Renewables in electricity markets and distributionally robust Bernoulli newsvendor problems

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Outline

1. **Renewables (wind) offering in electricity markets**

The newsvendor problem

- The newsvendor problem is one of the most classical problems in **stochastic optimization** (or **statistical decision theory**)
- It can be traced back to:

FY Edgeworth (1888). The mathematical theory of banking. *Journal of the Royal Statistical Society* **51**(1): 113–127

(even though in this paper the problem is about how much a bank should keep in its reserves to satisfy request for withdrawal (i.e., the *bank-cash-flow problem*))

- It applies to varied problems as long as:
  - one shot possibility to decide on the quantity of interest
  - outcome is uncertain
  - known marginal profit and loss
  - the aim is to maximize expected profit!

Optimal offering as a price-taker

- Let us focus on a given market time unit $t + k$ (e.g., 12-1pm tomorrow – and drop time-related notations)
- Write
  - π_s the day-ahead price
  - π_o the unit regulation cost if over-producing (referred to as *overage* penalty)
  - π_u the unit regulation cost if under-producing (referred to as *underage* penalty)
- The revenue for $t + k$ is given by

$$R = \pi_s \omega - \pi_o (\omega - y)_+ - \pi_u (y - \omega)_+$$

  where y and ω are for contracted and actual energy generation (random variable)

- One readily obtains that the expected utility maximization offer is

$$y^* = \operatorname*{argmin}_y \mathbb{E}_\omega \left[\mathcal{L}(y, \omega, \pi_o, \pi_u) \right] = \hat{F}_\omega^{\,-1} \left(\frac{\pi_o}{\pi_o + \pi_u} \right)$$

  where $\hat{F}_\omega$ is the predictive CDF for wind power generation

  In practice, π_o and π_u are replaced by estimates (/forecasts).

- **Issues? Is wind really a price-taker?**

Optimal offering as a price-maker

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- directly
- through population effects

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Pool Strategy of a Price-Maker Wind Power Producer

Marco Zugno, *Student Member, IEEE*, Juan M. Morales, *Member, IEEE*, Pierre Pinson, *Senior Member, IEEE*, and Henrik Madsen

***Abstract*—We consider the problem of a wind power producer trading energy in short-term electricity markets. The producer is a price-taker in the day-ahead market, but a price-maker in the balancing market, and aims at optimizing its expected revenues from these market floors. The problem is formulated as a mathematical program with equilibrium constraints (MPEC) and cast as a mixed-integer linear program (MILP), which can be solved employing off-the-shelf optimization software. The optimal bid is shown to deliver significantly improved performance compared to traditional bids such as the conditional mean or median forecast of wind power distribution. Finally, sensitivity analyses are carried out to assess the impact on the offering strategy of the producer's penetration in the market, of the correlation between wind power production and residual system deviation, and of the shape of the forecast distribution of wind power production.**

***Index Terms*—Electricity markets, mathematical programs with equilibrium constraints, offering strategies, price-maker, wind power.**

NOMENCLATURE

A. Sets

δ_ω Residual system deviation in scenario ω.

λ_ω^{DA} Day-ahead market price in scenario ω.

C^W Installed capacity for wind power producer.

C. Lower-Level Variables

$p_{k\omega}$ Up-regulation from block k in scenario ω.

$p_{j\omega}$ Down-regulation from block j in scenario ω.

λ_ω^B Balancing market price in scenario ω.

$\mu_{k\omega}^S$ Dual variable for capacity constraint at the balancing market for block k in scenario ω.

$\mu_{j\omega}^D$ Dual variable for capacity constraint at the balancing market for block j in scenario ω.

D. Upper-Level Variables

Population Dynamics for Renewables in Electricity Markets: A Minority Game View

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***Abstract*—The dominance of fluctuating and intermittent stochastic renewable energy sources (RES) has introduced uncertainty in power systems which in turn, has challenged how electricity market operate. In this context, there has been significant research in developing strategies for RES producers, which however typically focuses on the decision process of a single producer, assuming unrealistic access to aspects of information about the power system. This paper analyzes the behavior of an entire population of stochastic producers in an electricity market using as basis a minority game: *the El Farol Bar problem*. We illustrate how uncomplicated strategies based on a adaptive learning rules lead to the coordination among RES producers and a Pareto efficient outcome.**

I. INTRODUCTION

The deployment of stochastic renewable energy sources (RES) e.g. wind and solar power, has been

to be available to them, such as complete information of the power system's attributes and access to the predictive distributions of all other stochastic producers, while assuming that they accurately model real-time production. Second, they assume that only a single producer is strategic and capable of devising an optimal strategy, with the rest following without a reaction. That is, they do not capture the inherent dynamics of participating in a market, with the exception being the formulation of the competition among stochastic producers as an equilibrium program with equilibrium constraint [6]. However, such techniques are computationally demanding with reduced tractability, often too complex to be implemented in real-world real-time basis.

In this paper we analyze the behavior of a population of stochastic producers in electricity markets focusing

- Price-maker effect may be on:
  - day-ahead price
  - systems state (i.e., need for up- or down-regulation)
  - unit regulation costs
- Possible extensions for, e.g., wind-storage systems (optimizing both offering and control policies)

2. **The Bernoulli newsvendor problem**

- The overage π_o and underage π_u penalties are unknown!
- Only one of them is active in most cases (two-price imbalance settlement)

One can then define the penalization as a the outcome of the Bernoulli variable s (with chance of success τ):

$$s = \frac{\pi_o}{\pi_o + \pi_u}$$

where

$$\begin{aligned} \pi_o &= (\pi_s - \pi_b)\,\mathbf{1}_{\{s_L \geq 0\}} \\ \pi_u &= (\pi_b - \pi_s)\,\mathbf{1}_{\{s_L < 0\}} \end{aligned}$$

(π_b the balancing price, s_L the overall system imbalance)

s has only 2 potential discrete outcomes, i.e.

$$\begin{aligned} (i) \quad & \pi_o \neq 0,\ \pi_u = 0 \ \Rightarrow\ s = \frac{\pi_o}{\pi_o + \pi_u} = 1,\ 1 - s = 0 \\ (ii) \quad & \pi_o = 0,\ \pi_u \neq 0 \ \Rightarrow\ s = \frac{\pi_o}{\pi_o + \pi_u} = 0,\ 1 - s = 1 \end{aligned}$$

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Definition (Bernoulli newsvendor problem)

Based on a Bernoulli random variable s (with chance of success τ), and the uncertain parameter ω (with c.d.f. F_ω), the decision y^* minimizing the expected opportunity cost function is

$$y^* = \underset{y}{\operatorname{argmin}} \; \mathbb{E}_{\omega,s}\left[\mathcal{L}(y,\omega,s)\right] ,$$

where the opportunity cost is defined as

$$\mathcal{L}(y,\omega,s) = s(\omega - y)_+ + (1-s)(y-\omega)_+ ,$$

and where $(.)_+$ is for the positive part.

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and where $(.)_+$ is for the positive part.

And its solution:

Proposition

Consider F_ω the c.d.f. for the uncertain parameter ω and τ the chance of success for s. The optimal decision y^* for the Bernoulli newsvendor problem (**??**) *is*

$$y^* = F_\omega^{-1}(\tau) \, .$$

3. **Distributionally robust versions**

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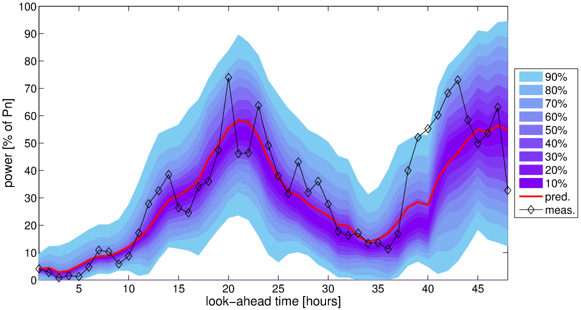
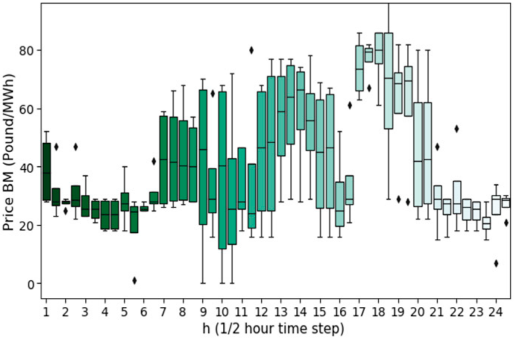
- F_ω and F_s are still unknown and need to be replaced by forecasts, $\hat{F}_\omega$ and $\hat{F}_s$
- These forecasts are necessarily imperfect...

Both renewable energy generation and balancing market outcomes are notoriously difficult to predict...



$\hat{F}_\omega$



(input to obtain) $\hat{F}_s$
(reproduced from Lucas *et al.* (2020))

We adapt here the solution approach of Fu *et al.* (2021) to the Bernoulli newsvendor problem...

Definition (distributionally robust Bernoulli newsvendor problem – ambiguity about $\hat{F}_\omega$)

Consider a Bernoulli random variable s with estimated chance of success $\hat{\tau}$, the uncertain production ω with predictive c.d.f. $\hat{F}_\omega$, and an ambiguity set $\mathcal{B}_{\hat{F}_\omega}(\rho)$ with radius ρ. The distributionally robust Bernoulli newsvendor problem, with ambiguity about $\hat{F}_\omega$, is that for which the decision y^* is given by

$$y^* = \operatorname*{argmin}_y \sup_{F_\omega \in \mathcal{B}_{\hat{F}_\omega}(\rho)} \mathbb{E}_{\omega,s}\left[\mathcal{L}(y,\omega,\tau)\right].$$

How to define $\mathcal{B}_{\hat{F}_\omega}(\rho)$?

Deformation operator and FSD-ambiguity set

A first-order stochastic dominance ambiguity set (FSD-ambiguity set) is such that

$$\underline{F}_\omega(x) \le F_\omega(x) \le \overline{F}_\omega(x), \quad \forall x,\ \forall F_\omega \in \mathcal{B}_{\hat{F}_\omega}(\rho)\,.$$

As an example, we introduce here a double-power deformation operator that fulfil the above definition.

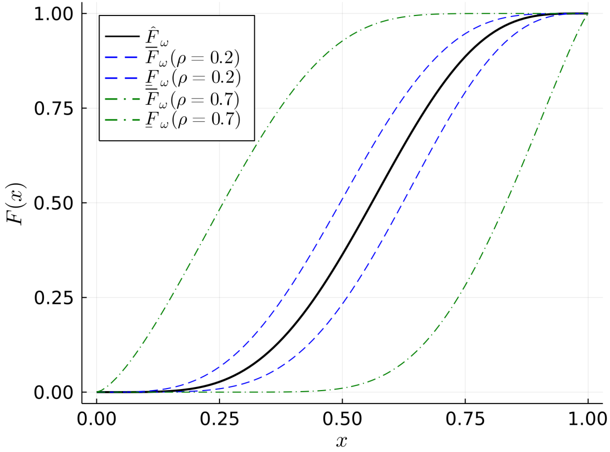
Definition (double-power deformation operator)

Consider a reference c.d.f. F_ω. The upper $\overline{\mathcal{O}}_\rho$ and lower $\underline{\mathcal{O}}_\rho$ double-power deformation operators are defined as

$$\overline{\mathcal{O}}_\rho(F_\omega) = \left(1-(1-F_\omega{}^{\frac{1}{1-\rho}})\right)^{1-\rho},$$

$$\underline{\mathcal{O}}_\rho(F_\omega) = 1-(1-F_\omega{}^{\frac{1}{1-\rho}})^{1-\rho},$$

with ρ the deformation parameter.



These deformation readily allow to define $\underline{F}_\omega$ and $\overline{F}_\omega(x)$.

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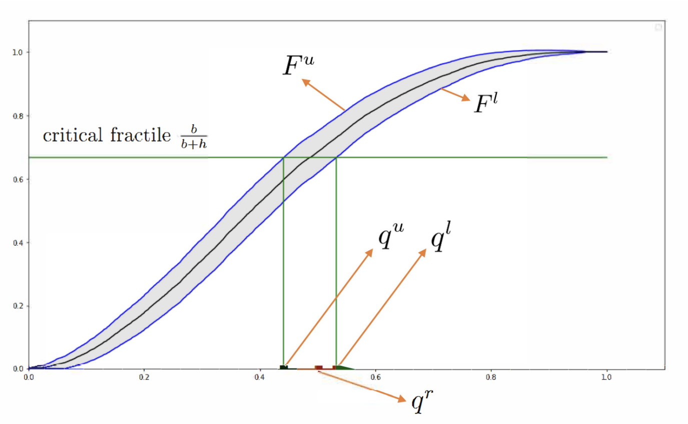
The worst-case distribution F^{ws} for this distributionally robust problem is defined by



(reproduced from Fu *et al.* (2021))

$$F^{\text{ws}}(x) = \begin{cases} \overline{F}_\omega(x), & x < \overline{F}_\omega{}^{-1}(\hat{\tau}) \\ \tau, & \overline{F}_\omega{}^{-1}(\hat{\tau}) < x < \underline{F}_\omega{}^{-1}(\hat{\tau}) \\ \underline{F}_\omega(x), & x > \underline{F}_\omega{}^{-1}(\hat{\tau}) \end{cases}$$

Distributionally robust solution

Theorem

Consider an FSD-ambiguity set defined by a ball $\mathcal{B}_{\hat{F}_\omega}(\rho)$ with radius ρ, yielding the two bounding distributions $\underline{F}_\omega$ and $\overline{F}_\omega$. For a predicted chance of success $\hat{\tau}$, the solution of the distributionally robust Bernoulli newsvendor problem (**??**) *is*

$$y^* = \hat{\tau}\underline{F}_\omega^{-1}(\hat{\tau}) + (1-\hat{\tau})\overline{F}_\omega^{-1}(\hat{\tau})\,.$$

Distributionally robust solution

Theorem

Consider an FSD-ambiguity set defined by a ball $\mathcal{B}_{\hat{F}_\omega}(\rho)$ with radius ρ, yielding the two bounding distributions $\underline{F}_\omega$ and $\overline{F}_\omega$. For a predicted chance of success $\hat{\tau}$, the solution of the distributionally robust Bernoulli newsvendor problem (**??**) *is*

$$y^* = \hat{\tau}\underline{F}_\omega^{-1}(\hat{\tau}) + (1-\hat{\tau})\overline{F}_\omega^{-1}(\hat{\tau}) .$$

For sufficiently large values of the radius ρ, we obtain the following (robust) limiting case:

Corollary

For sufficiently large values of ρ, the radius of $\mathcal{B}_{\hat{F}_\omega}(\rho)$, the solution of the distributionally robust Bernoulli newsvendor problem (**??**) *converges to the robust solution $y^* = \hat{\tau}$.*

Ambiguity about $\hat{F}_s$

The following solution is not available in the literature...

Definition (distributionally robust Bernoulli newsvendor problem – ambiguity about $\hat{F}_s$)

Consider a Bernoulli random variable s with estimated chance of success $\hat{\tau}$, the uncertain production ω with predictive c.d.f. $\hat{F}_\omega$, and an ambiguity set for $\hat{F}_s$ defined by the ball $\mathcal{B}_{\hat{\tau}}(\varepsilon)$ with radius ε. The distributionally robust Bernoulli newsvendor problem, with ambiguity about $\hat{F}_s$, is that for which the decision y^* is given by

$$y^* = \operatorname*{argmin}_y \max_{\tau \in \mathcal{B}_{\hat{\tau}}(\varepsilon)} \mathbb{E}_{\omega,s}\left[\mathcal{L}(y, \omega, \tau)\right]$$

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$$y^* = \operatorname*{argmin}_y \max_{\tau \in \mathcal{B}_{\hat{\tau}}(\varepsilon)} \mathbb{E}_{\omega,s}\left[\mathcal{L}(y,\omega,\tau)\right]$$

How to define $\mathcal{B}_{\hat{F}_s}(\varepsilon)$?

Definition (uniform and level-adjusted ambiguity sets for $\hat{F}_s$)

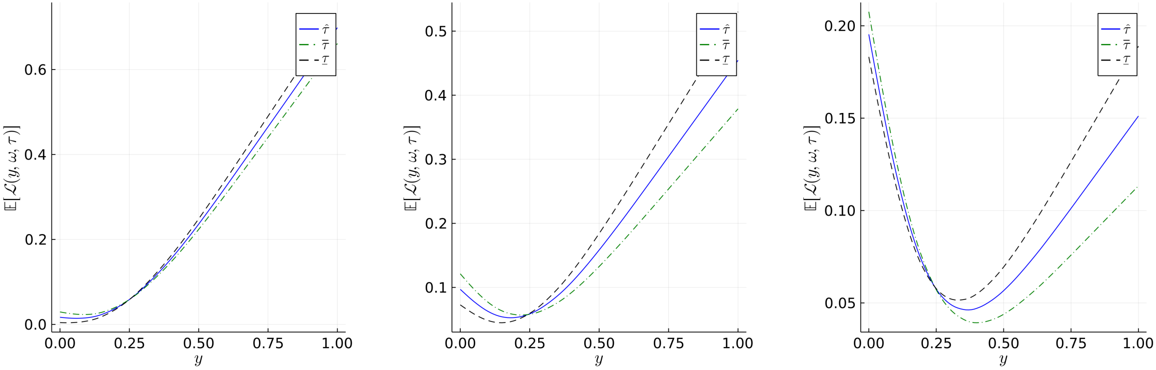
Given a ball radius ε, a uniform ambiguity set for $\hat{F}_s$ is defined by the ball $\mathcal{B}_{\hat{\tau}}$ with

$$\begin{aligned} \underline{\tau} &= \max(\hat{\tau} - \varepsilon, 0) \\ \overline{\tau} &= \min(\hat{\tau} + \varepsilon, 1) \end{aligned}$$

while a level-adjusted ambiguity set (with parameter $\theta \in [0,1)$) for $\hat{F}_s$ is defined by the ball $\mathcal{B}_{\hat{\tau}}$ with

$$\begin{aligned} \underline{\tau} &= \max(\hat{\tau} - \varepsilon(1 - 4\,\theta\,\hat{\tau}(1-\hat{\tau})), 0) \\ \overline{\tau} &= \min(\hat{\tau} + \varepsilon(1 - 4\,\theta\,\hat{\tau}(1-\hat{\tau})), 1) \end{aligned}$$

Intuition for the upcoming result



(a) $\tau = 0.1$, $\varepsilon = 0.05$ (b) $\tau = 0.5$, $\varepsilon = 0.1$ (c) $\tau = 0.8$, $\varepsilon = 0.05$

Figure: Expected opportunity cost, as a function the decision y, for different values of τ and ε (using uniform ambiguity sets). The uncertain parameter ω follows a Beta(2,6) distribution (with expected value $\mathbb{E}[\omega] = 0.25$), while the estimate $\hat{\tau}$ is based on 15 samples.

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Theorem

Consider an ambiguity set for $\hat{F}_s$ defined by a ball $\mathcal{B}_{\hat{\tau}}(\varepsilon)$ with radius ε, and the predictive c.d.f. $\hat{F}_\omega$ for the uncertain parameter ω. The solution of the distributionally robust Bernoulli newsvendor problem (**??**) *is*

$$
\begin{aligned}
y^* &= \hat{F}_\omega^{-1}(\overline{\tau})\,\mathbf{1}_{\{\hat{F}_\omega^{-1}(\overline{\tau})<\mathbb{E}[\omega]\}} + \hat{F}_\omega^{-1}(\underline{\tau})\,\mathbf{1}_{\{\hat{F}_\omega^{-1}(\underline{\tau})>\mathbb{E}[\omega]\}} \\
&\quad + \mathbb{E}[\omega]\,\mathbf{1}_{\{\hat{F}_\omega^{-1}(\overline{\tau})\geq\mathbb{E}[\omega]\}}\,\mathbf{1}_{\{\hat{F}_\omega^{-1}(\underline{\tau})\leq\mathbb{E}[\omega]\}}\,.
\end{aligned}
$$

Theorem

Consider an ambiguity set for $\hat{F}_s$ defined by a ball $\mathcal{B}_{\hat{\tau}}(\varepsilon)$ with radius ε, and the predictive c.d.f. $\hat{F}_\omega$ for the uncertain parameter ω. The solution of the distributionally robust Bernoulli newsvendor problem (**??**) *is*

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And, for sufficiently large values of ε, we obtain the following limiting (robust) case:

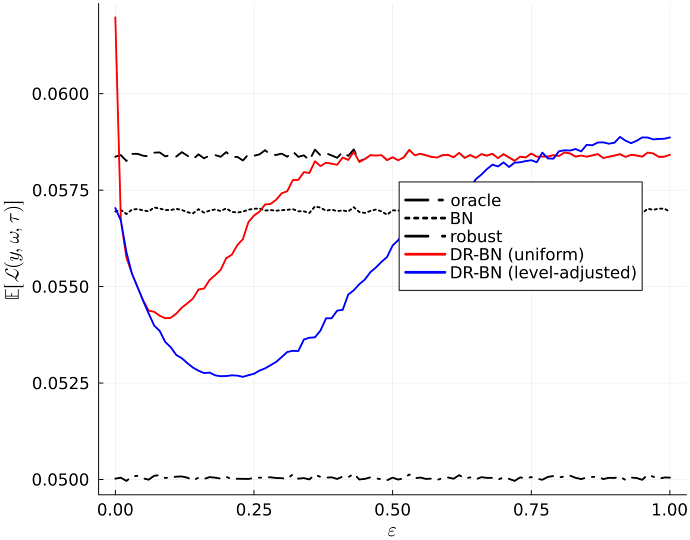
Corollary

For sufficiently large values of ε, the radius of $\mathcal{B}_{\hat{\tau}}(\varepsilon)$, the solution of the distributionally robust Bernoulli newsvendor problem (**??**) *converges to the robust solution $y^* = \mathbb{E}[\omega]$.*

4. **Simulations and case-study application**

Ambiguity about $\hat{F}_s$ - impact of ball radius ε

- Monte-Carlo simulation with $N = 10^7$ replicates
- $\omega \sim \text{Beta}(2, 6)$
- $\tau = 0.75$
- $m = 10$ (number of draws of a Bern(τ) to obtain estimate $\hat{\tau}$)
- $\theta = 0.9$ (for the level-adjusted ball)



The performance measure γ (in %) is then defined as $\gamma = \frac{L_{\text{BN}} - L^*_{\text{DR-BN}}}{L_{\text{BN}} - L_{\text{O}}}$, and can be expressed in percents. For the above example, $\gamma_{\text{U}} = 40.3\%$ and $\gamma_{\text{LA}} = 60.2\%$.

Ambiguity about $\hat{F}_s$ - impact of forecast quality

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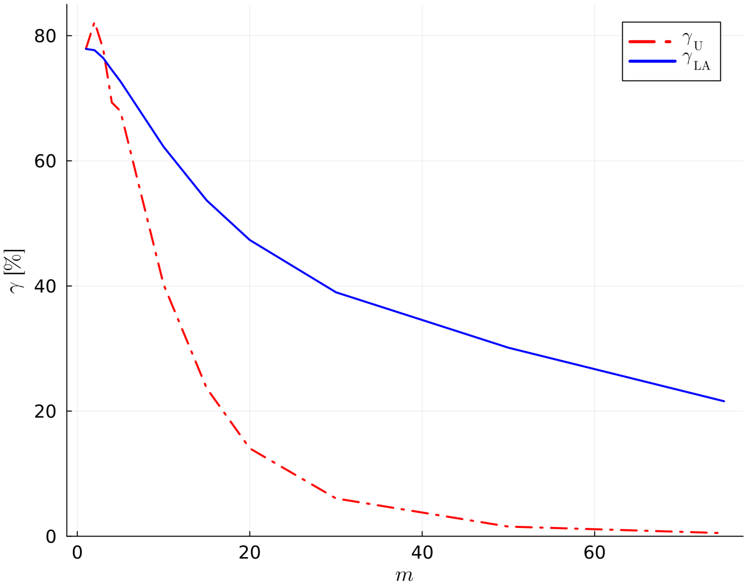
- Monte-Carlo simulation with $N = 10^7$ replicates
- m (number of draws of a Bern(τ) to obtain estimate $\hat{\tau}$) is a proxy for forecast quality
- $\omega \sim$ Beta$(2, 6)$
- $\tau = 0.75$
- $\theta = 0.9$ (for the level-adjusted ball)

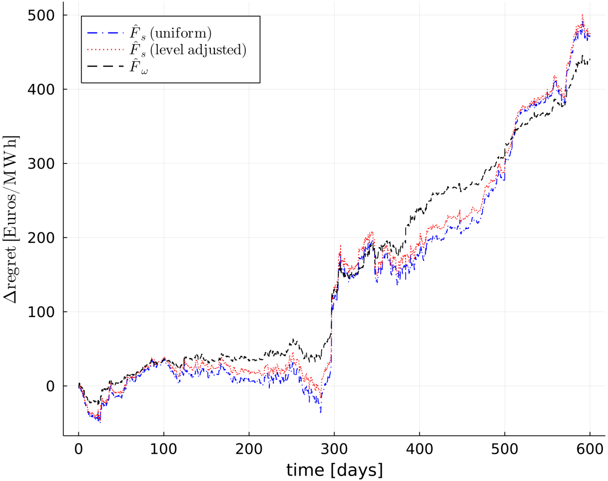


Application to French electricity market

Portfolio of wind farms from Midwest France (confidential) over a 2-year period

- first 131 days for warm start and cross-validation (to decide on m, ρ, ε, θ)
- remaining 600 days for genuine out-of-sample evaluation

Approach	R [€/MWh]	r [€/MWh]
Oracle	31.63	0
BN	29.25	2.38
DR-BN ($\hat{F}_w$)	29.34	2.29
DR-BN ($\hat{F}_s$, uniform)	29.35	2.28
DR-BN ($\hat{F}_s$, level-adj.)	29.36	2.27



Cumulative regret normalized per MWh produced

5. **Concluding thoughts and discussion**

Closing remarks

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Bernoulli newsvendor problems are a more general form suitable for renewables in electricity markets

- same expected utility maximization solution
- distributionally robust versions with ambiguity about $\hat{F}_\omega$ and $\hat{F}_s$

A natural next step is to look at the joint ambiguity!

Thanks for your attention!