Chemical ENGINEERING

CAPD CENTER

Decomposition Algorithms for Optimal Power Systems Infrastructure Planning

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Motivation Optimization Models Power Systems in Process Systems Engineering

- *Applications in Demand Side Management increasingly important*
- *Impact of Shale Gas on Energy Mix for Power Generation*

Goal: Models Long-Term Planning Power Systems
a) Deterministic: Mixed-integer linear programming (MILP)
b) Uncertainty: Multi-stage Stochastic Integer Programming (mSIP)

Challenge: Develop Computationally Efficient Methods
a) Nested Benders Decomposition Method for deterministic model
b) Stochastic Nested Decomposition Method for stochastic

Application: ERCOT region (Texas) in US

Mathematical Programming

MINLP: *Mixed-integer nonlinear programming*

$$\min Z = f(x,y)$$
$$s.t. \quad h(x,y) = 0$$
$$g(x,y) \le 0$$
$$x \in R^n, \quad y \in \{0,1\}^m$$

$$f(x): R^n \to R^1, h(x): R^n \to R^m, g(x): R^n \to R^q$$

MILP: *f, h, g* linear

LP: *f, h, g* linear, only *x*

NLP: *f, h, g* nonlinear, only *x*

MILP

$$\min Z = a^T y + b^T x \qquad \textit{Objective function}$$

$$st \qquad Ay + Bx \leq d \qquad \textit{Constraints}$$

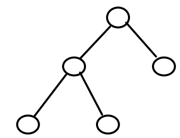
$$y \in \{0,1\}^m,\ x \geq 0$$

Theory for Convexification

Lovacz & Schrijver (1989), Sherali & Adams (1990),

Balas, Ceria, Cornuejols (1993)

Branch and Bound ***Beale (1958), Balas (1962), Dakin (1965)***



Cutting planes ***Gomory (1959), Balas et al (1993)***

Branch and cut ***Johnson, Nemhauser & Savelsbergh (2000)***

LP (simplex) based

"Good" formulation crucial! => *Small LP relaxation gap*

Drawback: exponential complexity

Major codes: CPLEX, GUROBI, XPRESS

SCIP (non-commercial)

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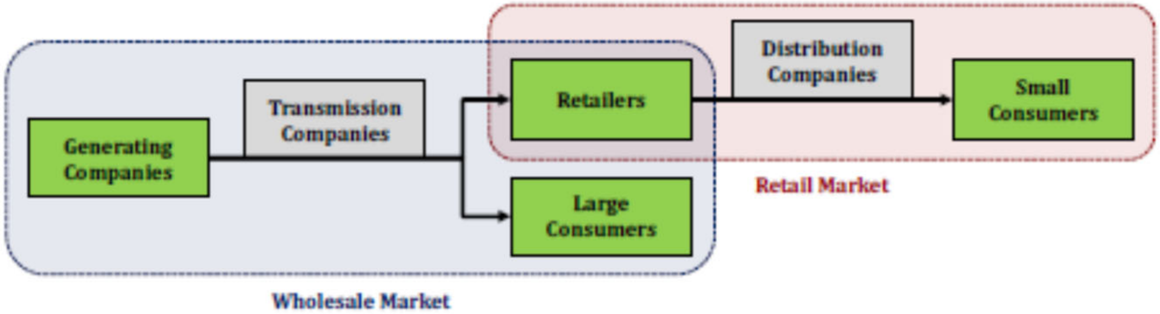
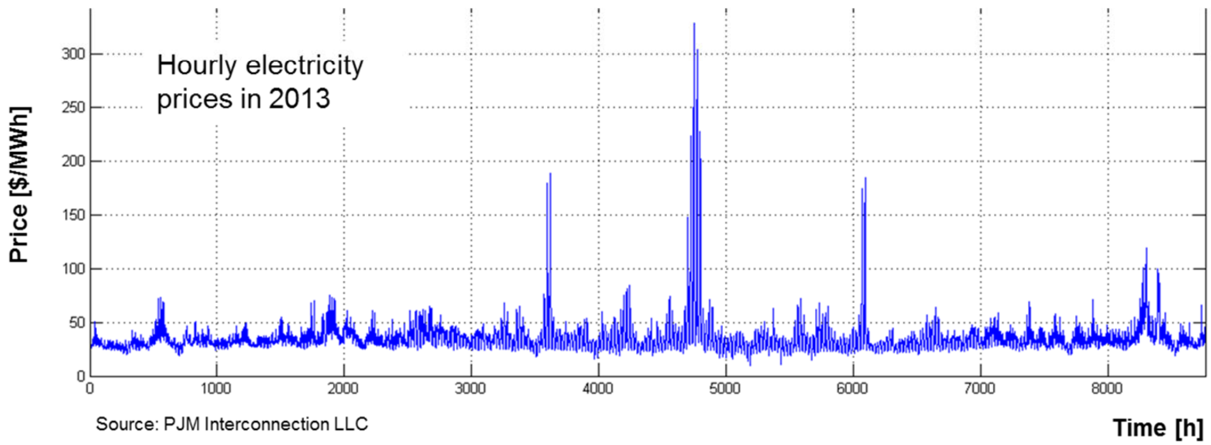
The main driver for Demand Side Management is time-sensitive pricing





Source: PJM Interconnection LLC

- Electricity prices change on an **hourly** basis (more frequently in the real-time market)
- Challenge, but also **opportunity for electricity consumers**

Chemical plants are large electricity consumers → high potential cost savings

2

Optimal Multi-scale Capacity Planning under Hourly Varying Electricity Prices

Carnegie Mellon

Mitra, Grossmann, Pinto, Arora (2012)

Given:

- Power-intensive plant
- Products $g \in G$ (Storable and Nonstorable)
- Product demands d_g^t for season $t \in T$
- Seasonal electricity prices on an hourly basis $e^{t,h}$, $t \in T$, $h \in H$
- Upgrade options $u \in U$ of existing equipment
- New equipment options $n \in N$
- Additional storage tanks $st \in ST$

Determine:

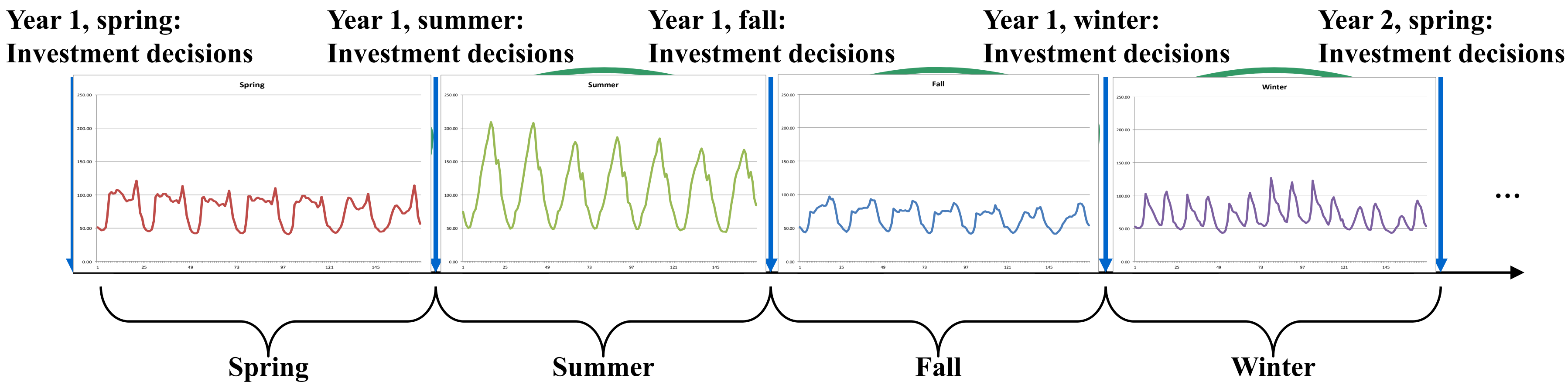
- Production levels $\Pr_g^{t,h}$
- Mode of operation $\tilde{y}_{m,o}^{t,h}$, $y_m^{t,h}$
- Sales $S_g^{t,h}$
- Inventory levels $INV_g^{t,h}$

for each season on an hourly basis

- Upgrades for equipment $VU_{m,u}^t$
- Purchase of new equipm. VN_n^t
- Purchase of new tanks $VS_{st,g}^t$

With minimum investment and operating costs

Incorporating design decisions: seasonal variations drive the development of a seasonal model



- Horizon: 5-15 **years**, each year has 4 **periods** (spring, summer, fall, winter)
- Each period is represented by **one week on an hourly basis**
- Each representative week is repeated in a **cyclic** manner (**13** weeks reduced to **1** week) (8736 hr vs. 672 hr)
- Design decisions are modeled by **discrete equipment sizes**

MILP model for multi-scale capacity planning

$$\min \quad OBJ = \sum_t (Cost^t_{ops} + Cost^t_{invest}) \qquad (37)$$

Operational
Disjunction over the modes that describe the feasible region

Operational
Logic constraints for transitions (e.g. minimum uptime/downtime)

Operational
Mass balances for inventory, constraints related to demand

Strategic
Additional storage

Strategic
Additional equipment

Idea: additional modes for which variables are controlled by the corresponding binary investment

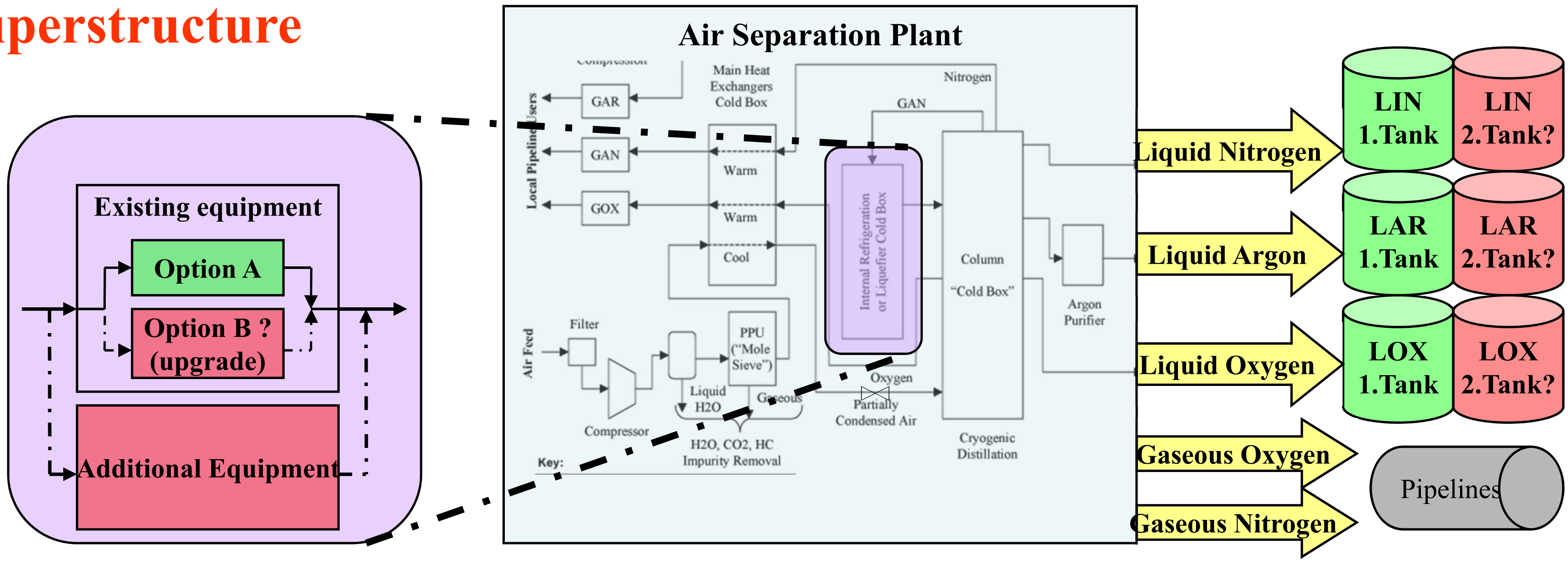
Strategic
Equipment replacement

Idea: the corresponding mode has an alternative feasible region

Terms for the objective functi

Retrofitting an air separation plant

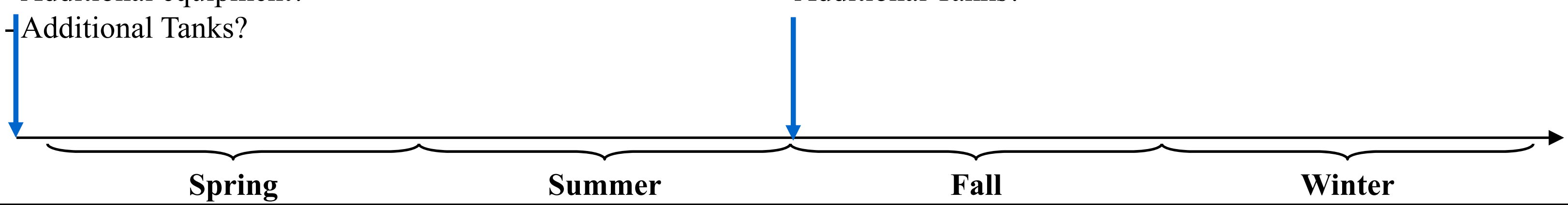
Superstructure



Time

Spring - Investment decisions: (yes/no)
- Option B for existing equipment?
- Additional equipment?
- Additional Tanks?

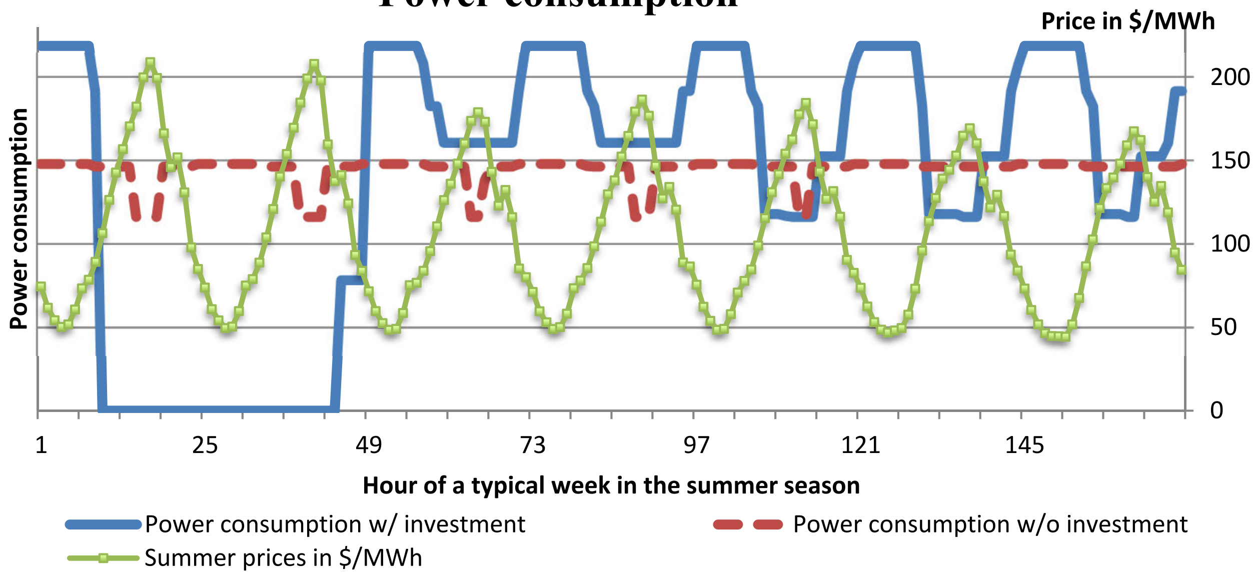
Fall - Investment decisions: (yes/no)
- Option B for existing equipment?
- Additional equipment?
- Additional Tanks?



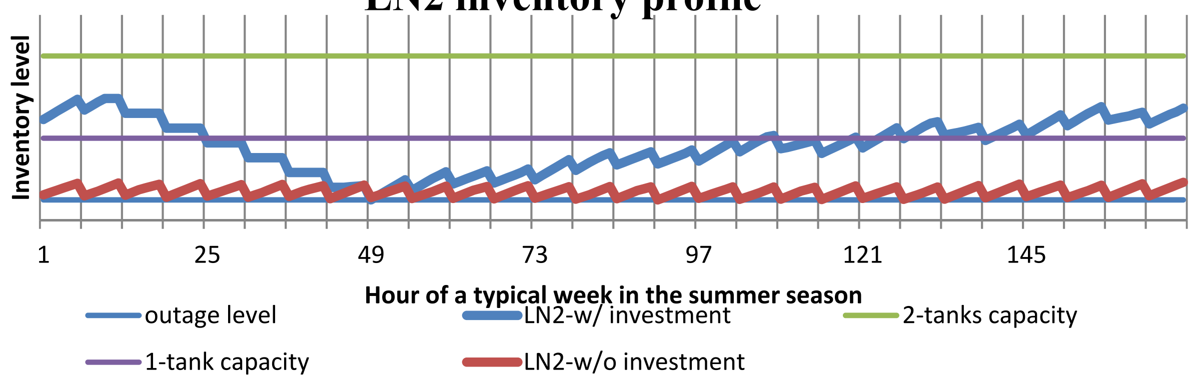
- The resulting MILP has **191,861 constraints** and **161,293 variables (18,826 binary.)**
- Solution time: **38.5 minutes** (GAMS 23.6.2, GUROBI 4.0.0, Intel i7 (2.93GHz) with 4GB RAM

Investments increase flexibility help realizing savings.

Power consumption



LN2 inventory profile



Remarks on case study

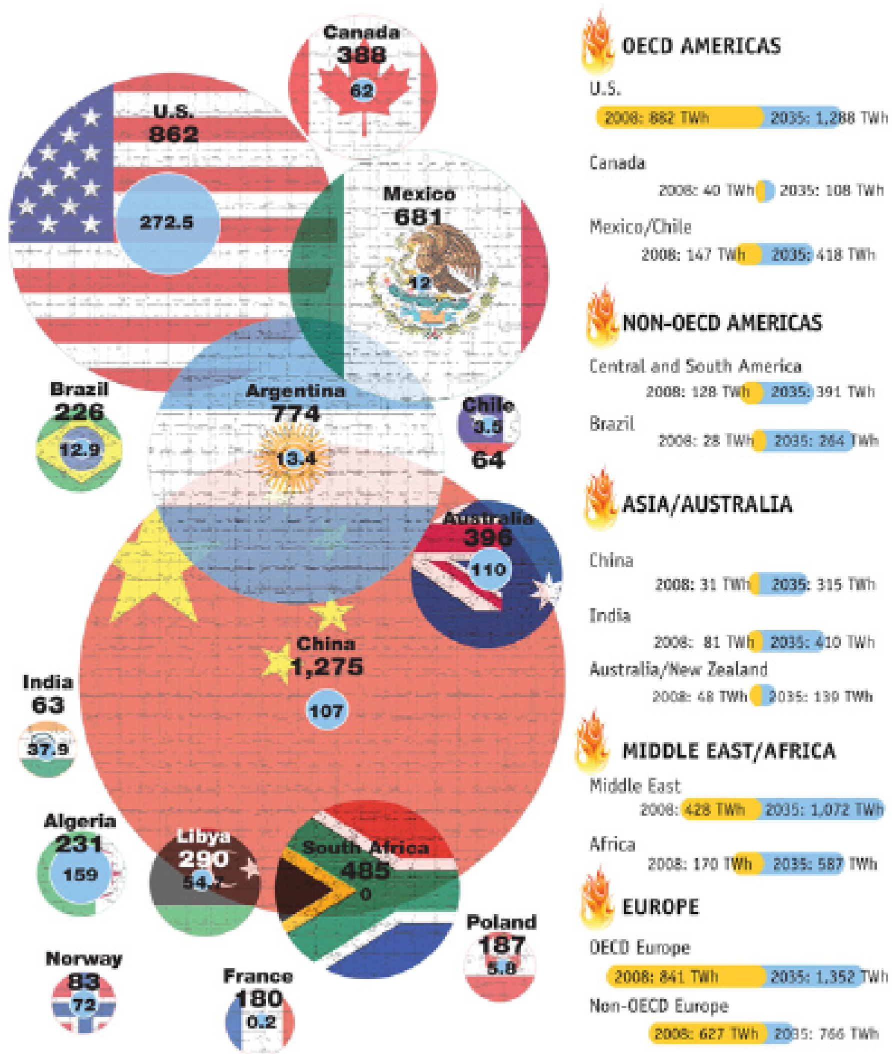
- **Annualized costs: $5,700k/yr**
- **Annualized savings: $400k/yr**

- Buy new liquefier in the first time period (annualized investment costs: $300k/a)
- Buy additional LN2 storage tank ($25k/a)
- Don't upgrade existing equipment ($200k/a) equipment: 97%.

Zhang, Q. and I.E. Grossmann, "Enterprise-wide Optimization for industrial demand side management: Fundamentals, advances and perspectives," *Chemical Engineering Research and Design* 116, 114-131 (2016).

Source: CAPD analysis; Mitra, S., I.E. Grossmann, J.M. Pinto and Nikhil Arora, "Integration of strategic and operational decision- making for continuous power-intensive processes", submitted to ESCAPE, London, Juni 2012

Shale Gas Reserves in World



units = trillion cubic feet

Larger circles = technical reserves
Smaller circles = potential reserves

yellow = current useage
blue = estimate for 2035

Sonal Patel, "THE BIG PICTURE: A Shale Gas Revolution", Power, June 2012

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Few facts about Shale Gas in US

Price of Natural Gas $12.69/MMBtu 6/2008 vs $1.97/MMBtu 4/2012
***Latest: $2.36/MMBtu* 5/2019**

Price Ethane $1.38/gal 6/2008 $0.85/gal 11/2012 $0.58/gal 5/2019

Perspectives Article: Jeff Siirola

The Impact of Shale Gas in the Chemical Industry
***AIChE Journal,* Volume 60, pp 810–819 (2014)**

CO_2 emissions from power plants decreased by 19.7% since 2011

U.S. Energy self-sufficiency by 2020

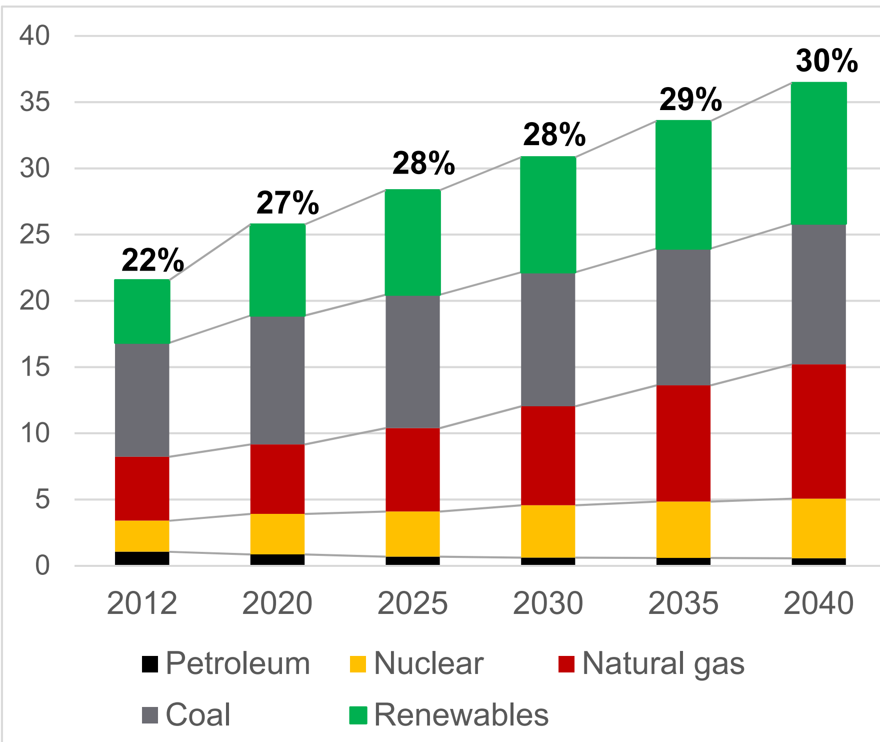
Planning of Electric Power Infrastructures

Electricity mix gradually shifts to lower-carbon options

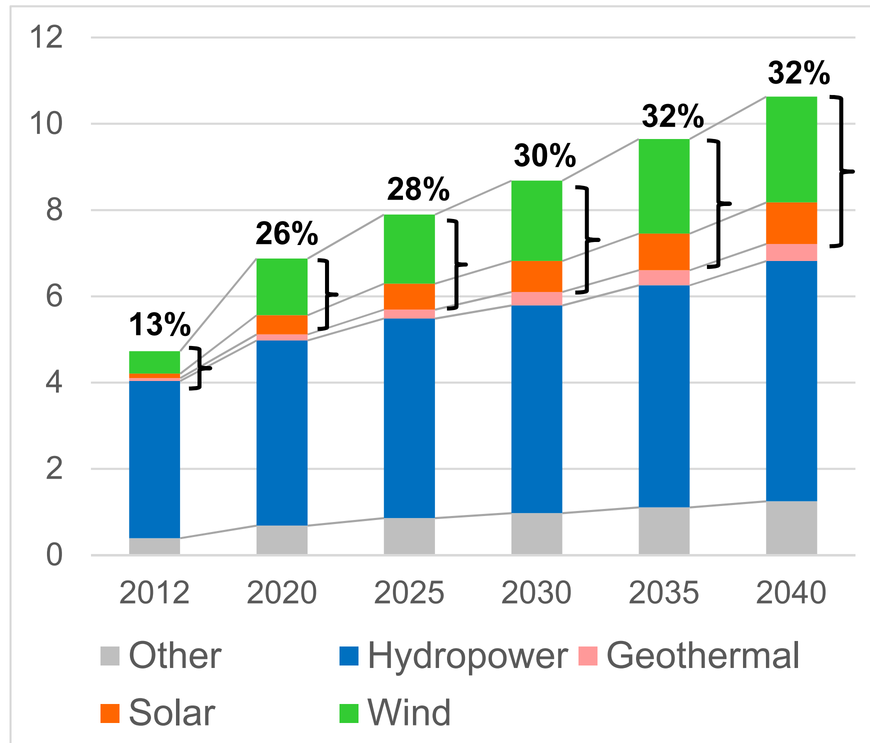
Cristiana Lara, Ignacio Grossmann (2017)

NETL ExxonMobil

World net electricity generation by fuel, 2012-40 (trillion kWh)[1]



World net electricity generation from renewable power by fuel, 2012-40 (trillion kWh) [1]

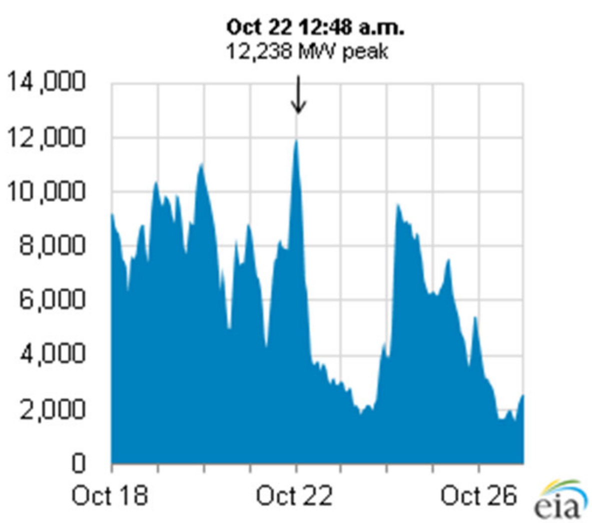


[1]EIA, *Annual Energy Outlook 2016*
[2]IRENA (international Renewable Energy Agency), *The Power to Change: Solar and Wind Cost Reduction Potential to 2025*, 2016

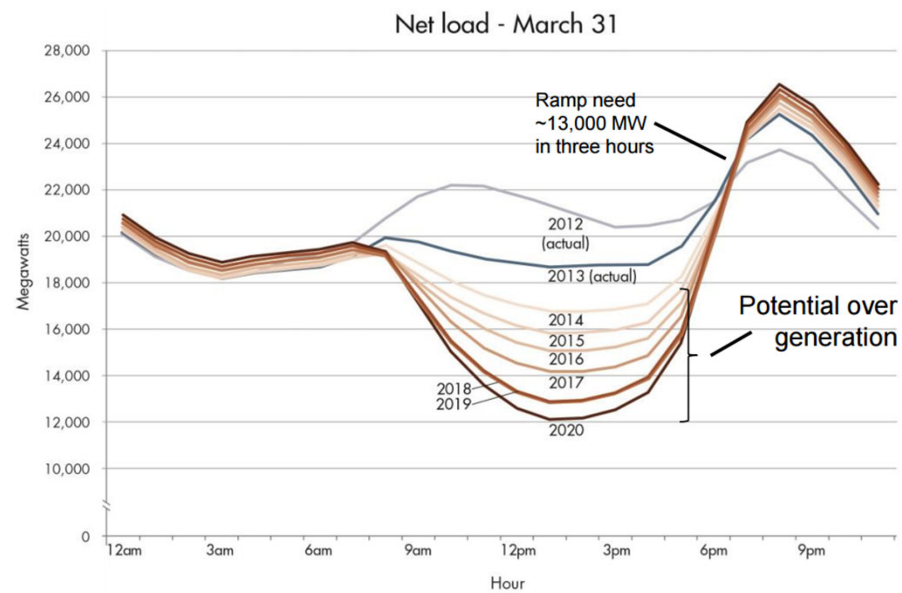
Challenge

High variability in the renewables capacity factor

Hourly generation in the ERCOT(Texas) electric, Oct 18-26, 2015 (MW) grid



CAISO duck curve - Net load in March 31



- Increasing contribution of intermittent renewable power generation in the grid makes it important to include **operational details** at the **hourly** (or sub-hourly) **level** in long term planning models to capture their variability

- U.S. Energy Information Administration, based on the Electric Reliability Council of Texas (ERCOT)
- California ISO (CAISO)

Problem Statement

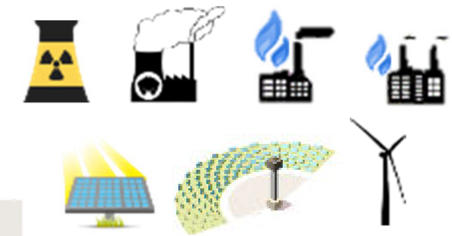
Given a region with:

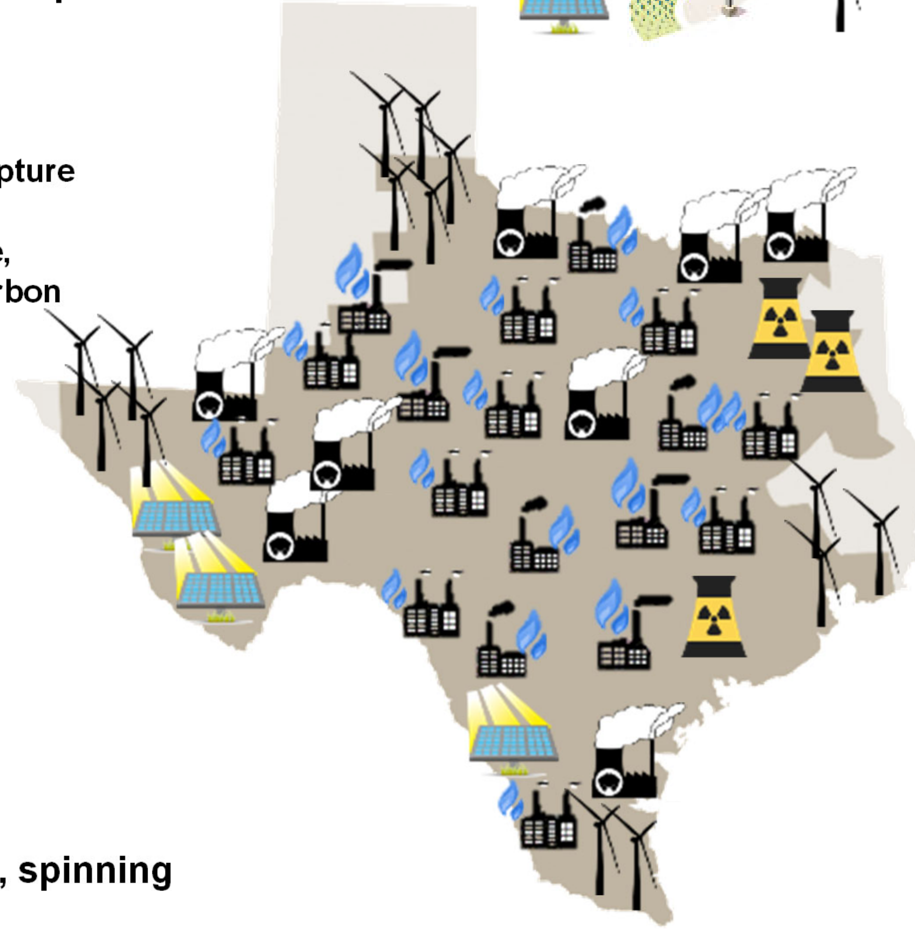
A set of existing and potential generators with the respective

- **generation technology***

if existing:	**if potential:**
<u>nuclear</u>: steam turbine	**<u>nuclear</u>: steam turbine**
<u>coal</u>: steam turbine	**<u>coal</u>: IGCC w/ or w/o carbon capture**
<u>natural gas</u>:	**<u>natural gas</u>:**
▪ **steam turbine,**	▪ **gas-fired combustion turbine,**
▪ **gas-fired combustion turbine,**	▪ **combined cycle w/ or w/o carbon capture**
▪ **and combined cycle**	**<u>solar</u>:**
<u>solar</u>: photo-voltaic	▪ **photo-voltaic**
<u>wind</u> turbines	▪ **concentrated solar panel**
	<u>wind</u> turbines

- **location, if applicable**
- **nameplate capacity**
- **age and expected lifetime**
- **CO_2 emission**
- **operating costs**
- **investment cost, if applicable**
- **operating data**
  - **if <u>thermal</u>: ramping rates, operating limits, spinning and quick-start maximum reserve**
  - **If <u>renewable</u>: capacity factor**





*** Assume no hydropower**

Problem Statement

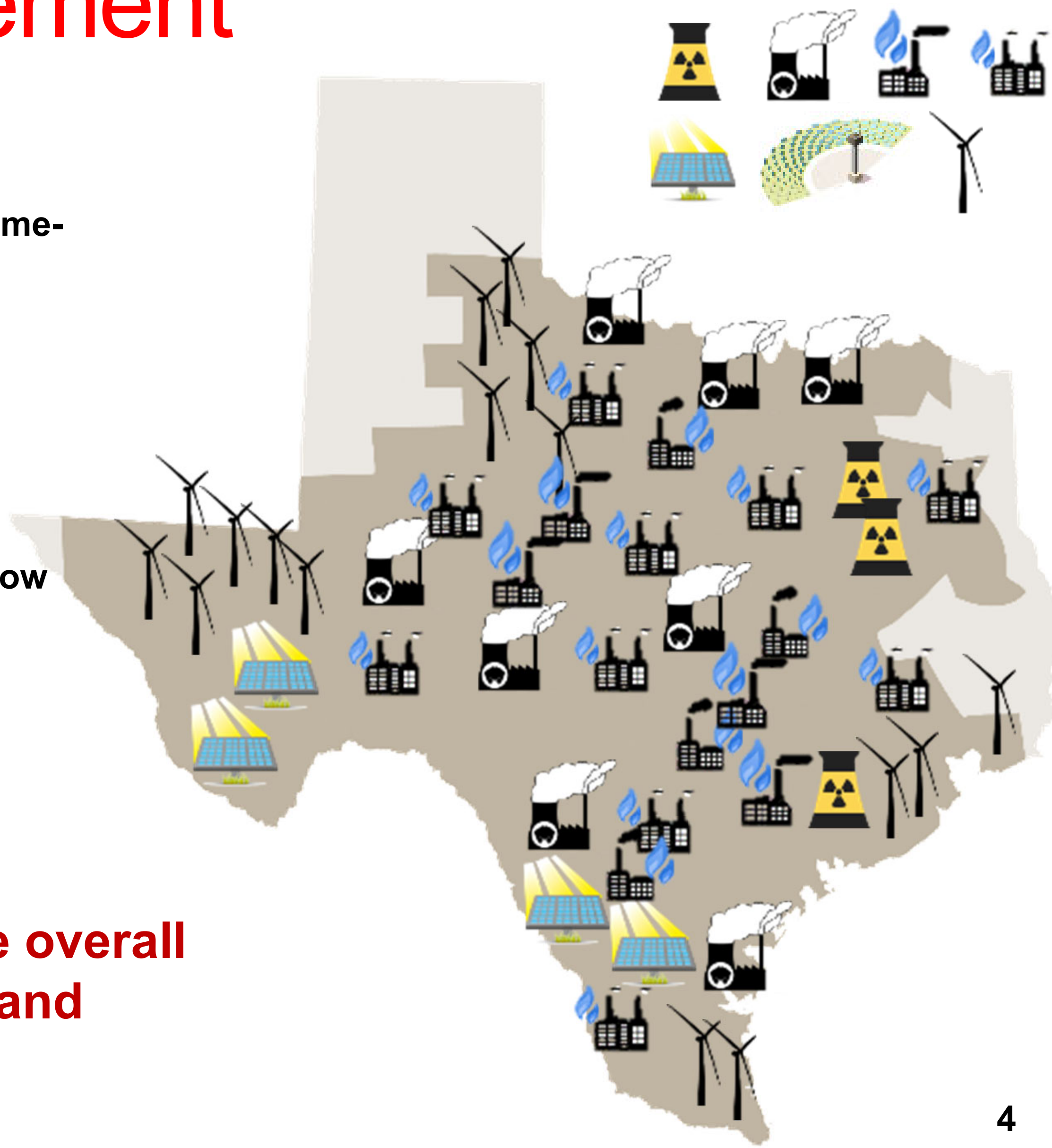
Given:

- **Projected load demand over the time-horizon at each location**
- **Distance between locations**
- **Transmission loss per mile**

Find:

- **When, where, which type and in how many generators to invest**
- **When to retire the generators**
- **Whether or not to extend their lifetime**
- **Power flow between locations**
- **Detailed operating schedule**

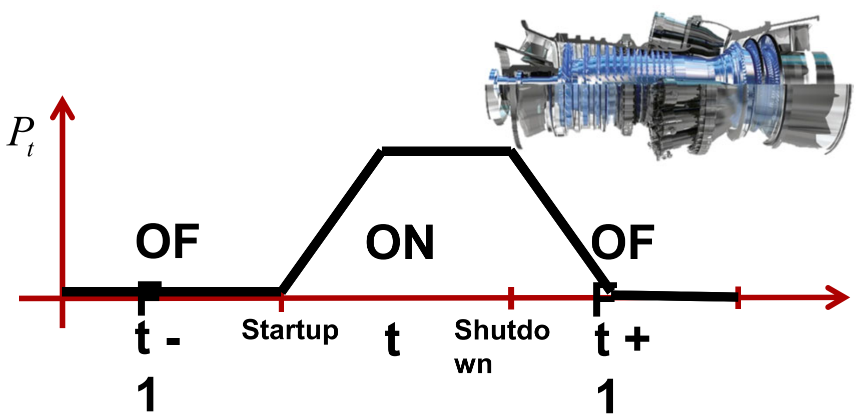
in order to minimize the overall operating, investment, and environmental costs



Unit commitment

Optimization unit commitment

- "Unit commitment (UC) is an optimization problem used to determine the operation schedule of the generating units at every hour interval with varying loads under different constraints and environments."



Why to include unit commitment in a planning model?

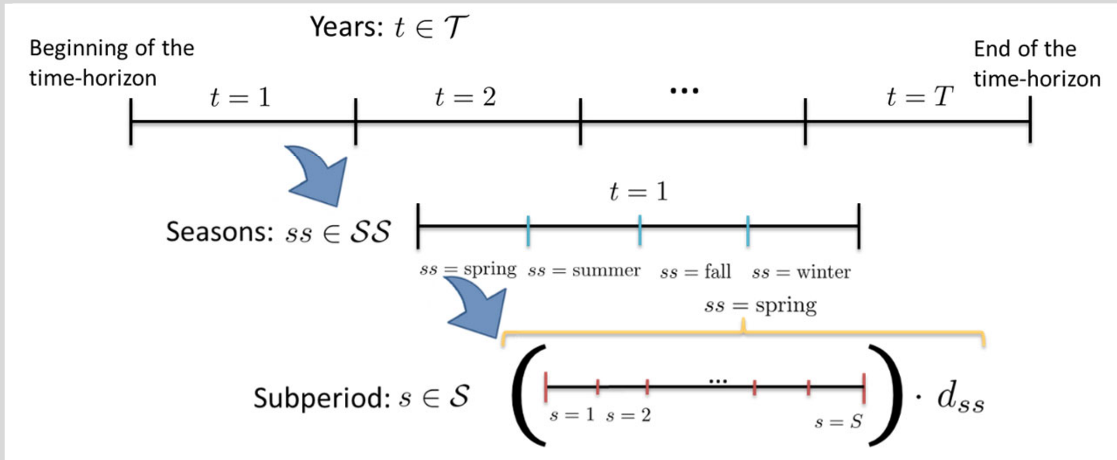
- Accounts for the need of fast ramping rates in a system with high renewable penetration.
- Helps ensuring flexibility and robustness of the system.
- Accounts for startup cost in the total cost.

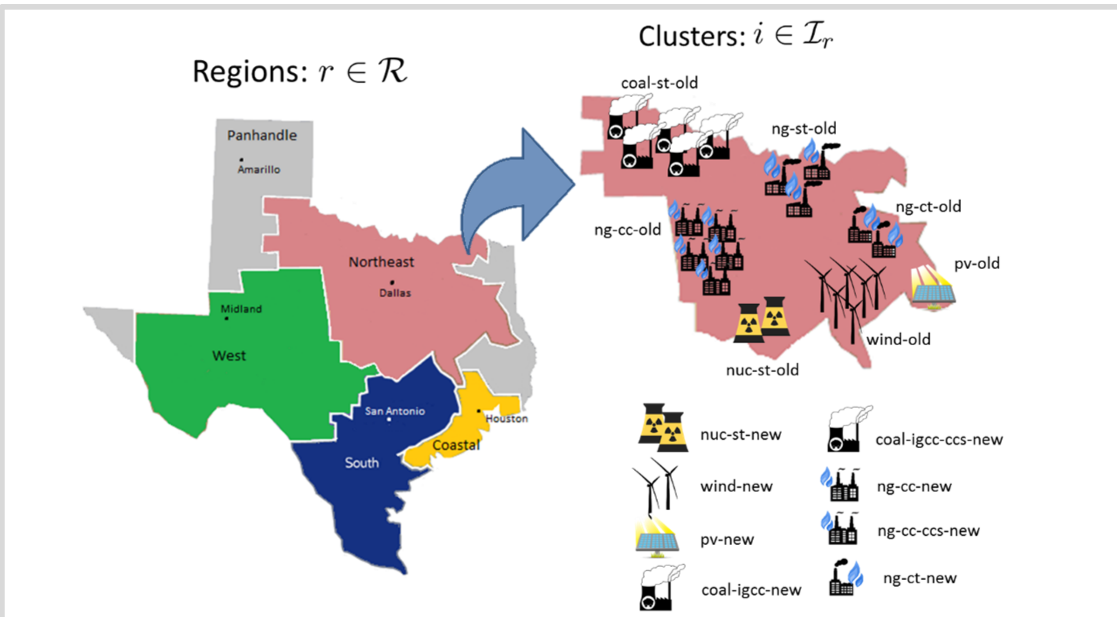
Very important for systems with **increasing share of renewables**

Modeling Strategies

To tackle the multi-scale aspect and reduce the size of the model

- Time scale approach:
  - 1 representative cycle per season (e.g., a day or a week) with hourly level information
- Region and cluster representation
  - Area represented by a few zones
  - Potential locations are the midpoint in each zone
  - Clustering of generators*
- Transmission representation
  - Flow in each line is determined by the energy balance between each region *r*.
  - This approximation ignores *Kirchhoff's Circuit Law*





*Palmintier, B.S., Webster, M.D., *Heterogeneous unit clustering for efficient operational flexibility modeling,* 2014

MILP Model

Continuous variables:
- Power output at sub-period *s*
- Curtailment generation slack at *s*
- Power flow between regions at *s*
- Deficit from renewable quota at *t*
- Spinning reserve at *s*
- Quick-start reserve at *s*

Discrete variables:
- no. of generators installed at period *t*
- no. of generators built at *t*
- no. of generators retired at *t*
- no. of generators with life extended at *t*
- no. of generators ON at sub-period *s*
- no. of generators starting up at *s*
- no. of generators shutting down at *s*

Summary of constraints:

- **Energy balance:** ensures that the sum of instantaneous power generated at region *r* plus the net power flow being sent to this region equal the load demand plus a slack for curtailment.
- **Capacity factor:** limits the generation of renewable generators to be less than or equal to a given fraction of the capacity in each hour.
- **Unit commitment constraints:** compute the startup and shutdown, operating limits and ramping rates for thermal generators.
- **Operating reserve constraints :** determine the maximum contribution per thermal generator for spinning and quick-start reserves, and the minimum total operating reserves.
- **Investment constraints :** ensure that the planning reserve and renewable energy contribution requirements are satisfied, and limit the yearly installation per generation type.
- **Constraints of number of generators:** define the number of generators that are operational, built, retired, and have their life extended at each time period *t*.

MILP Model

Objective function:

Minimization of the **net present cost** over the planning horizon comprising:

- **Operating** and **startup** costs;
- Cost of **investments** in **new generators** and to **extend the life** of generators that achieved their expected lifetime
- **Fuel** consumption;
- **Environmental** costs (carbon tax for CO_2 emission and penalty for not meeting the minimum renewable annual energy production requirement)

MILP concise representation:

$$\mathscr{P} : \Phi = \min_{x_t, y_t} \sum_{t \in \mathcal{T}} f_t(x_t, y_t)$$

$$\text{s.t.} \quad (x_t, y_t) \in \mathcal{X}$$

Objective function

Feasible region defined by the constraints

Linking variables (mixed-integer)

Non-linking variables (mixed-integer)

Linking variable, x_t , represents:

- Number of **operational generators** in cluster *i* of region *r* at year *t*.
- Number of **generators built** in cluster *i* of region *r* at year *t*.

We extend Nested Benders Decomposition for a class of multiperiod MILPs

- This algorithm decomposes the problem by time period, which in this case is **by year**.
- It consists of **Forward** and **Backward Passes**.
- The **Forward Pass** solves the problem in myopic fashion (1 year time horizon), and yields an **upper bound.**
- The **Backward Pass** projects the problem onto the subspace of the linking variables by adding cuts, and yields a **lower bound.**

MILP subproblem concise representation:

$$\mathscr{P}_{t,k} : \Phi_{t,k}(\hat{x}_{t-1,k}, \phi_{t,k}) = \min_{x_t, y_t, z_t} \quad f_t(x_t, y_t) + \phi_{t,k}(\hat{x}_{t,k})$$

$$\text{s.t.} \quad z_t = \hat{x}_{t-1,k} \qquad \leftarrow \mu_{t,k} \in \mathbb{R}^n$$

$$(x_t, y_t, z_t) \in \mathcal{X}_t$$

Cost-to-go function

State of the system at start of *t* (parameter)

State variables (mixed-integer)

Local variables (mixed-integer)

Duplicated state variables (continuous)

where the cost-to-go function is defined as:

$$\phi_{t,k}(\hat{x}_{t,k}) := \min_{x_t, \alpha_t} \{ \alpha_t : \alpha_t \geq \hat{\Phi}_{t+1,k} + \mu_{t+1,k}^{\intercal}(\hat{x}_{t,k} - x_t) \}$$

"Benders-like" cut

Birge, J.R., *Decomposition and Partitioning Methods for Multistage Stochastic Linear Programs*, 1985

Pereira, M.V.F., Pinto, L.M.V.G, *Multi-stage stochastic optimization applied to energy planning*, 1991

Zou, J., Ahmed, S., Sun, X.A., Stochastic Dual Dynamic Integer Programming, 2016

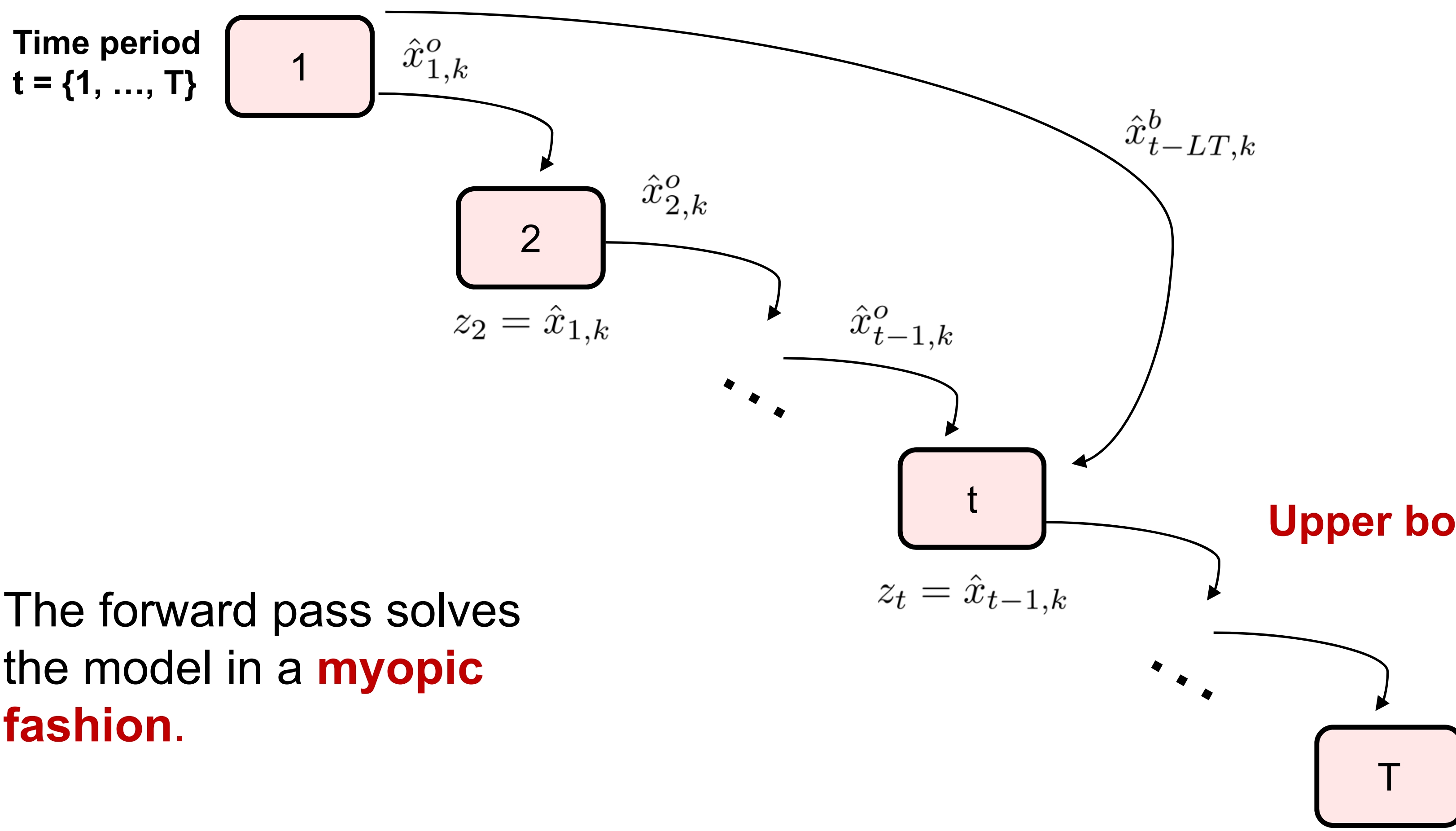
Nested Decomposition for Mixed-Integer Problems

Basic idea:

- This algorithm decomposes the problem by time period, which in this case is **by year**.
- The algorithm consists of **Forward** and **Backward Passes**.
- The **Forward Pass** solves the problem in myopic fashion (1 year time horizon).
- The **Backward Pass** projects the problem in the space of the previous time periods by adding cuts.

- Birge, J.R., *Decomposition and Partitioning Methods for Multistage Stochastic Linear Programs*, 1985
- Pereira, M.V.F., Pinto, L.M.V.G, *Multi-stage stochastic optimization applied to energy planning*, 1991
- Sun & Ahmed, *Nested Decomposition of Multistage Stochastic Integer Programs with Binary State Variables,* 2016

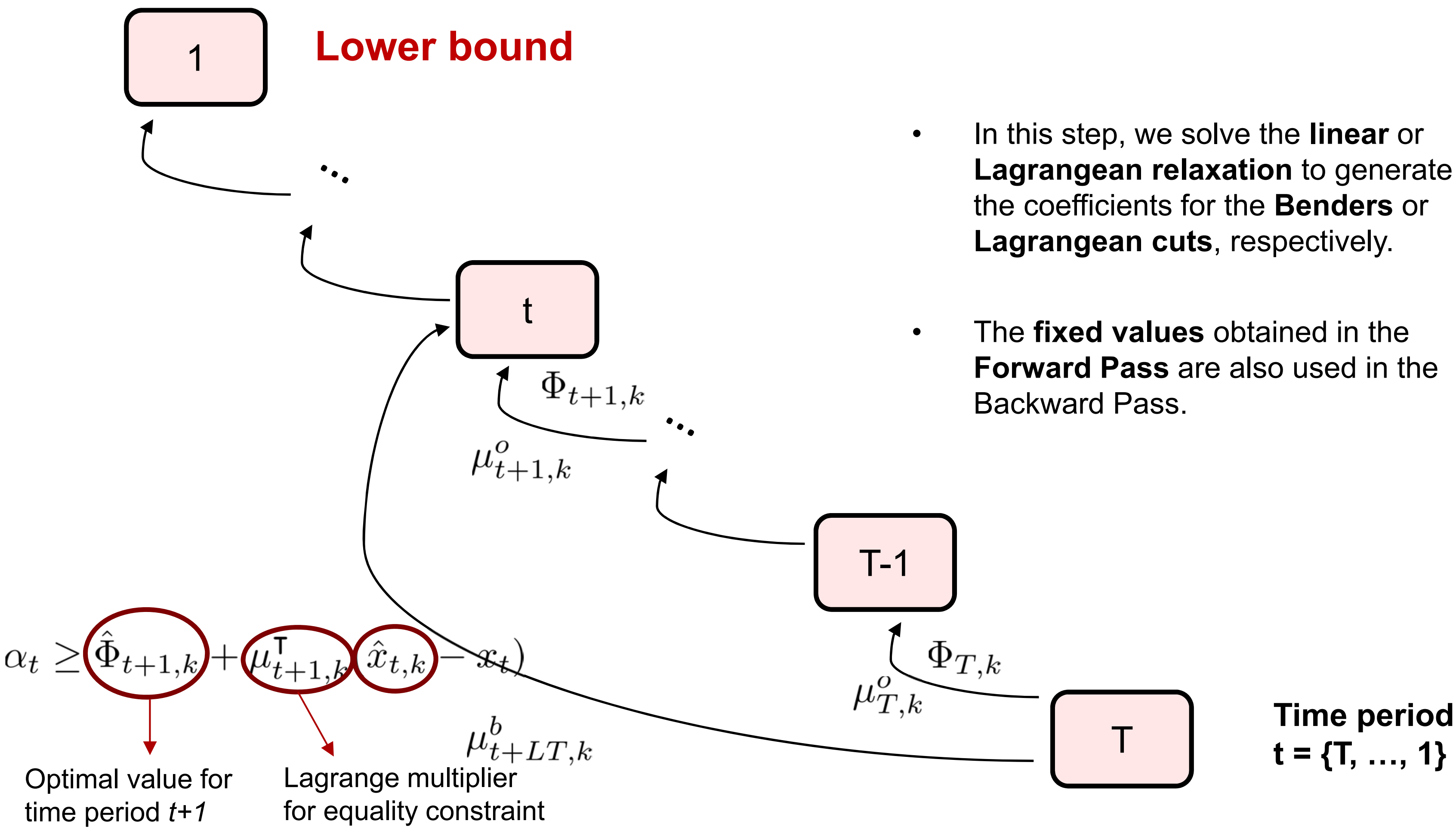
Forward Pass generates a feasible solution



Upper bound

The forward pass solves the model in a **myopic fashion**.

Backward Pass generates Benders cuts and improves the cost-to-go approximation



- In this step, we solve the **linear** or **Lagrangean relaxation** to generate the coefficients for the **Benders** or **Lagrangean cuts**, respectively.
- The **fixed values** obtained in the **Forward Pass** are also used in the Backward Pass.

Options for the cost-to-go function

Benders cut: The coefficients are obtained from the solution of the linear relaxation.

Lagrangean cut: The coefficients are obtained from the solution of the Lagrangean Dual (through subgradient method).

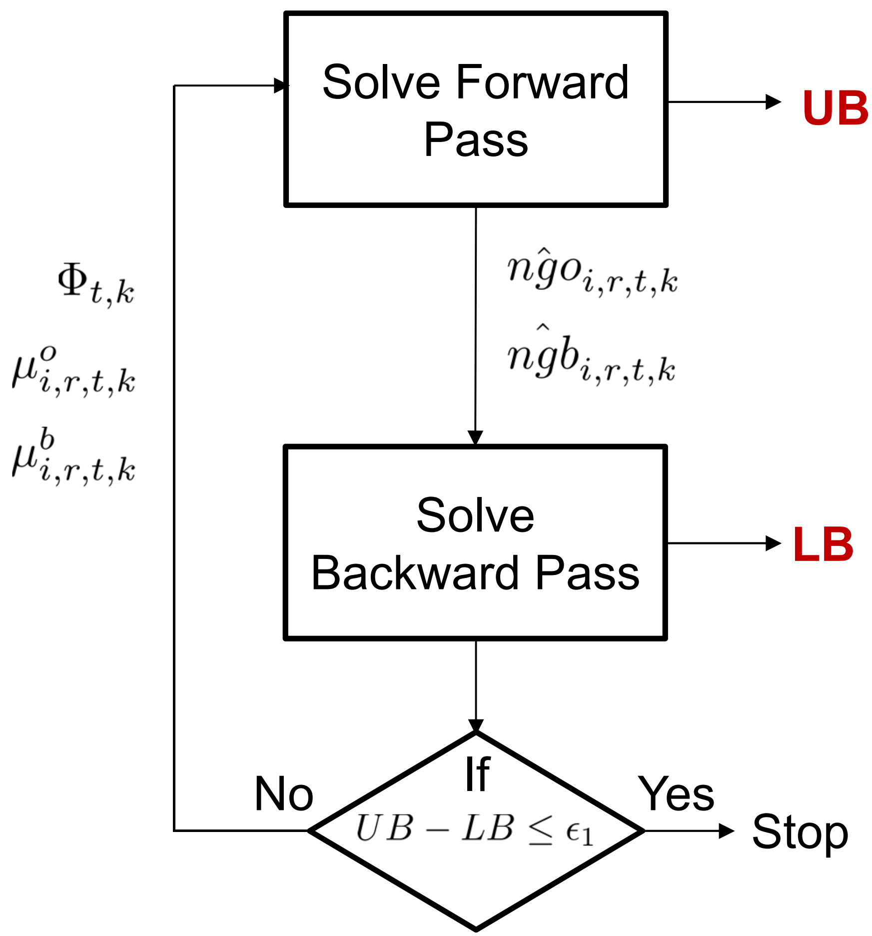
Strengthened Benders cut: The coefficients are obtained from the solution of the Lagrangean relaxation, solved after initializing the multiplier using the linear relaxation.

Potential duality gap disclaimer:
The Nested Decomposition Algorithm does not have guaranteed finite convergence with these cuts for the case of integer and continuous state variables

Zou, J., Ahmed, S., Sun, X.A., Stochastic Dual Dynamic Integer Programming, 2016

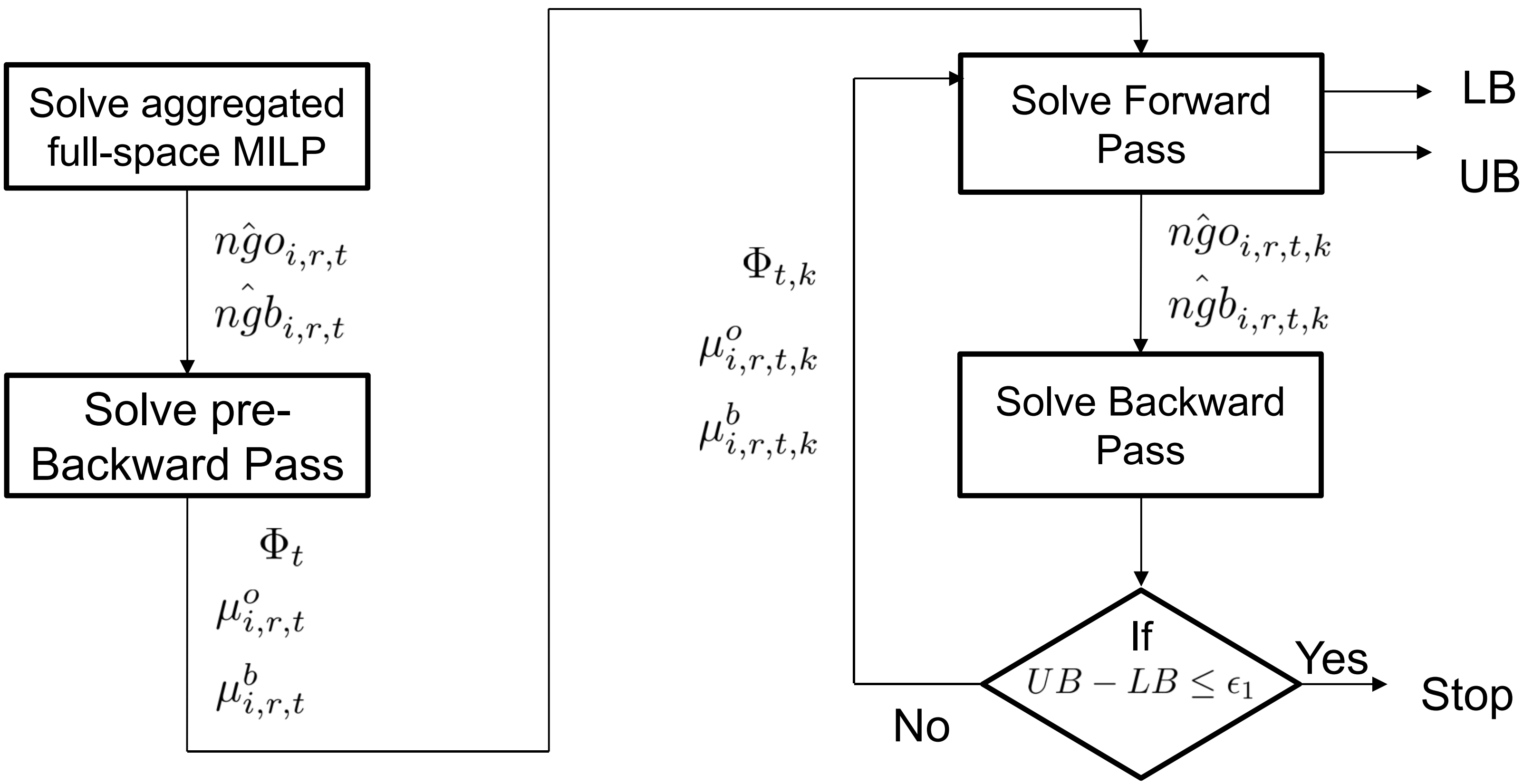
Nested Decomposition Algorithm

1. Set iteration k=1, and tolerance ϵ_1.
2. Solve the Forward Pass for time periods $t = 1, \ldots, T$, and store the fixed values for $\hat{ngb}_{i,r,t,k}$ and $\hat{ngo}_{i,r,t,k}$.
3. Compute **upper bound**.
4. Solve the Backward Pass for time periods $t = T, \ldots, 1$, and store the cuts' coefficients.
5. Compute **lower bound**.
6. If $UB - LB \leq \epsilon_1$, STOP.
7. If not, set k = k+1, go back to step 2.



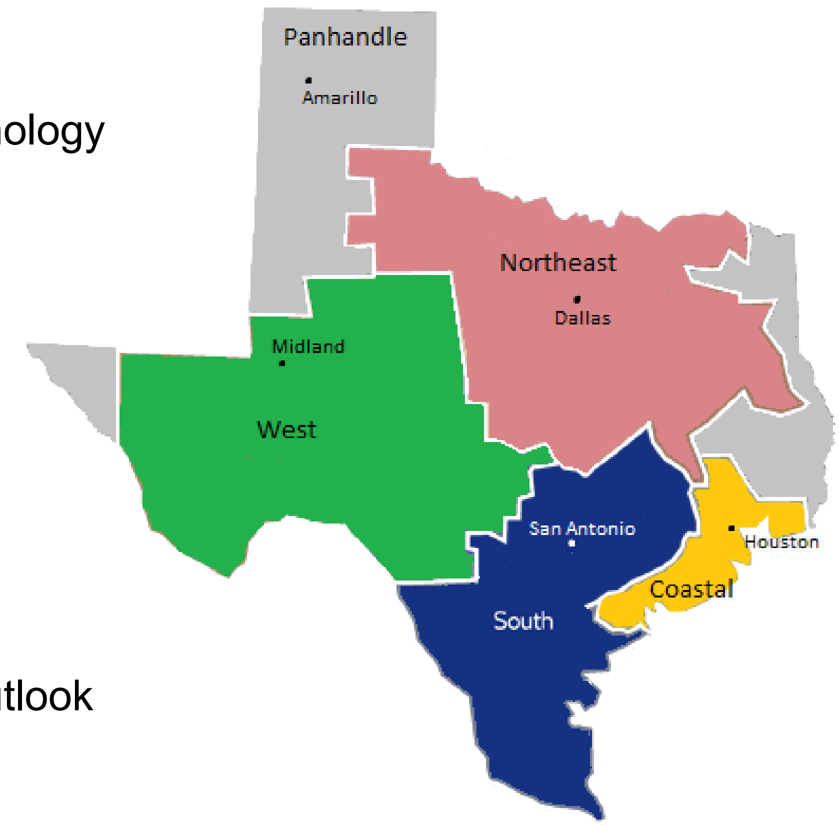
Improved Nested Decomposition Algorithm

- The idea is to solve an aggregated / simpler version of the full-space MILP model before starting the algorithm, and to use its solution to pre-generate cuts before entering in the first Forward Step.



Case Study: ERCOT (Texas)

- **30 year** time horizon (1st year is 2015)
- Data from **ERCOT database**
- Cost information from NREL (Annual Technology Baseline (ATB) Spreadsheet 2016
- All costs in **2015 USD**
- Regions:
  - Northeast (midpoint: Dallas)
  - West (midpoint : Glasscock County)
  - Coastal (midpoint: Houston)
  - South (midpoint : San Antonio)
  - Panhandle (midpoint : Amarillo)
- Fuel price data from EIA Annual Energy Outlook 2016 - Reference case
- No imposed carbon tax
- No RES quota requirement



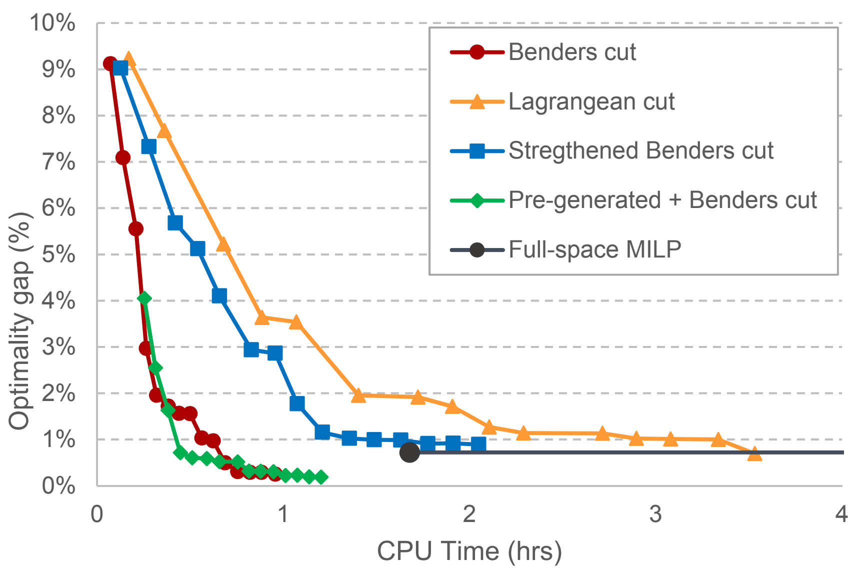
Algorithm Performance

1 representative day per season

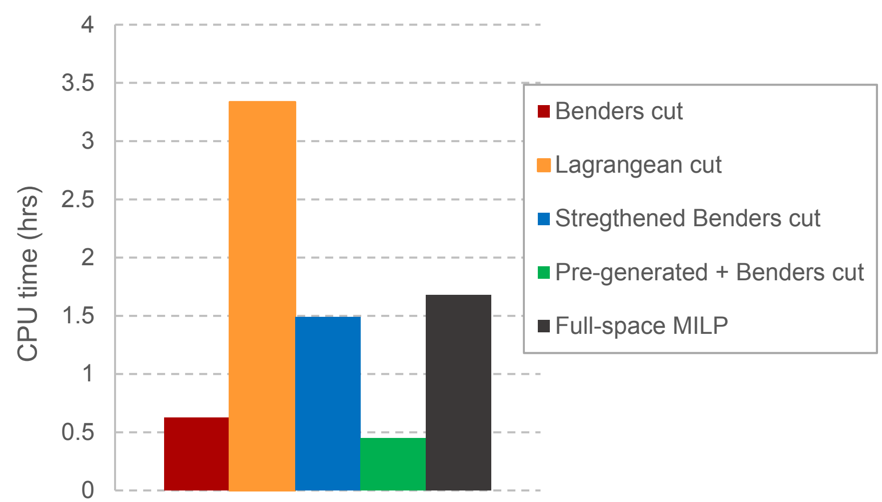
Full-space MILP Model

Integer variables: 413,644	optcr: **1%**
Continuous variables: 594,147	CPU Time: 1.7 hours
Equations: 1,201,761	Optimality gap: 0.72%
Solver: CPLEX	**Minimum cost: $186.7 billions**

Optimality gap over solution time

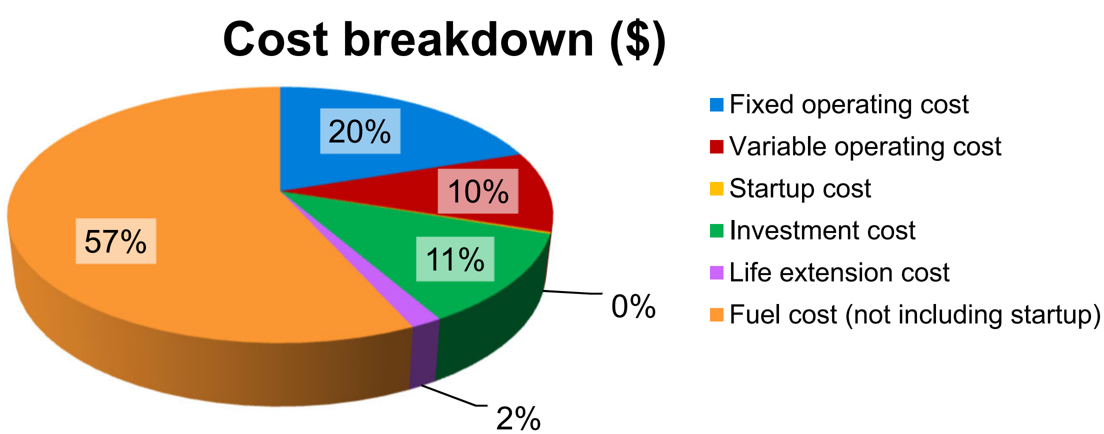


The improved algorithm with Benders cuts converges in 30 min within ≤1% gap

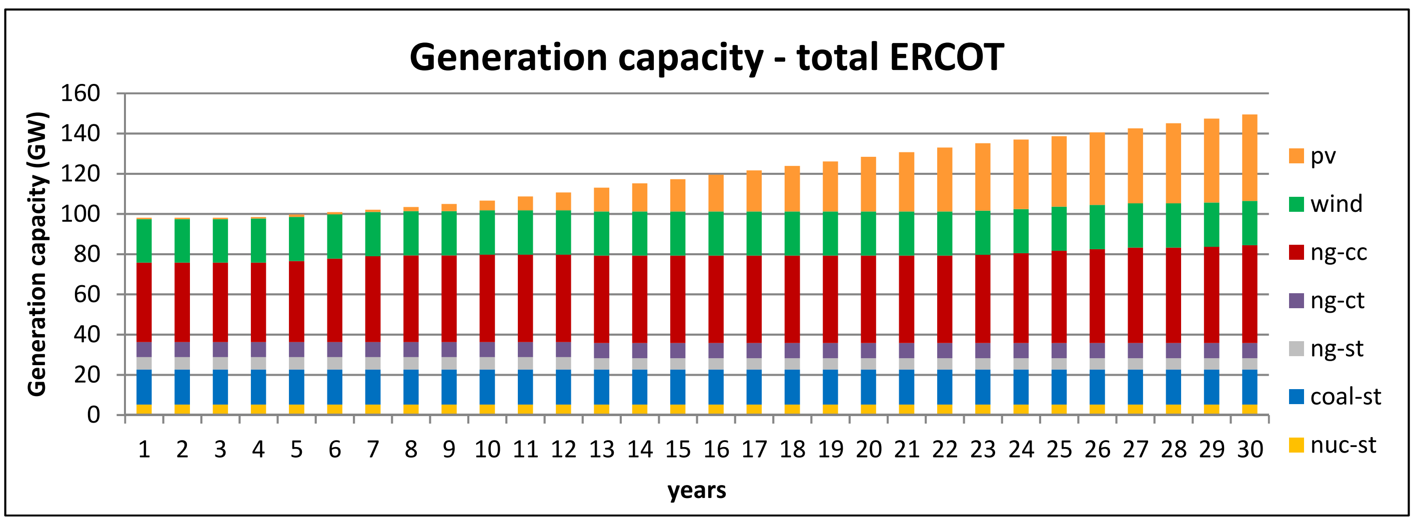


Results

1 representative day per season



Total cost: $186.7 billions



- **64-fold increase in PV-solar capacity**
- **2% increase in wind capacity**
- **23% increase in natural gas combined cycle capacity**

Algorithm Performance

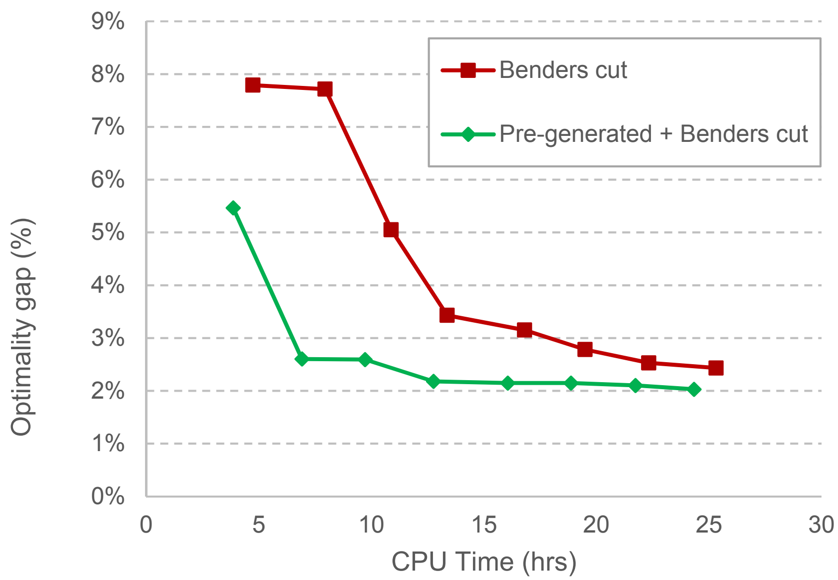
1 representative week per season

Full-space MILP Model

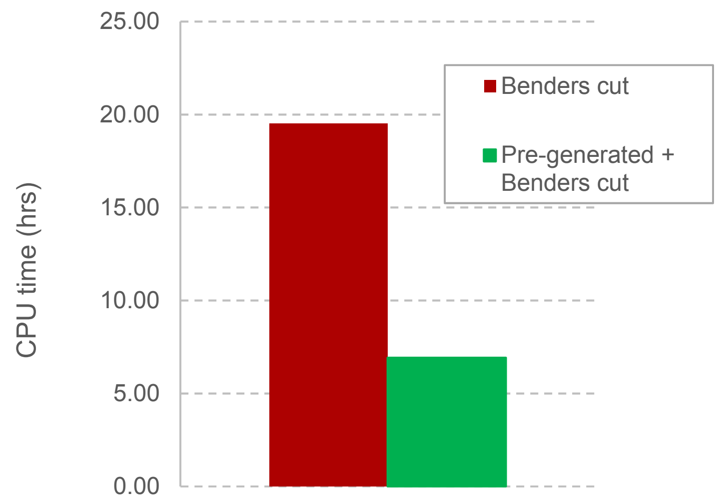
Integer variables: 2,901,964
Continuous variables: 4,136,547
Equations: 8,476,641

Solver: CPLEX
optcr: 1%
CPU Time: **Out of memory! (Does not solve)**

Optimality gap over solution time

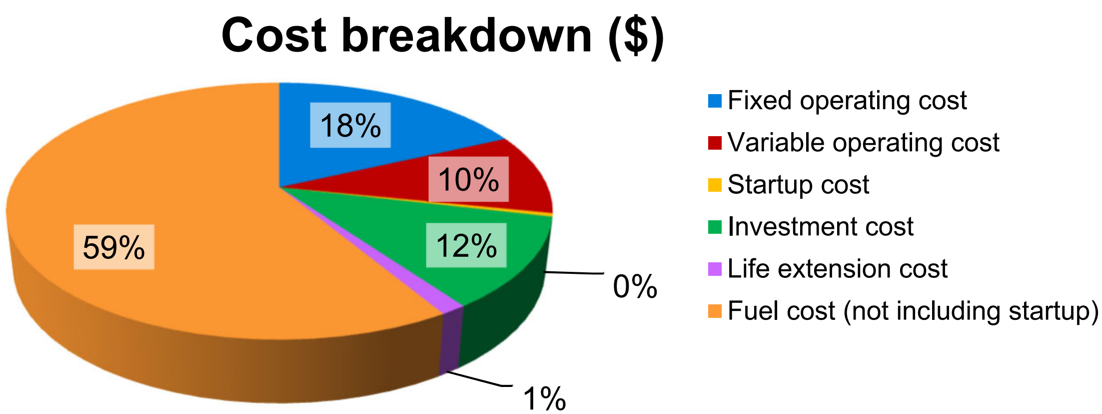


The **improved algorithm** with Benders cuts converges in **7 hrs** within **≤3% gap**



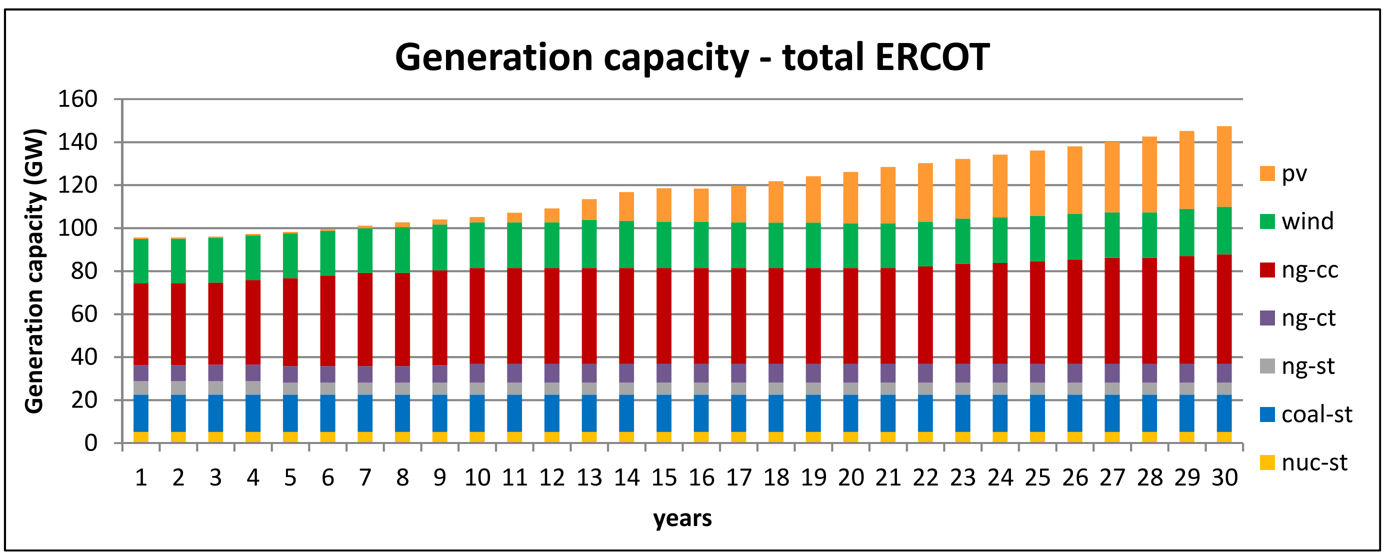
Results

1 representative week per season



Total cost: $195.8 billions

5% increase w r. t. one representative day



- **56-fold increase in PV-solar capacity**
- **8% decrease in wind capacity**
- **34% increase in natural gas combined cycle capacity**
- **17% increase in natural gas combustion-turbine capacity**

Special DOE Study: Scenario Description

Model assumptions

- Energy balance
- Capacity factor of the renewable generators
- Unit commitment constraints
- Operating reserve constraints
- Investment constraints.
- Generators balance

Objective function:

Minimization of the net present cost over the planning horizon comprising:

- Operating, startup, investment and retrofit costs
- Fuel consumption
- Environmental costs (if applicable)

New IDAES technology assumptions

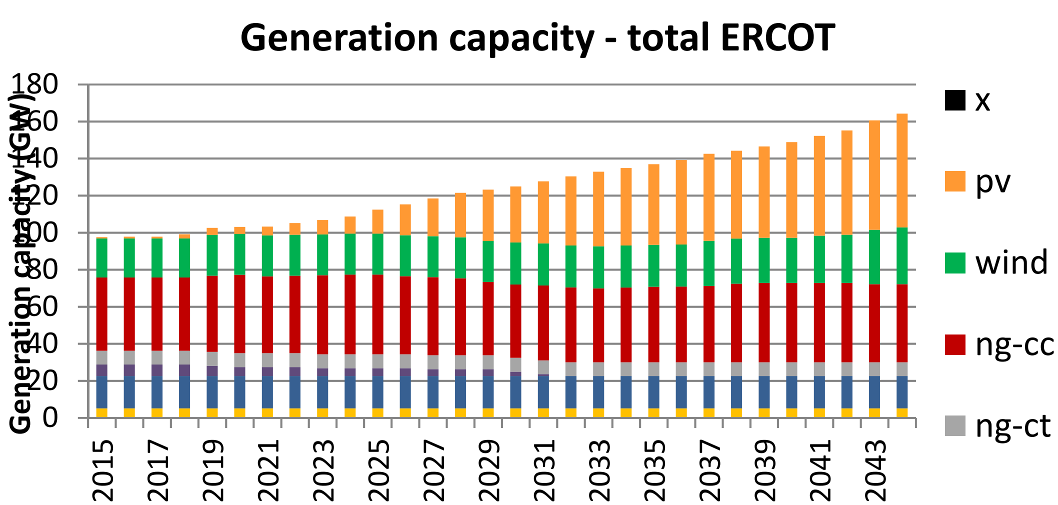
- New hypothetical coal based technology.
- Capital cost assumed to be the cost of a new coal generator from ATB Spreadsheet 2016: US$ **3,534,660/MW**
- Heat rate: **7.45 MMBtu/MWh**.
- Ramping rate is assumed to be **100% in one hour**.
- The first case assumes **no learning rate**
- Seconds case assumes that this new technology has the same **learning rate** as Concentrated Solar Panel (newest of the current technologies so it has the steepest learning rate: **1.42%**)

Learning rate: log-linear equation relating the unit cost of a technology to its cumulative installed capacity or electricity generated

IDAES Institute for the Design of Advanced Energy Systems · NETL National Energy Technology Laboratory · Berkeley Lab · Sandia National Laboratories · Carnegie Mellon · West Virginia University · U.S. Department of Energy

Performance / Cost target for hypothetical IDAES technology

Assuming **no annual learning rate** for X:



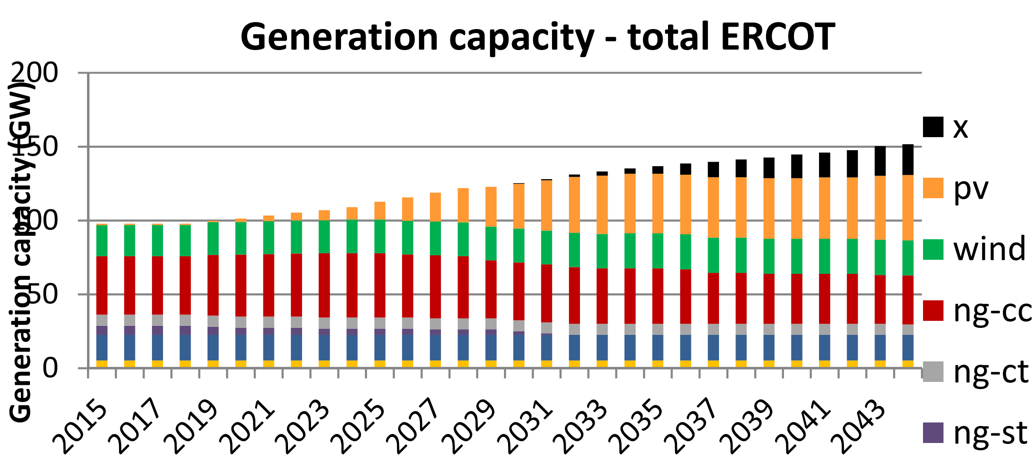
Net Present Cost: US$ 238.7 billions

The new technology X is **not** chosen by the optimization

Performance / Cost target for hypothetical IDAES technology

If we assume that technology X has the same **annual learning rate** as Concentrated Solar Panel (1.42%)

Not enforcing technology X selection



Net Present Cost: US$ 236.4 billions

↓ US$ 2.3 billion

By considering an annual learning rate, this new technology became **competitive** and the overall net present **cost dropped.**

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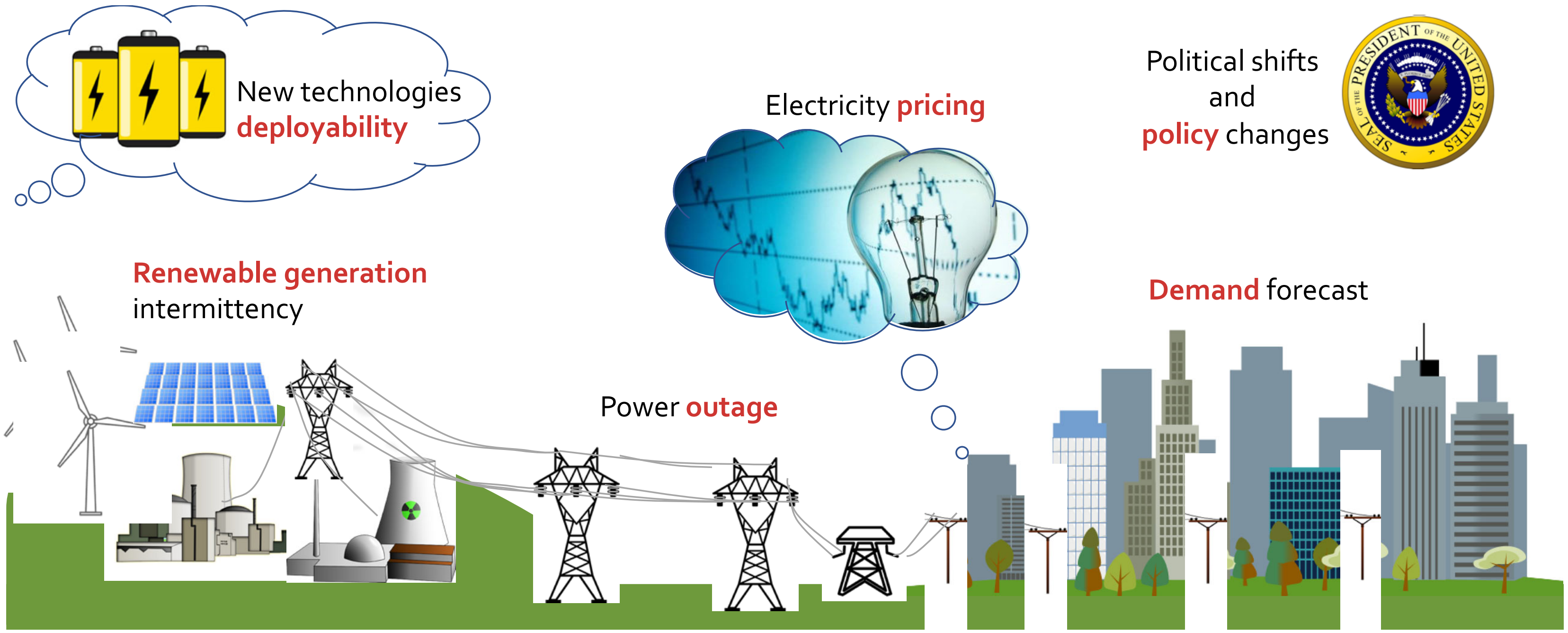
Electric Power Systems are Uncertain



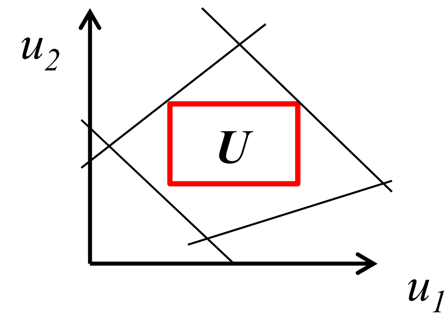
How to anticipate effects of uncertainty?

Approaches to Optimization under Uncertainty

Sahinidis (2004)

If deterministic uncertainty set

Robust Optimization: **Ensure feasibility over uncertainty set**



If probability distribution function

Stochastic Programming: **Expected value, recourse actions**

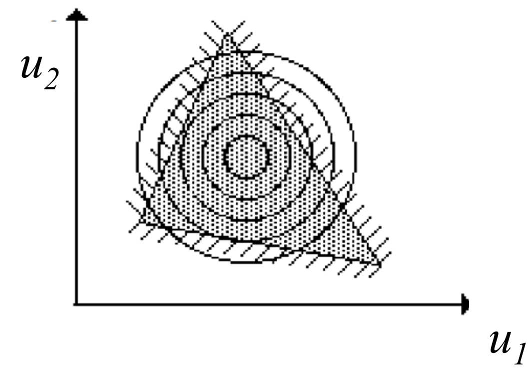
Option: add risk measure

Stage 1
Here & now

u

Recourse
Wait & see

Chance Constrained Optimization: **Ensure feasibility level confidence**



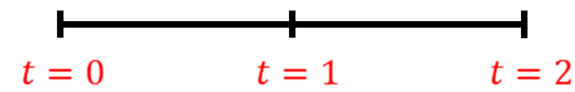
Impact of optimization under uncertainty has been limited in industrial practice

Major reasons: *ill-defined problem*, *computational expense*

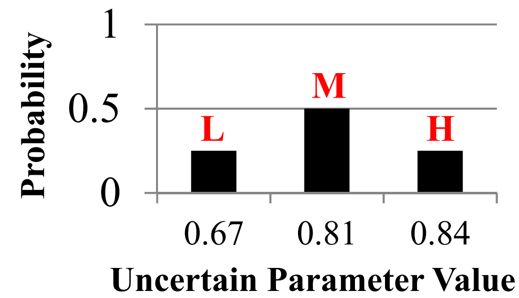
Stochastic Programming

- **Stochastic programming** is a scenario-based framework for optimization under uncertainty (Birge & Louveaux, 2011)

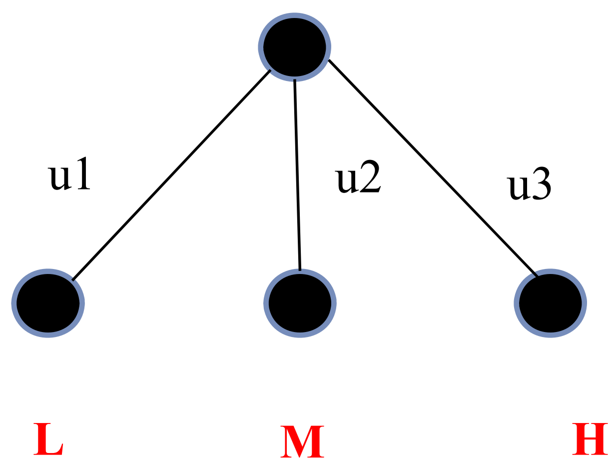
- Time horizon is divided into a set of **discrete time points**



- Uncertain parameters are described by a **discretized probability distribution**



- **Discretized distribution gives rise to Scenario Tree**



Multistage Stochastic Programming

Birge & Louveaux, 1997; Sahinidis, 2004

$$\min \quad z = c^1x^1 + E_{u^2}[c^2(u)x^2(u^2) + \ldots + E_{u^N}[c^N(u)x^N(u^N)]\ldots]$$

s.t.

$$W^1x^1 = h^1$$

$$T^1(u)x^1 + W^2x^2(u^2) = h^2(u)$$

$$\vdots$$

$$T^{N-1}(u)x^{N-1}(u^{N-1}) + W^Nx^N(u^N) = h^N(u)$$

$$x^1 \geq 0, x^t(u^t) \geq 0, t = 2, \ldots, N-1$$

u ***Exogeneous uncertainties (e.g. demands)***

Special case: two-stage programming (N=2)

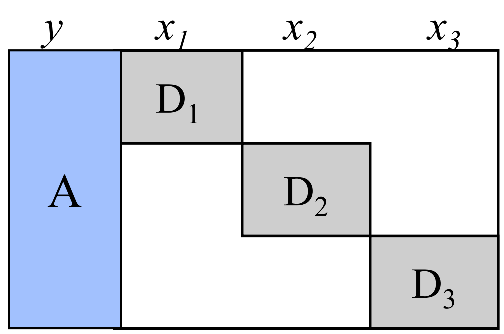
x^1 stage 1 ***u*** ***x^2 recourse (stage 2)***

Decomposition Techniques

Benders decomposition

Benders (1962), Magnanti, Wong (1984)

Complicating Variables



complicating variables

$$\max \ a^T y + \sum_{i=1,..n} c_i^T x_i$$

$$st \ \ Ay + D_i x_i = d_i \ \ i = 1,..n$$

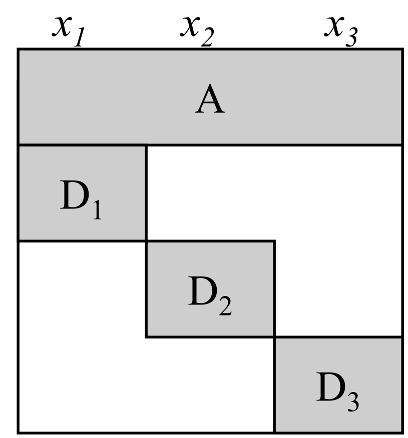
$$y \ge 0, \ x_i \ge 0, i = 1,..n$$

Two-stage Stochastic Programming

Lagrangean decomposition

Geoffrion (1972) Guinard (2003)

Complicating Constraints



complicating constraints

$$\max \ c^T x$$

$$st \ \ Ax = b$$

$$D_i x_i = d_i \ \ i = 1,..n$$

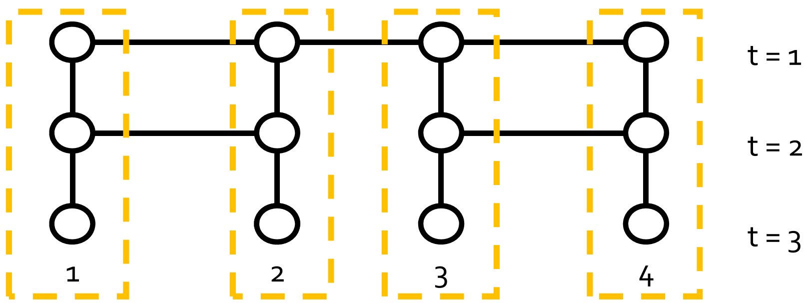
$$x \in X = \{x | x_i , i = 1,..n, | x_i \ge 0\}$$

Multistage Stochastic Programming

Decomposition Algorithms:
Mitigating the combinational explosion

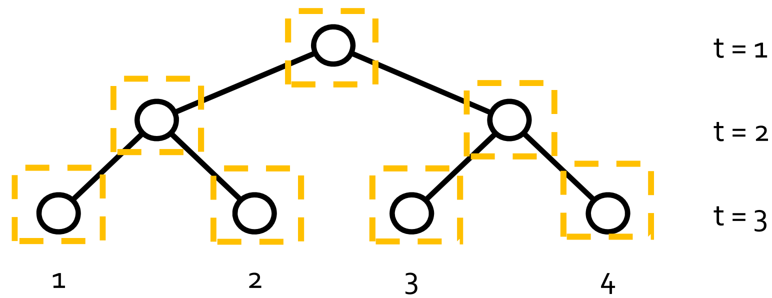
- The decomposition algorithms for multistage stochastic programming models can be classified as:

Scenario-based Decomposition



Each subproblem has **the size of the deterministic single scenario** version of the problem.

Stage-based Decomposition

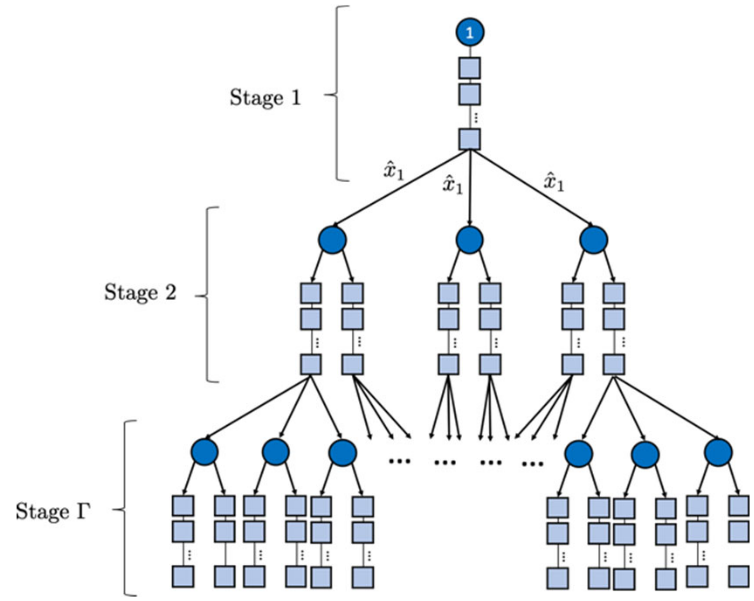


Thus, if **deterministic problem** is already **computationally expensive, stage-based** decomposition is the **best hope**!

Extension to Multi-stage Stochastic Integer Programming (MSIP)

Optimization over the **expected-value**

- Number of scenarios **grows exponentially** with the **number of stages**.
- Both strategical, , and operational, , uncertainties.
- **Combinatorial explosion**!



MSIP subproblem concise representation:

$$\mathcal{P}_{n,k} : \Phi_{n,k}(\hat{x}_{P(n),k}, \phi_{n,k}) := \min_{(x_n, y_n)} \quad f_n(x_n, y_n) + \sum_{m \in C(n)} q_{nm} \phi_{m,k}(\hat{x}_{n,k})$$

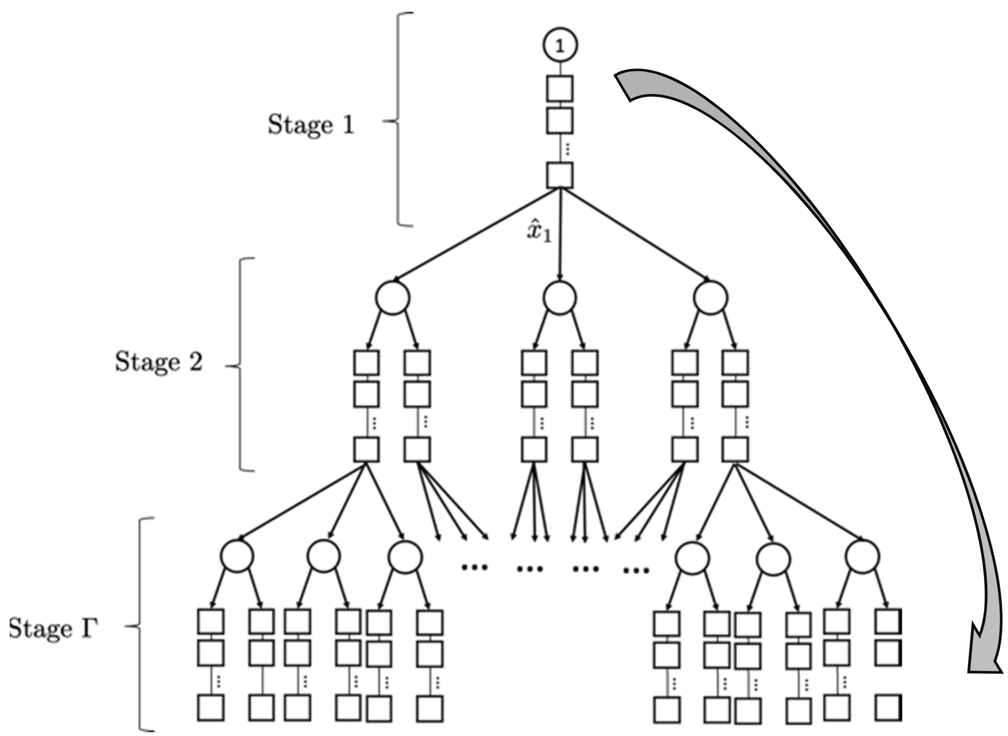
$$\begin{aligned} s.t. \quad & (z_n, x_n.y_n) \in \chi_n \\ & z_n = \hat{x}_{P(n),k} \quad \leftarrow \mu_{n,k} \in \mathbb{R}^{\ell} \\ & x_n \in \mathbb{Z}_+^{\ell_1} \times \mathbb{R}_+^{\ell_2}, \quad y_n \in \mathbb{Z}_+^{o_1} \times \mathbb{R}_+^{o_2}, \quad z_n \in \mathbb{R}^{\ell} \end{aligned}$$

where the cost-to-go function is defined as:

$$\phi_{n,k}(\hat{x}_{n,k}) := \min_{x_n, \alpha_n} \left\{ \alpha_n : \; \alpha_n \geq \sum_{m \in C(n)} q_{nm} \cdot \left(\hat{\Phi}_{m,k'} + \mu_{m,k'}^{\intercal} (\hat{x}_{n,k'} - x_n) \right) \; \forall \, k' \in \mathcal{K} | k' < k \right\}$$

Stochastic **N**ested **D**ecomposition

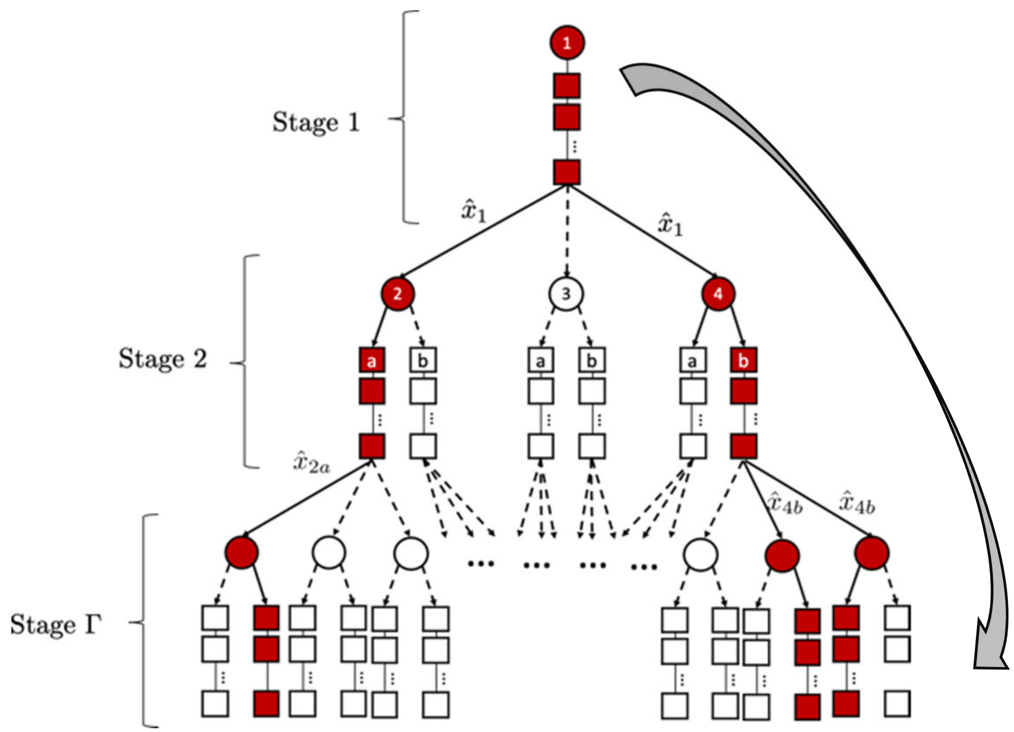
Decompose **by node** in the scenario tree



Forward Pass

Stochastic Nested Decomposition

Decompose **by node** in the scenario tree

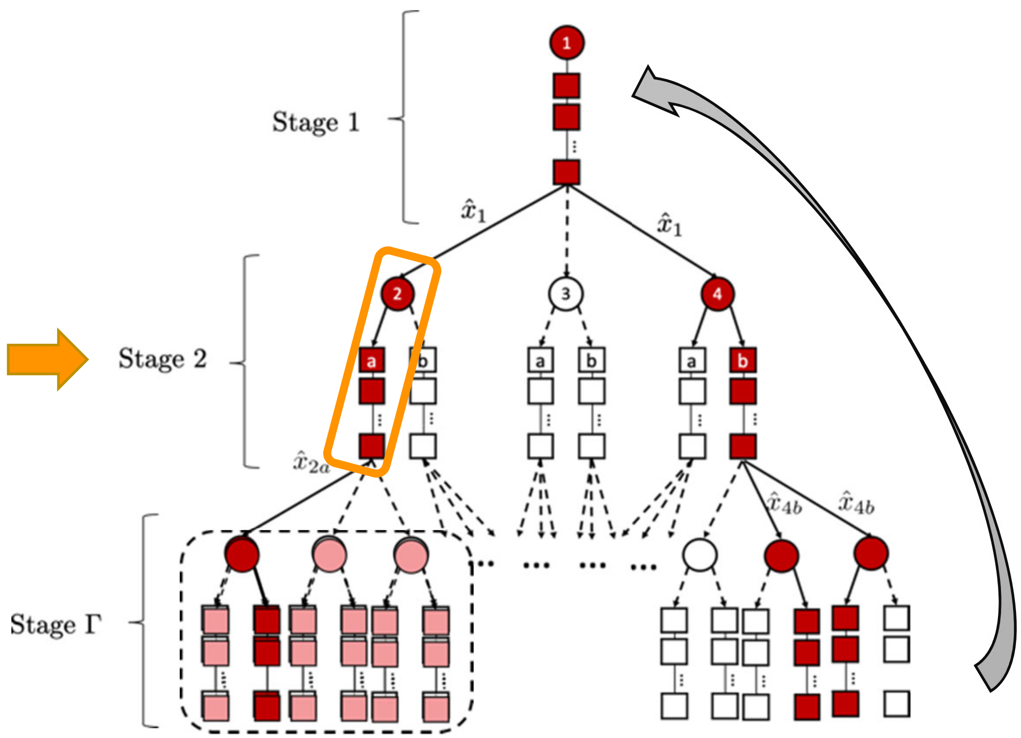


Forward Pass

- **Scenario Sampling**
- Solve subproblems for a **subset of the scenarios.**
- Get the **Statistical Upper Bound** **(**for a certain **confidence level**)

Stochastic Nested Decomposition

Decompose **by node** in the scenario tree

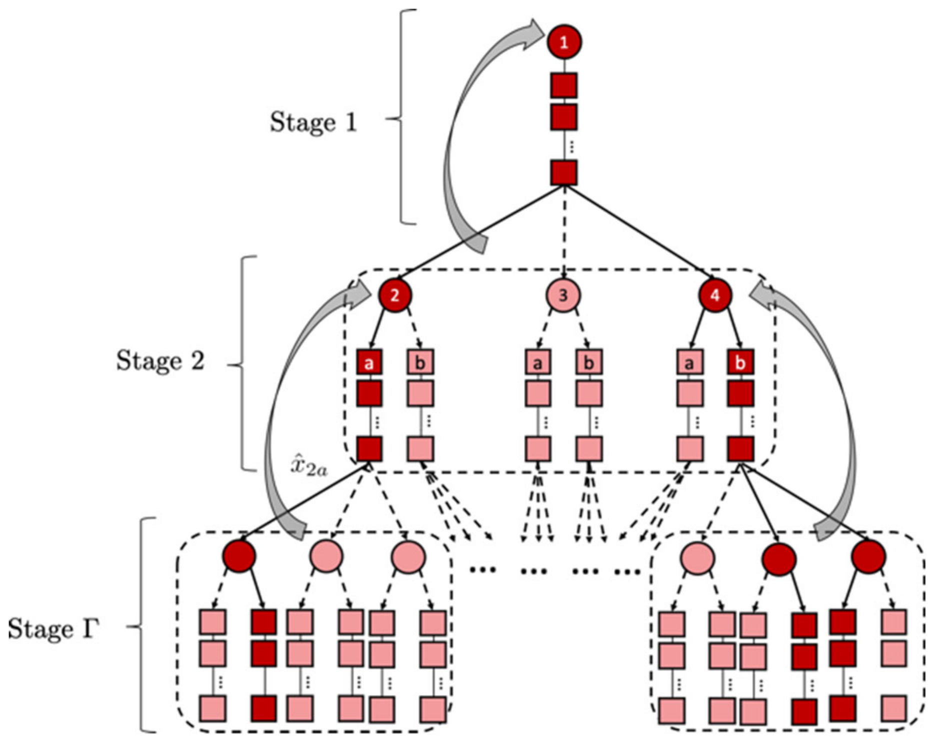


Backward Pass

- Solve for **all children nodes** of the nodes in the sampled scenarios.

Stochastic Nested Decomposition

Decompose **by node** in the scenario tree



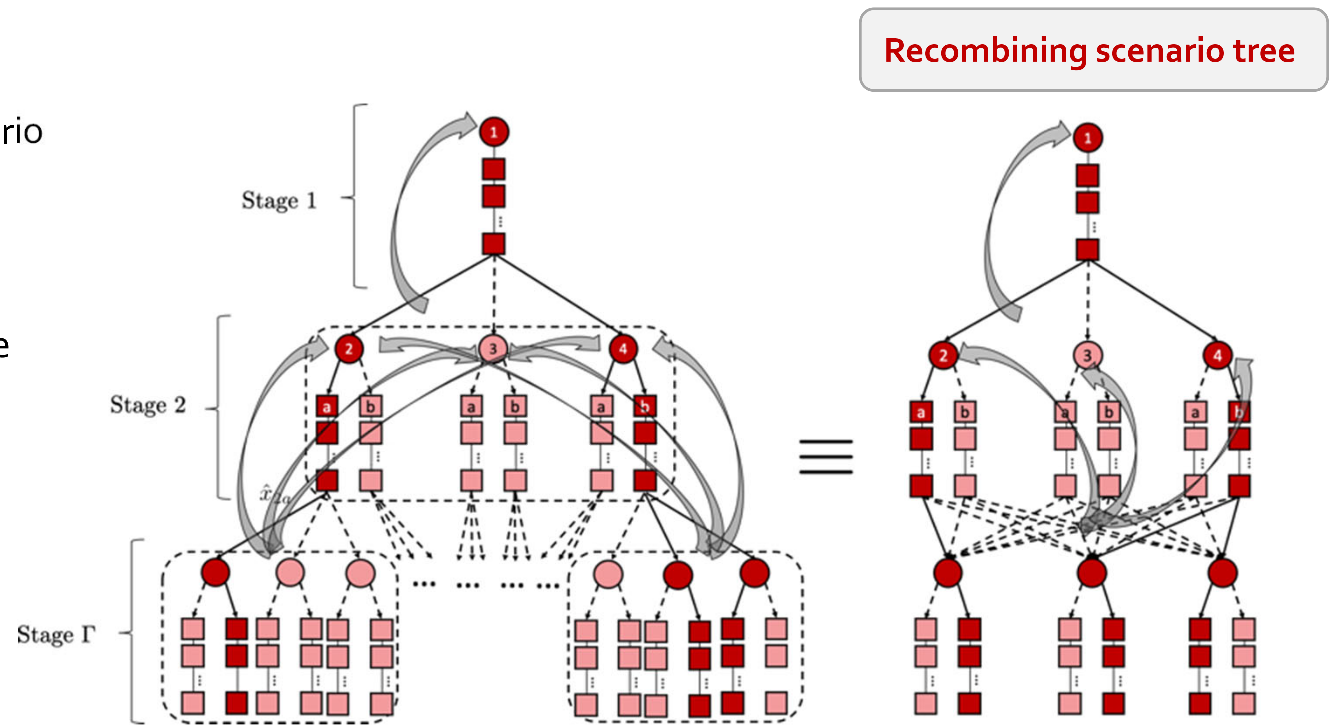
Backward Pass

- Solve for **all children nodes** of the nodes in the sampled scenarios.
- Relaxed subproblems.
- Benders cut takes the **weighted average of the coefficients for the cut** based on the conditional probability
- Solution of node 1 is the **Lower Bound**

Stochastic Dual Dynamic integer Programming

A **special case** of the SND

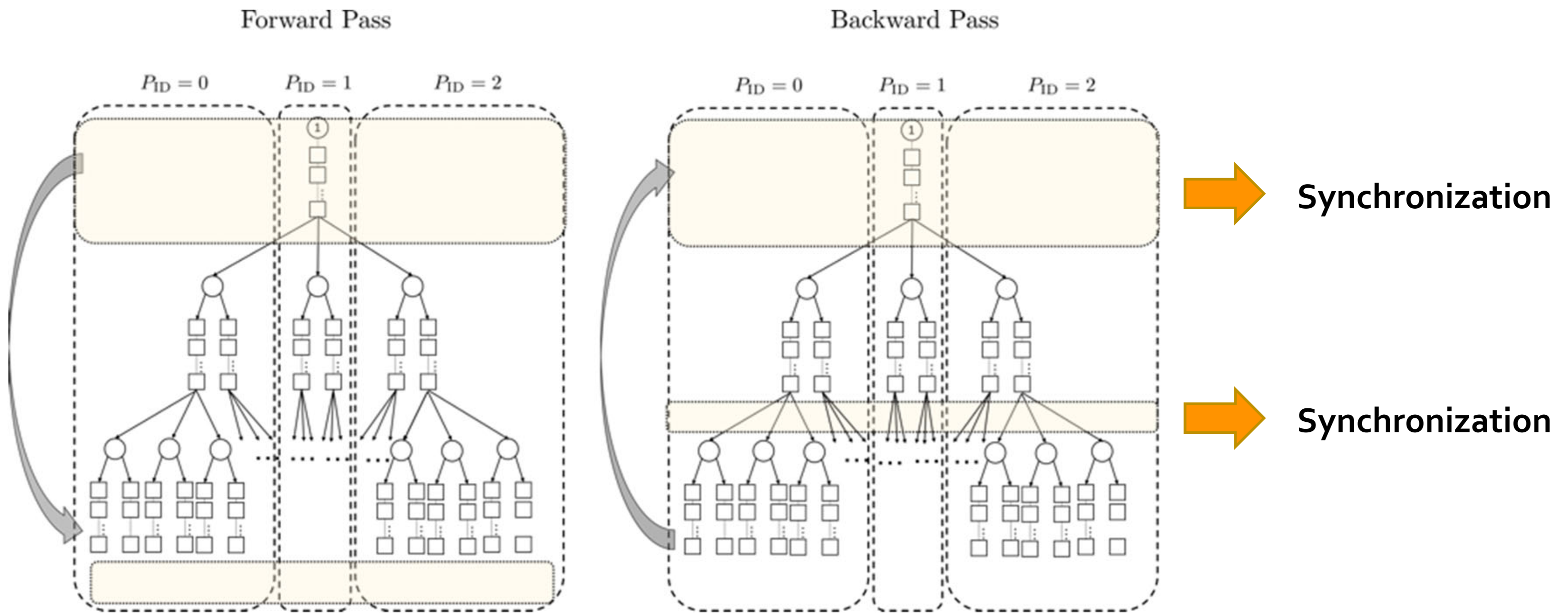
- Assumes that the scenario tree is **stage-wise independent**.
- **Cuts can be shared** between all nodes in the same stage
- Avoid the "curse of dimensionality"



Zou et al.., (2018) *Stochastic Dual Dynamic Integer Programming,*, Mathematical Programming.
Lara et al. (2019) *Electric Power Infrastructure Planning Under Uncertainty: Stochastic Dual Dynamic Integer Programming (SDDiP) and parallelization scheme,* Manuscript in preparation.

SDDiP has potential for **parallelization**

- Subproblems of the nodes within stage are **independent** from each other.
- **Synchronization** is required to share the Benders cuts



Lara et al. (2019) *Electric Power Infrastructure Planning Under Uncertainty: Stochastic Dual Dynamic Integer Programming (SDDiP) and parallelization scheme,* Manuscript in preparation.

First hypothetical case study in the ERCOT region

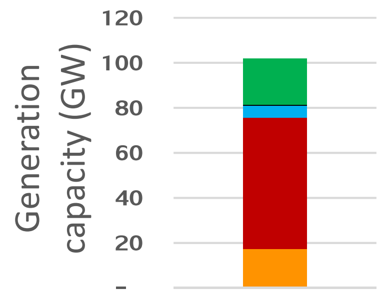
- Strategic uncertainty (**3 realizations** per stage): **carbon tax.**
- **Lithium-ion, lead-acid and flow batteries**
- Operational uncertainty(**2 realizations** per stage): **different profiles** for representative days.
- **15** scenario **samples** per iteration.
- **4 representative days** per year.
- **Node Size** (before cuts):
  - 50,042 constraints
  - 13,746 integer variables
  - 22,755 continuous variables

- MILP subproblems solved using **Gurobi 8.0.1**
- **Processor: 2.3 GHz Intel Core i5**
- **Stopping criteria: 1% optimality gap**
- Number of parallel processes: 3
- Sampled scenarios per iteration: 15
- 95% confidence interval in the statistical upper bound

5 year planning under uncertainty

First hypothetical case study in the **ERCOT** region

Stochastic solution
Minimum expected cost over the 5 years:
$61.7 billion

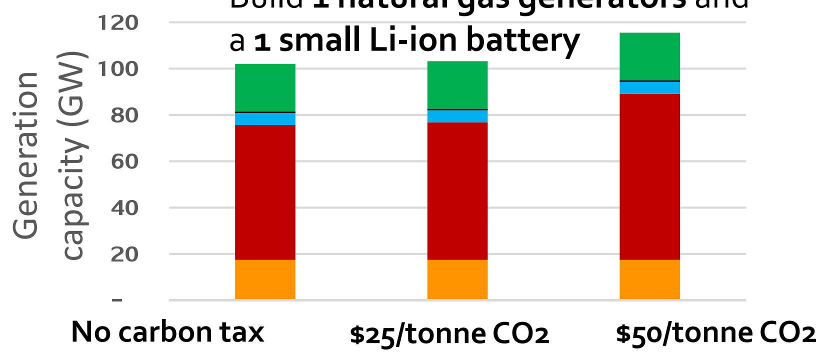


First stage:
Build **3 new natural gas** generators

Build **2 natural gas generators** and a **small Li-ion battery**

Build **1 natural gas generators** and a **1 small Li-ion battery**

Build **1 natural gas generators** and a **2 small Li-ion batteries**



Second hypothetical case study in the **ERCOT** region

- Strategic uncertainty (**3 realizations** per stage): **carbon tax.**
- Operational uncertainty(**2 realizations** per stage): **different profiles** for representative days.
- **5 stages** (with 1 year per stage)
- $(2 \cdot 3)^4 =$ **1,296 scenarios** → 1,555 nodes.
- **15** scenario **samples** per iteration.
- **4 representative days** per year.
- **Node Size** (before cuts):
  - 50,042 constraints
  - 13,746 integer variables
  - 22,755 continuous variables
- **Deterministic-equivalent** size:
  - **77.8 million** constraints
  - **21.4 million** integer variables
  - **35.4 million** continuous variables
- MILP solved using **Gurobi 8.0.1**
- **Processor: 2.3 GHz Intel Core i5**
- **Stopping criteria: 2% optimality gap**
- Number of parallel processes: 3

Solution time: 1.4 hours

Parallel SDDiP allows the solution of instances with quadrillions of variables and constraints

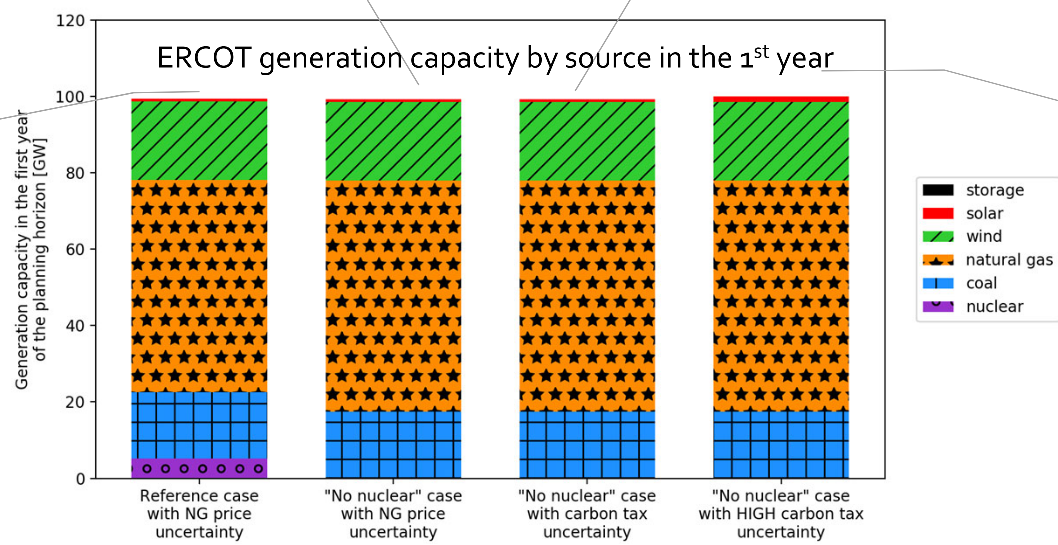
To test the capabilities of our algorithm, we solve the problem for 5 stages, 10 stages and 15 stages

For most of the scenario trees tested, the *here-and-now* decisions consist of investing in new **natural gas (NG) power plants**

***No nuclear* case** (with **natural gas price uncertainty**) shows nuclear plants being **replaced by NG plants**.

***No nuclear* case** (with **carbon tax uncertainty**) shows nuclear plants being **replaced by NG plants**.

Reference case (with **natural gas price uncertainty**) shows **no significant expansion** in 1st year.



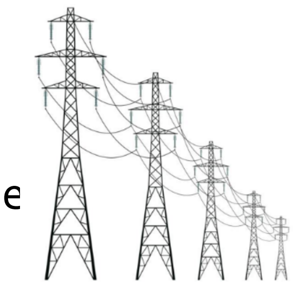
Risk of having **steep carbon tax fees** makes the optimization **invest less in NG** and **more in renewable** sources in the 1st year.

The value of stochastic programming

- To evaluate the **potential gain of handling uncertainty:**
  - We solve the **MSIP formulation** for the **no nuclear case** with **high carbon tax uncertainty**
  - We solve the **deterministic version** of the ***no nuclear*** **case** with **high carbon tax** uncertainty **using averages of carbon tax realizations**. Then, we **re-solve the MSIP** formulation for the ***no nuclear*** **case** with **high carbon tax** uncertainty **fixing the 1st year investment decisions** from **deterministic solution**.

By considering carbon tax as an uncertain parameter, the **value of stochastic programming** is **$2.18 billion**, which is the **savings one can achieve in the long term**.

Future work



- Improve the **transmission representation** in the model and include the option for **transmission expansion**[1].
- Adapt the parallel SDDiP algorithm to handle **stage-wise dependent** parameters[2,3].
- Include **construction lead time** in the multistage stochastic programming formulation.
- Adapt parallel SDDiP to address **risk-averse**[4] GEP problems.
- Evaluate and improve the parallel scalability of the SDDiP algorithm.

[1] N. Alguacil, A. L. Motto, and A. J. Conejo. Transmission expansion planning: a mixed- integer LP approach. *IEEE Transactions on Power Systems*, 2003.
[2] G. Infanger and D. P. Morton. Cut sharing for multistage stochastic linear programs with interstage dependency. *Mathematical Programming*, 1996.
[3] S. Rebennack. Combining sampling-based and scenario-based nested benders decomposition methods: Application to stochastic dual dynamic programming. *Mathematical Programming*, 2016.
[4] A. Shapiro, W. Tekaya, J. P. da Costa, and M. P. Soares. Risk neutral and risk averse stochastic dual dynamic programming method. *European Journal of Operational Research*, 2013.