  

(3.2)

Where \(i\) indicates the battery, \(\sum_{t=1}^{t_p} \text{charge}(i, t)\) the sum of the charging operation [kWh] during time period \(t_p\) and \(B_C\) is the battery storage capacity. A usage of 1 would therefore mean that the battery is completely charged and discharged once per day (or similarly the equivalent of one (dis-)charging operation is executed by buffering consumption during a day).

Figure 3.8: Distribution of battery usage for houses with and without PV for a time period \(t_p\) of one year.

Figure 3.8 shows that the variation of the battery usage of a house equipped with PV has a much higher variation and a lower median than the one without PV. When PV is installed, complete cycles are less important, since buffering is done during the day during high cost hours for excessive electricity production. Without PV, charging during night and discharging during the day lead to more complete charge and discharge cycles.

The lower median might be due to the resulting lower net load through the electricity supply from the PV. Most households with a battery only have a usage of about 0.93. The battery is used to its full extent because load during the day is satisfied with night time electricity. Some of the households have a lower energy need than the battery capacity could satisfy, others might still need the full capacity. Therefore, the variation in battery use increases.
3. Results

Comparison of the deterministic model-predictive control vs. the multi-period deterministic approach

Motivation For a test run, a deterministic MPC, minimizing costs with a horizon ($t_{pred}$) over 3 days, updating ($t_{real}$) after 24 hours$^1$ was compared with the full multi-period deterministic approach, for which complete information for the whole year are assumed. The SOC of the battery was practically zero before new information was fed into the system to update the optimization. The SOC is the only memory which is passed over from one day to the other.

Therefore, from a modeling perspective, this shows that an update horizon $t_{real}$ of 24 h or longer is not justified. The update horizon determines how frequently the MPC problem is solved. Between two consecutive MPC executions, the prediction of the most recently executed MPC is applied on an open loop fashion. The computational time is inversely proportional to the length of the updating horizon and a SOC of zero indicates that the outcome of the next time step is independent of the optimization of the one before.

Consequently, in order to minimize computational time, the multi-period deterministic approach is useful. If the result should be more realistic, it is recommended to reduce the updating time to a lower number of hours.

The influence of the prediction horizon is evaluated in the first paragraph. The impact on the length of the updating cycles on the SOC, the estimated costs and the consumption are evaluated in the second paragraph.

Prediction horizon ($t_{pred}$) Figure 3.9 depicts the average electricity costs as a function of the prediction horizon. The updating cycle is kept constant on an hourly basis ($t_{real} = 1h$, varying $t_{pred}$).

The average battery usage decreases, and thus its leveraging effect diminishes when the prediction horizon is shorter than 18 hours and drastically decreases, when the prediction horizon is shorter than 12 hours. At the same time, the average electricity costs increase correspondingly. Consumption does not change much, as the household needs to consume anyways.

Therefore, it is concluded that realistic estimations for the future net load (load and solar production) especially for the next 12 hours are of outmost importance if a battery is controlled to minimize electricity costs.

The reason is straightforward: a daily cycle takes 24 h and when used, the battery is not used optimally, since day/night cycles are not accounted for. Predictions longer than 24 hours do not significantly increase the battery’s control action either, since most of the control is done for one day.

---

$^1$Radiation uncertainty was taken into account only.
3.1. Single household optimization

Figure 3.9: Costs, battery usage and consumption (net grid exchange) for changing the prediction time.

**Updating cycles** \( (t_{\text{real}}) \)  Figure 3.10 shows the incurred costs and the respective battery usage, when keeping the prediction horizon constant at 24 hours and changing the update time, after which an update was carried out (varying \( t_{\text{real}}, t_{\text{pred}} = 24h \)). The prediction horizon of 24 hours was chosen, as it seems realistic to have sufficiently precise information about the consumption of the next 24 h (and is in accordance to the results of the evaluation above).

Increasing the time after which an update for the battery control is made increases costs, lowers the battery usage and increases consumption. This is the case, as some battery control is made incorrectly considering changing weather conditions. The relative change is small (especially in comparison with altering the prediction horizon): the difference from an updating cycle of 1 hour and 16 hours for the average electricity costs is about 2.6 percent and the same orders of magnitude apply for the differences for the average battery usage and consumption, only the average SOC declines by about 20 percent. The changes are so small because this analysis accounted for changing radiation only. No uncertainty in consumption or other weather parameters that influence the consumption were considered. Incorporating these uncertainties, the relative influence of \( t_{\text{real}} \) would surely
rise but not change the key message.

Figure 3.10: Costs, battery usage and consumption (net grid exchange) for changing the realization time.
3.1.3 Stochastic optimization – comparison to deterministic model

The cost optimization was conducted given the current state of net demand and the expectation for future net demand (demand minus the electricity production through solar PV). After operation, the expectation was replaced by the actual realization and a new optimization was executed. Only a one week optimization was carried out, since the multiple time-step and scenario approach is very time consuming.

Cost optimization with hourly update and expected value

Figure 3.11b depicts the realized load profile with PV and battery usage for the second week of August for 3 different households. The black-white line represents the outcome of the deterministic model, the colored lines are the realized scenarios of the scenario-based stochastic optimization.

Figure 3.11a depicts the aggregated normalized grid exchange and net load for all households and the aggregated normed SOC. The normalization was done by diving the summed load or the summed grid exchange by its maximum and by dividing the summed SOC by the battery’s storage capacity.

It can be seen that the grid exchange of the stochastic model is extreme than for the deterministic model. The stochastic optimization uses the battery to a lower extent, as the SOC drops to the same level for both models, but the peak SOC is lower for the stochastic model. Also the battery action in the stochastic model lags behind the deterministic model’s battery action.

This results in a higher cost profile for the more realistic, stochastic model, as perfect knowledge is never given. The average electricity costs are listed in Table 3.1. The stochastic model results in electricity costs are on average 3% higher per week than for the deterministic model. This value would increase if the temporal resolution was smaller, i.e. if the time series is not given in hours but in minutes for example. Then, abrupt changes would not average out over the hourly record and the battery action of the stochastic model would lead to higher costs compared to the deterministic version.

The stochastic model seems to be the realistic version of a battery: the decision of the control is made at the actual point in time, based on expectations for the future which themselves are based on the current state and historical data. The control is executed, and for the next time step, based on changing weather conditions or consumption, the control strategy is changed. This model should therefore be used with the highest possible time resolution and a high number of scenarios. It then gives the best indication for possible
3. Results

Table 3.1: Overview on average costs for different models.

<table>
<thead>
<tr>
<th></th>
<th>Load\textsuperscript{a}</th>
<th>Net load\textsuperscript{b}</th>
<th>Deterministic model\textsuperscript{c}</th>
<th>Stochastic model\textsuperscript{d}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average costs [USD/week]</td>
<td>61</td>
<td>37</td>
<td>24.7</td>
<td>25.3</td>
</tr>
<tr>
<td>Computational time</td>
<td>–</td>
<td>–</td>
<td>20 min</td>
<td>10 days</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Cost of the original load with TOU.
\textsuperscript{b}Cost of the net load (incl. PV) with TOU and grid export at FIT, without storage.
\textsuperscript{c}As \textsuperscript{b}, with storage steered by multi-period deterministic approach.
\textsuperscript{d}As \textsuperscript{b}, with storage steered by stochastic model.

savings if a battery is used under uncertainty. The computational time however is tremendous and about seven-hundred times higher than for the one shot deterministic optimization. This huge difference is due to the needed calculation of the expected value based on the ARIMA model before the optimization problem is compiled and solved, the higher number of optimizations (as the model is updated after each time step) and due to the number of scenarios.

The results of the deterministic model-predictive control approach was – due to limitations on the simulation server – not taken into account in the comparison, since for the analyses of Section 3.1.3 and Section 3.1.2 different samples than in this analysis were used. It can however be expected that the deterministic model-predictive control approach results in cost savings that are higher than the stochastic model, since for the MPC only one future scenario without incorporating uncertainty is taken into account.

Therefore, it can be concluded that the deterministic model with full knowledge gives a good benchmark for the evaluation of potential savings when the temporal resolution of the time series is hours. This, because it is a little too optimistic approximation for real world behavior but needs a computational time that is extremely lower.
3.1. Single household optimization

(a) The normed sum of the grid exchange and SOC differ for both models while the aggregated net load without battery usage is the same.

(b) Three exemplary realized load profiles, the thick black/white line represents the deterministic model, the colored lines are the stochastic realized scenarios.

Figure 3.11: Results of the stochastic and deterministic optimization.
3. Results

3.1.4 Cost of anarchy

The results are depicted in the Appendix in Fig.A.4 for the case without PV and Fig.A.5 for the case with PV implementation.

The costs in terms of storage capacity amount to about 350 kWh capacity for 2'000 customers in the case of no PV implemented (equivalent to 0.175 kWh storage capacity per customer). Given that the Powerwall capital costs with a storage capacity of 7 kWh are 3'500 USD, this equates to $90 USD capital cost savings per customer. This value would increase, when taking into account the 13 million residential customers in California.

If PV is implemented on a household level, the storage capacity costs amount to ca. 950 kWh which is about 0.475 kWh per customer (potential capital savings of $250 USD/customer).

Besides all limitations concerning grid constraints, most importantly the feedback on the wholesale market is not considered. In the model, the decentralized and the centralized storages are facing the same TOU-tariff which is not plausible. Firstly, due to the impact on load shifting, the tariff scheme would adapt. Secondly, a centralized storage would be managed by a utility that faces different electricity costs.
3.2 Cost minimizing battery size for a single household and multiple households

3.2.1 Cost minimizing battery sizes for a single household with and without PV

The cost minimizing battery sizes for a single household were calculated based on the EAC of Table 2.3. Figure 3.12 shows the optimal cost minimizing battery storage capacity as a function of the EAC applied for all households. The trend is clear and the average optimal storage capacity smoothly decreases with an increasing EAC until it reaches zero for all households for an EAC of 76 USD/kWh*a. For all cases with an EAC higher than this number, no household needs storage.

![Figure 3.12: Optimal cost minimizing storage capacity as a function of the EAC for a household without PV.](image)

The resulting average battery sizes as a function of EAC, which itself is a function of the discount rate, assumed investment and lifetime, are depicted in Figure 3.13. Discount rates are chosen to be between 1-5 %, since at the production of the thesis, the general interest rate level was very low (for USD, EUR and CHF the base rate was lower than 0.5 %).
For most cases the size required for the Powerwall\textsuperscript{2} is not reached. When assuming the battery costs to only consist of capital costs, the optimal battery size reaches the size of the Powerwall for the EAC, in which the lifetime is assumed to be 15 years and the discount rate is less or equal to 2\% only. When taking into account installation costs, the cost minimizing battery size never reaches the size of the Powerwall.

![Sizing based on capital costs](image1)

![Sizing based on total costs](image2)

Figure 3.13: Cost minimizing average battery size as a function of discount rate, assumed investment and lifetime for a household without PV.

The same analysis was undertaken for a household equipped with a 2 kW PV installation. The PV installation was assumed to be there already, so these investment costs were not taken into account. Remuneration for grid export was set to zero to catch the sole effect cost saving effect of storage.

The optimal cost minimizing storage capacity was decreased for a given EAC and the curve, analogous to Figure 3.12 was much flatter. The resulting average needed battery size is shown in Figure 3.14. For none of the scenarios the optimal battery size reaches the storage capacity of the Power-\textsuperscript{2}Size in this case refers to storage capacity as well as to charging rate as both were considered to be linearly dependent from each other. In the following pages the cost minimizing storage capacity is represented.
3.2. Cost minimizing battery size for a single household and multiple households

Figure 3.14: Cost minimizing average battery size as a function of discount rate, assumed investment and lifetime for a household with PV.

It can be summarized that the cost minimizing size of the Powerwall is reached quicker for houses without PV, when only accounting for capital costs because a lot of energy has to be stored at night to be consumed during the day. When accounting for total costs, small battery sizes are optimal for households equipped with PV since batteries are able to store energy for time periods between sunshine and subsequent electricity production.

3.2.2 Savings for single households with and without PV

The average electricity cost savings were calculated for the battery sizes determined above.

The base cost for the evaluation were households without any storage (depending on the setting, with or without PV) and without the possibility to sell back to the grid. Thus, e.g. if the resulting battery size is zero (such as in the case of an assumed life time of 8 years for PV being installed beforehand), no savings occur as no battery is installed.

Figure 3.15 depicts the savings that occur if optimally sized batteries are
3. Results

installed. Much of the actual electricity savings are directly amortized by
the EAC of the battery and only for some cases (when the resulting EAC is
low) additional cost savings can be achieved. This is most prominent when
it is accounted for capital costs only and a low interest rate as well as a long
lifetime is assumed. Cheaper batteries with higher storage capacities are
installed and thus additional savings are higher.
3.2. Cost minimizing battery size for a single household and multiple households

Figure 3.15: Cost savings for optimal battery sizings, derived as a function of assumed EAC.

(a) Cost savings for households without PV installed, as a function of battery size.

(b) Cost savings for households having 2 kW of PV installed, as a function of battery size.
3. Results

3.2.3 Cost minimizing battery sizes for multiple households

It was analyzed to which extent electricity costs can be reduced if more households are combined to share a battery and minimize their total costs (costs for electricity and the battery).

**Simple aggregation** Figure 3.16b and 3.16a show the electricity, battery and total costs and the respective costs minimizing battery sizes as a function of the number of houses aggregated and the assumed EAC. The costs depicted are the average values per house, once for houses with PV equipped, once for houses without PV. The results are based on 150 randomly drawn house clusters for each cluster size and EAC (150 times 5 EACs used times 5 cluster sizes analyzed). The results were then averaged for each EAC/cluster size combination. This means that the results for a given cluster size do not compromise the same houses, which is a shortcoming of this analysis.

The effect of reducing total costs by combining houses together can only be observed for houses with PV installed. The reasoning might be traced back to Figure 3.8, where the average usage of the Powerwall is shown for households equipped with and without PV. The distribution of the battery usage is much broader for the houses with PV. As the variety of usage is larger, combining different households together opens the opportunity to profit from aggregation. Also, smaller capacities are needed, as surplus electricity from solar radiation needs less storage as it can directly be consumed by other households.

This effect however is undetectable for households without PV. Surplus electricity cannot be transferred to the battery for the consumption by other houses and thus, the usage of the battery is distributed more uniformly and charging/discharging takes place within a complete 24 hours cycle.
3.2. Cost minimizing battery size for a single household and multiple households

Figure 3.16: Cost minimizing battery sizes and costs as a function of different EACs and house cluster sizes.
3. Results

Pareto constraint – comparison to simple aggregation  Figure 3.17 shows the results for implementing the Pareto constraint in $n$ aggregated households with PV. The Pareto constraint was once implemented with FIT (see Eq. 2.14) and once without FIT (see Eq. 2.15). The costs are depicted as the proportion of the total costs for single houses that are not in the community.

$$\text{Proportion of costs} = \frac{\sum_{house=1}^{n} \text{costs for single house}_{FIT/noFIT}}{\text{costs for cluster of n houses}_{FIT/noFIT}}$$ (3.3)

The sample size was reduced to 20 iterations due to the extremely long computational time when running with a Pareto constraint. One iteration corresponds to the simulation of each case shown in Fig. 3.17 and the calculation of the single house optimization with FIT and without FIT. Therefore, the same sample of randomly chosen houses was used for one iteration. To calculate the average total costs per house, the single house optimization was done for each house of the community (for which the total costs with and without FIT, with and without Pareto constraint were calculated) and summed for each cluster to have a benchmark to compare to.

From Figure 3.17 it is concluded that the Pareto constraint leads to an increase of total costs compared to the simple aggregation. It does not let costs to be as high as the sum of the costs for single houses. This might also be due to excluding the battery costs from the Pareto constraint. The application of a FIT leads to a lower costs reduction, either because costs are lower anyways or because the Pareto constraint with the FIT is a tighter requirement that reduces synergy effects.

The proportion of the costs of the community drop significantly from 2-6 houses aggregated. Afterwards, no strong additional reduction is shown. Therefore, using the same sample of houses to benchmark for each iteration, the cost reducing effect of the community can better be observed than always using a new random sample.

3.2.4 Optimal charging rate and storage capacity combination for single houses

Only the total costs for the Powerwall are known but it is decided how much of the costs can be allocated to the storage capacity and the charging rate.

Therefore, different cost weightings $\alpha$ from 0.05 – 0.95 (in steps of 0.05) were chosen, whereby the costs for the storage capacity $C_{sc}$ [USD/kWh*a] are given by
3.2. Cost minimizing battery size for a single household and multiple households

![Graph showing cost proportions across different household aggregations.]

Figure 3.17: Costs incurred on a community level in proportion to the costs at a single house level.

\[ C_{sc} = \frac{EAC_{bat}}{\text{max. storage capacity of Powerwall}} \times \alpha. \]  \hspace{1cm} (3.4)

and analogously are costs for the charging rate \( C_{cr} \) rate

\[ C_{cr} = \frac{EAC_{bat}}{\text{max. charge rate of Powerwall}} \times (1 - \alpha) \]  \hspace{1cm} (3.5)

In this analysis, one single \( EAC_{bat} \) of 700 [USD/a*battery]\(^3\) was used, because only considering one \( EAC_{bat} \) was feasible due to the long computational time. Also, the scope was to analyze the effects of aggregation and not to investigate the influence of different EACs. 150 iterations were done for each weighting, and PV/battery combination (= 4500 simulations). For better readability, the cost minimizing ratio of charging rate to storage capacity is defined as \( r_{rc} \)

In Figure 3.18, \( r_{rc} \) and the total occurring costs are depicted as a function

\(^3\) lifetime of 12 years and a discount rate of 3 %.
of the cost weighting of the storage capacity. Two conclusions can be drawn from this analysis.

Firstly, regarding the upper subplot, depending on the PV installation, the $r_{rc}$ of the Powerwall is reached for cases for which the storage capacity accounts for about 75-85% of the costs of the sized battery (resulting, the charging rate corresponds to 15-25%). If in the process of construction, the cost structure corresponds to this ratio, the Powerwall is appropriately designed in terms of $r_{rc}$ to satisfy the consumers’ needs.

Secondly, regarding the lower subplot, in order to minimize total costs, the storage capacity has a much higher impact in the process of cost reduction than the charging rate. Thus, if an extra Dollar can be invested either in increasing the charging rate or storage capacity, increasing the latter one seems to have a higher impact in terms of total cost reduction.

![Figure 3.18: Cost minimizing ratio of charging rate to storage capacity and total costs (electricity plus EAC_{bat}) as a function of the cost weighting of the storage capacity on the battery’s costs.](image-url)
3.3 Impact on the grid level

The impact on the distributional grid level was evaluated for four TS, five battery sizes and four sizes of PV installed on the houses. At each node, a random number and sample of houses between 1-200 was taken. Even though the nodes are designed to serve up to 1000 houses, this conservative approach was chosen, in which the load of the houses themselves not violate the grid constraints so that the only possible violation rises through the implementation of PV/ battery systems. The grid topology is depicted in Figure A.6 in the Appendix.

The notation introduced in Section 2.1.1 is used to differentiate for the TS. Battery sizes used cannot all be purchased in the given sizes used here. Therefore, the charging and discharging rate was used as a function of the battery storage capacity, following the specifics of the Powerwall (the storage capacity is equal to 7 kWh and the peak (dis-)charging rate 3.3 kW). The (dis-)charging rate was adapted linearly to the used storage capacity (i.e. a storage capacity of 1 kWh is 1/7 of the Powerwall capacity, therefore the peaking (dis-)charging rate corresponds to $3.3 \times 1/7 \approx 0.47$ kW). Battery sizes used correspond to storage capacities of 0 kWh (no battery), 1 kWh, 4 kWh, 7 kWh (one Powerwall) and 10 kWh.

In California August is the month with the highest load (mostly due to cooling), but also the one with the strongest radiation and consequently potential PV production. Therefore, the evaluation was done for a day in August (August 2nd), the model for the households and the grid impact was run for 3 consecutive days, August 1st-3rd, to exclude all transitions from the evaluation (i.e. complete discharging at end of period, or charging in the beginning) and to only focus on the steady state operation.

3.3.1 Violation of grid constraints

For analyzing violations of constraints, Matpower produces a $24 \times 123$ matrix representing the hours of the day and the nodes in the test grid for each tariff profile and PV/ battery combination (four TS $\times$ 4 PV sizes $\times$ 5 battery sizes $= 80$ matrices). Depicting these information would go beyond the scope of this thesis. Therefore, for each combination, the proportion of violating grid constraints was calculated.

$$\text{Proportion constr. violations [\%]} = 100 \times \frac{\sum \text{number of violations}}{123 \times 24}$$ (3.6)
Figure 3.19: Proportion of overvoltages for test day, different TS and PV/battery combinations.

For all PV/battery combinations and all TS, the proportion of power and voltage angle constraints violations was lower than \( \approx 0.5 \% \) and minimally varying throughout the combinations.

The number of violations was much higher when considering voltage magnitudes and the proportion of violations for the upper bound (‘overvoltages’) is depicted in Figure 3.19. Overvoltages occur for all TS, as soon as PV is implemented. For the cases in which batteries are implemented, a battery storage capacity of 4 kWh seems to be enough to dampen the effects of a 2 kW PV installation. TS 2-4 seem also to reduce grid constraint violations, but to a small degree only. PV capacities greater than 2 kW lead to a systematic violation of voltage magnitude constraints. Therefore, the implementation of PV/battery systems is not recommended on this scale. The summed infeed to the grid of surplus energy, also with a FIT equal zero, would surpass the distribution grid capabilities. Thus, the TS plays a minor role when it comes to mitigate overvoltages.
3.3. Impact on the grid level

3.3.2 Power consumption

Figure 3.20 depicts the active power usage and the reactive power usage for the case examined.

Considering Fig. 3.20a, an increase in the PV size leads to a decrease in real power consumption. A kink of decreasing consumption occurs when installing PV with a capacity of 2 kW. Afterwards, the additional reduction in consumption decreases and flattens out. Changing the TS has a minor impact on the consumption of houses with different PV installations. Only for the case (TS 2) for which no FIT is applied, the consumption seems to be systematically reduced compared to TS 1, since the battery is used to the maximal extent.

For the TS 1 and 2 and large PV capacities, the consumption decreases when increasing the battery size and keeping the PV size constant. Large batteries can take better advantage of large PV installations (electricity derived from PV is consumed during the night as well) and smaller batteries are already completely discharged before the night. Thus, they recharge during the night and therefore need to totally withdraw more electricity from the grid. For large PV capacities, the same reasoning applies for TS 3 and 4 while for small PV sizes no trend is visible when increasing the battery size.

Considering Fig. 3.20b, using PV results in negative reactive power consumption, which means that power flows from the consumer to the producer to maintain voltage below the maximum threshold.

No explicit trend can be spotted for the reactive power consumption when altering the PV or battery size. The reactive power consumption is on a low level (≈ 40 times lower than for the real power). Thus random behavior in the load or in the sampling of households for the nodes might lead to a higher magnitude in variability in the reactive power demand compared to the magnitude of the change of the PV/ battery sizes.
3. Results

Figure 3.20: Power consumption of the whole test grid for different test cases.
3.3. Impact on the grid level

3.3.3 Cost effects

In Figure 3.21, the total costs for the whole distribution grid are shown. Costs in this case are the costs on the generator level and not on a household level. The pricing is done with the PGE TOU pricing.

The overall trend is that increasing the battery and the PV size leads to decreasing costs. For TS 1, 2 and 4 this holds for all cases. For TS 3, in which a high peak load is penalized, a small battery capacity without PV implementation increases the costs on the generator level. The battery is used to balance out peaks but in total more electricity is consumed (since the round-trip efficiency of the battery leads to an increased withdraw from the grid).

![Figure 3.21: Total costs for different test cases.](image-url)
3. Results

3.4 Impact on power markets

3.4.1 Market model

Production costs and clearing price

The model was run for the whole month of August. In the following, depending on the scope of the analysis, different time scales to depict the results are chosen. When results for one day are shown, it is August first, when two consecutive days are shown, it is August first and second.

The daily median electricity wholesale market price for California in August 2013 and 2014 (most recent data available) and the median wholesale price for August of the model output (without any PV/ battery system implemented) are shown in Figure 3.22. For most hours, the median of the modeled prices is very close to the median of the realized prices. During 1 am - 6 am however, the median modeled price sharply drops, which is explained in the following paragraphs.

The marginal cost curves (red) and the respective demand curves (blue) are
3.4. Impact on power markets

depicted in Figure 3.23. The influence of a deviation in demand of 22 GW generation is large: a 3% deviation of the daily maximal demand is linked to a change of the resulting clearing price of 30 USD/MWh.

The underlying assumptions of the model are that only one wholesale market for electricity exists (neither a day-ahead, hour-ahead differentiation is done nor a LMP-based market exists). Also it is assumed that generators are dispatched only according to the merit-order of their marginal costs and not to a combination of location (as a result of congestion) and costs (no OPF for California was run).

Thus, in this model it is not accounted for overproduction to ensure grid stability and to compensate for losses. This might be the driver that the median modeled prices are dropping – in contrast to the realized prices – to zero USD/MWh.\(^4\)

![Figure 3.23: Merit order of marginal generation costs and demand for an exemplary day in August.](image)

\(^4\)During night, for the whole month, in the model as well in reality negative prices appear.
3. Results

Emissions arising from electricity generation

In Figure 3.24, the average cumulated emissions\(^5\) for a given generating capacity are depicted and the graphic reads as follows: If 30 GW of capacity in California are dispatched for one hour, the total emissions for producing this amount of electricity account to ca. 2.8 million tonnes of CO\(_2\) emissions (and analogous for the other pollutants).

The emission curve for August 2nd without PV/battery systems is depicted in Fig. A.8 in the Appendix.

Figure 3.24 and Figure 3.23 show that especially power plants dispatched at a capacity of 22 GW have the highest marginal emissions, whereby the increase in emissions for power plants with extremely high marginal costs is comparably small. In these Figures, the unit [MWh/h] is used to emphasize the hourly resolution which is important for the chosen scale of the cumulated emissions.

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\(^5\)These are averaged values on the basis of a MWh/h production. The total time for averaging is a year, and so all additional emissions (that might be marginally higher, e.g. ramp-up and -down emissions) are averaged over the total yearly production.
3.4. Impact on power markets

3.4.2 Feedback of PV/ battery systems

Demand and price

The potential feedback of the PV/ battery systems on the power markets was quantified by first running the single-household optimizations with the PGE TOU pricing (in the following called run 1), then evaluating the changed demand pattern based on the penetration depth of the systems and deriving the resulting wholesale and retail price (in the following called run/iteration 2) and iterating several times for the changing demand and price patterns (run/iteration >2).

Feed-in-tariff of 0 cents

Figure A.9 in the Appendix depicts the change of the wholesale price (z-axis) depending on the hour of the simulation (x-axis, starting at 12 at night on August 1st 2010), the penetration depth of the PV/ battery systems as well as the iteration of the feedback loop (both on the y-axis).

In iteration 1, the base case was always carried out, thus the change from run 1-3 to iteration 2-4 is depicted here. The retail price is not shown, since it is a linear function of the wholesale price with a price floor of 0.1 cents/kWh. The price changes and the respective changes of demand (shown in Figure A.10 in the Appendix) occur mainly during night times between 1 am - 5 am. This is coherent with Figure 3.23. As prices are low during night, battery systems designed to minimize the costs of the consumers are charging in these hours. Since a small change in demand at the baseload level leads to a dispatch of power plants which bid into the market with marginal costs much higher than zero, this lets the price increase over-proportionally.

The higher the level of PV deployed, the lower the impact on the market. Batteries can charge during the day, when PV lead to a surplus of electricity and do not need to charge during night. The higher the PV capacity for a household with a battery, the lower the price responsiveness of the household is, as most of the electricity demand can be covered by the PV/battery system. The change in price and demand between systems with 4 and 6 kW PV installations is small: the potential of the battery is reached and the consumption is mostly satisfied with electricity directly derived from PV or from the battery. Only the consumption for the few hours before radiation is strong is served by charging the battery during night.

Feed-in-tariff of 4, 8 and 12 cents

Exemplary results for a penetration depth of 30 % are presented in Figure 3.26 and 3.25 (penetration depth is the number of households from the population possessing PV/battery systems). Also in these examples, small changes in demand from 1 am to 5 am lead to large changes in the realized price. The magnitude decreases with
increasing PV installation. Varying the FIT does not have an influence on price and demand. Only for the first run of the case, in which infeed is remunerated with 12 cents, large deviations occur, since the FIT is higher than the actual tariff for night time hours. In the following runs, the residential tariff scheme adapts and deviations are as small as before.

The adaptation process is shown exemplary in Figure 3.27. The higher the FIT is, the longer it takes for the retail price to adapt. For a FIT of 12 cents the price profile converges already for the third iteration and only little changes occur to the fourth iteration.

Table 3.2 lists the summed, relative costs incurred of the fourth run, the one considered to be closest to a steady state (at the wholesale market and for residential customers). For normalization, the costs of the base case (without residential PV or battery, only default load) are chosen. Therefore, the normalized costs (ncw) at the wholesale market for each PV/battery combination (PV/bat) and penetration depth (pd) are calculated based on the realized price (rp) and realized demand (rd).
3.4. Impact on power markets

\[ ncw_{PV/bat, pd} = \frac{rp_{PV/bat, pd}}{rp_{0/0,0}} \times rd_{PV/bat, pd} \times rd_{0/0,0} \]  

(3.7)

Calculation of costs for the customers include the revenue from feeding electricity into the grid (similar to objective function in Equation 2.7a) and normalization is done analogously to Equation 3.7.

Figure 3.26: Wholesale prices for 2 exemplary days, for a PV/battery system penetration depth of 30% and different FITs and PV sizes.

Costs in the wholesale market exclude costs for the FIT, as this is a policy measure paid by taxes. Differences between the costs for the wholesale market and for customers can therefore be seen as a subvention the customer receives.

The results show that an increasing FIT does not change much of the costs for residential customers or those incurred at the wholesale market. An increase of the FIT from 0-12 cents/kWh leads to an average decrease of about 2% of the total costs for a given PV/battery combination for residential customers. Therefore, for better readability, FIT of 0 and 8 cents are excluded in Table 3.2.

It is concluded that the FIT plays a minor role in the costs for residential customers if residential tariff schemes underly changes of the markets.
3. Results

These observations all together suggest that the impact of battery storages are apparent, especially if implemented on a large scale. Most importantly, utilities need to react and change their pricing scheme. Smoother TOU pricing and faster adaption to consumption patterns may be key for it. Given that changes can quickly establish in the markets without having strong negative externalities (e.g., congestion). It rather gives the chances to reduce overall costs incurred on the market by reducing the dispatch of expensive units through intelligent electricity pricing for consumers with batteries.

Figure 3.27: Retail price for 2 exemplary days, for a PV/battery system penetration depth of 30% and different FITs and PV sizes.
### 3.4. Impact on power markets

Table 3.2: Normalized summed costs in the wholesale market for the last, most converged run, for different FITs, PV/ battery system penetration depths and PV sizes.

<table>
<thead>
<tr>
<th>PV/Bat. Pen. depth [%]</th>
<th>FIT [cents/kWh]</th>
<th>2 kW PV no bat.</th>
<th>2 kW PV &amp; 1 bat.</th>
<th>4 kW PV &amp; 1 bat.</th>
<th>6 kW PV &amp; 1 bat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>92</td>
<td></td>
<td>58</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>92</td>
<td></td>
<td>58</td>
<td>92</td>
</tr>
<tr>
<td>20</td>
<td>12</td>
<td>89</td>
<td></td>
<td>58</td>
<td>96</td>
</tr>
<tr>
<td>30</td>
<td>12</td>
<td>87</td>
<td></td>
<td>58</td>
<td>97</td>
</tr>
<tr>
<td>40</td>
<td>12</td>
<td>85</td>
<td></td>
<td>58</td>
<td>98</td>
</tr>
</tbody>
</table>

\(a\) For houses with a PV/ battery system, not considering profit through FIT in order to see effect of FIT on residential electricity costs.
3. Results

Total emissions

In Figure 3.28, the summed pollution for a PV/ battery system penetration depth of 30% is depicted. The fourth iteration with the highest degree of convergence to a steady state is shown. As the metric is the mass, different scales are used to account for different pollutants’ levels.

![Figure 3.28: Accumulated emissions for the 2 example days, fourth iteration and a penetration depth of the PV/ battery system of 30 %.

Increasing the PV size, the level of pollution is falling. The FIT seems to play a minor role in reducing pollution, since houses are feeding in surplus electricity independent of the level of remuneration. For other cases and penetration depths, Table 3.3 gives an overview on the summed and normed emissions, where the sum of the pollutants weighted with the scale analogous to [2] is listed (similar to Figure’s 3.28 depiction):

$$\sum p = \text{NOx (kg)} + \text{O}_3 (\text{kg}) + \text{SO}_2 (\text{kg}) + \text{CO}_2 (\text{t}) + \text{CH}_4 (\text{kg}) + \text{N}_2\text{O} (100\text{g}) \quad (3.8)$$

6The scale used in the data source [2] is in pound, only the conversion to kg and tonnes was done.
3.4. Impact on power markets

The sum of pollutants is given as a ratio of the emissions of a specific FIT/PV size/penetration depth combination and the emissions incurred in a market without residential PV or battery.

\[
\text{depicted fraction} = \frac{\sum p(\text{given combination})}{\sum p(\text{no PV or battery})} \quad (3.9)
\]

Table 3.3: Normalized pollutants emissions in weight for the last, most converged run, for different FITs, PV/ battery system penetration depths and PV sizes.

<table>
<thead>
<tr>
<th>PV/Bat. Pen. depth [%]</th>
<th>FIT [cents/kWh]</th>
<th>2 kW PV no bat.</th>
<th>no PV &amp; 1 bat.</th>
<th>2 kW PV &amp; 1 bat.</th>
<th>4 kW PV &amp; 1 bat.</th>
<th>6 kW PV &amp; 1 bat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>12</td>
<td>99</td>
<td>96</td>
<td>92</td>
<td>91</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>99</td>
<td>95</td>
<td>92</td>
<td>91</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>98</td>
<td>99</td>
<td>91</td>
<td>89</td>
<td>87</td>
</tr>
<tr>
<td>20</td>
<td>12</td>
<td>98</td>
<td>98</td>
<td>91</td>
<td>88</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>97</td>
<td>99</td>
<td>92</td>
<td>86</td>
<td>84</td>
</tr>
<tr>
<td>30</td>
<td>12</td>
<td>97</td>
<td>95</td>
<td>92</td>
<td>88</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>94</td>
<td>98</td>
<td>94</td>
<td>83</td>
<td>81</td>
</tr>
<tr>
<td>40</td>
<td>12</td>
<td>94</td>
<td>96</td>
<td>99</td>
<td>83</td>
<td>82</td>
</tr>
</tbody>
</table>
3. Results

3.5 Sizing of PV/battery systems

3.5.1 Overview on PV/battery sizes

The mean and the upper 95% confidence interval (the lower one is equal to zero) for the raw output of the required battery size for a given PV size is given in Figure 3.29. The requirement of the battery size was calculated based on the method explained in Section 2.5. Here, for each household the battery requirement to stay below a given peak load for a varying PV size is shown. In the next subsections, limitations of this results are shown, as the data quality sharply drops with an increasing maximum peak load. Low peak size allowances in combination with low PV capacities require large battery sizes which are unrealistic to use. For a maximal peak size allowances greater than 1 or 2 kW, the PV/battery requirements flattens out. The marginal costs to reduce the peak load (which can be assumed to be a linear function of the system size) is strongly decreasing after this threshold. Thus, if the objective of a planner is to reduce the peak load, after an initial investment, the peak load is reduced over proportionally to the extra costs.

Figure 3.29: The required battery storage capacity as a function of the allowed maximum peak load and PV capacity.
3.5. Sizing of PV/battery systems

3.5.2 Proportion of houses without need of storage

For different PV sizes, the proportion of houses that do not need storage to never exceed a given maximum peak load for the entire year is depicted in Figure 3.30. Holding the maximum peak load constant, the PV size has little influence on the proportion of households that do not need storage at all. The peak load of most households does not occur during the time when radiation and PV production is highest.

![Figure 3.30: Proportion of households that do not need storage to stay below a given peak load.](image)

3.5.3 Isoquants of PV/battery systems to reduce peak size

**Fitting of isoquants**

Figure A.1 in the Appendix exemplary shows the derived iso-quants $I(PV)_{B/PV}$ for each allowed maximum peak load size ($\in [0, 1, 2, 3, 4, 5]$) for two different households. A maximum load, $m_l$, of zero is equal to the self sufficiency case and the higher the allowed peak size, the smaller the required PV/battery size combination is. In order to derive the curves, a function (red line) of the type

$$f(x) = a \cdot x^{-b} + c$$

(3.10)
was fitted to all points (blue dots) of iso-quants. $a, b$ and $c$ are constants, $x$ the PV size and $f(x)$ the battery size.

**Quality of fits**

**Fitting iso-quants for single households**  Figure A.1 exemplary depicts the iso-quant fitting to two households for maximum peak load allowed from 0-5 kW. Figure 3.31 shows the distribution of loads and the proportion of houses for which an iso-quant could be fitted (this means that the values of the power function do not turn negative at any point). Fitting the model of Equation 3.10 gives sufficient results, the higher the occurring load of a household compared to the maximum allowed peak size is. This is due to two reasons. First, data quality drops as the load is crossing less often the maximum allowed peak load. For example, only some households regularly exceed a load higher than 3 kW. In this case, fitting a proper power function to the data points is not possible. Second, the higher the maximum peak size is chosen, the flatter the resulting iso-quants are. Thus power functions are less suited for the fitting. In the following it is referred to this methods as *fit direct*.

**Fitting iso-quants for average PV/ battery requirement**  Consequently, fitting iso-quants to the average required PV/ battery system leads to much better results. An example is depicted in Figure 3.32. For higher peak load allowances this still leads to better results, since the averaging takes the requirements of all households into account before fitting and is thus supported by a larger data basis. In the following it is referred to this method as *fit average*.

**3.5.4 Derivation of optimal PV/ battery sizing to reduce maximum peak**

To find the corresponding optimal PV/ battery size combination for every peak shave, the slope of the iso-cost curve has to be equal to the iso-quants’ slope for a PV/ battery combination (see Section 2.5.4).

For all households, the derivative of the fitted curve $(a * b * x^{b-1})$ was set to be equal the slope of the iso-quant, $\frac{d[I(PV)_{b/pv}]}{dPV}$. The respective PV/ battery size combinations for all households were calculated for each shaving size ($x$ represents the PV size in the fitted curve).
3.5. Sizing of PV/ battery systems

For the maximal peak size of 1 kW and 2 kW, the result of fitting the average battery need for all PV sizes and setting its slope equal to the iso-cost curve is shown in Figure 3.32. The point in which both slopes are equal is the cost minimizing PV/ battery combination.

Table 3.4 gives an overview on the results of optimal system sizes and costs for the iso-quants derived from averaged PV/ battery sizes of all households.

The sizing results for all households are depicted in Figure 3.33. Note that here also, the sample size is reduced with an increased maximum peak load analogous to Figure 3.31.
3. Results

![Graphs showing fit of average iso-quant for maximal peak load of 1 kW and 2 kW](image)

Figure 3.32: Exemplary iso-quant fits, iso-cost curves and derived intersection for the average household (fit average) for two different max peak loads.

Table 3.4: Average optimal sizes for PV/battery systems derived with two different methods.

<table>
<thead>
<tr>
<th>Max peak load [kW]</th>
<th>Optimal PV size [kW]</th>
<th>Optimal battery size [kWh]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fit direct</td>
<td>fit average</td>
</tr>
<tr>
<td>0</td>
<td>10.7</td>
<td>14.2</td>
</tr>
<tr>
<td>1</td>
<td>5.3</td>
<td>7.7</td>
</tr>
<tr>
<td>2</td>
<td>3.4</td>
<td>3.9</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>2.1</td>
</tr>
<tr>
<td>4</td>
<td>1.3</td>
<td>1.1</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

For the averaged iso-quants, fit to average, the adj. R² was in the range of 0.98 for max. peak load of 0-5 kWh. The adj R² are this close to 1, as the required battery size for each PV/max load combination was averaged for each max. peak load. The averaged results of the fitting to the household was done on the data basis depicted in Figure 3.31.
3.5. Sizing of PV/ battery systems

Figure 3.33: Costs and PV/ battery sizes for different maximum peak loads. Total costs, inclusive installation are depicted.
3. Results

**Charging rate** To derive the optimal charge rate for a given maximum peak load, the same procedure was carried out as for the storage capacity. The iso-quant curves were fitted to the charging rate requirement for each PV size and the slope of the iso-cost curve was set equal to the one of the iso-quant. To derive the iso-cost curve, the charge rate costs [USD/kW] that result from dividing the investment of the battery by its maximal charge rate were used.

Table 3.5 lists the average optimal charging rates for different peak loads. It can be seen that they do not seem as large as the storage capacity of the battery for the equivalent maximum peak size. The storage capacity can easily get very large when – for a long time – no charging action occurs and sufficient electricity has to be stored for future operations. This is not the case for the rate.

In this approach, the charging rate is evaluated separately from the storage capacity (here the storage capacity can take any value) and thus low PV sizes result. This is the case as an increase in PV size (and a resulting increase in PV costs) is less cost effective than increasing the charging rate itself.

Table 3.5: Average optimal charging rate and PV size, derived analogous to fit average.

<table>
<thead>
<tr>
<th>Maximum peak load [kW]</th>
<th>Optimal charging rate [kW]</th>
<th>Optimal PV size [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.4</td>
<td>0.21</td>
</tr>
<tr>
<td>1</td>
<td>5.2</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>4.0</td>
<td>0.22</td>
</tr>
<tr>
<td>3</td>
<td>2.8</td>
<td>0.22</td>
</tr>
<tr>
<td>4</td>
<td>1.6</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The shortcoming of this approach is that the optimal battery storage capacity and the optimal charging rate are evaluated separately. For the optimal charging rate, the storage capacity is neglected and can take any value and vice versa. Thus, the optimal PV size for the optimal charge rate is much lower as for the storage capacity (the iso-cost curve can be shifted closer to the origin of the PV/battery combination). In order to account for the more-dimensionality of the optimization, for further work it is suggested that the three dimensional intersection for the iso-quant of charging rate, storage capacity and PV capacity is set equal to their iso-cost plane. This could not done in this work, as more data points would have been needed and computational time to produce these was not given.
3.5. Sizing of PV/ battery systems

3.5.5 Design of self sufficient PV/ battery system as a function of reliability

The sizing and the costs of a PV/ battery combination for a self sufficient system depend significantly on its reliability. Therefore, mostly it is not cost efficient to implement a 100 % reliable system (max peak load 0 kW, see row 1 in Table 3.4).

In this Subsection the sizes of such a system are contrasted to the potential savings for lower reliabilities. In Subsection 3.5.7, the investment costs are taken into account and the NPV is derived as a function of the reliability.

Table 3.6 gives an overview on the insufficient electricity supply and the optimal PV/ battery sizes, depending of the degree of reliability. Analogously, Figure 3.34 depicts the average optimal sizes and the average electricity cost savings for the cases considered.

The values for PV/ battery sizes and the costs are much lower compared to those depicted in Table 3.4 for the 100 % self sufficient case. The advantage of this approach is that it puts per definition more weight on the consumption time than the maximum peak load approach. In the latter one, the accumulated consumption for loads higher than a given threshold is decisive, which can easily let the required battery increase to unrealistic sizes.

In this approach data quality was higher, since for 97 % of the cases fitting of the curves was successful and the fitting quality did not decrease with a decreasing rate of reliability.
Table 3.6: Overview on optimal PV/ battery size combinations, costs and savings as a function of reliability.

<table>
<thead>
<tr>
<th>Reliability [%]</th>
<th>Annual insufficient supply¹ [MWh/a]</th>
<th>Optimal Battery size¹ [kWh]</th>
<th>Optimal PV size¹ [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>0/0.04/5.4</td>
<td>3.5/13.7/27.8</td>
<td>0.06/6.4/13.4</td>
</tr>
<tr>
<td>97</td>
<td>0/0.13/5.8</td>
<td>4.2/10.5/19.3</td>
<td>0.05/5.2/10.8</td>
</tr>
<tr>
<td>95</td>
<td>0/0.25/5.9</td>
<td>3.9/10.1/18.2</td>
<td>0.04/4.7/9.7</td>
</tr>
<tr>
<td>90</td>
<td>0/0.57/5.7</td>
<td>3.3/9.1/15.8</td>
<td>0.02/3.9/8.5</td>
</tr>
<tr>
<td>85</td>
<td>0/0.95/5.7</td>
<td>2.6/8.1/14.2</td>
<td>0.02/3.2/7.7</td>
</tr>
<tr>
<td>80</td>
<td>0/1.36/5.6</td>
<td>2.1/7.1/13.1</td>
<td>0.02/2.4/7.0</td>
</tr>
</tbody>
</table>

Original electricity costs without PV/ battery per customer based on TOU ¹ 522/ 1,550/ 3,480 [USD/a].
¹ 5 % percentile/ Median /95 % percentile.

Figure 3.34: Average PV and battery sizes and resulting costs as a function of reliability.
3.5. Sizing of PV/ battery systems

3.5.6 Load dependent electricity pricing

Results for the load dependent electricity pricing

The results of the load dependent electricity costs, derived with the methods described above are listed in Table 3.7. The values are comparable to the values of the PGE TOU-pricing profile, as the values of low demand are close to the price of low-peak hours.

Table 3.7: Load dependent electricity prices.

<table>
<thead>
<tr>
<th>Load [kW]</th>
<th>0 – 1</th>
<th>1 – 2</th>
<th>2 – 3</th>
<th>3 – 4</th>
<th>&gt;4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price [cents/kWh]</td>
<td>16.11</td>
<td>23.21</td>
<td>28.59</td>
<td>39.94</td>
<td>67.28</td>
</tr>
</tbody>
</table>

3.5.7 Net present value of PV/ battery systems for peak load shav- ing and self sufficiency

For all maximum peak loads and reliability cases that were analyzed, the NPV of the PV/ battery installation was calculated based on expected electricity cost savings and investment costs for different assumed lifetimes and discount rates.

\[ \text{NPV} = \sum_{t=1}^{t=n} \frac{S_t}{(1 + r)^t} - C_0 \]  (3.12)

Where \( S_t \) stands for the electricity cost savings incurred in time period \( t \) in USD by using the PV/ battery system, \( r \) the discount rate, \( n \) is the number of time periods in years (assumed lifetime) and \( C_0 \) the investment costs of the PV/ battery system [USD].

The optimal maximum peak load

In Figure 3.35, the average NPV (NPV) of optimally sized PV/ battery systems is depicted as a function of the maximal allowed peak load. For the case, for which total costs were considered, the NPV increases sharply when starting from the case of complete self sufficiency to allowing a maximum peak load of 1 kW. When further increasing the maximal allowed peak load, the NPV increase flattens out and for the case of a maximum peak load of 5 kW decreases slightly. For no case however, the NPV turns positive. It is analogous for the case in which capital costs were considered only. For

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Footnote: The average electricity bill is the same as with TOU-pricing.
3. Results

the cases of a lifetime of 10 years and 20 years with a discount rate of 5% the NPV is negative and reaches its maximum at a maximum peak load of 3 or 4 kW. For the case with a lifetime of 20 years and a discount rate of 1%, the NPV is even positive for the self sufficiency case. This is due to the high savings in comparison to the low investment. The investment is spread over a long lifetime and a low discount rate highly values future savings.

For the considered cases it is concluded that PV/battery systems are not cost efficient yet, even if they are sized optimally.

![Figure 3.35: Average NPV of optimally sized PV/battery systems as function of maximal allowed peak load, depicted for TS 4, load dependent electricity pricing.](image-url)
The optimal scale of reliability for the self sufficient case

For the self sufficient case, to find the optimal PV/ battery size combination, load dependent electricity pricing (TS 4) was not a requirement. The objective of increasing the degree of reliability is not to reduce the peak load, since it defines the numbers of hours that a household needs electricity from the grid. Therefore savings with both TS were used to calculate the NPV for all reliability requirements.

In Figure 3.36, the average NPV of optimally sized PV/ battery systems is depicted as a function of the reliability for tariff structure 1 and 4. For tariff structure 1, the maximum NPV based on total costs is at the lower end of the evaluated reliability. It is at the the upper end of reliability for the calculation based on capital costs. Here no optimal reliability can be derived.

For tariff structure 4, the maximum NPV is at a reliability of 95 %. At this point, the NPV is negative when considering total cost and positive when only accounting for capital costs.

Therefrom it is concluded that sizing based on reliability is a good way to quantify the capability of a PV/ battery system, since the user can marginally increase and decrease the degree of expected self sufficiency from the grid. Since the costs for PV/ battery systems are still high compared to the electricity costs and thus potential savings, also this approach does not lead to a defined optimal degree of reliability and thus PV/ battery size.
3. Results

(a) Savings calculation based on tariff structure 4, load dependent electricity pricing.

(b) Savings calculation based on tariff structure 1, PGE TOU pricing with FIT.

Figure 3.36: Average NPV of optimally sized PV/ battery systems.
Chapter 4

Conclusion

This master thesis explored the economic impact of the usage of residential photovoltaics with battery storage. In the following, conclusions and results are summarized.

**Single household electricity cost savings and payback-time for PV/-battery systems**

Electricity cost savings through the installation of PV/battery systems can make up a big portion of the original electricity bill for individual households. When using a standard size for the PV installation of 2 kW combined with a Powerwall however, the largest part of the savings is due to the PV installation.

For the investment for a PV/battery system, the financial viability is important. Therefore, the payback time of such a system for different tariff structures was calculated. This was done based on the potential annual electricity cost savings for each customer and the investment costs for a PV/battery system. This resulted in payback times for all tariff structures analyzed that were much higher than the guaranteed lifetime of the Powerwall. The shortcomings of this approach are that maintenance and replacement costs for the battery and the PV installation were neglected and the tariff structure just as the electricity consumption for future years were assumed to be fixed. Discount rates for the future are unknown and are highly important for the financial viability.

Beside limitations and shortcomings of this estimation it is concluded that potential electricity cost savings through PV/battery systems can be substantial for individual households. In most cases however, the investment is still not financially viable, since the costs are too high in comparison to potential electricity cost savings in the future.
4. Conclusion

Comparison of different modeling approaches

In this thesis a multi-period deterministic, a deterministic model predictive control and stochastic modeling approach were evaluated.

The advantage of the multi-period deterministic model is that it gives good estimations in a reasonable computational time. These results however are too optimistic, since in reality the battery control algorithm does not have perfect information for the load and PV production.

The stochastic model was used to benchmark the results of the deterministic one, since it operates based on the current state and historic data but is not aware of future net load. The data used in this thesis have a time resolution of one hour. Over this time period, much of the uncertainty cancels out. Thus, the deterministic model only resulted in slightly too optimistic results. It can certainly be assumed that an increase in the time resolution leads to an increased difference in the performance of both models.

The deterministic model predictive control approach was used to find the needed prediction horizon and updating cycle for the future net load. In this analysis, only uncertainty in radiation was assumed. It is concluded that the prediction horizon for adequate battery control should be at least 12 hours, but best 24 hours, since the influence on the resulting operation is large. The updating cycle should be as short as possible and not longer than 24 hours.

Optimal battery sizing for single and multiple households

Optimal battery sizing based on a yearly load profile and the equivalent annual costs of a battery were analyzed.

The optimal sizing of a battery for a single household, based on PG&E time of use pricing and its load profile, is able to reduce its total expected costs. It was shown that for single households the cost minimizing battery size is not reached if realistic investment costs, battery lifetime and discount rates are assumed. For these cases, a large part of the electricity cost savings of a single household are needed for the investment in the battery.

It is concluded that at the moment, the costs of the Powerwall are too high to be cost efficient for this size. Total costs, compromising electricity costs and battery investment can be reduced if smaller sizes of batteries can be installed. This requires the costs for the implementation of a battery to linearly decrease when smaller sizes are installed.
For **aggregated households** it was shown that clustering houses can reduce total costs. This was observed for clusters of households with PV, since surplus electricity is not fed into the grid but shared in the community. When however implementing a Pareto constraint which ensures that houses do not pay more when being in the community, electricity cost savings are reduced.

Thus, it is concluded that increasing the number of houses sharing a battery can reduce total costs for every household. A smart way to account for the contribution in savings for every household is however an important research question for further work.

**Impact on the grid level**

In this analysis, the impact on a distribution grid level from the aggregated usage of PV/battery systems from households was evaluated. It was shown that overvoltages due to the total infeed of electricity from PV are the most problematic constraint violation. The total load of batteries however did not lead to problems. For certain cases, the cost minimizing battery control lead to a reduction in the number of overvoltages from PV.

A conservative number of houses per node was chosen and an optimal power flow instead of a power flow model was run. The PV capacity of the installation was increased to up to 6 kW, which is unrealistic for each house to have. Combined with batteries which have a lower charge rate than the power output of the PV panels, this necessarily leads to a surplus infeed of electricity for some hours.

It is concluded that for a distribution grid, the impact of PV installations can have highly distorting effects. If households minimize their costs with the usage of a battery, this can reduce the impact of PV systems, if the right PV/battery capacity ratio is chosen.

**Impact of PV/battery systems on power markets**

The evaluation of the impact of PV/battery systems on the power markets shows that a simple market model can lead to results that replicate some of the market behavior.

It was shown that thresholds in the marginal cost curve of the generators exist and a small change in demand can significantly increase the price on the wholesale market. The same holds for the aggregated emissions. The dispatched capacity that leads to increased emissions falls together with an increased marginal price for one of the thresholds of the marginal cost curve.
4. Conclusion

only.

When it comes to the implementation of PV/battery systems, it seems that the biggest impact in total cost reduction on the market are due to PV installations and not to batteries. The price responsiveness of houses equipped with batteries leads to a varying behavior of prices especially during night time hours. Since the electricity tariff scheme adapts to the changing demand the oscillatory behavior of the price is reduced quickly.

Thus, it is concluded that if the effects of the implementation of PV/battery systems are supposed to be mitigated, a quickly adapting tariff scheme is useful. Also, batteries do not necessarily lead to a cost reduction on the power markets. They can increase the demand during the time that is normally served by cheap power plants up to a demand for which the first non-renewable units are dispatched. Despite of this, they can be used to flatten out demand so that expensive, flexible units are not needed any more.

**Optimal PV/battery sizing for peak load reduction and self sufficient house design**

An algorithm was developed to define the optimal sizing of PV/battery systems for different maximum peak loads and self sufficiency rates. It was shown that for small maximum peak loads and small PV sizes, large battery sizes are required which are unrealistic to implement.

With an increased maximum peak load, the precision of the derivation of an optimal PV/battery size is reduced since the data quality sinks. No optimal maximum peak load for a household that is charged based in a load dependent electricity pricing scheme could be identified.

If sizing based on the optimal degree of self-sufficiency is chosen, it seems that a system with $\approx 95$% reliability minimizes total costs. In this case, total costs compromising installation are still negative for the scenarios analyzed, but have the highest net present value. For a reduced reliability, no differences in data quality are observed.
Appendix A

Appendix

This master thesis was written with TexStudio and an ETH-Template available at ETH’s webpage. Graphics were made by Powerpoint, Excel and Matlab.
For modeling, Matlab 2012 a, installed on a remote server of Stanford University was used. For data exploration, R-Studio, also installed on a remote server was used.

A.1 Appendix – Formulas

A.1.1 Estimation of beta-distribution parameters

Based on Equation (2) and (3) of [12]:

$$
\beta = (1 - \mu)(\frac{\mu(1 + \mu)}{\text{sigma}^2} - 1)
$$

and

$$
\alpha = \frac{\mu \beta}{1 - \mu}
$$

for scaling purposes, one can replace

- $\mu_r : \text{for } \mu_r$
- $\sigma_r : \text{for } \sigma_r$

and after rearranging, one comes to the solution of

$$
\alpha = (\frac{(\mu_r - a) \ast (b - \mu_r)}{\sigma_r^2} - 1) \ast \frac{\mu_r - a}{b - a}
$$

$$
\beta = (\frac{(\mu_r - a) \ast (b - \mu_r)}{\sigma_r^2} - 1) \ast \frac{b - \mu_r}{b - a}
$$
A.2 Overview of models used

In the following, an overview on the models that were used in this thesis is given. The simulation length was altered in some simulations to a shorter time than 1 year to reduce total computational time (stochastic model) and/or because information for a day are already sufficient to be shown (impact on the distribution grid level). The same holds true for the number for the sample size, which was only reduced in cases for which the simulation of the whole sample was not feasible (i.e. simulation longer than approximately a week). For all models, results, conclusions and specifications worth mentioning are given in one sentence. The model for the optimal PV/battery size combination is not discussed since no optimization formulation similar to the other formulations is used and the methods part deeply discusses the implications of the model.
### Table A.1: Different specifications for single house optimization.

<table>
<thead>
<tr>
<th>Model used in</th>
<th>Section 3.1.1</th>
<th>Section 3.1.2</th>
<th>Section 3.1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Multi-period deterministic approach with modelled load data</td>
<td>Multi-period deterministic approach with PG&amp;E Data</td>
<td>Deterministic model-predictive control approach</td>
</tr>
<tr>
<td>Optimization formulation</td>
<td>Analogous to Equation 2.7a</td>
<td>Analogous to Equation 2.7a</td>
<td>Analogous to 2.7a, uncertain solar radiation</td>
</tr>
<tr>
<td>PV/battery specifications</td>
<td>One Powerwall and 2 kW PV per house</td>
<td>One Powerwall and 2 kW PV per house</td>
<td>One Powerwall and 2 kW PV per house</td>
</tr>
<tr>
<td>Sample size</td>
<td>3 example loads: Low, Base, High</td>
<td>1923 households</td>
<td>100 households</td>
</tr>
<tr>
<td>Simulation length</td>
<td>One year</td>
<td>One year</td>
<td>One week</td>
</tr>
<tr>
<td>Other specifications</td>
<td>Deterministic optimization with complete knowledge for one year done in one shot</td>
<td>Deterministic optimization with complete knowledge for one year done in one shot</td>
<td>Deterministic optimization for realization horizon, taking net load of prediction horizon into account, uncertainty in radiation only</td>
</tr>
<tr>
<td>Outcome</td>
<td>Model load data do not sufficiently represent consumption patterns and uncertainty but give insights on the results of cost minimizing battery control</td>
<td>Cost savings for individual costumers with PV/battery can be very high when original load is low and a FIT is applied, contrasted to investment costs, only for a few costumers the implementation is cost efficient during Powerwall lifetime</td>
<td>The prediction horizon for cost minimizing battery control does not need to be higher than 24h, but should also not be smaller than 12h</td>
</tr>
</tbody>
</table>
Table A.2: Specifications stochastic optimization and optimal sizing.

<table>
<thead>
<tr>
<th><strong>Model used in</strong></th>
<th><strong>Section 3.1.3</strong></th>
<th><strong>Section 3.1.2</strong> Optimal battery sizing for single households</th>
<th><strong>Section 3.1.2</strong> Optimal battery sizing, aggregated multiple households</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization formulation</td>
<td>Analogous to Equation 2.10a</td>
<td>Analogous to Equation 2.12a</td>
<td>Analogous to 2.12a, but with aggregated load curves</td>
</tr>
<tr>
<td>PV/battery specifications</td>
<td>One Powerwall and 2 KW PV per house</td>
<td>Cost minimizing battery size (with Powerwall specifications), houses with and w.o. PV</td>
<td>Cost minimizing battery size (with Powerwall specifications), for aggregated houses with and w.o. PV</td>
</tr>
<tr>
<td>Sample size</td>
<td>500 households</td>
<td>1923 households</td>
<td>150 iterations for the aggregation, one iteration $\equiv$ one random sample of houses for one EAC</td>
</tr>
<tr>
<td>Simulation length</td>
<td>One week</td>
<td>One year</td>
<td>One year</td>
</tr>
<tr>
<td>Other specifications</td>
<td>Stochastic optimization based on estimation for future net load, based on ARIMA model</td>
<td>Deterministic Optimization with complete knowledge</td>
<td>Deterministic Optimization with complete knowledge</td>
</tr>
<tr>
<td>Outcome</td>
<td>Given stochastic model replicates possible real battery control, deterministic approach is well suited for data with hourly resolution, since computational is much smaller and results differ only marginally</td>
<td>Given the equivalent annual costs of the Powerwall, the size of the Powerwall is optimal for houses only when assuming long a long battery lifetime</td>
<td>The total (capital plus electricity) costs per house sink when increasing the number of houses aggregated, when aggregating houses equipped with PV</td>
</tr>
</tbody>
</table>
A.2. Overview of models used

Table A.3: Specifications for multiple house battery sizing and optimal charging rate to storage capacity ratio.

<table>
<thead>
<tr>
<th>Model used in</th>
<th>Section 3.2.3 Multiple house battery sizing with Pareto constraint</th>
<th>Section 3.2.4 Optimal charging rate to storage capacity ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization formulation</td>
<td>Analogous to 2.12a, but with constraints of Section 2.2.3</td>
<td>Analogous to the formulation in Section 2.2.4</td>
</tr>
<tr>
<td>PV/battery specifications</td>
<td>Houses with (2 kW capacity) and without PV</td>
<td>Houses with (2 and 4 kW capacity) and without PV</td>
</tr>
<tr>
<td>Sample size</td>
<td>20 iterations for each cluster size of houses and each case (aggregation, Pareto,...)</td>
<td>150 households for each PV/battery combination and each cost weighting</td>
</tr>
<tr>
<td>Simulation length</td>
<td>One year</td>
<td>One year</td>
</tr>
<tr>
<td>Other specifications</td>
<td>Cost savings of Pareto constraint house cluster is compared to aggregated loads and single house optimization, same sample of houses for each iteration for base-lining</td>
<td>With specifics of the Powerwall, different costs weighting on charging rate and storage capacity are used</td>
</tr>
<tr>
<td>Outcome</td>
<td>Pareto constraint leads to lower savings of cluster of houses than aggregated loads model, but total costs are still lower than for the sum of all single houses</td>
<td>If the storage capacity makes up to about 75 - 85 % of and the charging rate the rest of the total costs, the charging rate to storage capacity ratio of the Powerwall is optimal</td>
</tr>
</tbody>
</table>
## A. Appendix

Table A.4: Different specifications for distribution grid and market model.

<table>
<thead>
<tr>
<th>Model used in</th>
<th>Section 3.3 Implications on distribution grid</th>
<th>Section 3.4 Implications on the wholesale market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization formulation</td>
<td>According to OPF formulation of Section 2.3.1, with differing tariff structures for the battery optimization</td>
<td>Wholesale market model according to Figure 2.6, with houses optimizing according to Equation 2.7a</td>
</tr>
<tr>
<td>PV/battery specifications</td>
<td>Differing battery (0-10 kWh storage capacity) and PV capacities (0, 2, 4, 6 kW) per house</td>
<td>Differing battery (0 and 7 kWh storage capacity) and PV capacities (0, 2, 4, 6 kW) per house</td>
</tr>
<tr>
<td>Sample Size</td>
<td>For each iteration 85 loads, with random number of houses between 1-200</td>
<td>For each iteration 1923, extrapolated to the market</td>
</tr>
<tr>
<td>Simulation length</td>
<td>Three days, second day evaluated</td>
<td>One month, depending on focus results for different time scales are shown</td>
</tr>
<tr>
<td>Other specifications</td>
<td>Deterministic Optimization with complete knowledge as basis</td>
<td>Deterministic Optimization with complete knowledge as basis, but changing prices per iteration</td>
</tr>
<tr>
<td>Outcome</td>
<td>Max. voltage constraint is mostly violated, highly dependent on the PV capacity, can be reduced by battery usage and – to a very limited extend – by the tariff structure</td>
<td>PV/battery implementation will mostly change prices during 1 am - 5 am, especially for high battery storage capacities and low PV capacities, reduction of total emissions possible when using large PV capacities</td>
</tr>
</tbody>
</table>
A.3 Figures

A.3.1 Optimal PV/battery sizing

Figure A.1: Exemplary fitted iso-quants for two households and different allowed maximum peak loads.

A.3.2 Methods–PV

Figure A.2: Exemplary qq-plot for temperature distribution.
A. Appendix

A.3.3 Results savings

Figure A.3a and A.3b show the savings in USD per household contrasted to parameters of the load. The maximum of the load seems to have only little impact on the savings of the household for the model (see Fig. A.3a). Obviously, the higher the mean and the minimum of the load are, the higher the savings are.

The normalized standard deviation is used to only account for the deviation and exclude the general load level. For both households with and without PV, the normalized standard deviation seems to be negatively correlated to the savings. It implies that the battery capacity is not able to retain big jumps and thus savings are lower for higher deviations of the net load.
A.3. Figures

(a) Parameters of the load (excl. PV) vs. savings.

(b) Parameters of the net load (incl. PV) vs. savings.

Figure A.3: Savings of the two models contrasted to load parameters.
A. Appendix

A.3.4 Cost of Anarchy

Figure A.4: Cost of Anarchy without PV.

Figure A.5: Cost of Anarchy with PV.
A.3.5 Impact on the grid

Figure A.6: The grid topology of the 123 node test feeder used, source: ieee.org.
A.3.6 Results wholesale market

Figure A.7: Marginal costs and capacities of power plants in California by their location.

Figure A.8: Typical total emissions per hour in California for a day in August.
Figure A.9: Change of price [USD] to previous run, depending on iteration, penetration depth and sizing of the PV/battery system, feed-in-tariff equal to zero.
Figure A.10: Change of demand [MW] to previous run, depending on iteration, penetration depth and sizing of the PV/battery system, feed-in-tariff equal to zero.

(a) Households only with a battery.  
(b) Households with addit. 2 kW PV.  
(c) Households with additionally 4 kW PV.  
(d) Households with addit. 6 kW PV.
Bibliography


Bibliography


