Dispatching Stochastic Heterogeneous Resources Accounting for Grid and Battery Losses

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Operation Control of Active Distribution Grids

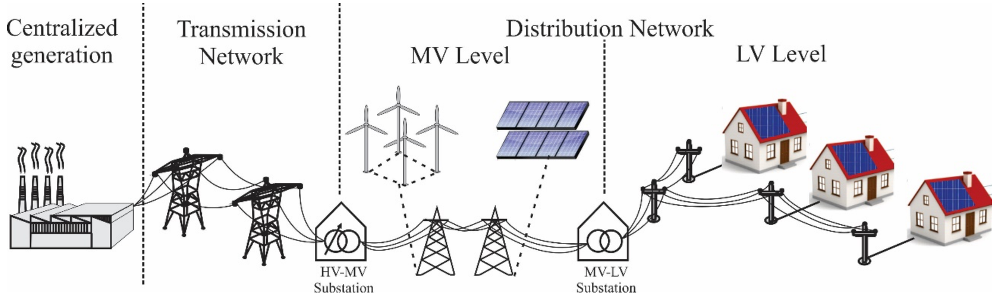
Proliferation of uncertain distributed energy resources (DERs)

+

Generation and demand should be balanced

→

Increasing amount of reserves required



Reserves are costly!

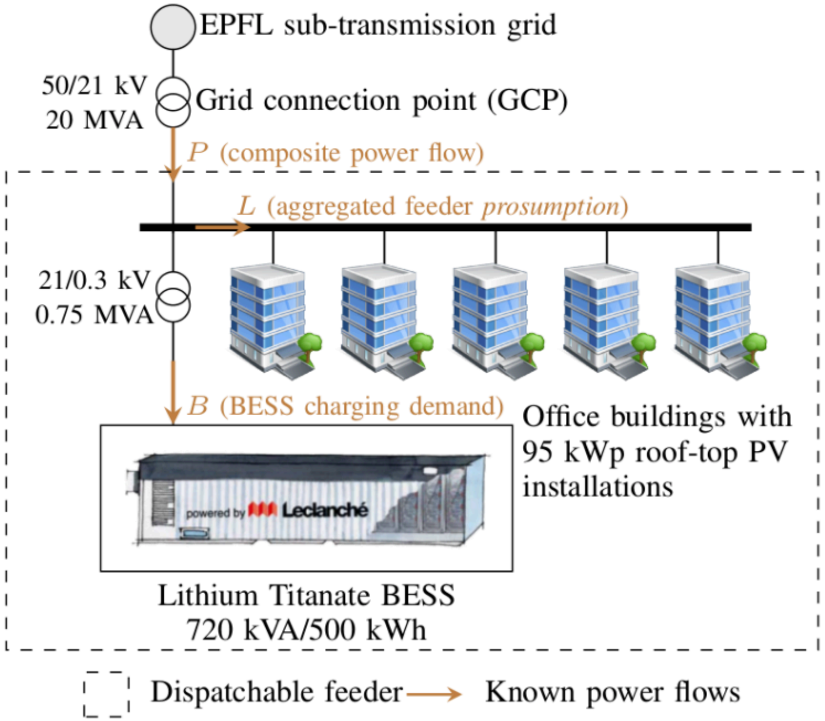
✓ stochastic DERs if adopted passively may raise grid investment and operational costs

Energy Storage for Absorbing Uncertainties

- Energy storage and storage control can allow a high penetration of DERs in the power grid

+ Energy storage cost is expected to drastically reduce

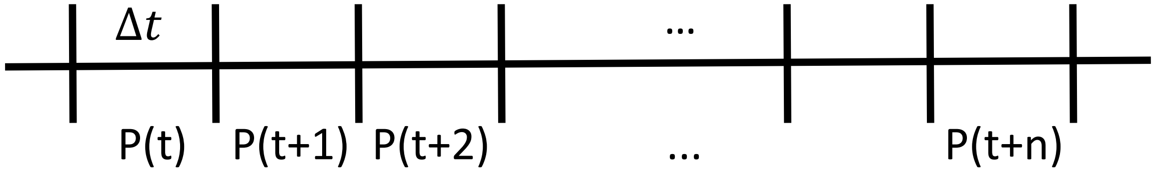
➢ We focus on **radial distribution grids with DERs and batteries**

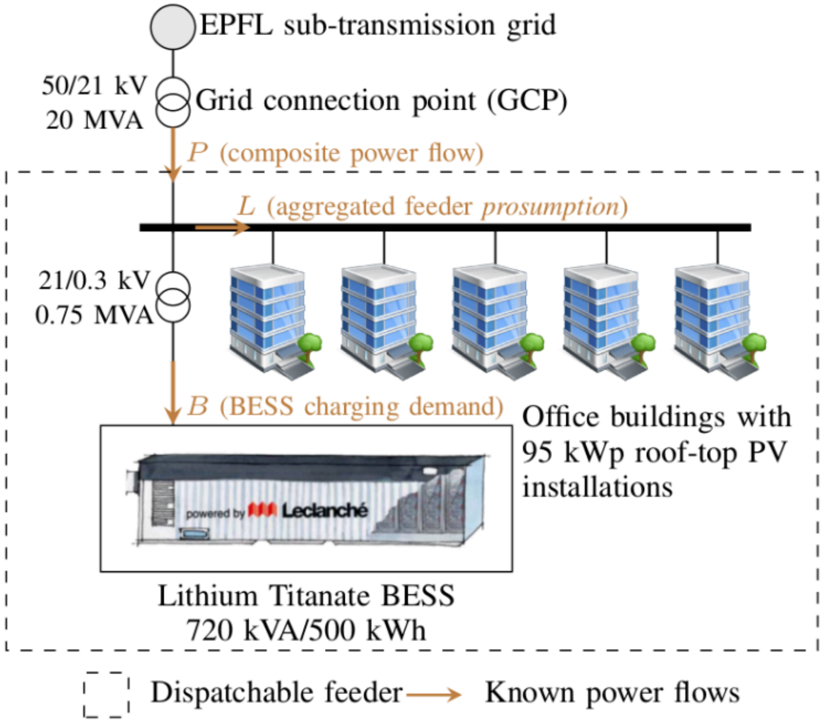


Dispatch and Re-dispatch Feeders for Compressing the Amount of Reserves

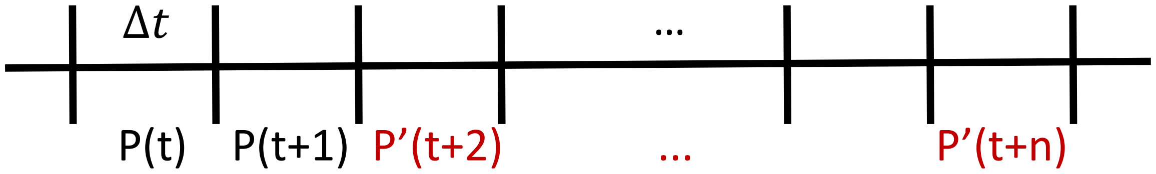
Aggregate batteries, DERs and loads into a single point to provide dispatchability

Dispatch: Commit day-ahead for the next day a plan of positive or negative power values at the Point of Common Coupling (PCC) with the main grid





Re-dispatch: Based on new information change the plan intra-day



[Bozorg et al 2018]

Criteria for a Successful Dispatch Plan

- Three-stage procedure: day-ahead, intra-day and real-time operation

When is a dispatch plan good?

- The real-time control algorithm can take feasible battery charge/discharge decisions with which:

✓ the realized PCC power is very close to the planned one

✓ the grid constraints are satisfied

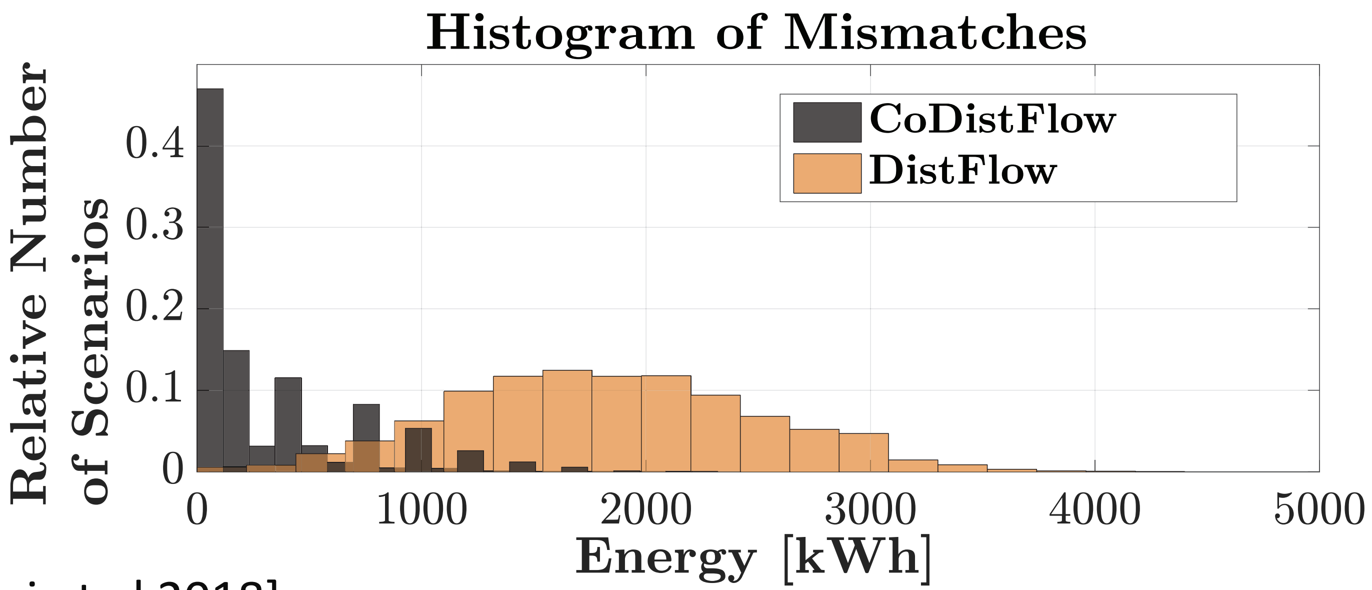
- **Dispatch plan should consider**
  1. uncertainty of DERs and loads
  2. grid and batteries losses, grid and batteries constraints

     ⇨ need of solving a non-convex AC OPF

Dispatching with CoDistFlow

- ✓ it solves iteratively a multi-period scenario-based AC OPF
- ✓ the solution satisfies the exact AC power flow equations for all scenarios
- ✓ it is computationally efficient

Example in Figure: Dispatching with CoDistFlow vs Simplified DistFlow



[Stai et al 2018]

Overview

- Scenario-based multi-period AC OPF in radial distribution grids with DERs and batteries for computing a day-ahead dispatch plan

- Solution with CoDistFlow [Stai et al 2018], [Wang et al 2019]
  - ✓ it is developed both for single-phase and for three-phase unbalanced radial distribution grids
  - ✓ for simplicity we focus on single-phase

- Receding Horizon Control over CoDistFlow for re-dispatching [Stai et al 2019]

- Demonstration of evaluations using real data on a real-life Swiss grid

Scenario-based Multi-period AC OPF for Computing a Dispatch Plan

Dealing with Uncertainty: Scenario-based Optimization

Multiple ways, e.g.,

- robust optimization
- chance-constrained optimization
- scenario-based optimization

We choose **scenario-based optimization** since it allows for …

✓ … proper modeling of the uncertainty of stochastic resources (e.g., non-parametric)

✓ … inclusion of general convex constraints

✓ … accounting for any existing time correlations

[Pinson et al 2009]

Day-ahead Dispatch Plan: Multi-Period Scenario-based AC OPF Formulation

Minimize

Objective **for *T* time-slots ahead**, expected value over scenarios, a weighted sum of:

1. *penalty for each scenario that cannot follow dispatch plan*
2. *penalties on batteries state-of-energy*
3. *penalties on power exchanged with main grid*

subject to (for every scenario & time-slot ahead)

1. Exact AC power flow equations
2. Voltage constraints
3. Ampacity constraints
4. Battery power and battery state-of-energy constraints

1. **Dispatch plan common for all scenarios**
2. One battery power trajectory for every scenario and battery
3. One electrical state (line power flows, bus voltage magnitudes) for every scenario

AC Power Flow Equations for the Branch Flow Model using Angle Relaxation and with Shunt Elements

DistFlow Equations

Active Power

$$P_\ell^d(t) = \sum_{k:\mathbf{G}_{lk}=1} P_k^d(t) + p_\ell^d(t) + p_{B,\ell}^d(t) + r_\ell f_\ell^d(t),$$

Reactive Power

$$Q_\ell^d(t) = \sum_{k:\mathbf{G}_{lk}=1} Q_k^d(t) + q_\ell^d(t) + q_{B,\ell}^d(t) - (v_{\mathrm{up}(\ell)}^d(t) + v_\ell^d(t))b_\ell/2 + x_\ell f_\ell^d(t),$$

Current Square Magnitude

$$f_\ell^d(t) = \left\| S_\ell^d(t) + j\frac{v_{\mathrm{up}(\ell)}^d(t) b_\ell}{2} \right\|^2 / v_{\mathrm{up}(\ell)}^d(t).$$

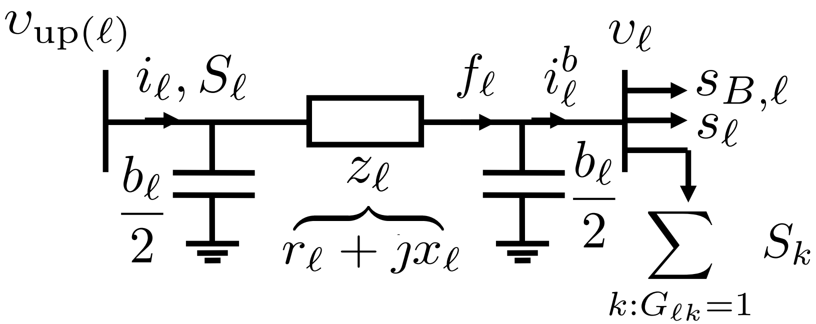
Voltage Square Magnitude

$$v_\ell^d(t) = v_{\mathrm{up}(\ell)}^d(t) - 2\Re\left\{ z_\ell^* \left(S_\ell^d(t) + j v_{\mathrm{up}(\ell)}^d(t)\frac{b_\ell}{2} \right) \right\} + \|z_\ell\|^2 f_\ell^d(t),$$

Non-convex

[Farivar Low 2013], [Nick et al 2016], [Baran Wu 1989]

line π model



Complex power flow:

$$S_\ell = P_\ell + jQ_\ell$$

Complex power injection:

$$s_\ell = p_\ell + jq_\ell$$

Complex battery power injection:

$$s_{B,\ell} = p_{B.\ell} + jq_{B,\ell}$$

ℓ: line index

d: scenario index

Voltage and Ampacity Constraints

Voltage Constraints

$$v^d_{\mathrm{up}(1)}(t) = 1,\ \underline{v}^2 \le v^d_\ell(t) \le \overline{v}^2$$

Bounds on voltage square magnitude at each bus

Current Constraints

$$\Re\{i^d_\ell(t)\} = \frac{P^d_\ell(t)}{\sqrt{v^d_{\mathrm{up}(\ell)}(t)}},\quad \Im\{i^d_\ell(t)\} = \frac{Q^d_\ell(t)}{\sqrt{v^d_{\mathrm{up}(\ell)}(t)}},$$

$$\Re\{i^{b,d}_\ell(t)\} = \left(P^d_\ell(t) - r_\ell f^d_\ell(t)\right)/\sqrt{v^d_\ell(t)},$$

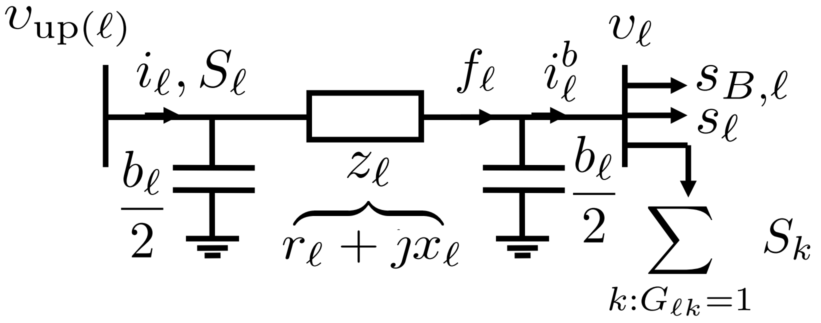
$$\Im\{i^{b,d}_\ell(t)\} = \left(Q^d_\ell(t) - x_\ell f^d_\ell(t)\right)/\sqrt{v^d_\ell(t)} + \left(v^d_{\mathrm{up}(\ell)}(t) + v^d_\ell(t)\right)b_\ell/\left(2\sqrt{v^d_\ell(t)}\right),$$

Non-convex

$$\|i^d_\ell(t)\| \le \overline{I}_\ell, \|i^{b,d}_\ell(t)\| \le \overline{I}_\ell.$$

Bounds on ampacity at each endpoint of the line

line π model



Complex power flow:

$$S_\ell = P_\ell + \jmath Q_\ell$$

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Complex battery power injection:

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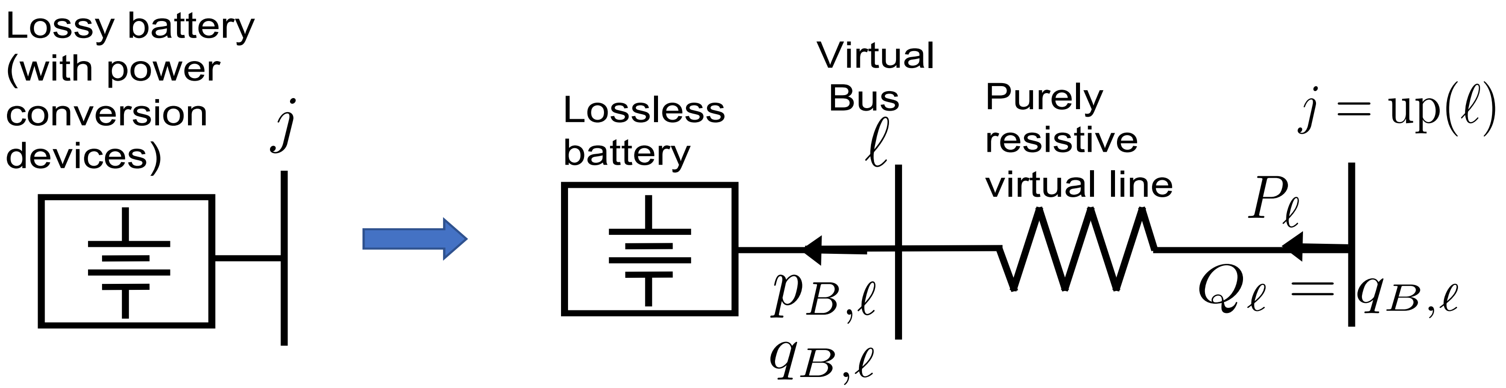
d: scenario index

Modeling Lossy Battery Using the AC Single-Port Circuit Equivalent



- **For each battery connected to bus *j* a new virtual bus ℓ is added and connects to bus *j* via a virtual purely resistive line**
- Battery losses = active power losses of this resistive line
- Battery state-of-energy = one of the lossless battery, with same capacity and rated power
- No voltage constraints to the virtual bus, no ampacity constraints to the virtual line

Battery Power and State-of-Energy Constraints

$$\mathrm{SoE}^d_{B,\ell}(t+1) = \mathrm{SoE}^d_{B,\ell}(t) + p^d_{B,\ell}(t)\Delta t,$$
$$a_B \overline{\mathrm{SoE}}_{B,\ell} \le \mathrm{SoE}^d_{B,\ell}(t) \le (1-a_B)\overline{\mathrm{SoE}}_{B,\ell},$$

State-of-energy constraints

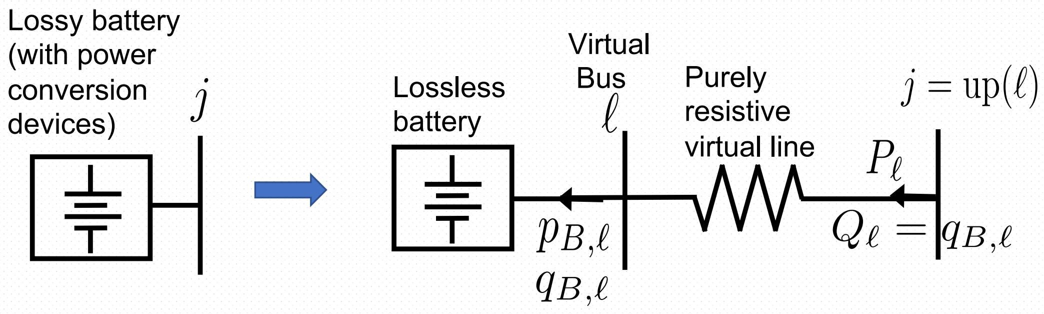
$$\left(P^d_\ell(t)\right)^2 + (q^d_{B,\ell}(t))^2 \le (s^R_{B,\ell})^2,$$

Power constraints, taken at the real bus the battery connects

$$\mathrm{SoE}^d_{B,\ell}(0) = \mathrm{SoE}^I_{B,\ell},$$

The initial state-of-energy is known

Battery model



Objective Function

λ_d : probability of each scenario
d : scenario index
w : weights of objectives

Weighted sum of partial objectives, expected value over scenarios:

1. Minimize a penalty on the state-of-energy of the batteries
2. Minimize the reactive power exchanged at the PCC

$$w_1 \sum_{d,t,i} \lambda_d \phi(\mathrm{SoE}^d_{B,i}(t)) + w_2 \sum_{d,t} \lambda_d |Q^d_1(t)| + w_3 \sum_{d,t} \lambda_d |P^d_1(t)|$$
$$+ w_4 \sum_{d,t} \lambda_d P^d_1(t) + w_5 \sum_{d,t} \lambda_d \|S^d_1(t) - S^{DP}(t)\|^2.$$

3. Minimize the active power exchanged at the PCC
4. Maximize the power export to the main grid
5. Minimize the error between the dispatch plan and the optimal power at the PCC

Why Existing Relaxation Methods with Exact Solutions Do not Apply?

- Relaxation techniques with exact solutions: **relax the current square magnitude equality into inequality** [Li Chen Low 2012], [Nick et al 2016]
- Requires objective increasing with losses for exactness
- **Observation**: Not compatible with scenario-based optimization, i.e., the solution may not be exact for all the scenarios

✓ *E.g., if relaxing the last equality we do not obtain exact solution for the 2nd scenario*

Example with 2 scenarios

$$\min_{S,S_B,v,S^{DP},f} w_2 \sum_d \lambda_d |Q_1^d| + w_3 \sum_d \lambda_d |P_1^d|$$

$$\begin{aligned} s.t.\ \forall d \in \{1,2\}, \\ P_1^d = p_1^d + p_{B,1}^d + r_1 f_1^d, \\ Q_1^d = q_1^d + q_{B,1}^d + x_1 f_1^d, \\ P_1^1 = P_1^2 = P^{DP}, \\ Q_1^1 = Q_1^2 = Q^{DP}, \\ v_1^d = 1 - 2(r_1 P_1^d + x_1 Q_1^d) + \|z_1\|^2 f_1^d, \\ (p_{B,1}^{+,d})^2 + (p_{B,1}^{-,d})^2 + (q_{B,1}^d)^2 \leq (s_{B,1}^R)^2, \\ v_0^d = 1, \underline{v}^2 \leq v_1^d, \\ \boxed{f_1^d = \|S_1^d\|^2,} \end{aligned}$$

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Example with 2 scenarios

$$\begin{aligned}\min_{S,S_B,v,S^{DP},f} \; & w_2 \sum_d \lambda_d |Q_1^d| + w_3 \sum_d \lambda_d |P_1^d| \\ & s.t.\ \forall d \in \{1,2\}, \\ & P_1^d = p_1^d + p_{B,1}^d + r_1 f_1^d, \\ & Q_1^d = q_1^d + q_{B,1}^d + x_1 f_1^d, \\ & P_1^1 = P_1^2 = P^{DP}, \\ & Q_1^1 = Q_1^2 = Q^{DP}, \\ & v_1^d = 1 - 2(r_1 P_1^d + x_1 Q_1^d) + \|z_1\|^2 f_1^d, \\ & (p_{B,1}^{+,d})^2 + (p_{B,1}^{-,d})^2 + (q_{B,1}^d)^2 \le (s_{B,1}^R)^2, \\ & v_0^d = 1, \underline{v}^2 \le v_1^d, \\ & f_1^d \ge \|S_1^d\|^2, \end{aligned}$$

Proposed Algorithm: CoDistFlow

Main Idea of Proposed CoDistFlow

- Replace the terms that are responsible for non-convexities with constants
- Solve iteratively the new optimization problem with updated constants for each iteration
- Initialize with Simplified DistFlow, i.e., constants=0
- Battery losses are treated similarly with the non-linearities of the power flow equations

DistFlow Equations

$$P_\ell^d(t) = \sum_{k:\mathbf{G}_{lk}=1} P_k^d(t) + p_\ell^d(t) + p_{B,\ell}^d(t) + r_\ell f_\ell^d(t),$$

$$Q_\ell^d(t) = \sum_{k:\mathbf{G}_{lk}=1} Q_k^d(t) + q_\ell^d(t) + q_{B,\ell}^d(t) - (v_{\mathrm{up}(\ell)}^d(t) + v_\ell^d(t)) b_\ell/2 + x_\ell f_\ell^d(t),$$

~~$$f_\ell^d(t) = \left\| S_\ell^d(t) + j \frac{v_{\mathrm{up}(\ell)}^d(t) b_\ell}{2} \right\|^2 / v_{\mathrm{up}(\ell)}^d(t).$$~~

$$v_\ell^d(t) = v_{\mathrm{up}(\ell)}^d(t) - 2\Re\left\{ z_\ell^* \left(S_\ell^d(t) + j v_{\mathrm{up}(\ell)}^d(t) \frac{b_\ell}{2} \right) \right\} + \|z_\ell\|^2 f_\ell^d(t),$$

Replace by constants =correction terms

Main Idea of Proposed CoDistFlow

Voltage Constraints

$$v^d_{\mathrm{up}(1)}(t) = 1,\ \underline{v}^2 \le v^d_\ell(t) \le \overline{v}^2$$

Current Constraints

$$\Re\{i^d_\ell(t)\} = \frac{P^d_\ell(t)}{\sqrt{v^d_{\mathrm{up}(\ell)}(t)}}, \quad \Im\{i^d_\ell(t)\} = \frac{Q^d_\ell(t)}{\sqrt{v^d_{\mathrm{up}(\ell)}(t)}},$$

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$$\|i^d_\ell(t)\| \le \overline{I}_\ell, \|i^{b,d}_\ell(t)\| \le \overline{I}_\ell.$$

Replace by constants=correction terms

CoDistFlow Modules

1. **Improved DistFlow (iDF):** Solves a convex scenario-based multi-period AC OPF based on DistFlow with correction terms; Outputs the dispatch plan and the battery trajectories

2. **Load Flow (LF):** Updates the correction terms

Improved DistFlow (iDF) Module

Observes batteries state-of-energy

- Power prosumption forecast scenarios: $\boldsymbol{s}$
- Correction terms

iDF

- Dispatch plan: $\boldsymbol{S^{DP}}$
- Battery trajectories: $\boldsymbol{s_B}$
- Electrical state: **E'**

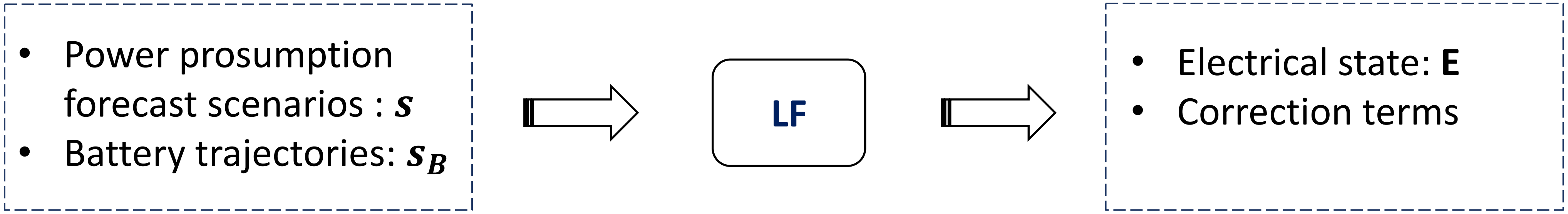
Solves a convex multi-period scenario-based AC OPF with

i. quadratic objective

ii. quadratic grid security constraints

iii. linear power flow equations - obtained by DistFlow with correction terms

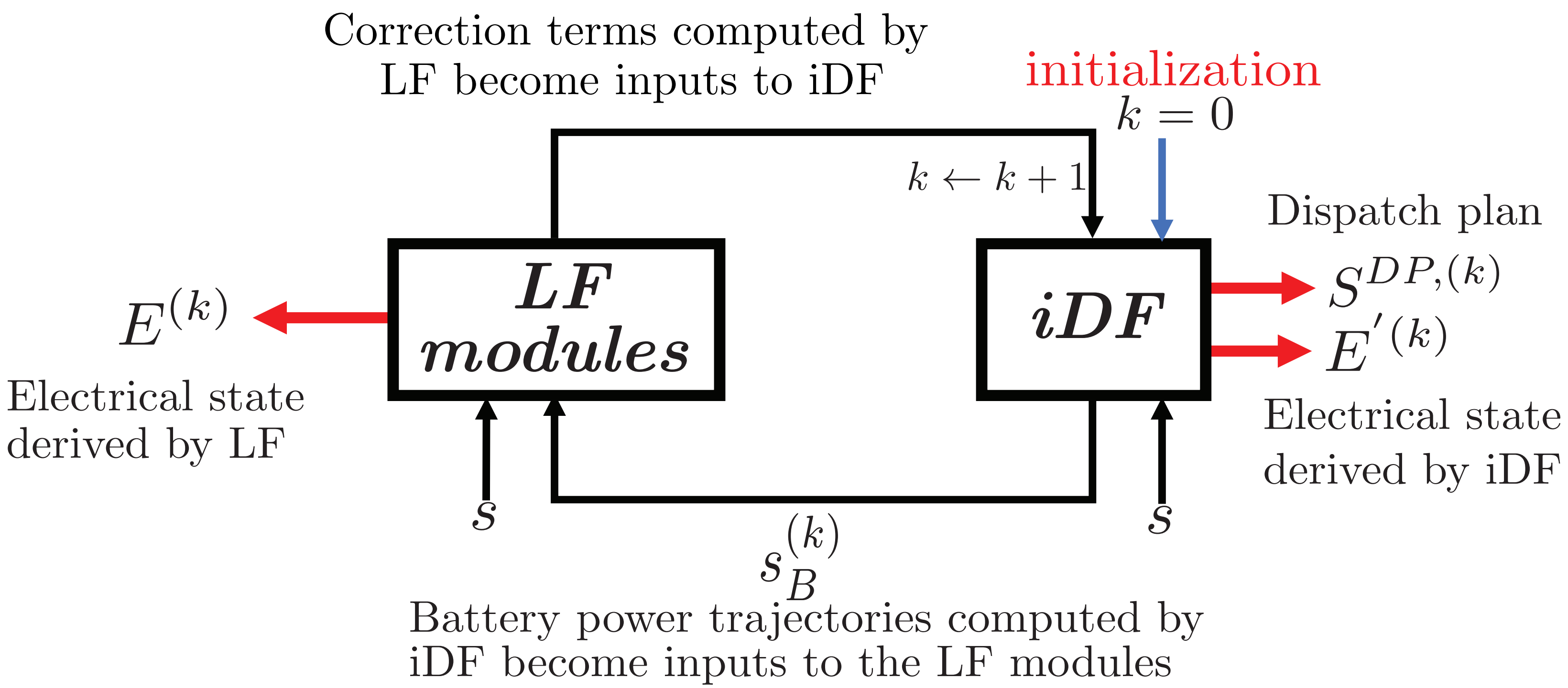
Load Flow (LF) Module



Solves a load flow for every scenario and time interval

- Active power correction terms <- active power losses
- Reactive power correction terms <- reactive power losses
- Voltage square magnitude correction terms <- square magnitude of the voltage drop on the line impedance
- Current correction terms <- voltage magnitudes

CoDistFlow Algorithm



Fixed point of CoDistFlow: when the exchanged quantities (battery trajectories, correction terms) do not change

✓ In practice convergence occurs after a few iterations

Correctness of CoDistFlow

- ✓ iDF uses external correction terms
- ✓ Not obvious the solution of iDF satisfies the exact AC power flow equations and grid constraints

Theorem: Under a mild condition on the admittance matrix, the electrical states by LF (E) and iDF (E’) coincide at a fixed point of CoDistFlow

Implications:

- ➢ Dispatch plan is computed based on electrical state E’ that satisfies the exact AC power flow equations

- ➢ The exact ampacity and voltage constraints are satisfied; their correction terms computed based on E’ are exact

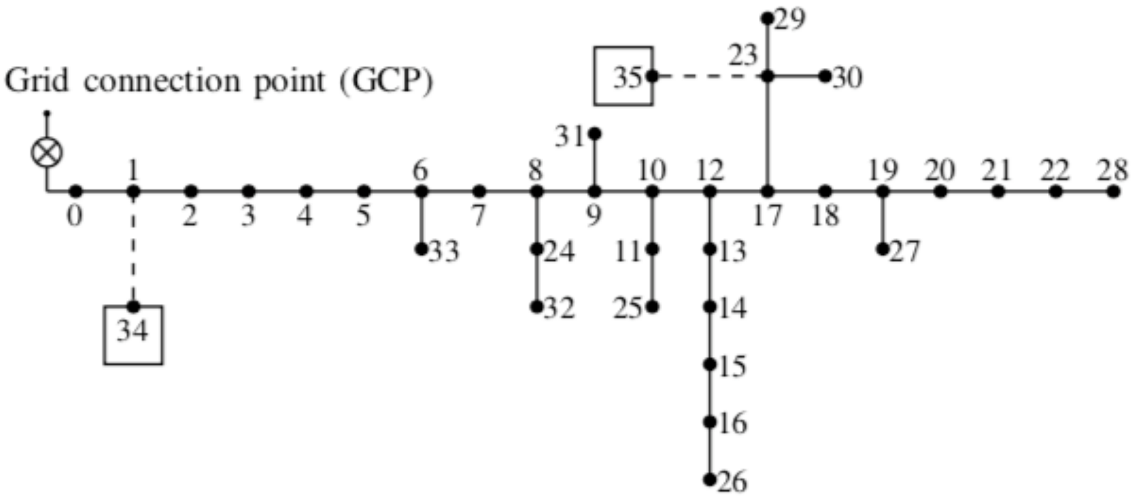
Comparison with Sequential Linearization of Constraints (SLC) with Sensitivity Coefficients

Iterative linearization of the power flow equations as well as the voltage and current equations around the optimal operating point updated at each iteration [Schmidli et al 2016]

✓ We compared the two schemes with numerical evaluations

CoDistFlow	SLC
Initialized with Simplified DistFlow	Hard to find a good initialization
Takes a few iterations to converge (typically 3-5)	Takes a large number of iterations to converge (more than 100) Often does not converge
Solution close to optimal: small dispatch plan tracking errors	Not optimal solution: large dispatch plan tracking errors

Evaluating CoDistFlow on a Real-Life Swiss Grid



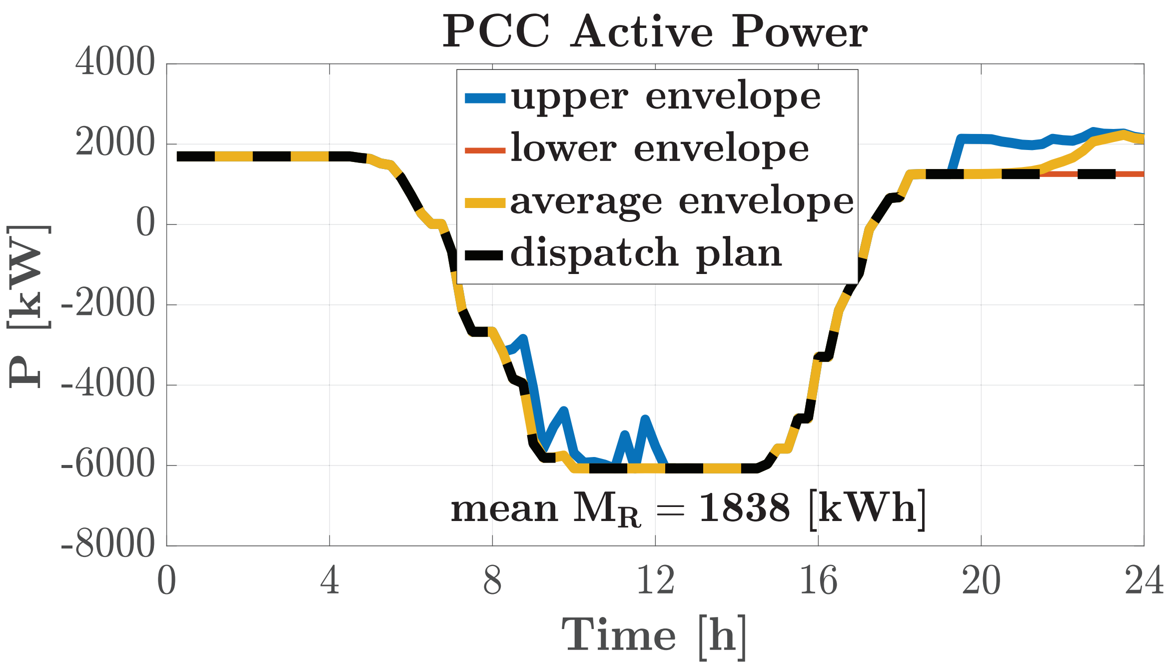
- 6 scenarios obtained after scenario-reduction
- 1 battery, 6 MWh, 6MW at node 23
- T=96, 15 min time intervals

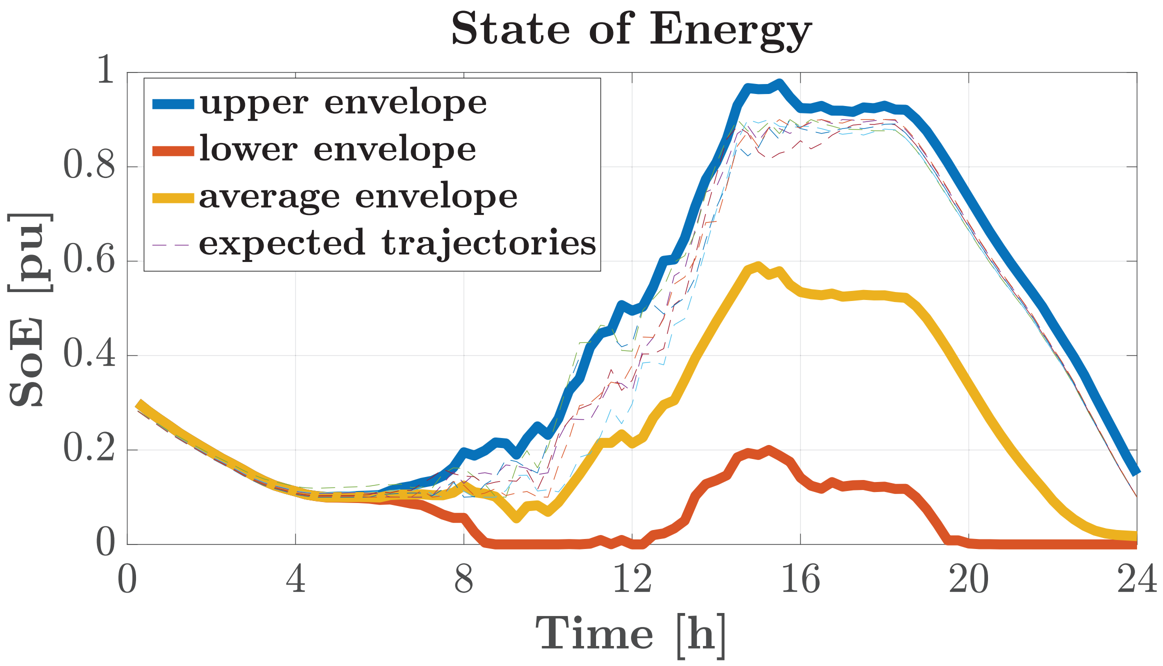
Dispatch plan energy error (DE_E): difference of dispatched and realized energy per 15 min
M_R**:** daily sum of absolute values of DE_E

We show that

- Accurately accounts for the grid and battery losses
- The security constraints are not violated

Simplified DistFlow Leads to Significant Dispatch Plan Tracking Errors

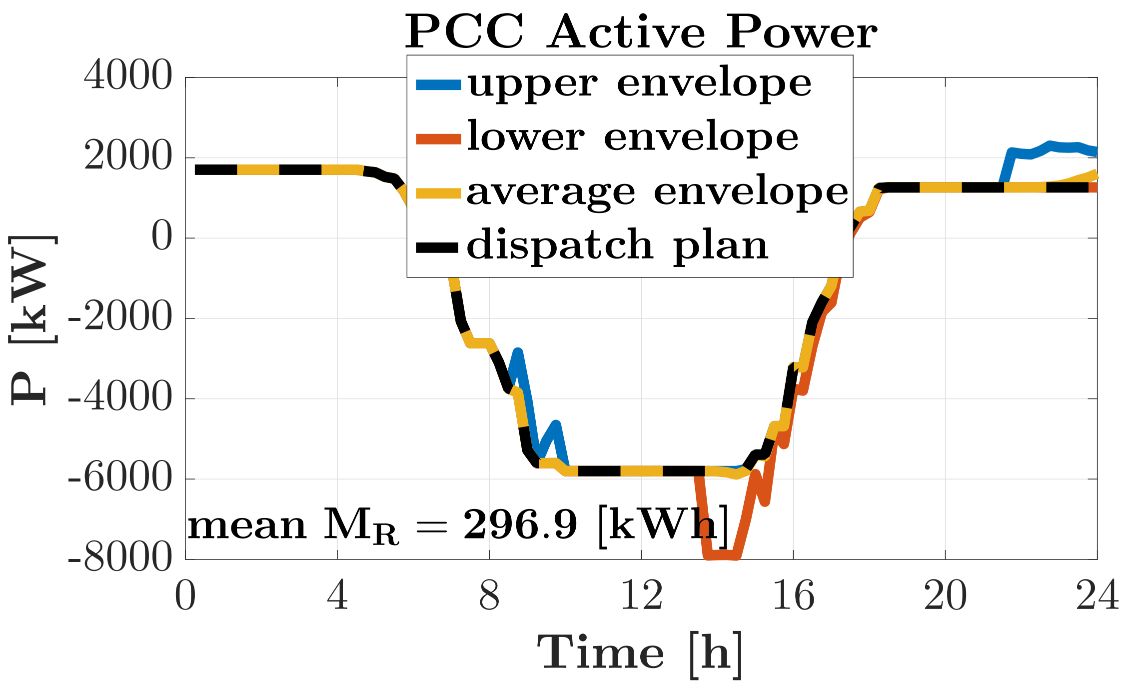




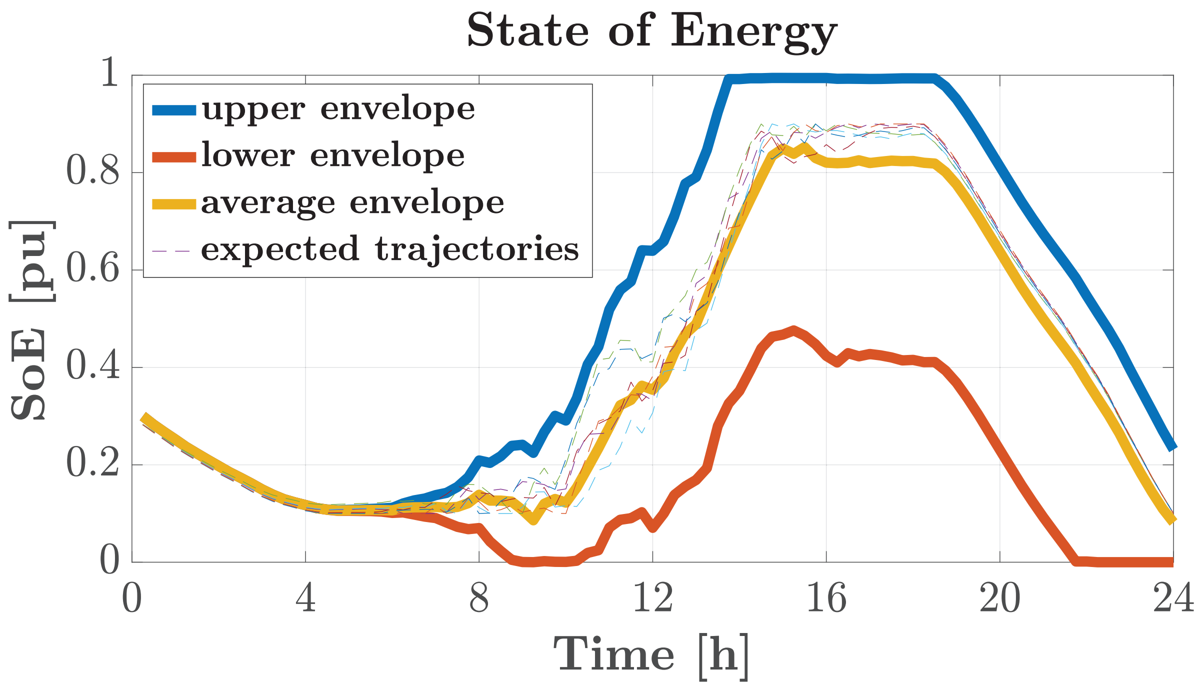
Failures from above: extra power is required due to not accounting for losses

The battery discharges more than expected

CoDistFlow Significantly Reduces the Dispatch Plan Tracking Errors



Failures due to uncertainty



The battery trajectories are close to the expected ones

CoDistFlow Satisfies All Grid Constraints for All Scenarios at Convergence

- Comparisons of CoDistFlow with a scheme that iteratively corrects the power flow equations but not the voltage/current constraints (‘Naive’)

IEEE 13-bus test feeder - Convergence after 3 iterations

Number of violated constraints per iteration

Iteration Index	1	2	3
CoDistFlow	2	0	0
Naive	16	36	40

IEEE 37-bus test feeder - Convergence after 5 iterations

Number of violated constraints per iteration

Iteration Index	1	2	3	4	5
CoDistFlow	87	66	4	7	0
Naive	87	183	103	73	78

Intra–day Re-dispatch

- ❑ Update the dispatch plan for next time horizons using more recent information (observed state-of-energy of batteries, new forecasts)

Why Re-dispatching?

✓ When tracking a day-ahead dispatch plan during operation…

…there may exist **better forecasts** for the remaining part of the day

…the realization might have not been close to the predicted day-ahead scenarios, leading to depleted flexibility in the batteries

Proposed Approach for Intra-day Re-dispatch: RHC over CoDistFlow (1)

Our approach consists of

- **Receding Horizon Control (RHC)** for re-dispatching
- **CoDistFlow** solves the multi-period scenario-based AC OPF at each RHC round every $\Delta\tau$

Most complete re-dispatch scheme in the literature that …

1. … accounts for multiple DERs and batteries
2. … handles the prosumption uncertainties
3. … provides a solution that satisfies the exact AC power flow equations and grid constraints
4. … accounts for an accurate model of battery losses and constraints

Proposed Approach for Intra-day Re-dispatch: RHC over CoDistFlow (2)

✓ Every $\Delta\tau$ CoDistFlow is applied:

- updated prosumption forecast scenarios
- observed state-of-energy of the batteries
- fixed dispatch plan for the first time steps

→ **CoDistFlow** →

- dispatch plan for the next time horizon
- corresponding battery trajectories

Evaluation Measures

- **Dispatch plan power error (DP_E):** difference of dispatched and realized power at PCC per real-time interval

- **Dispatch plan energy error (DE_E):** difference of dispatched and realized energy per hour

- **Cost per hour (DP_{Cost}):** up and down-regulation costs and costs for frequency containment reserves

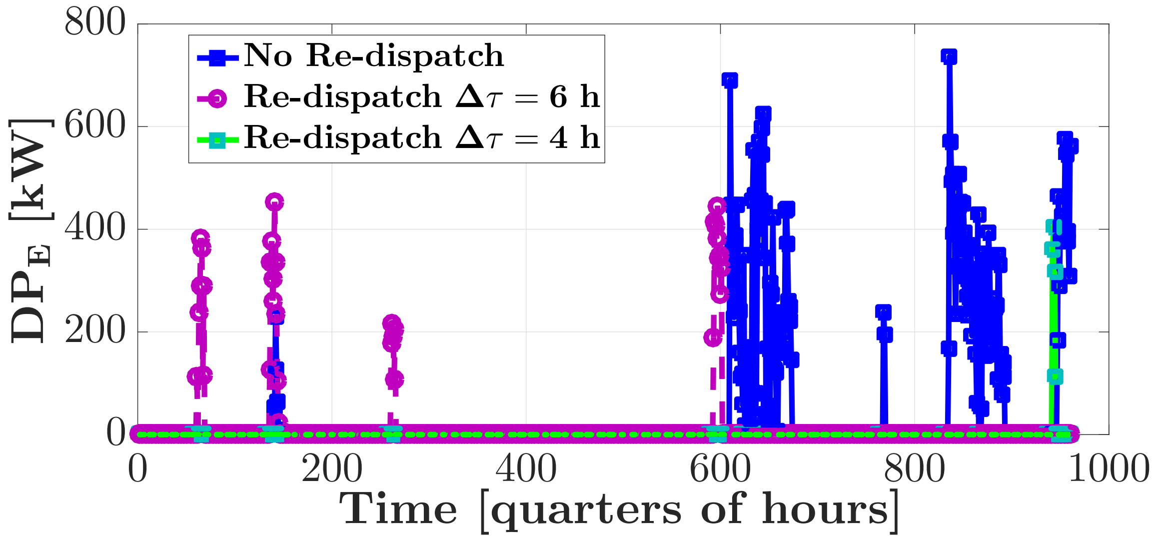
$$DP_{Cost}(t_h) = \left|\max\{\chi^+ \cdot DE_E(t_h), \chi^- \cdot DE_E(t_h)\}\right| + \Delta t_r \cdot \chi^C \sum_{t_r = t_h/\Delta t_r}^{t_h/\Delta t_r + 1/\Delta t_r} |DP_E(t_r)| \, .$$

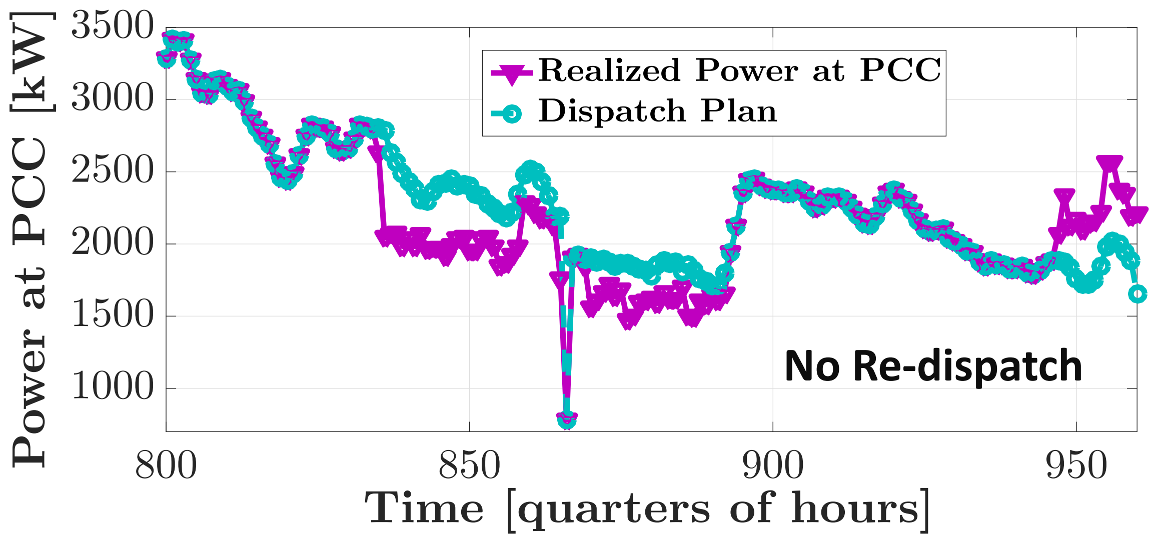
Prices from Fingrid:

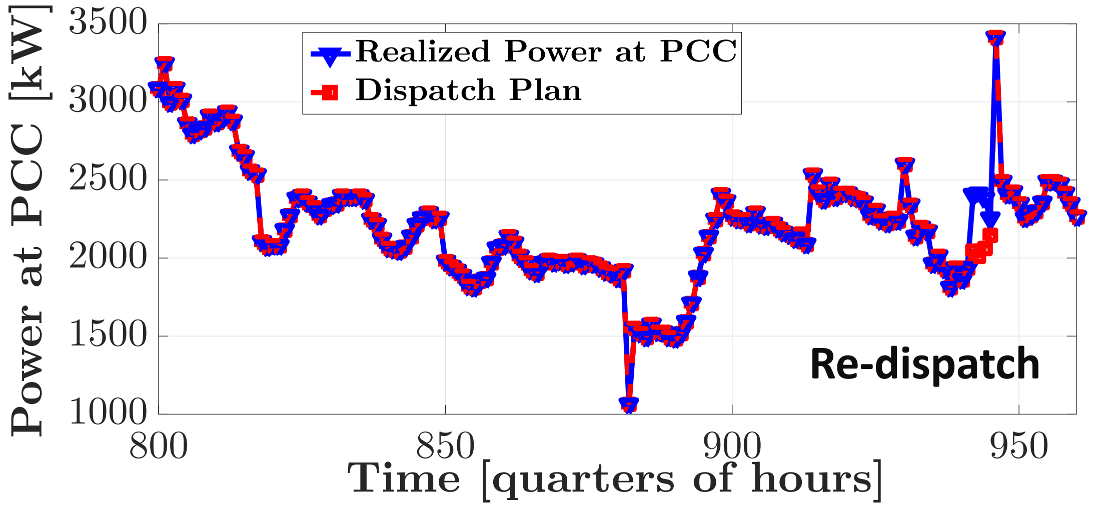
$\chi^C = 18.10$ €$/MWh$
$\chi^+ = 56.22$ €$/MWh$
$\chi^- = 45.97$ €$/MWh$

- **Averages of Daily Aggregates:** $CDE_E = \frac{24}{\overline{T}} \sum_{t_h=0}^{\overline{T}-1} |DE_E(t_h)| \, , \; CDP_{Cost} = \frac{24}{\overline{T}} \sum_{t_h=0}^{\overline{T}-1} DP_{Cost}(t_h).$

Re-dispatch is Efficient for Reducing Dispatch Plan Tracking Errors and Costs (1)







- 80 scenarios created from real data of a real-life Swiss grid
- Two batteries, 3MWh each
- T=96

Re-dispatch is Efficient for Reducing Dispatch Plan Tracking Errors and Costs (2)

TABLE I: Comparisons of CDE_E, DE_E [kWh], CDP_{Cost} [€].

Scheme	CDE_E	98% perc. DE_E	CDP_{Cost}
No Re-dispatch	978.8	400.91	68.6
$\Delta\tau = 6$ h	209.53	249.51	15.57
$\Delta\tau = 4$ h	29.95	0.0325	2.22
$\Delta\tau = 2$ h	0.15	0.0242	0.0095

TABLE IV: CoDistFlow vs. No iterations. CDE_E in kWh, CDP_{Cost} in €.

Scheme	CDE_E Re-disp.	CDP_{Cost} Re-disp.	CDE_E No Re-disp.	CDP_{Cost} No Re-disp.
CoDistFlow	209.53	15.57	978.8	68.6
(A)	270.11	20.07	1031.3	72.9
(B)	270.83	20.13	1032.3	72.98

- ✓ If re-dispatching every 6 h, DE_E and cost reduce by 80%, if re-dispatching every 4 h more than 30× and if re-dispatching every 2 h they eliminate to zero
- ✓ A small city (e.g., Lausanne) has 50 – 60 feeders: if No Re-dispatch yearly cost may reach **1,234,800 – 1,481,760 € >> 39, 960 – 47, 952 €** if re-dispatching with Δτ=4 h
- ✓ Not accounting for the losses may increase the yearly cost by **108,000 €**

Conclusive Remarks

- CoDistFlow and RHC over CoDistFlow are proposed towards a stable and predictable global grid

- CoDistFlow solves a scenario-based multi-period AC OPF
  - ✓ a fixed point satisfies exact power flow equations and exact operational constraints
  - ✓ incorporates a realistic battery modeling at no extra cost in complexity

- CoDistFlow and RHC over CodistFlow are key
  - ✓ Not considering the grid/battery losses -> increased dispatch plan tracking errors
  - ✓ Not iteratively correcting the grid constraints -> solutions do not satisfy the exact grid constraints
  - ✓ Not re-dispatching intra-day -> dispatch plan tracking cost may become considerably large

References

Thank you!

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